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MATHS

STUDY PACK

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CHAPTER 1

GENERAL ARITHMETIC

SYLLABUS OBJECTIVES

Learner should be able to :

a) State with example the various categories of the number system

b) Carryout operations involving decimals

c) Round off numbers to required significant figures

Decimals

The value of a figure depends upon what place it occupies in the number system. The decimal comma divides the whole number from the fraction.

.

Hundreds	Tens	Units		Tenths	Hundreds	Thousands
4	1	9	,	8	7	5

Decimal fractions represent fractions whose denominators are powers of 10

e.g. 8 tenth = 8 tenth $= \frac{8}{10}$

7 hundredths $= \frac{7}{10^2} = \frac{7}{100}$

A decimal may be approximated to fewer decimal places by rounding up if the next digit is greater than or equal to 5. Also by rounding down if the next digit is less than 5 .

Example 1

Simplify the following correct to 3 decimal places

21,059 + 0,4031 + 0,009 – 19,83741

```
  21,059
  0,4031          on addition and subtraction the decimal commas
+ 0,009           should be in a vertical straight line downward
21,4711

  21,4711
19,83741
  3,18369
```

21,059 + 0,4031 + 0,009 – 19,83741	=	1,63369
	=	1,634 to 3 d.p

Example 2

Simply $\dfrac{20,4\times 2,8}{9,6}$

$$\frac{20,4\times 2,8}{9,6} = \frac{\cancel{57,12}}{9,6}\ \frac{multiply.numerator}{and.denominator by.10}$$

$$= \frac{5\cancel{7}1,2}{96}$$

$$= \underline{5,95}$$

Working

20,4

<u>x 2,8</u>
4 080

<u>+1 632</u>

<u>57,12</u>

<u>EXERCISE 1.1</u>

1. Simplify the following to 3 decimal places

a). 2,634 + 0,0051 + 0,009

b) 5,098 – 0,32

c) 0,065 + 0,403 + 0,998 – 0,21

d) 16,5 – 17,431 + 12,34

2) Simplify the following

a) 1,64 × 60,4

b) 0,05 × 0,36

c) $\dfrac{18,19}{0,17}$

d) 0,072÷0.9

e) $\dfrac{26,4x0,1}{0,296}$

f) $\dfrac{(0,3)^2x30}{(0,9)^2}$

g) $\dfrac{3,6\text{ x }3,24}{0,72}$

h) $\dfrac{0,93 + 0,53}{0,93 – 0,9}$

3. Write 3,62817 correct to two decimal places

Significant figures (SF)

These are all non-zero digits in a number

A zero between non-zero digits is significant e.g 3<u>0</u>2; 4<u>0</u>1
Zeros after the last non zero digit are significant e.g 3<u>0</u>0; 3,42<u>0</u>
Zeros before the first non-zero digit are non significant e.g. 0,<u>00</u>52 ; <u>0</u>,1

Example 3
Write the following correct to 3 s.f

a) 7 285 b) 71,82365 c) 0,0026792

a) 7 285 correct to 3 s. f is 7 290
b) 71,82365 correct to 3 s.f is 71,8
c) 0,0026792 correct to 3 s.f is 0,00268

Fractions

Operations on fractions
When more than one operation to a problems is given the mnemonic BODMAS must be appli

Example 4

Simplify $\frac{3}{5}$ of $(1\frac{7}{9}-\frac{2}{3})$

= $\frac{3}{5}$ $_5$ of $(\frac{16}{9}-\frac{2}{3})$ brackets first followed by of

= $\frac{3}{5}\times\frac{10}{9}$

= $\frac{2}{3}$

Example 5

Simplify $\frac{5}{7}\div\frac{2}{3}=\frac{5}{7}\times\frac{3}{2}$ (when changing from division to multiplication invert)

$=\frac{15}{14}$

$=1\frac{1}{14}$

<u>EXERCISE 1,2</u>

1. Write the following correct to two significant figures

a) 8,369 b) 23 8470 c) 0,058964
d) 0,000851 e) 401,25 f) 2,081

2. Simplify the following

a) $\frac{13}{14}$ b) $\frac{2}{3}-\frac{3}{5}$ c) $4\frac{3}{5}-3\frac{1}{3}$

d) $5\frac{1}{7}-1\frac{12}{3}$ e) $\frac{1}{4}+\frac{3}{8}-\frac{5}{6}$ f) $9\frac{1}{3}+5\frac{3}{4}-6\frac{1}{2}$

g) $\frac{8}{10}\times\frac{5}{4}$ h) $\frac{8}{9}\div\frac{1}{3}$ i) $2\frac{1}{4}\times 9\frac{1}{2}$

j) $1\frac{3}{5}\div 2\frac{2}{15}$ k) $2\frac{3}{4}\times 5\frac{2}{3}\div\frac{15}{12}$ l) $\frac{1}{9}\div 1\frac{2}{3}\times 3\frac{3}{5}$

3. Arrange in ascending order

Hint put under a common denominator

a) $\frac{2}{3},\frac{5}{7},\frac{7}{9}$ b) $\frac{7}{9},\frac{3}{4},\frac{10}{13}$

4. Simplify the following Hint: Use BODMAS

a) $(\frac{43}{5}-2\frac{1}{2})\div 3\frac{2}{5}$ b) $((\frac{51}{3}-4\frac{1}{2})-(6\frac{2}{3}+5\frac{1}{4})$

c) $2\frac{1}{2}+2\frac{1}{3}\times\frac{3}{4}-\frac{1}{2}$ d) $\frac{3}{4}\times 18-6\div\frac{1}{3}$

e) $\frac{16\div 4}{16-4}$ f) $\frac{2\frac{1}{3}+3\frac{1}{4}}{4\frac{1}{2}-3\frac{1}{4}}$

Approximations

Approximation are done in practice for all measurements of qualities such as length, time mass etc.
True values need to be stipulated in order to make a visual picture of the original data as specified to a certain degree of accuracy. For continuous variables like length, there is no true or actual value hence the need to approximate.

Example 4

The length of a rectangle is 8,5cm and the breadth is 6,5 cm. Each measurement has been given to the nearest cm. Calculate the largest and smallest possible areas of this rectangle.

Largest possible area – use upper limits (L $\times$ W)
= 8,54 $\times$ 6,54
= 55,8516 cm^2
= 55,85cm^2

Smallest possible area – use lower limits
= 8,45 $\times$ 6,45
54,50 cm^2

EXAMINATION QUESTION

1. a) $2\frac{3}{4}+\frac{51}{8}-3\frac{1}{3}$ - b) $4\frac{3}{8}\times\frac{1}{15}\div 11\frac{2}{3}$

c) $(2\frac{1}{4})\div 2\frac{1}{2}-1\frac{4}{5})+\frac{3}{4}$

2. a) Find the value of $(1\frac{1}{2}+\frac{2}{3})\times 1\frac{1}{5})$ giving your answer as a fraction in its lowest term

b) Evaluate 5,4 + 4 $\times$ 0,3

c) Calculate 6% of 5 450

d) Express 42cm as a percentage of 1,05m (c 1984)

3. Write 0,4997 correct to

a) 3 Significant figures

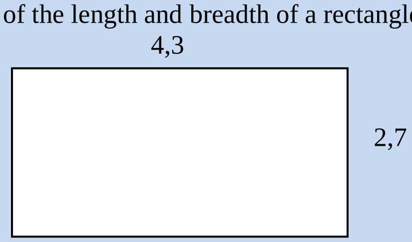
b) 2 decimal places

4. Arrange these fractions in ascending order

$\frac{3}{4},\frac{3}{11},\frac{1}{3},\frac{1}{6}$

5. Write to the nearest whole number, the value of √549

6. The measurements of the length and breadth of a rectangle are given as shown in the diagram.



a) Calculate the least area of the rectangle

b) The greatest area of the rectangle

7) Showing the answer as a decimal, find the exact value of

a) 0.36 $\times$ 0,02

b) 9,15 ÷ 3

c) 8,6 – 8,9

CHAPTER 2

NUMBER BASES

This system represents numbering where the place values are in terms of power of given base. Ordinary, numbers are written in the base ten or denary system

Syllabus Objectives

Learner should be able to :

a) Expand number in the powers of their bases
b) Convert base ten numbers to other bases and vice-versa
c) Carryout operations of numbers in their bases

Base ten or Denary system

All counting numbers are in base ten. The placing of the digit shows their value.

For example 23 and 548

tens
Units — mean 2 tens, 3 units

2 3

$23 = 2\times 10 + 3 \times 1$
$= 2 \times 10 \times 3 \times 1$

Hundreds
tens — 5 hundreds, 4 tens , 8 units
Units

5 4 8

$548 = 5 \times 100 + 4 \times 10 + 8 \times 1$
$= 5 \times 10^2 + 4 \times 10 + 8 \times 1$

Generally the place value of the digits is shown by the ascending powers of the base.

$10^1\ 10^0$
$2\quad 3 = 2 \times 10^1 + 3 \times 10^0$
$= 2 \times 10^1 + 3 \times 1$
Value of 2 is $2 \times 10^1 = 20$

$10^3\ 10^2\ 10^1\ 10^0$
$5\quad 4\quad 8\quad 2 = 5\times10^3 + 4 \times 10^2 + 8\times10^1+2\times10^0$
Value of 5 is 5×10^3
$= 5000$

Other Bases

The same applies to numbers in other bases

For example base five system is based on powers of five5^2 5^1 5^0

$2\ 3\ 4 = 2 \times 5^2 + 3 \times 5^1 + 4 \times 5^0$

(but 5^0= 1twenty, 3 fives, 4 units

Base two is based on power two

2^2 2^1 2^0

$1\ 1\ 0 = \quad 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$ (but 2^0=1

two 1 fours, 1 twos, 0 units

Take note that all digits forming base five are less than five e.g. 234, also all digits forming a number in base two are less than two e.g 1011_2

The notation 234 $_{five}$ means 234 in base five, also 1001_2 mean 1011 in base two.

Example 1

Expand a) 521 b) 6 728 c) 1101

10^2 10^1 10^0

a)	521	=	5 2 1
	10	=	$5 \times 10^2 + 2 \times 10^1 + 1 \times 10$
		=	$5 \times 10^2 + 2 \times 10^1 + 1$

8^3 8^2 8^1 8^0

b)	6 728 $_{eight}$	=	6 7 2 8
		=	$6 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 + 8 \times 8^0$
		=	$6 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 \times 8$
c)	1101 $_{two}$	=	1101
		=	$1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
		=	$1 \times 2^3 + 1 \times 2^2 + 1$

EXERCISE 2.1

Expand the following in the powers of the given bases

a) 314 $_{ten}$	b) 344 $_{five}$	c) 7 642 $_{eight}$
d) 11 $_{two}$	e) 2 344 $_{five}$	f) 1011_2
g) $71\,40_5$	h) $2\,112_3$	i) 267_8
j) xy_2	k) abc_5	

Changing to other bases from base ten

Convert 342 to a number in

a) base 5 b) Base 2

a)

5	342
5	68 r 2
5	13 r 3 take remainders
5	2 r 3 ↑ upwards

2	342
2	171 r 0
2	85 r 1
2	42 r 1

5	0 r 2	2	21 r 0
		2	10 r 1
		2	5 r 0
		2	2 r 1
		2	1 r 0
		2	0 r 1

$324_{10} = 2\,332_5$ $\quad\quad\quad\quad$ $342_{10} = 101010110_2$

<u>To change from base ten to another base</u>

a) Divide the number in base ten number by the required base.
b) Continue dividing until O is reached, writing down the remainder each time.
c) Take the remainders upwards,

Example 3

Convert the following to base ten

a) 342_5 $\quad\quad$ b) 1101_2

$5^2\ 5^1\ 5^0$

a) $3\ \ 4\ \ 2 = 3 \times 5^2 + 4 \times 5^1 + 2 \times 5^0$

$= 75 + 20 + 2$

$= 97_{10}$

$2^3\ 2^2\ 2^1\ 2^0$

b) $1\ 1\ 0\ 1_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 1$

$= 13_{10}$

To change from other bases to base ten

a) Expand the given number
b) Solve

Exercise 2,2

1. Convert the following from base 10 to given base

a) 13 to base 5 $\quad$ b) 68 to base 2

c) 161 to base 8 $\quad$ d) 64 to base 2

e) 570 to base 5 $\quad$ f) 671 to base 8

g) 69 to base 3 $\quad$ h) 1 056 to base 8

i) 1130 to base 8

2. Convert the following to base ten.

a) 345_8 $\quad$ b) 1235_8 $\quad$ c) $1\,2445_8$

d) 112_5 $\quad$ e) 101_2 $\quad$ f) 111012_5

g) 10112_5 $\quad$ h) 2678_9 $\quad$ i) 7768_9

j) $2\ 6438_9$

Convert 1223_5 to a number in base two. **Hint first change to base ten**

Adding and subtracting number bases

$$\begin{array}{r} 288 \\ +\ \underline{979} \\ \underline{1\ 267} \end{array}$$

The following steps were followed in adding the numbers in base ten.

Step 1: 9+8 = 17 (1r7) Put down the remainder and carry 1
Step 2: 8+7 + 1 = 16 (1r6) put down 6 and carry 1
Step 3: 2 + 9 + 1 = 12 (1 r2) put down 2 and carry 1
Step 4: 1+0 = 1 put down 1 and carry 0

The same method applies for adding in other base.

$$\begin{array}{l} 344_5 \\ \underline{444_5} \\ \underline{1343_5} \end{array}$$

Step 1 4+4 = 8 (i.e. 1 five + 3) Put down remainder 3 and carry 1

Step 2 4+4+1= 9 (I five + 4 or 1r4) Put down remainder 4 and carry 1

Step 3 3+4+1 = 8 (five + 3 or 1r3)

Example 4
Add the following.

a) $\begin{array}{r} 43_5 \\ \underline{+\ 41_5} \\ \underline{134_5} \end{array}$ $\begin{array}{l} 1101 \\ \underline{+\ 111^2} \\ \underline{10100_2} \end{array}$

Exercise
Add the following numbers

a) $\begin{array}{r} 125 \\ \underline{+189} \\ \underline{\quad} \end{array}$ b) $\begin{array}{r} 343_5 \\ \underline{+\ 321_5} \\ \underline{\quad} \end{array}$ c) $\begin{array}{l} 234_5 \\ \underline{330_5} \\ \underline{\quad} \end{array}$

d) $\begin{array}{l} 432^5 \\ \underline{444^5} \\ \underline{\quad} \end{array}$ e) $\begin{array}{l} 132_5 \\ \underline{233_5} \\ \underline{\quad} \end{array}$ f) $\begin{array}{l} 11 \\ \underline{1_2} \\ \underline{\quad} \end{array}$

g) $\begin{array}{l} 101_2 \\ \underline{111_2} \\ \underline{\quad} \end{array}$ h) $\begin{array}{l} 1111_2 \\ \underline{11_2} \\ \underline{\quad} \end{array}$ i) $\begin{array}{l} 1001_2 \\ \underline{111_2} \\ \underline{\quad} \end{array}$

j) $\begin{array}{l} 212_3 \\ \underline{211_3} \\ \underline{\quad} \end{array}$ k) $\begin{array}{l} 313_4 \\ \underline{213_4} \\ \underline{\quad} \end{array}$

Subtraction

The same rules for subtraction in base ten also apply in other bases e.g.

$$\begin{array}{rrr} 9 & 1^{+10} & 2^{+10} \\ -\ 4^{+1} & 3^{+1} & 8 \\ \hline 4 & 7 & 4_{10} \end{array} \qquad \begin{array}{rrr} 4 & 1^{+5} & 2^{+5} \\ -2^{+1} & 4^{+1} & 3^{5} \\ \hline 1 & 1 & 4_5 \end{array}$$

When subtracting you give the base of numbers being subtracted. In base 10, 10 is given, in base 5, five is given, in base two, two is given etc

Example 5

subtract the following

a)
$$\begin{array}{rrr} 3 & 3^{+5} & 2^{+5} \\ -2^{+1} & 4^{+1} & 4_5 \\ \hline & 3 & 3_5 \end{array}$$

b)
$$\begin{array}{rrrr} 1 & 0^{+2} & 0^{+2} & 1_2 \\ -\ {}^{+1} & 1^{+1} & 1 & 1_2 \\ \hline & 0 & 1 & 0 \end{array}$$
$= 10_2$

Exercise 2.4

Solve the following

a) $\begin{array}{r} 122_5 \\ -\ 44_5 \\ \hline \end{array}$ b) $\begin{array}{r} 401^5 \\ -\ 234_5 \\ \hline \end{array}$ c) $\begin{array}{r} 2\ 1\ 1_3 \\ 1\ 2\ 2_3 \\ \hline \end{array}$

d) $\begin{array}{r} 1\ 0\ 0_2 \\ -\ 11_2 \\ \hline \end{array}$ e) $\begin{array}{r} 1\ 0\ 0\ 1_2 \\ -\ 1\ 1\ 1_2 \\ \hline \end{array}$ f) $\begin{array}{r} 1\ 0\ 1\ 1_2 \\ -\ 1\ 0\ 0_2 \\ \hline \end{array}$

g) $\begin{array}{r} 2\ 0\ 1_5 \\ -\ 4\ 4_5 \\ \hline \end{array}$ h) $\begin{array}{r} 6\ 5\ 1_8 \\ -1\ 7\ 6_8 \\ \hline \end{array}$ g) $\begin{array}{r} 6\ 7\ 5_8 \\ -\ 7_8 \\ \hline \end{array}$

SUMMARY

All ordinary numbers are in base ten.
All digits forming a number in a certain base should be less than that base e.g. all digits forming a number in base 2 are less than 2 (i.e. 1 or 0).

EXAMINATION QUESTION

a). Express $2^4 + 2^3 + 2$ as a number in base two
b) Convert 21 to a number in base two
c) Evaluate $432_5 + 414_5$ giving your answer in base five

2 i) Convert 11001_2 to a number in base 10
ii) Evaluate $204_5 + 243_5 - 21_5$' giving your answer in given bases

3 a) Express 4638_8 as a number in base 10
b) Evaluate
i) $211_3 + 102_3$, giving your answer in base 3

4a) If $22_{10} = 34_m$, find the value of m
b) if $27_{10} = 123_x$ find the value of x

5.i) Write down $1 \times 2^4 + 1 \times 2^2 + 1 \times 2^2 + 1 \times 2^1 + 1$ as a number in base 2

ii) If $103_x = 67$, find the value of x

6 Solve the equation

$$2^{2n-1} = 64$$

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CHAPTER 3

STANDARD FORM

Standard form refers to a number with one digit before the comma e.g. 2,51; 3,87; 9,38 etc.

Syllabus Objectives

Leaner should be able to

a) Express numbers in standard form
b) Carry out operations of numbers in standard form
c) Find reciprocals using calculations and reciprocal tables
d) Carryout calculations using reciprocals

Standard form for scientific notation

In general any number in standard form is written in the form A x 10^n where I $\leq$ A < 10 where n is an integer (i.e any positive or negative whole number including zero)

Example 1

Express the following in standard form

a) 709 b) 6 318,2 c) 0,00356

a) $709 = 7{,}09 \times 100$
$= 7{,}09 \times 10^2$

b) $6\,318{,}2 = 6{,}3182 \times 1000$
$= 6{,}3182 \times 10^3$

c) $0{,}00356 = \dfrac{3{,}56}{1000}$

$= \dfrac{3{,}56}{10^3}$

$= 3{,}56 \times 10^{-3}$

For numbers less than zero the value of n is always negative as in example c

Example 2

Express the following numbers as decimal fractions

a) $3{,}041 \times 10^2$ b) 5×10^{-3} c) $8{,}2 \times 10^{-4}$

a) $3{,}041 \times 10^2 = 3{,}041 \times 100$ $= 304{,}1$	b) $5 \times 10^{-3} = \frac{5}{1000}$ $= 0{,}005$	c) $8{,}2 \times 10^{-4} = \frac{8{,}2}{10000}$ $= 0{,}00082$

Exercise 3,1

1. Express the following in standard form

a)	20	b)	301	c)	1 255
d)	3	c)	65,2	d)	452,2
e)	0,1	f)	0,543	g)	0,00893
j)	0000987	k)	0,0000895	l)	0,011

Express the following as a decimal fractions

a)	$6{,}54\ 10^1$	b)	$4{,}582 \times 10^2$	c)	3×10^{-2}
c)	3×10^{-2}	d)	$5{,}43 \times 10^{-2}$	e)	$9{,}87 \times 10^{-6}$
f)	$5{,}1 \times 10^{-4}$	g)	$2{,}8 \times 10^{-1}$		

Addition and subtraction in standard form

Both operations can be carried out either by the method of changing to ordinary form or by the method of factorization.

Example 3

Find the value of the following giving the answer in standard form.

$3{,}43 \times 10^3 + 5{,}27 \times 10^4 = 3\ 430 + 52\ 700$
$= 56\ 130$
$= 5{,}613 \times 10^4$

Or by factorization

$3{,}43 \times 10^3 + 5{,}27 \times 10^4 = 10^3\ (3{,}43 + 5{,}27 \times 10)$
$= 10^3\ (3{,}43 + 52{,}7)$
$= 10^3\ (56{,}13)$
$= 10^3 \times 56{,}13)$
$= 10^3 \times 5{,}613 \times 10^1$ $(56{,}13 = 5{,}613 \times 10^1)$
$= 5{,}613 \times 10^4$

b) By changing to ordinary form

$3{,}7 \times 10^{-3} – 5{,}2 \times 10^{-4} = \frac{3{,}7}{10^3} - \frac{5{,}2}{10^4}$

$= 0{,}0037 – 0{,}00052$

$= 0{,}00318$

$= 3{,}18 \times 10^{-3}$

Or by factorisation

$3{,}7 \times 10^{-3} - 5{,}2 \times 10^{-4}$ = $10^{-3}(3{,}7 - 5{,}2 \times 10^{-1})$
= $10^{-3}(3{,}7 - 5{,}2 \times 10^{-1})$
= $10^{-3}(3{,}18)$
= $3{,}18 \times 10^{-3}$

Exercise 3,2

Simplify the following using a) the method of charging to ordinary form
b) the method of factorizing

Give all answers in standard form

a) $4{,}5 \times 10^{3} + 3{,}2 \times 10^{2}$ b) $6{,}8 \times 10^{4} + 5{,}1 \times 10^{5}$

c) $4{,}8 \times 10^{2} - 3{,}2 \times 10^{2}$ d) $9{,}85 \times 10^{4} - 9{,}75 \times 10^{3}$

e) $403{,}2 + 520{,}4$ f) $2{,}76 \times 10^{-2} + 8{,}72 \times 10^{-1}$

g) $6{,}7 \times 10^{-2} - 8{,}53 \times 10^{3}$ h) $4{,}16 \times 10^{-2} + 8{,}72 \times 10^{-1}$

j) $2{,}2 \times 10^{-3} - 7{,}8 \times 10^{-4}$ k) $6{,}8 \times 10^{-4} - 3{,}1 \times 10^{-5}$

Multiplication and Division

Example 4

Simplify a) $8 \times 10^{4} \times 7 \times 10^{5}$ b) $1 \times 10^{4} \div 4 \times 10^{-3}$
c) $3{,}5 \times 10^{3} \times 3{,}4 \times 10^{-5}$

a) $8 \text{ x } 10^{4} \text{ x } 7 \text{ x } 10^{5}$ = $8 \times 7 \times 10^{4} \times 10^{5}$ (by laws of indices $10^{4} \times 10^{5} = 10^{4+5}$
= $56 \times 10^{4+5}$
= 56×10^{9}
= $5{,}6 \times 10^{1} \times 10^{9}$
= $5{,}6 \times 10^{10}$

b) $1 \text{ x } 10^{4} \div 4 \text{ x } 10^{-3}$ = $\frac{1\text{x}10^{4}}{4 \times 10^{-3}}$ (by laws of indices when dividing subtract
= $0{,}25 \times 10^{4--3}$
= $0{,}25 \times 10^{7}$
= $2{,}5 \times 10^{-1} \times 10^{7}$
= $2{,}5 \times 10^{-1+7}$
= $2{,}5 \times 10^{6}$

c) $3{,}5 \text{ x } 10^{3} \text{ x } 3{,}4 \text{ x } 10^{--5}$ = $3{,}5 \times 3{,}4 \times 10^{3} \times 10^{-5}$
= $11{,}90 \times 10^{3} + (-5)$
= $11{,}9 \times 10^{-2}$
= $1{,}19 \times 10^{1} \times 10^{-2}$
= $1{,}19 \times 10^{1+(-2)}$
= $1{,}19 \times 10^{-1}$

<u>EXERCISE 3,3</u>

1. Simplify the following, giving your answer in standard form.

a) $(4 \times 10^5) \times (7 \times 10^2)$ b) $(8 \times 10^5) \div (4\times 10^3)$

c) $(3 \times 10^{-3}) \times (6 \times 10^{-2})$ d) $(9 \times 10^{-3}) \times (3 \times 10\text{-}^4)$

e) $(3 \times 10^{-3}) \times (6 \times 10^{-2})$ f) $(3,5 \times 10^{-2}) \div 5 \times 10^{-1}$

g) $(1,44 \times 10^5) \times (5 \times 10^3)$ h) $(1,21 \times 10^{-3-}) \div (1,1\times 10^{-2})$

i) $(1,404 \times 10^4) \times 2,6 \times 10^{-3})$ j) $5,6 \times 10^{2\,-} \times 1,3 \times 10^{-1}$
k) $7,28 \times 10^3 \div 1,3 \times 10^{-1}$ l) $5,52 \times 10^4 \div 2,4 \times 10^2$

2. Given that $E= mc^2$ express the value of the E in standard form given that $m= 3 \text{ x } 10^{-3}$ and $c= 2 \text{ x } 10^9$

3) A tobacco sales room measures $24\text{m} \times 20\text{m} \times 15,75\text{m}$. Calculate its volume in cm^2 in standard form
4) A field of maize has on area of $37\,500\text{m}^2$. What
(a) Is its area in hecters
(b) If 6 tonnes of maize were gathered, what is the yield per hecters of maize.

5. 950 metal sheets are staked together in a warehouse. (Answers in standard form)
(a) If each is of thickness of 6,5mm. Calculate the height of the pile in mm
(b) Express the height of the pile in km.

Reciprocals of numbers

The reciprocal of any number x = $\frac{1}{x}$

Examples

Find the reciprocal of the following

a) 2 b) 20 C) $4\frac{1}{2}$ d) 0,05

a) Reciprocal of 2 $= \frac{1}{2}$

$= 0,5$

b) Reciprocal of 20 $= \frac{1}{20}$

$= 0,05$

c) Reciprocal of 4 ½ $= \frac{1}{4\frac{1}{2}}$

$= \frac{1}{\frac{9}{2}}$

$$= \quad 1 \times \frac{2}{9}$$

$$= \quad \frac{2}{9}$$

d) Reciprocal of 0,05 $= \dfrac{1}{\frac{5}{100}}$ (change 0,05 to a fraction)

$$= \quad 1 \div \frac{5}{100}$$

$$= \quad 1 \times \frac{100}{5}$$

$$= \quad 20$$

Exercise 3.4

Find the reciprocal of the following. Give each answer as a decimal correct to 3 s. f

a)	4	b)	5	c)	0,05	d)	0,3
e)	900	f)	$\frac{1}{4}$	g)	$2\frac{1}{2}$	g)	$1\frac{3}{4}$
h)	$7\frac{1}{10}$	i)	0,225	j)	0,48	k)	0,018

Use the reciprocal tables
The tables give the reciprocals correct to 4.s.f.
The differences in the right-hand columns must be subtracted. The decimal comma must be placed by inspection.

EXAMPLE 6

Use the reciprocal tables to find the reciprocals of
a) 2,51 b) 251 c) 0,418

a) Reciprocal of 2,51 $= \dfrac{1}{2,51}$

$= 0,3984$ (across 2,5 and under 6)

b) Reciprocal of 251 $= \dfrac{1}{251}$

$= \dfrac{1}{2,51 \times 10^2}$ (in standard form)

$= \quad 0{,}3984 \times \dfrac{1}{100}$ ($^{1}/_{2,51}$ is a reciprocal of 2,51)

$= \quad \dfrac{0{,}3984}{100}$

$= \quad 0{,}003984$

c) Reciprocal of 0,418 $= \dfrac{1}{0{,}418}$

$= \dfrac{1}{4{,}18} \times 10^{-1}$

$= \dfrac{1}{4{,}18} \times \dfrac{1}{10}$

$= 0.2392 \times 10$ $\quad (10^{-1} = \dfrac{1}{10^{1}})$

$= 2.392$

Calculations using reciprocal

Example 7

Find the value of $\dfrac{4}{225}$

$\dfrac{4}{225} = 4 \text{ x } \dfrac{1}{225}$ (but $\dfrac{1}{2{,}25}$ is the reciprocal of 2,25

$= 4 \times 0{,}4444$

$=$ 1,778 to 4 s.f

EXERCISE 3.5

1 a) Use table to find the reciprocals of

s)	1,5	b)	4,9	c)	1,52	d)	4,98
f)	2,556	g)	9,83	h)	3,065	e)	3 329

2. Solve the following

a) $\frac{7}{1,52}$ b) $\frac{8}{0.35}$ c) $\frac{5}{170}$ d) $\frac{4}{3.38}$

e) $\frac{3}{0.477}$ f) $\frac{7}{1.34}$ g) $\frac{7}{134}$

EXAMINATION QUESTIONS

1) Simply the following. Express the answers in standard form.

a) $3,8 \times 10^{-4}) \times (2 \times 10^{-5})$

b) $(9,53 \times 10^{3}) - (7,9 \times 10^{2})$

c) $(4 \times 10^{-3}) \times (9 \times 10^{5})$

d) $8,2 \times 10^{-5}) \div (4,1 \times 10^{-3})$

2) If U = $3,2 \times 10^{2}$ and v= $2,56 \times 10^{1}$ find the values of

a) U + V b) UV c) $\frac{U}{V}$ d) $\frac{V}{U}$

3) Calculate the following using tables leave the answer to 3.s. f.

a) $\frac{1.35}{2,38}$ b) $\frac{45,8}{\sqrt{6,31}}$

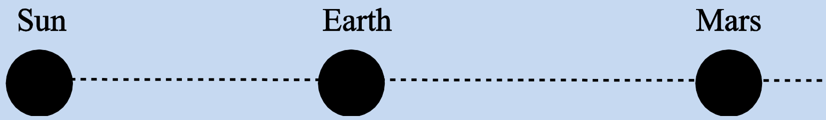
4. Find the value of x if $\frac{1}{X} = \frac{1}{12} + \frac{1}{21}$

5 Given that y = $\frac{a}{a+b}$, if a = $2,1 \times 10^{-2}$ and b = $1,8 \times 10^{-1}$. Calculate the value of y and leave the answer in standard form

6. Given that m = 3×10^{2} and n= 5×10^{-4} express in standard form

i) mn ii) $\frac{m}{n}$

7a)



In the diagram, the sun, Earth and Mars are in a straight line. It is given that the earth is 1,496 x 10^8 km from the sun and mars is 2,279 x 10^8 km from the sun

i) Write down 1,496 $\times 10^8$ in ordinary from
Find, the standard form, the difference of Mars from the Earth.

(ZIMSEC NOV 2008)

CHAPTER 4

SET

A set is a collection of well defined objects

diagram

Picture shows a set of school pupils

SYLLABUS OBJECTIVES

Learner should be able to:
a) Define a set
b) Identify all the required set notations and state their meaning
c) Represent given information in a venn diagram
d) Apply the concept of sets to solve problems

There are 3 ways of writing down sets.
a) By listing its elements e.g. A {1;3;5;7;9}
b) By description e,g A = {odd numbers less than 10}
c) By using the set builder notation
If A = {1;2;3;4}, then A = $\{x:1\leq x\leq 4\}$. In set builder notation.

Symbols	Meaning
Finite Set A = { a,b,c,d .,.,.,y,z}	The set is large hence its element can be placed in chronological order With the first few and last few elements
Infinite set A= {2,4,6.,.,.}	Sets which never end
A=B	Set A and set B have equal number of elements which are similar e.g. A {1,2,3} and B {1;3;2}
Ø or {}	Empty set means the set has no elements e.g. (odd numbers divisible by 2}
$\in$	Member of means the elements belongs to a set
$\notin$	Not a member means does not belong to a set
$A\subseteq B$ $A\subset B$	A is a proper subset of B A is a subset of B

A ⊃ B	A contains B
A u B	A union B means all elements in set A and set B grouped together
A n B	A intersection B means only elements which are common to both sets are grouped together.
n (A)	Number of elements in set A. Repeated elements are counted once.

Universal set ڠ

The universal set is the mother of all elements being considered. It is shown by the symbol ڠ

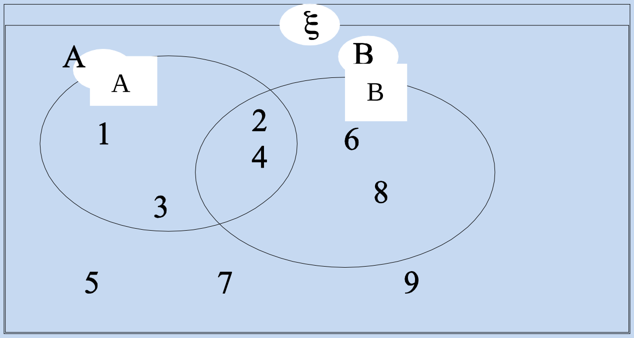
Other small set can be formed by drawing elements from the universal set. For example, if the whole numbers from 1 to 9 are being considered, then ڠ = {1,2,3,4…,9}, then a smaller set A= {even numbers} is restricted to {2;4,6,8} , B = {odd numbers} is restricted to {1;3,5,7,9}. Many sets can be formed from the universal set.

Venn Diagrams

The venn diagrams is a helpful means of illustrating the mother set (universal set) and the smaller sets formed from it. In a Venn diagram the universal set is usually represented by a rectangle.

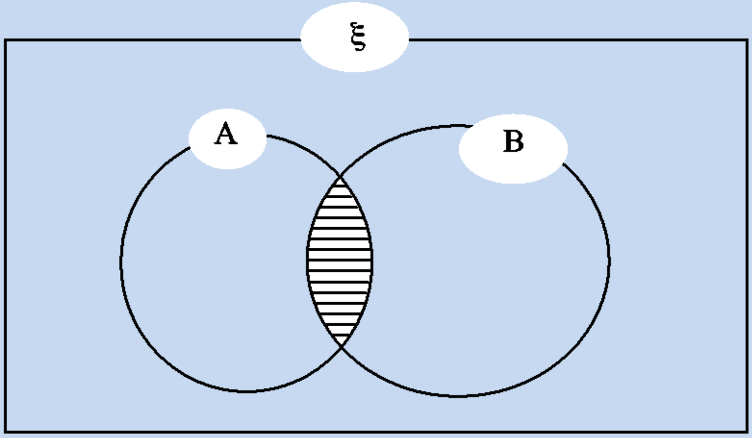
For example if ڠ = { 1;2;3;4;.;.; 9}, A = {1,2,3,4,}
B = { 2;4;6;8;}

The venn diagram will illustrate these sets

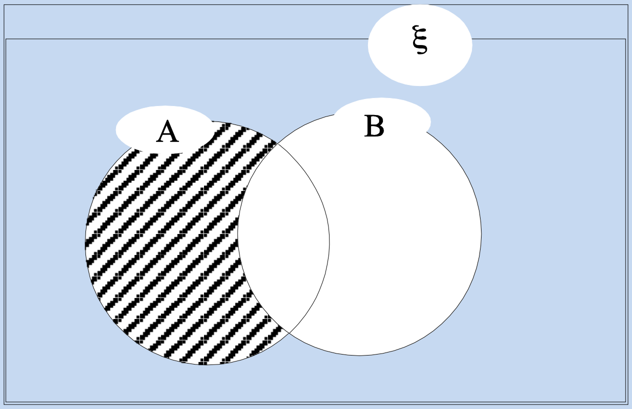


Each of the regions in the Venn diagram can explained by shading as follows.

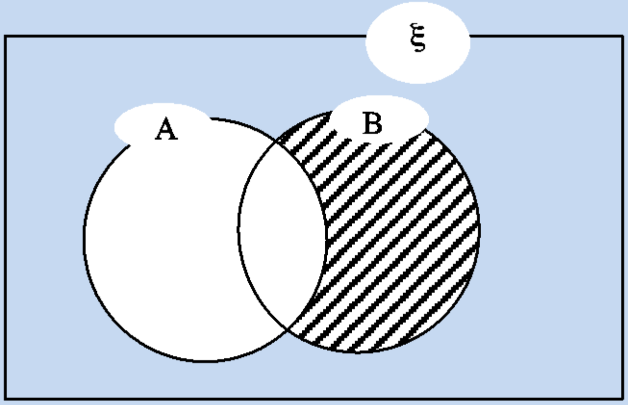
a)



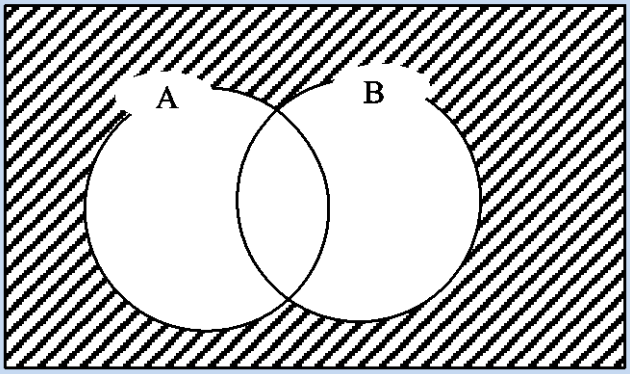
A n B

(c) 

Elements found in set A only

d) 

Elements found in set B only

e) 

Elements in the universal set but not in both A and B

COMPLEMENT

A complement of a set contains all the elements of the universal set, which are not in the referred set. For example A^1 is the complement of set A, i.e. all the elements of ἐ which are not in A .

Example 1

Given that $\quad$ غ = {1;2;3;4.;.;.;9}
A = { 1;2;3;4;}
B = {2;4;6;8}

Find	i)	A n B	ii)	Au B	iii)	A^1
	iv)	A^1 u B^1	v)	$(A \text{ u } B)^1$	vi)	n(A u B)

i) An B = {2, 4} $\quad$ ii) A u B = {1;2;3;4;6;8}

iii) A^1 = {5,6,7,8,9}

(iv) A^1= {5;6;7;8;9}, B^1= {1;3;5;7;9}
then A^1 u B^1 = {1;3;5;6;7;8;9}

v) A u B = {1;2;3;4;5;6;8;}
then $(\text{ A u B})^1$ = {5;7;9}

vi) (A u B) = {1;2;3;4;6;8}
Then n (AuB) = 6

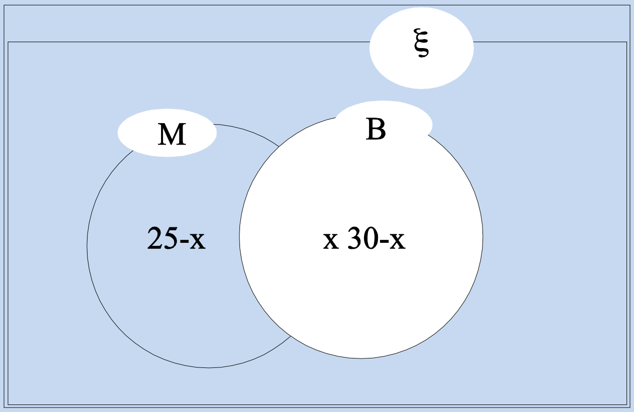
Example 2

50 students were asked which mode of transport they used to school. Thirty said they used bus and 25 said they used motor vehicles. Draw a venn diagram to illustrate this and find how many use both bus and motor vehicle.

Let $\quad$ غ = {all the students}
M= {students who use motor vehicle}
B= {students who use buses}

It is required to find n(MnB). Since its unknown; let it be x in Venn diagram.

REVISIT THE DIAGRAM



$$(25-x) + x + (30 - x) = 50$$
$$(25-x) + x + 30-x = 50$$
$$55 - x = 50$$
$$55 - 50= x$$
$$x = 5$$

:- Student who use motor vehicle and buses = 5.

Commonly used term

"Neither nor" means none of the stated. For example if 10 students used neither bus nor motor vehicle, it. means they used other mode of transport like walking. This would be illustrated in the Venn diagram as shown below.

Either .. or means from the two option person uses one or both. For example, if 40 students use either motor vehicle or bus to school it means a) Students who do not use motor vehicles use buses.

b) Those who do not use buses use motor vehicles.

c) Some use both modes of transport and are found in the intersection.

Exercise 1.1

1. List the elements of the following sets
   a) A = { x ; x <10; x is a factor of 20}
   b) B = {Interger x :5 < x <10}
   c) C = { x :0 < x $\leq$ 9, x is even}
   d) D= {Integer: y > 0}
   e) E = { x :-4 ¾ < x < 3 ½}

2. If ع = {1; 2;3; 4, …9}
   P = {2, 4, 6, 8}
   Q= {1; 2; 3; 4}

Find a i) P n Q ii) P u Q iii) P^1 iv) Q^1
v) $(P \text{ n } Q)^1$ vi) P^1 u B^1 vii) $(P \text{ u } Q)^1$ viii) P^1n Q^1

b) Illustrate the set in a venn diagram.

3. Draw venn diagrams to illustrate the following by shading the required regions
   i) PuQ ii) P n Q iii) $(P \text{ n } Q)^1$
   iv) P^1 u Q^1 v) $(P \text{ u } Q)^1$ vi) P^1n Q^1

4. If ع ={1;2;3;4;.;.;.;20} List the members of the following sets.
   a) { x:x is an even number, x Є ع}

   b) { x : x is a factor of 30, x Є ع}

   c) { x : x +1 10, x Є ع}

   d) { x :y : y= x +1, x Є ع,y Є ع}

   e) { x :y :y= x^2 +1, x Є ع,y Є ع }

5. If ع = { x ; 1$\leq$ x $\leq$ 20, x is an integer}, A= { x :: x is a perfect square} and B = { x : x is a factor of 40}
   a) fInd i) n(B) ii) n(B^1) b) n $(AUB)^1$

   b) List the numbers of the set A^1n B^1

6. 50 pupils took examinations in history and commerce. 44 passed the history examination, 41 passed the commerce examination and 39 passed both
   i) Show the information on a venn diagramm
   ii) How many passed neither examination.

7. In a class, 40 pupils play either soccer or tennis. If 19 pupils play tennis and 21 pupils play soccer. find the number of pupils who play both sports given that there are 45 pupils in the class.

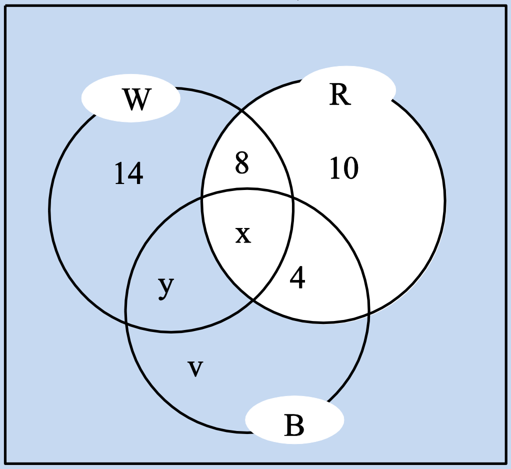
VENN DIAGRAMS WITH THREE SUBSETS

Examples 3

Every family in a village owns at least a wheelbarrow, a radio or a bicycle. 30 own a wheelbarrow, 25 own a radio and 18 own a bicycle. 14 own a wheelbarrow only, 10 own a radio only, 8 own wheel barrow and radio only and 4 own radio and bicycle only. How many own

a) All the three b) bicycle and wheelbarrow only
c) bicycle only

Let خ {all students}, w= {those who own wheel barrow}
R= {those who own radios}, B= {those who own radios}



a) Since n(R) = 25
Let x families own all three
$x + 8 + 4 + 10 = 25$
$x + 22 = 25$
$x = 3$
:. 3 families own all three

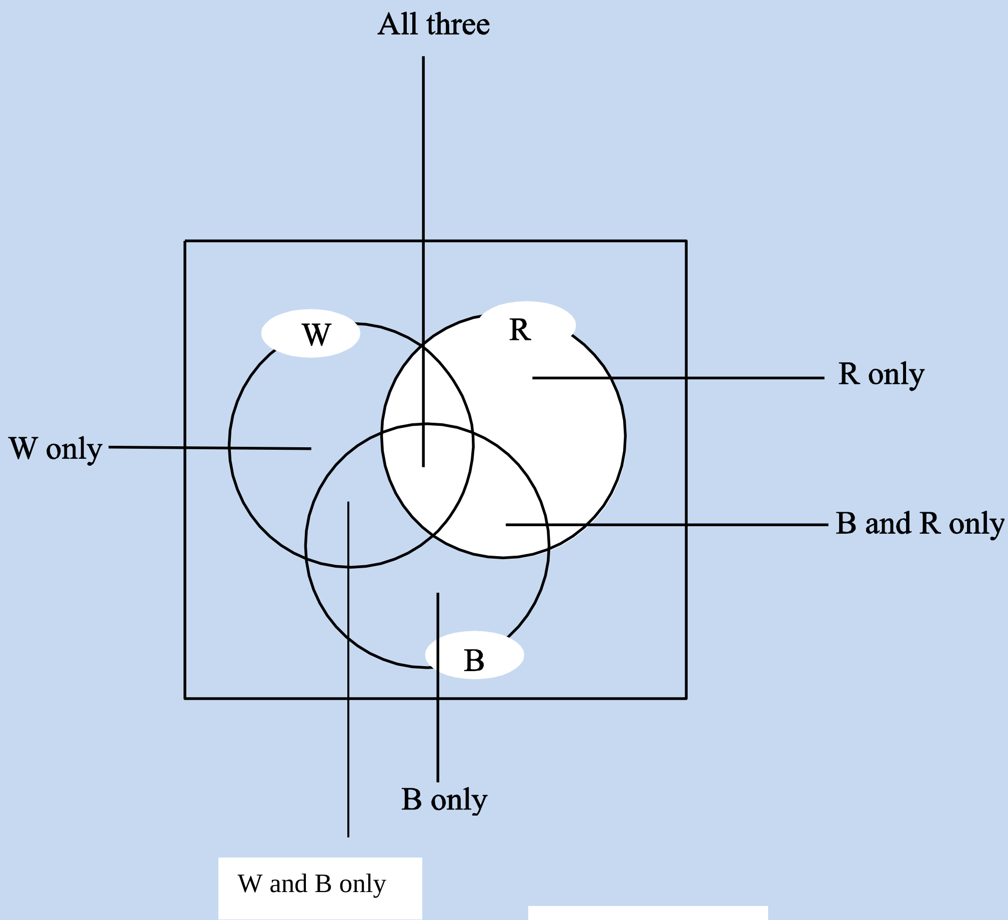
b) Since n(w) = 30
let y families own bicycle and wheel barrow only
$14 + 8 + x + y = 30$
But x = 3
$14 + 8 + 3 + y = 30$
$25 + y = 30$
$y = 5$

c) Since n(B) =18
Let V families use bicycles only
$y + v + x + 4 = 18$
but x = 3 and y=5
$5 + v + 3 + 4 = 18$
$v = 6$
:. 6 own bicycles only.

Summary
Given three set in a venn diagram. The diagram below shows the meaning of the region.

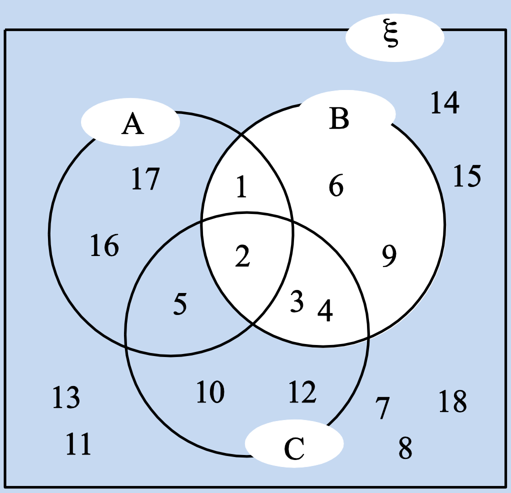
All three W,R and B

R= {Radios}
B= {bicycles}
W = {wheel burrows}



Exercise **1,2**

From the Venn diagram in fig 1,01 list the following sets.



a)	A	b)	A n B	c)	B UC	d)	A n B n C
e)	A n C n B^1	f)	$(A \text{ u } B)^1$	g)	C u (AnB)	h)	A^1 n B^1
j)	A n B^1 n ^{1}						

3. From the Venn in fig 1,01 Find

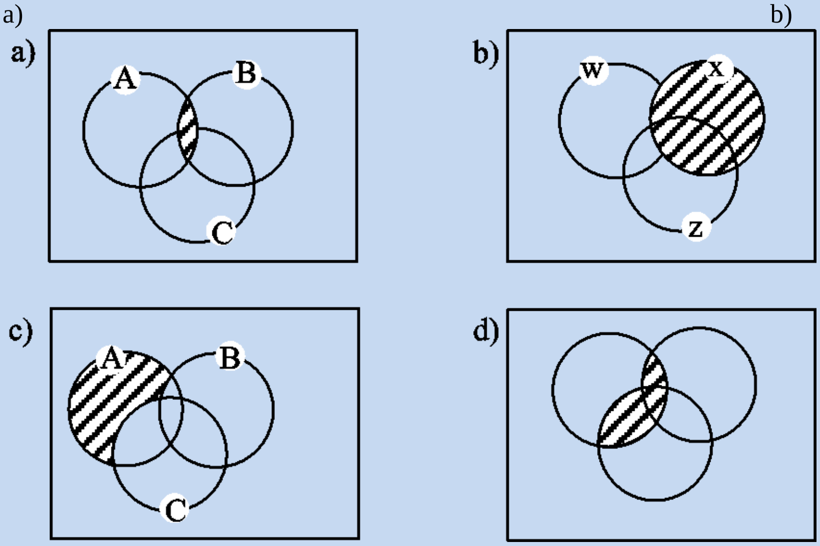
a) n (A) $_1$ b) n (B UC) n(An BnC)

d) n (An Bn C) e) n (A^1nB^1nc)

4. Use the set notation to describe the shade region in each of the Venn diagram below.

REVISIT THE DIAGRAM

a) b)



5.

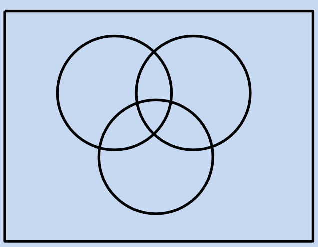


Fig 1,03

Copy fig, 1,07 for each of the following and shade in.

a) P n Q n R b) Q n R c) P u R

d) A n B n C e) A n B^1 n c^1 f) A^1 u B^1

g) An (BuC) h) A u (BnC)

6. 30 people were each asked to indicate which fruit trees they had in their orchard from amongst mango, peaches and apple. 23 people had mango, 27 had peaches, and 19 had apples. 20 people had both mango and peaches, 18 had both peaches and apple and 17 had mango and apple.
How many people had all the fruit trees in their orchard?

7. 25 % of the pupils use cars to come to school, 35% use train and 40% walk to school. 5% use both cars and train, 3% walks and use cars and 2% walks and use train. If 1% use three modes, what percentage of the pupils use none of the three mode of transport.

EXAMINATION QUESTIONS

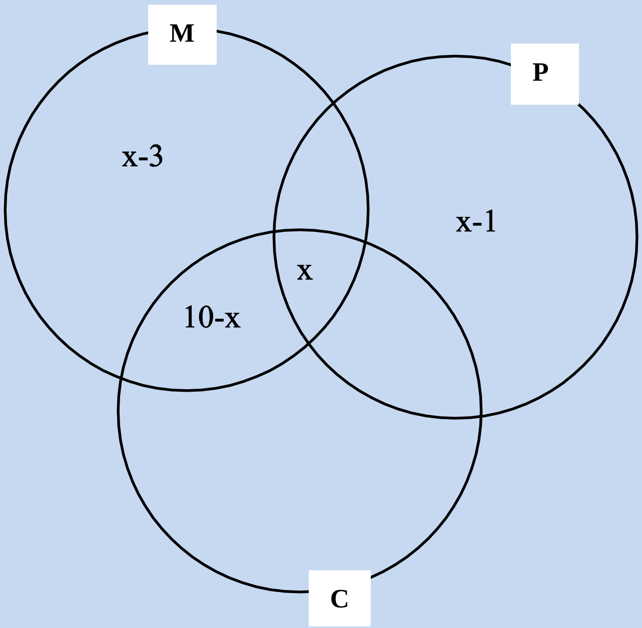
1a) In a group of 25 students, 15 study Mathematics, 12 Physics and 18 chemistry. It is given that

8 Study Mathematics and Physics
5 Study Physics and chemistry
10 Study chemistry and mathematics and
x study all the three subjects

Each students studies at least one of these subjects

i) Copy and complete the venn diagram to show the number of elements in each subsest in terms of x

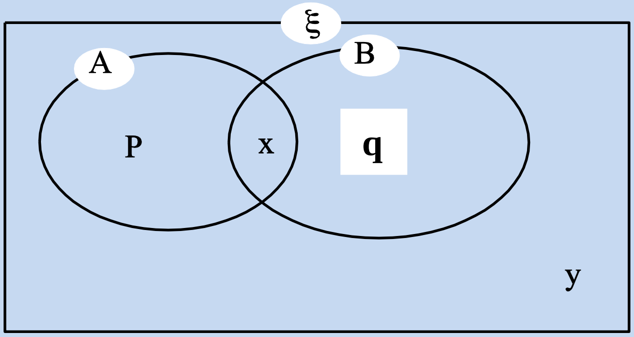
REVISIT THE DIAGRAM



ii) From an equation in x and solve it. Hence or otherwise find the number of students who study mathematics and chemistry only. (ZIMSEC: JUNE 2006).

2a) ڠ = {1;3;5;7;9,11}, x = {1;5;9} and Y= {3,9,11}
List the elements of i) i) x^1 n Y ii) (X u Y)^1

b) In Fig 6.22 is the set of children in a certain chosen group. A = {children in youth club A} and B= {children in youth club B}. The letters p,q, x and y in the figure represent the number of children in each subset.



Given that n (ḃ) = 200, n (A) = 75 and n(B) =35,

i) Express p in terms of x

ii) Find the smallest possible value of y

iii) Find the largest possible value of x

iv) Find the value of q if p=45 (C1981)

3.a) ḃ= {x:x is an integer 10< x $\leq$100}

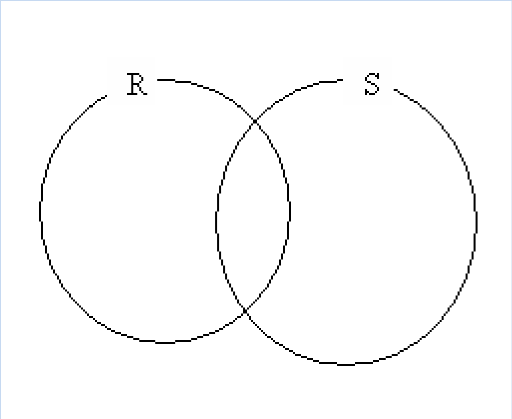
A = {x:x is a multiple of 17}

B = {x;x divided by 15 leaves a remainder 7}

Find

(i) n(A) ii) the largest number of B iii) n(AuB)

b)



The venn diagram shows sets R and S such that R u S =ḃ

On separate copies of the diagram, shade

i) R^1 u S ii) R^1 u S

ZIMSEC NOV 2002

CHAPTER 5

LOGARITHIMS

In number 10^2, 10 is the base and 2 is the power

i) $10^2 = 100$ $\log_{10} 100 = 2$

ii) $2^3 = 8$ $\text{Log}_2\ 8 = 3$

From the above examples, the logarithm of a number (e.g. 100) is the power to which a given base (i.e. 10) can be raised to give that number. In General $x^y = a$ then $\log_x a = y$

Syllabus objectives
Learner should be able to

a) State and apply all the required laws of logarithms
b) Use tables to find the logarithms of number
c) Use logarithms tables
d) Carryout multiplication and division using logarithms
e) Carryout calculation of powers and roots using logarithms

Law of logarithms

1. $\text{Log}_a (x\,y) = \text{Log}_a x + \log_a y$

2. $\text{Log}_a (x/y) = \text{Log}_a x - \log_a y$

3. $\text{Log}_a x^n = n \log_a x$

4. $\log_a a = 1$

Example 1
Express the following as logarithms of single numbers

i) log 5 + log 4 ii) log 27 = log 3
iii) 1-log 2 iv) ¾ log 64

i) $\text{Log } 5 + \log 4 = \log (5 \times 4)$ Law I
$= \text{Log } 20$

ii) $\text{Log } 27 - \text{Log } 3 = \text{Log}\left(\frac{27}{3}\right)$ Law 2

$= \text{Log } 9$

iii) $1 - \text{Log } 2 = \text{Log } 10 - \text{Log } 2$
$= \text{Log}\left(\frac{10}{2}\right)$
$= \text{Log } 5$

iv) ¾ Log 164 = Log $16^{\frac{3}{4}}$

= Log $(\sqrt[4]{16})^3$

= $\log (2)^3$

= Log 8

Example 2

a) Given log 2 = 0,3010 and log 3= 0,4771, evaluate log 5

b) Solve the following for x $\log_{10} x = -1$

c) Evaluate, $\log_2 64$

a) Log 5 = log $\left[\frac{10}{2}\right]$

= $\log_{10} 10 - \log 2$ but $\log_{10} 10 = 1$

= log 1-0,3010

= 0,6990

b) $\log_{10} x = -1$

$10^{-1} = x$

$x = \frac{1}{10}$

c) $\log_2 64$

Let N = $\log_2 64$

Then $2^N = 64$

$2^N = 2^6$

:-N= 6

$\log_2 64 = 6$

Remember the following

$\log_{10} 1 = 0$

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

etc

$X^y = x^z$

Means y = z

Log 0,001= 1/100

Log 1 – log 100

= 0-2

= - 2

Exercise 1,1

1) Simplify

a) $\text{Log}_{10} 20 + \text{Log}_{10} 4$ b) $\text{Log}_{10} 100 - \log_{10} 30$

c) $\text{Log}_5 75 - \log_5 5$ d) 2 log 5

e) ½ Log 81 f) log 9-Log 3

g) 1 + log 81 h) 2- 2log 2

2. Given that log 2=0, 3010 and log 3=0,4771, find the value of :

a) Log 0,002 b) Log 6 c) Log 27

3. Given that $\log_{10}9$ = 0,9542 evaluate

a) Log_{10} 81

b) Log_{10} 90

c) $\text{Log}_{10} \left(\dfrac{100}{9}\right)$

4. Find the values of a orb

$\text{Log}_{10}\, a = \log_{10} 8 + \log_{10} 5$ b) 3 log b=log 27

5) If $27^a \times 9^{(2a-3)} = 3^4$ show that the values of, a are given by the equation $2a^2+3a-10 = 0$. Hence, calculate the positive value of a, correct to 2 decimal places.

Logarithms as an aid to calculations +++++++

Logarithms are very useful in making cumbersome calculations e.g. 2,385 x 0, 9879

2,7890

Integer fractional part

Use of logarithm tables

The logarithm of a number has two parts, the integer part and the fractional part.

Finding the integer part

The inter part is found by expressing the number in standard form, then the power of ten becomes the integer part.

For example on finding integer parts of the following numbers.

i) Log_{10} 4200 ii) $\log_{10}$ 0, 0310

$\log_{10}$ 4200 = 3, (4 200 = $4{,}2 \times 10^3$ in standard form, therefore 3 forms the integer part since it is the power of ten in standard form.

Log 0, 0310=2 0, 0310 = 3, 10 x 10^{-2} in standard form, therefore. -2 forms the integer part it is written as $\bar{2}$ reads bar 2.

Finding the fractional part

Taking a cross-section of the tables

X	0	1	2	3	4	5	6	7	8	9	Differences 1 2 3 4 5 6 7 8 9
35	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 3 4 5 6 7 8 9

To find the fractional part the first two digits of your number gives you the row across which to look under the first column x. The third digit gives you the column still on that row. Then the last digit under the differences column in that raw gives you the difference which is added.

For Example

Log $_{10}$ 3,533 Fractional part is found across
35 under 3 (7427) difference 3 which is 2. Then 2 is added, 7427 +2 = 7429 so, 7429 is the fractional part

:- log 3,533 = 0,7429
Also log 0,3533= $\bar{1}$, 74279 (3,533x10-1 in standard form)

Example 3
Write down the integer parts of the following

i) 4,008 ii) 4 960 iii) 0,064 iv) 0,000047

i) 4,008 = 0, 4,008 = 4,008 $\times 10^{0}$ in standard form

ii) 4 960 = 3, 4 960 = 4,960 $\times 10^{3}$ in standard form

iii) 0,064 = $\bar{2}$, 0,064 = 6,2 $\times 10^{-2}$ in standard form

iv) 0,00047 = $\bar{5}$, 0,000047 = 10^{-5} in standard form

Example 4
Find the logarithms of the following

i) 6,51 ii) 0,0824

i) Log 6,51 = 0,8326
ii) Log 0,0824 = $\bar{2}$,9731

Brief Summary
When finding logarithms first find the interger part and then the fractional part. On the fractional part add the differences.

Try these
Find the logarithms of the following

a) 3,071 b) 30,71 c) 0,03071

d) 7,89 e) 0,789 f) 0,00789

g) 3248 h) 0,3248 i) 0,0003248

j) 83,51 k) 0,008351 l) 0,0008351

Antilogarithm

When given that log a= 0,8326. How do you find the number a?

From theory logarithm $\log_{10} a = 0,8326$

$a = 10^{0,8326}$

Since its difficult to simplify $10^{0,8326}$ the solution can be easily be found by looking for the antilogarithm on such tables.

When finding an antilog, we look up the fractional part only in antilog tables. Then use the integer to place the decimal correctly in the final number. How? We simply add 1 to the integer to get the number of digit before the comma thus 1+1 = 2 digits before comma. When the integer is for example 2. All bars are treated as negative numbers thus -2+1= -1 thus 0,0 non-zero digit, -3 means 0,00 non-zero digit which could be 0,01250, 0,00125 and 0,000125 respectively.

Like in finding logarithms add the difference.

Example 5

Find the antilogarithms of the following

i) 1,5638 ii) 0,4623 iii) 1,2113

i) Antilog 1,5638 = 36,63 (1+1 =2 digit before the comma)

ii) antilog 0,4623 = $10^{0,4623}$
= 2,900 (0+1 =1 digit before the comma

iii) antilog $\bar{1}$,2113 = $10^{\bar{1},2113}$ ($\bar{1}$+1=0)
= 0,1627

Try these

Write down the values of the following

a) $10^{2,9517}$ b) $10^{2,8572}$ c) $10^{3,8575}$

d) $10^{0,2340}$ e) $10^{\bar{1},2348}$ f) $10^{\bar{1},2654}$

g) $10^{2,5672}$ h) $10^{3,7271}$ i) $10^{4,5672}$

Multiplication and division

Example 6

Use logarithms

i) $307 \times 7,83$ ii) $\dfrac{0.3426}{0.9235}$

iii) $\dfrac{3.07 \times 0.783}{9.235}$

ii) 307 x 7,83

No	Log
307	2,4871
7,83	+ 0,8938

2 404 3,3809

(Find antilog of 3,3809)
:- 303 × 7,83 = 2 404

ii) $\frac{0.3426}{0.09235}$

No	Log
0,3426	1,5348
0,09235	2,9655
3,710	0,5693

:- $\frac{0.3426}{0.09235}$ = 3,710

iii) $\frac{3.07 \times 0.783}{9.235}$

No	Log
3,07	0,4871
0,783	+ 1,8938
Numerator	0,3809
9,235	- 0,9655
0,2602	1,4154

simplify the numerator

change the denominator to logarithms

$\frac{3.07 \times 0.783}{9.235}$

=0,2603

NB. When multiplying add the logarithms when dividing subtract the logarithms

Exercise 1,2

1. Evaluate the flowing

a) 29,34 × 4,432 b) 4934 ÷ 37,92

c) 0,6343 × 0,03482 d) 3,42 ÷ 35,94

e) 0,0561 ÷ 0,00342 f) $\frac{0.531 \times 0.436}{0.0649}$

g) $\frac{4.345 \times 9.143}{24.32}$ h) $\frac{2.64 \times 0.00921}{0.05738}$

i) $\frac{28.86 \times 105.2}{785.3}$ j) $\frac{0.523 \times 18.85}{8.447}$

POWERS AND ROOTS

Example 7

Evaluate

i) $(0,06827)^2$ ii) $\sqrt[3]{0,06827}$

i) $\sqrt[5]{\dfrac{0,6827}{0,4351}}$

i) $(0,06827)^2$

No	Log
0,0 6827	$\bar{2},8342 \times 2$
0,004660	$\bar{3},6684$

(remember bar is treated as a negative)

$(0,06827)^2 = 0,004660$

ii) $\sqrt{0,06827} = (0,06827)^{½}$

No	Log
0.06827	$\bar{2},8342 \times {}^1/_2$
8,786	0,9447

$\sqrt{0,06827} = 8,786$

ii) $\sqrt{\dfrac{0,6827}{0,4351}} = \left(\dfrac{0,6827}{0,4351}\right)^{1/5}$

No	Log
0,6827	$\bar{1},8342$
0,4351	$\bar{1},6386$
	$0,1956 \times {}^1/_5$
1,094	0,03912

(subtract)

$\sqrt[5]{\dfrac{0,6827}{0,4351}} = 1,094$

<u>Exercise 1,3</u>

Evaluate the following

a) $1,54^3$ b) $\sqrt[4]{243,4}$ c) $3,412^5$

d) $0,5623^3$ e) $0,0632^2$ f) $0,021^3$

g) $\sqrt{26,8} \times (3,5)^2$ h) $\sqrt[3]{587,5}$ i) $\sqrt[4]{0,06354}$

j) $\dfrac{1,483^3-1}{1,487^3+1}$ k) $\dfrac{26,7}{9,562}$ l) $\sqrt[3]{\dfrac{349}{2608}}$

EXAMINATION QUESTIONS

1) Evaluate

i) $0{,}358 \times 1{,}26$ ii) $\frac{0{,}0762 \times 0{,}98}{0{,}328}$

2. A cube of edge 6cm is smelted to form a right cylinder of height 4,2. Calculate the radius of the cylinder. (Volume of a cylinder is given by $V=\pi r^2h$, where r is the radius and h is the height. Use π 0=3, 12.

3. The value of Young's modules, a measure of the elasticity of a material may calculated from the formula

$$\frac{4WL^3}{3\pi d a^4}$$

Find Young's modulus for a beam for which W=35,6 L=14,2 a=0,40 d= 4,37

4. Given that 94 x 152 = 14 288

a) Find the value of N if 95 x 152 = 14 288 + N

b) Write down the exact value of

i) $0{,}094 \times 1520$

ii) $0{,}14\ 288 \div 0{,}0094$

(ZIMSEC NOV 2008)

5. Given that $\log_5 2 = 0{,}431$ and $\log_5 3 = 0{,}683$

Find the value of

a) Log 1 ½ b) $\log_5 \sqrt{3}$

c) Evaluate $\frac{\log 8 - \log 9}{\log 64 - \log 81}$

CHAPTER 6

BASIC ALGEBRA AND IRRATIONAL NUMBERS

Algebra uses letters of the alphabet to represent general numbers.

Syllabus objectives
Leaner should be able to
a) Simplify given algebraic terms
b) Factorise given algebraic terms
c) Simplify and solve irrational numbers

Simplification

Like terms (Grouping)
Terms of the same degree in the same variables are called like terms e.g. $2x$, $3x$, $-4x$ or $2x^2$, $3x^2$, $-4x^2$ or $2xy$, $-3xy$, $-4xy$
Like terms can be group together and can be added and subtracted.

Example1
Simplify i) $10x - 7x - 3x + 2x$
ii) $-5y + 6x + 6y - 8x$

i) $10x - 7x - 3x + 2x = 10x + 2x - 7x - 3$ (by grouping positive and negative terms together)
$= 12x - 10x$
$= 2x$

ii) $-5y + 6x + 6y - 8x = -5y + 6y + 6x - 8x$ (by grouping together like terms)
$= y - 2x$

Removing brackets

If any quantity multiplies the terms inside a bracket every term inside the bracket must be multiplied by that quantity when the bracket is removed..
In general, $a(x + y) = ax + ay$
And $a(x - y) = ax - ay$
When these two brackets multiply each other the terms in the first bracket multiply each term in the second bracket.

$(a + b)(x + y) = ax + ay + bx + by$
$(a + b)(x - y)$ $ax - ay + bx - by$

Example 2

Simplify i) $2(2y + 3x - 1)$ ii) $2(a - 3b) + 3(3a + b)$
iii) $(6m + n)(5m - n)$

i) $2(2y + 3x - 1) = 4y + 6x - 2$
ii) $2(a - 3b) + 3(3a + b) = 2a - 6b + 9a + 3b$ (Collecting like terms together)
$= 2a + 9a - 6b + 3b$
$= 11a - 3b$
iii) $(5m - n)(6m + n) = 30m^2 - 6mn + 5mn - n^2$

$= 30m^2-mn-n^2$

Exercise 1,1
Simplify

1) a) $6(a+2)$ b) $4(x-3y)$ c) $y(y^2+3)$

d) $5(x-2)-2(x+3)$ e) $3(2x-3y-1)$ f) $\frac{1}{4}(8u-4)+3$

f) $4(2-3a-1)-3a(2a-5)$ g) $5d-(e+2d)$

h) $4(x-3y-3t)-2(2x-5y-4t)$

i) $4(a^2-3a-1)-3a(2a-5)$

j) $4x(3x+2y)+3y(x+y)-2xy$

2) Expand and simplify

a) $(a-b)(a+b)$ b) $(5+a)(5+z)$

c) $(4z+1)(3z-1)$ d) $(5x-2)(x+4)$

e) $(x-5)(x+6)$ f) $(3c+4)(c-2)$

g) $(2x+1)^2$ h) $(2x-5y)^2$

i) $(2k-7)(4k+5)$ j) $(3n+t)(n+3t)$

k) $(K+2/3)(K-2/3)$ l) $\left(t+\frac{2}{5}\right)^2$

Addition and subtraction of algebraic fractions

Example 3

Simplify

i) $\frac{3x}{5}+\frac{3x}{5}$ ii) $\frac{x+1}{3}-\frac{x-3+2x}{4}$

iii) $\frac{x}{3}+\frac{2}{3x}-\frac{4}{3x^2}$ iv) $\frac{3}{x-1}-\frac{2}{x+4}$

v) $\frac{3}{(x-1)(x+4)}+\frac{2}{(x+4)}$

Solutions.

i) $\frac{3x}{5}=\frac{3x}{5}=\frac{3x+3x}{5}$

$= \dfrac{6x}{5}$

ii) $\dfrac{x+1}{3} - \dfrac{x-3}{4} + 2x = \dfrac{x+1}{3} - \dfrac{x-3}{4} + \dfrac{2x}{1}$

$= \dfrac{4(x-1) - 3(x-3) + 12(2x)}{12}$ Find the LCM of 3,4 and which is 12

$= \dfrac{4x + 4 - 3x + 9 + 24x}{12}$

$= \dfrac{4x - 3x + 24 + 4 + 9}{12}$

$= \dfrac{25x + 13}{12}$

ii) $\dfrac{3}{x} + \dfrac{2}{3x} - \dfrac{4}{3x^2}$ $\dfrac{= 9x(3) + 3x(2) - 3(4)}{9x^2}$ Find the L.C.M of x ,3 x and 3 x^2 which is 9 x^2

$= \dfrac{27x + 6x - 12}{9x^2}$

$= \dfrac{33x - 12}{9x^2}$ Divide every term by 3

$= \dfrac{11x - 4}{3x^2}$

iii) $\dfrac{3}{x-1} - \dfrac{-2}{x+4}$ $= \dfrac{3(x+4) - 2(x-1)}{(x-1)(x+4)}$ Find the L.C.M of (x -1) and (x +4) which in (x -1) (x +4)

$= \dfrac{3x + 12 + 12 + 2}{(x-1)(x+4)}$

$= \dfrac{3x - 2x + 12 + 2}{(x-1)(x+4)}$

$$=\frac{x+14}{(x-1)(x+14)}$$

v) $\frac{3}{(x-1)(x+4)}+\frac{2}{(x+4)}$ $=\frac{3+2(x-1)}{(x-1)(x+4)}$ Find the L.C.M of (x -1)(x +4) and x +4 which is (x -1) (x +4)

$$=\frac{3+2x-2}{(x-1)(x+4)}$$

$$=\frac{2x+1}{(x-1)(x+4)}$$

Exercise 1,2

Express each of the following as a single fraction.

a) $\frac{2x}{5}+\frac{4x}{5}$ b) $\frac{7y}{3}-\frac{2y}{12}$ c) $\frac{3}{x}-\frac{2}{y}$

d) $\frac{1}{3x}+\frac{1}{7x}$ e) $\frac{4}{3a}-\frac{5}{4b}$ f) $\frac{2}{3ac}+\frac{4}{2bc}$

g) $\frac{a}{bc}-\frac{e}{cd}$ h) $\frac{1}{3b}+\frac{1}{4b}-\frac{1}{b}$

i) $\frac{1}{a}+\frac{1}{2a}-\frac{1}{3a}$ j) $\frac{5}{y}-\frac{2}{y^2}+\frac{1}{y^2}$

2) Simplify each of the following

a) $\frac{x-1}{2}+\frac{x+1}{3}$ b) $\frac{x-1}{2}+\frac{x+1}{3}$ c) $\frac{2x+1}{2}+\frac{x-1}{3}$

d) $\frac{(c-2)}{3}-\frac{(c-3)}{5}$ - e) $\frac{1}{2}(2y+1)+\frac{1}{3}(y+2)$ f) $\frac{2(3x-2)}{5}+\frac{x-5}{6}$

g) $\frac{3x-1}{2}+\frac{x-2}{3}-\frac{7}{12}$ h) $\frac{2x+1}{x}+\frac{3y-2}{y}$

3. Simplify the following

a) $\frac{3}{x+1}-\frac{4}{x+1}$ b) $\frac{3}{(x+1)}-\frac{4}{x+1}$ c) $\frac{1}{4-c}+\frac{1}{3+c}$

d) $\frac{4}{d-1}+\frac{1}{d-2}$ e) $\frac{1}{n}+\frac{3}{n+2}$ f) $\frac{a}{a-b}+\frac{b}{a+b}$

g) $\frac{c}{d-5}+\frac{c}{d+5}$ h) $\frac{x-2}{x+3}+\frac{x-1}{x-2}$ i) $\frac{x}{(x+2)(x+1)}-\frac{2x}{x+1}$

j) $\frac{1}{(n-6)(n-4)}-\frac{2}{n-6}$

Factorization

Common factors
$2(x+y) = 2x + 2y$

Since, when x+y is multiplied by 2 the result is $2x+2y$, it means that 2 and $(x+y)$ are both factors of $2x+2y$
We can factorise $2x + 2y$ by taking out the common factor.

$$2x+2y=2\left(\frac{2x}{2}+\frac{2y}{2}\right)$$

$$=2(x+y)$$

By taking out or factoring 2 means every term inside the bracket is being divided by 2

Example 4

Factorise i) $2m + 6n$ ii) $px^2 - x^2 - qx$
iii) $12a^2bc^2 + 4a^2c^2 - 8a^3cx$

i) $2(m+3n)$

ii) $px^2 - x^2 - px = x(px - x - q)$

iii) $12a^2bc^2 + 4a^2c^2 - 8a^3cx = 2a^2c(6bc + 2cx - 4ax)$

Exercise 1,3
Factorise the following

a)	$3a + 3b$	b)	$ab + ca$	c)	$6x - 6y$
d)	$10 + 2y$	e)	$4x - x^2$	f)	$4c^2 - 2ce$
g)	$3x + 3y - 6Z$	h)	$3x^2 - 9x + 3$	i)	$6n^3 - 2n^2 x + n$
j)	$5ab + 4a^2b - 6ab^2$	k)	$t^2 - tm + 3t$	l)	$eq - ef - eh$

Difference of two squares

This applies to expressions of the form a^2-b^2 . On factorisation any expression $a^2-b^2 = (a-b)(a+b)$

Examples 5
Factorize the following

i) $x^2-y^2 = (x-y)(x+y)$

ii) $(a^2-4) = (a-2)(a+2)$

iii) $16-x^2 = 4^2 - x^2$
$= (4-x)(4+x)$

iv) $4h^2 - 9 = 2^2 h^2 - 3^2$
$(2h-3)(2h+3)$

Example 5

Factorise

i)	$6y^2 - 54$	ii)	$x^2 - 2\,^1/_4$	iii)	$x^6 - 81$
iv)	$11^2 - 7^2$				

i) $6y^2 - 54 = 6(y^2-9)$
$= 6(y^2-3^2)$
$= 6(y-3)(y+3)$

ii) $x^2 - 2\,^1/_4 = x^2 - {}^9/_4$

$= x^2 - \frac{3^2}{2^2}$

$= (x - 3/2)(x + {}^3/_2)$

(iii) $x^6 - 81 = (x^3)^2 - 9^2$
$= (x^3-9)(x^3+9)$

(iv) $11^2 - 7^2 = (11-7)(11+7)$
$= (4)(18)$
$= 72$

Exercise 1,4

Factorise the following

a)	a^2-4	b)	y^2-1	c)	$64b^2$-1
d)	x^4-1	e)	R^2-r^2	f)	$25x^2$-9
g)	100-z^2	h)	xy-xb^2	i)	$4x^2$-$9y^2$
j)	2-$8y^2$	k)	$3x^2$-$12y^2$	l)	$16x^2$-1
m)	f^2-2 $^1/_4$	n)	g^2 -25	p)	h^2-0,09

2. Solve

a)	2^2-1^2	b)	83^2-19^2	c)	102^2-13^2
d)	$0,8^2$-$0,7^2$	e)	$4,3^2$-$2,3^2$	f)	$5,2^2$-$4,8^2$

Factorization of trinomials

Expressions with three terms, like $5t^2 +5t -10$,y^2-y-42 are called trinomials
To factorize them it is advisable to separate them into types.

Type 1 Coefficient of a^2 is one
These are trinomials of the form x^2+bx+c where b and c are numbers

Example 7
Factorize i) $x^2 + 8x +15$ ii) $x^2-7x-18$

In general when factorizing trinomials of the form x^2+bx+c
1. Find the factors of +c, whose product is +c and whose sum is b
2. Replace b with the sum of those factors and factorize
3. Take note of the signs.

i) $x^2+8x+15 = x+5x+3x+15$ Factors of 15 are +3 and +5
$= x(x+5)+3(x+5)$
$= (x+3)(x+5)$
Check $= (x+3)(x+5)$
$= x^2+8x+15$

ii) $x^2-7x-18 = x^2-9x+2x-18$ Factors of -18 are -9 and 12
$= x(x-9)+2(x-9)$
$= (x-+2)(x-9)$
Check $(x+2)(x-9)= x^2-9x+2x-18$
$= x^2-7x-18$

If the product of the factors gives the original expression on expansion. it means the factors are correct.

<u>Exercise 1,5</u>
Factorise completely

a)	a^2-5a+ 4	b)	$x^2-3x-108$
c)	$x^2-7x-18$	d)	p^2+5p-84
e)	y^2-4y-12	f)	y^2+2y-8

g) $6-y-y^2$ h) $7+6d-d^2$

i) $n^2+4nm+3m^2$

Type 2 coefficient of x^2 is not one.
These are trinomials of the form. a x^2+b x +c
In the general when factorizing trinomials of the form a x^2+b x +c
1. First multiply coefficient of x^2 (i.e. a) by c
2. Find the factors of your product (ac) whose product is ac and whose sum is b

Example 8

Factorise i) $3x^2 + 16x + 5$ ii) $3d^2-5d-2$
iii) $xm-7x-5m+35$

$3\times 2+16x+5$ $= 3x^2+15+1x+5$ $3x5=15$ factors of 15 whose product are 15 and sum +16 are 15 and +1
$=3x(x+5)+1(x+5)$
$=(3x+1)(x+5)$

ii) $3d^2-5d-2 = 3d^2-6d+1d-2$ $3x-2=6$. Factors of -18 whose product is -6 and their sum is -5 are -6 and +1
$= 3d(d-2)+(d-2)$
$=(3d+1)(d-2)$
$xm-7x-5m+35 = x(m-7)-5(m-7)$
$=(x-5)(m-7)$

Exercise 1,6

Factorise completely

a) $2a^2-9a+5$ b) $4x^2-7x+3$

c) $7x^2+13x-2$ d) $12x^2-28x-5$

e) $2+13x-7x^2$ f) $2x^2-9x-5$

g) $10x^2-21+9$ h) $7d^2+2d+5$

i) $3c^2+7c+2$ j) $3p^2+p-10$

k) $ab+3a+2b+6$ l) $7v+7y-vy-y^2$

m) $15-5c+3d-cd$ n) $2gh-12g+3h-18$

Irrational numbers

Irrational numbers are numbers that cannot be expressed in the from a/b where a and b are integers and b≠0
Their values cannot be found exactly, hence their approximations are accepted e.g $\sqrt{3}$
The number in irrational numbers are non terminating decimals and non-repeating.
Rational numbers are numbers which are donated by Q and are in the form $^a/_b$ (fractions where b=). They include integer like $^2/_1$. The rational number is a non terminating decimal e.g 1/3=0,333
All integers, terminating or recurring decimals are rational numbers.

Surds

A surd is a root which will give a terminating or recurring decimal when the root is extracted e.g$\sqrt{2}$ surds are therefore examples of irrational numbers.

Example 9

Solve i) $\sqrt{36}$ ii) $\sqrt{144}$

i) $\sqrt{36} = \sqrt{9 \times 4}$ express as product of perfect squares

$= \sqrt{9} \times \sqrt{4}$

$= 3 \times 2$

$= 6$

ii) $\sqrt{144} = \sqrt{9 \times 16}$

$= \sqrt{9} \times \sqrt{16}$

$= 3 \times 4$

$= 12$

Example 10

Simply the following leaving the answer in surd form.

a) $\sqrt{63}$ b) $\sqrt{27} - \sqrt{18} + \sqrt{63}$ c) $\sqrt{12}+\sqrt{27}$

a) $\sqrt{63} = \sqrt{9 \times 7}$ Express the number as a product so that one of the numbers is a perfect square

$= \sqrt{9} \times \sqrt{7}$

$= 3\sqrt{7}$

b) $\sqrt{27} - \sqrt{18} + \sqrt{63} = \sqrt{9 \times 3} - \sqrt{9 \times 2} + \sqrt{9 \times 7}$

$= \sqrt{9} \times \sqrt{3} - \sqrt{9} \times \sqrt{2} + \sqrt{9} \times \sqrt{7}$

$= 3\sqrt{3} - 3\sqrt{2} + 3\sqrt{7}$

d) $\sqrt{12} + \sqrt{27} = \sqrt{4 \times 3} + \sqrt{9 \times 3}$

$= \sqrt{4} \times \sqrt{3} + \sqrt{9} \times \sqrt{3}$

$= 2\sqrt{3} + 3\sqrt{3}$

$= 5\sqrt{3}$

Example 11

Express each of the following as the square root of a single number.

i) $7\sqrt{2} = \sqrt{7^2} \times \sqrt{2}$

$= \sqrt{49} \times \sqrt{2}$

$= \sqrt{49 \times 2}$ putting under one square root sign

$= 98$

$13\sqrt{2} = \sqrt{13^2} \times \sqrt{2}$

$= \sqrt{169 \times 2}$

$= \sqrt{383}$

Exercise 1,7

Simplify

1	a)	$\sqrt{27}$	b)	$\sqrt{45}$	c)	$\sqrt{100}$
	e)	$\sqrt{245}$	f)	$\sqrt{243}$	g)	$(\sqrt{3})^2$
	h)	$(3\sqrt{7})^2$	i)	$2(2\sqrt{3})^2$	j)	$(\sqrt{3/4})^2$

2) Express as a square root of a single number

a)	$2\sqrt{3}$	b)	$3\sqrt{5}$	c)	$2\sqrt{7}$
d)	$\sqrt{75} \times \sqrt{12}$	e)	$\sqrt{3} \times \sqrt{5} \times \sqrt{75}$	g)	$(1+2\sqrt{3})^2$

3) Simplify the following

a)	$\sqrt{50} + \sqrt{18}$	b)	$\sqrt{27} + \sqrt{12} - \sqrt{3}$	c)	$\sqrt{45} + \sqrt{20}$
d)	$\sqrt{75} - \sqrt{12} + \sqrt{48}$	e)	$\sqrt{28} + \sqrt{63} + \sqrt{112}$		f) $\sqrt{112} - \sqrt{567}$

Rationalizing the Denominator
This is the process of making the denominator a rational number when dividing

Example 12
Rationalize the denominator

i) $\frac{5}{\sqrt{2}}$ ii) $\frac{3}{7+\sqrt{2}}$

i) $\frac{5}{\sqrt{2}} = \frac{5 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$ To make $\sqrt{2}$ rational we simply multiply by $\sqrt{2}$

$= \frac{5\sqrt{2}}{2}$

ii) $\frac{3}{7+\sqrt{2}} = \frac{3}{(7+\sqrt{2})} \times \frac{7-\sqrt{2}}{7-\sqrt{2}}$ Multiply both numerator and denominator by $7-\sqrt{2}$

$= \frac{3(7-\sqrt{2})}{49-7\sqrt{2}+7\sqrt{2}-(\sqrt{2})^2}$

$= \frac{3(7-\sqrt{2})}{49-2}$

$= \frac{3(7-\sqrt{2})}{47}$

Note- in general, a surd of the form at $a+b\sqrt{c}$ in the denominator is rationalized by multiplying it by $a-b\sqrt{c}$. (note change of sign).

Exercise 1,8

1. Simplify the following by rationalizing the denominator

a) $\frac{3}{\sqrt{2}}$ b) $\frac{5}{\sqrt{3}}$ c) $\sqrt{\frac{3}{7}}$

d) $\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}}$ e) $\frac{1}{1+\sqrt{3}}$ f) $\frac{7}{\sqrt{5+\sqrt{2}}}$

g) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ h) $\dfrac{\sqrt{7}}{\sqrt{3}}-\dfrac{\sqrt{8}}{\sqrt{6}}$ c) $\dfrac{\sqrt{35}}{\sqrt{245}}-\dfrac{\sqrt{21}}{\sqrt{48}}$

j) $\dfrac{2}{5+\sqrt{2}}$ k) $\dfrac{5}{5+\sqrt{3}}$

Summary of surds involving trigonometric ratios of 30º, 45º and 60º

Sin 30º = Cos 60º = $\dfrac{1}{2}$

Sin 60º = Cos 30º = $\dfrac{\sqrt{3}}{2}$

Sin 45º = Cos 45º $\dfrac{1}{\sqrt{2}}$

Tan 30º = $\dfrac{1}{\sqrt{3}}$

Tan 45º = 1

Tan 60º = √3

EXAMINATION QUESTION

1. **Simplify**
   (i) √648 ii) √1 1445

2) (a) Factorise $\times^2$-y^2
   (b) Given that $\times$-y = 4 and $\times^2$-y^2=20, find the value of x and the value of y

(ZIMSEC NOV 2008)

3) Factorise completely
   a) 49 $\times^2$ –y^2
   b) $3m^2$-7m-6

4) Factorise completely
   i) $2\times^2$ +a$\times$ -2b$\times$-ab
   ii) 3-12y^2

(ZIMSEC NOV 2008)

Simplify

i) $(^1/3\ \sqrt{5})^2$ ii) $(\sqrt{^{x/y}})^2$

CHAPTER 7

TRIGONOMETRIC RATIOS

Sine, Cosine and Tangent of angles are known as trigonometrical ratios. Trigonometry means the measurement of lengths and angles.

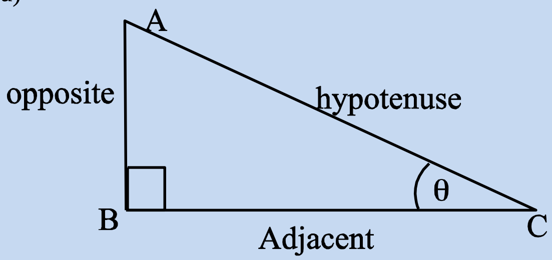
Syllabus objectives

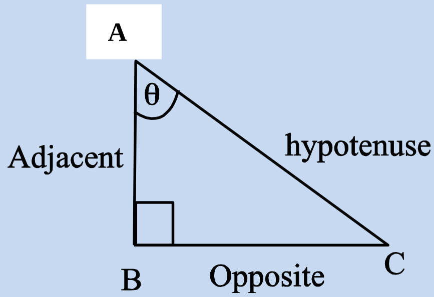
Leaner should be able to

a) Use tables to find the sine, cosine and tangents of angles.
b) Use table to find the angles whose values of sine, cosine of tangents are given.
c) Change from degrees to minutes
d) Calculate using trigonometric ratios

Given a right angle triangle ABC with angle C=⊖. The sides of the triangle are as follows

a)





Using diagram (a)

AC the hypotenuse is the longest side and is identified by being opposite the right angle.

AB the opposite. It is opposite to angle ⊖.

BC the adjacent. It is adjacent to angle ⊖. It is identified by forming part of the right angle and the angle ⊖.

When the position of ⊖ move to A as shown in diagram (b) the opposite and the adjacent also change. Using diagram A

The ratio $\frac{AB}{BC}$ is called the tangent of angle ⊖

The ratio $\frac{AB}{AC}$ is called the Sine of angle ⊖

The ratio $\frac{BC}{AC}$ is called the cosine of angle ⊖

Writing in short.

tan ⊖ = <u>Opposite</u> AB is opposite

adjacent BC is adjacent

Sin ⊖ = $\frac{\text{Opposite}}{\text{Hypotenuse.}}$

Cos ⊖ = $\frac{\text{Adjacent}}{\text{Hypotenuse}}$

Shortening further.

Tan ⊖ = $\frac{O}{A}$ Sin ⊖ = $\frac{O}{H}$ cos ⊖ = $\frac{O}{A}$.

The mnemonic SOHCAHTOA to assist in recall.

Degrees and minuets

In the previous explanation ⊖ is an unknown for an angle. Angles are usually measured to the nearest degree. However, it is possible to calculate with angles which contain fractions of a degree.

$1^0 = 60^1$

Example 1

a) Convert the following to degrees and minutes
i) $0{,}7^0$ ii) $20{,}7^0$
b) Convert, the following to degrees only
i) 20^1 ii) $100^0\ 20^1$

a) i) $1^0 = 60^1$
$0{,}7^0 = 0{,}7^0 \times 60^1$
$= 42^1$

ii) $20{,}7^0 = 20^0 + 0{,}7^0$
$= 20^0 + (0{,}7^0 \times 60^1)$
$= 20^0 + 42^1$
$= 20^0\ 42^1$

b) i) $20^1 = \frac{20^1}{60^1}$
$= 0{,}33^0$

ii) $100^0\ 20^1 = 100^0 + \frac{20^1}{60^1}$
$= 100^0 + 0{,}33^0$
$= 100{,}3^0$

Brief Summary

a) To change degrees to minutes, multiply by 60:
b) To change minutes to degrees, divide by 60

Exercise 1,1

a) Change the following to minutes
a) 3^0 b) $5^1/_2{}^0$ c) 40^0

2) Change the following into degrees and minutes

a) 20^1 b) 30^1 c) 600^1

3) Change the following into degrees and minutes

a) $50{,}4^0$ b) $100{,}8^0$ c) $4\frac{1}{2}\,^0$

4) Express as a decimal number of degrees leaving the answer to the nearest 0,1 of a degree.

a) $40^0\ 30^1$ b) $9^0\ 45^1$ c) $75^0\ 34^1$

<u>Using tangent tables and since tables</u>

<u>Similar manner</u>

a) The table gives the tangent or sine of any angle from 0^0 to 90^0 in intervals of $0{,}1^0$ or 6^1.
b) As angles increase towards 90^0, the sizes of their tangents or sine increase rapidly.
c) The difference column gives increment in intervals of 1. These are added.

<u>Cross section of a sine table</u>

	0^1	6^1	12^1	18^1	24^1	30^1	36^1….	Add Differences
⊖	$0{,}0^0$	$0{,}1^0$	$0{,}2^0$	$0{,}3^0$	$0{,}4^0$	$0{,}5^0$	$0{,}6^0$…..	$1{,}2^1{,}3^1{,}4^1{,}5^1$
46	0,7193	7206	7218	7230	7242	7254	7266…..	

<u>Example</u>

Find a) Sin $46^0\ 26^1$ b) $46{,}5^0$

a) Sin 46^0 26^1 = 0,7246
(i.e. 0,7242+4)

1) Look across 46^0 under 24^1 since it is nearest to 26^1. That give us a difference of $26^1 - 24^1 = 2^1$
2) Look under 2^1 on differences and add

b) Sin46,50 = 0,7254

<u>Using Cosine tables</u>

1) In the sine table, as angles increase from 0^0 to 90^0, their sine increase from 0 to 1.
2) In the cosine table, as angles increase from 0^0 to 90^0, their cosines decrease from 1 to 0.
3) In the cosine table, because cosines of acute angels decrease as the angles increase, the differences are subtracted.

<u>Exercise 1,2</u>

For each of the following use tables or calculator to find the following i) tangent ii) Sine iii0 cosine
a) 15^0 b) 44^0 c) $24{,}1^0$ d) 376^1

e) $45,1^0$ f) $32\ ^1/_2{}^0$ g) $57^0\ 12^1$ h) $48,6^0$

i) $85^0\ 36^1$ j) $71,8^0$ k)$4,2^0$ L) $45^0\ 54^1$

Using tables to find the angles whose tangents, sine or cosines are given

1) In the tables for given trig ratio. Identify the row and column under which the given value lies. When the particular value is absent find the nearest find the difference and under the difference column add the value for sine and tangent and subtract for cosine.

Example 3

(use tables)

a) Find the angles whose tangents are 0,4452

b) Find the angles whose sine are 0,4067

c) Find the angles whose cosine are 0,4067

a) Let the angle be A

COSA = 0,4452 Comparing

A = $63^0\ 34^1$ 4452- 4446= 6

In the difference column the value 5 (in the 2^1

Column is closest to 6.

Subtract 2^1 from $63^0\ 36^1$

Let the angle be A

b) Sin A = 0,4067

A = 24^0

c) Let the angle be A comparing

tan A = 0,4067 4067- 4061=6

A = $22^0\ 8^1$

Exercise 1,3

1) Use tables or calculator to find the angles whose tangents are as follows:

a) 0,7325 b) 0,8947 c) 0,5543

d) 0,3581 e) 5/8 f) 2/5

Hint for fractions reduce to a decimal first.

2) Use tables or calculator to find the angles whose

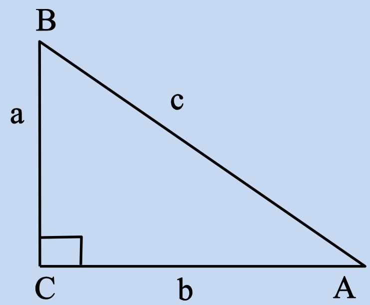
i) Sine ii) Cosine are as follows

a) 0,6878 b) 0,5299 c) 0,4747

d) 2/3 e) ½ f) 0,2000.

Calculations involving trigometrical ratios

Right angled triangles can be solved by using Pythagora's theorem and the trigonometrically ratios.



i) $c^2 = b^2 + a^2$

ii) $\text{Sin } A = \frac{\text{Opposite}}{\text{Hypotenuse}}$

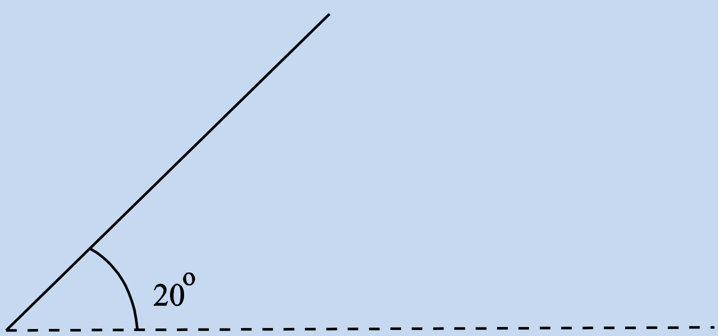
$= \frac{a}{c}$

$\text{Cos } A = \frac{b}{c}$ $\quad$ $\tan A \quad \frac{a}{b}$ $\quad$ Find Sin B, Cos B and tan B

Remember the mnemonic SOHCAHTOA.

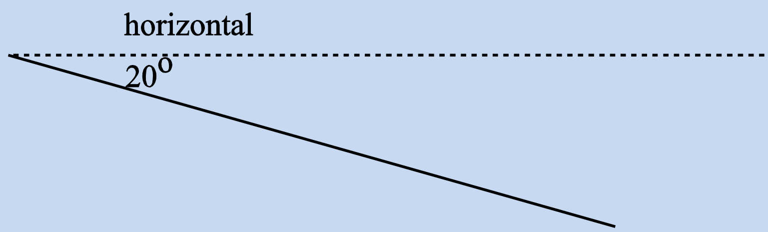
Angles of elevation and Depression

An angle of elevation is an angle measured upwards from the horizontal. For example 200 is the angle of elevation below:



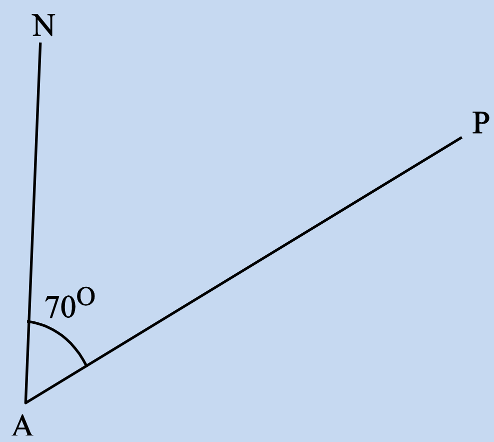
horizontal

Angle of depression is an angle measured downwards from the horizontal. In the diagram below 20^0 is an angle of depression.

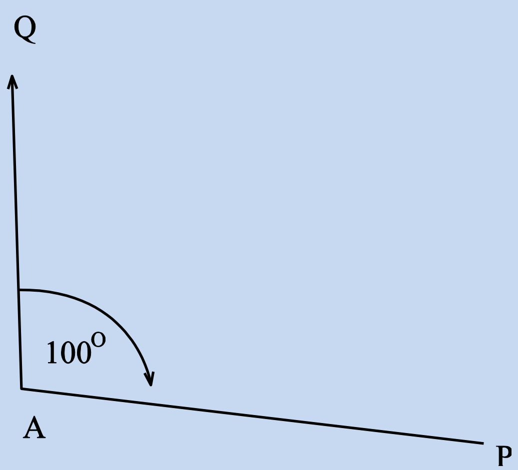


Bearing
Bearing is a clockwise measure from the north.

Bearing of P from A is 070^0

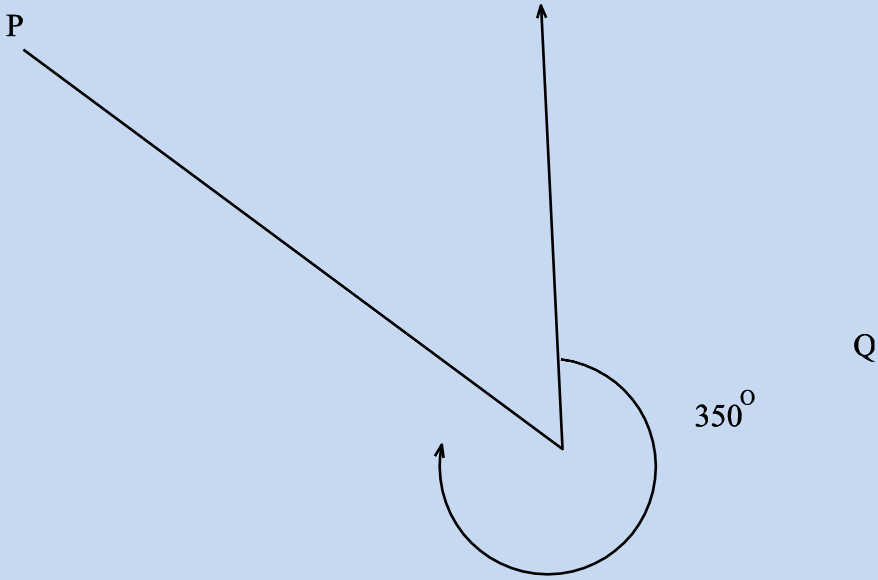


Bearing of P from A is 100^0



Bearing

Bearing of Q from P is 350^0



Example 4
Calculate the lengths marked x in the triangles shown below.

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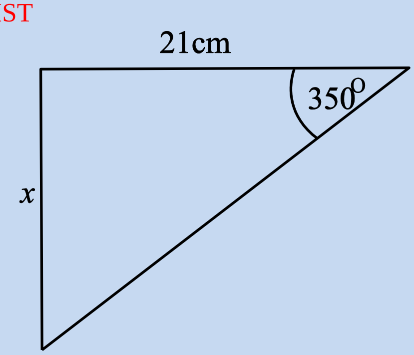
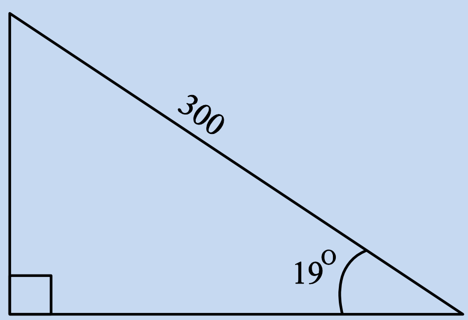


a)



b)

a) Using tan

Tan Ø = $^{O}/_{A}$

$$\text{Tan } 35^o = \frac{x}{21}$$

x = 21 tan 35
= 21 (0,7002)
= 14,7042
= 14,70 to 2.d.p

F

b) using cosine

Cos Ø= $^{A}/_{H}$

$$\text{Cos } 19^o = \frac{f}{390}$$

f = 390 Cos 19^o
= 390 (0,9455)
= 368,75 to 2 d.p

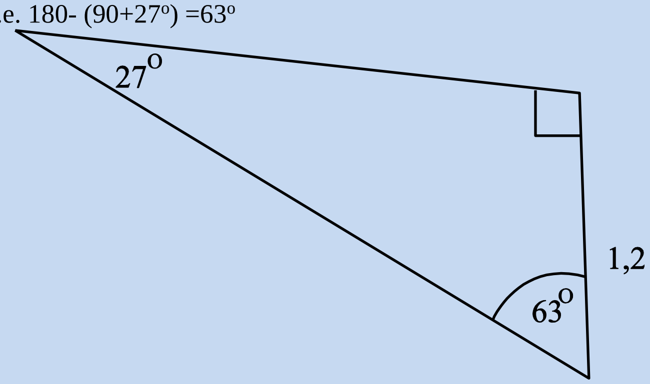
c) Using

Tan Ø = $^{O}/_{A}$

Tan 27º = $\dfrac{1.2}{b}$

b = $\dfrac{1,2}{\tan 27^{o}}$

When you get the above situation calculate the third angle in your triangle and use it i.e. 180- (90+27º) =63º



tan 63^{0} = $\dfrac{b}{1,2}$

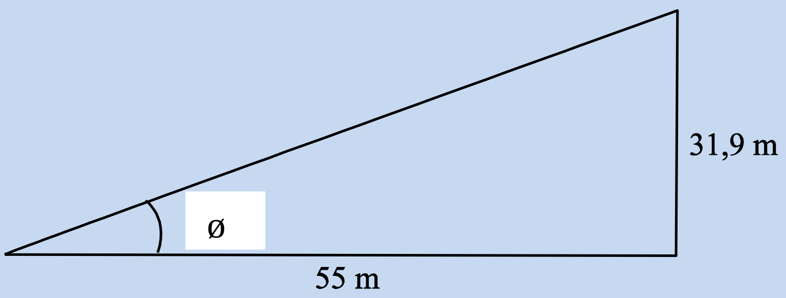
b = 1,2 tan 63
= 1,2 (1,9626)
= 2,36 to 3.s.f

Example 5

1) Find the angle of elevation of the top of a flag pole 81,9m high from a point 55m away on level ground

2. A ship sails 8km due east and then 13km due north. Find the bearing of the new position from the old.

1)

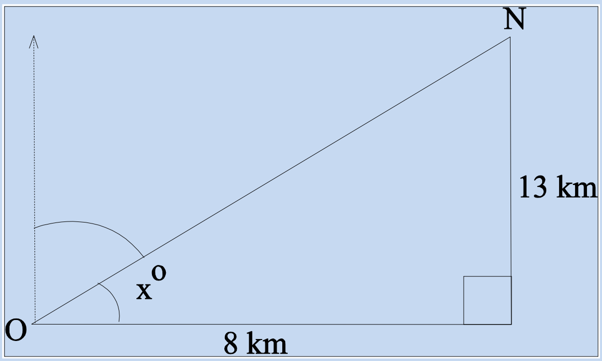


Tan Ø = $\dfrac{31,9}{55}$

Tan Ø = 0,58
Ø = 30º 7^{1}

Angle of elevation is $30^o\ 7^1$

2) Let the old position be O
Let the new position be N



To find the bearing we need to find angle x^o first.

$$\text{Tan } x^o = \frac{13}{8}$$

$$\text{Tan } x^o = 1{,}625$$

$$x = 58^0\ 24^1$$

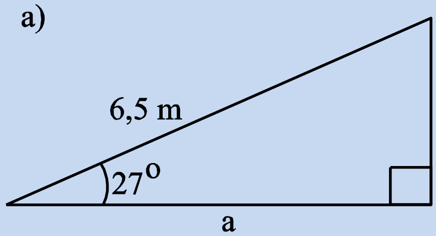
Bearing of N from O $= 90^o - Ø$
$= 90^o - 58^o 24^1$

$= 31^o\ 36^1$

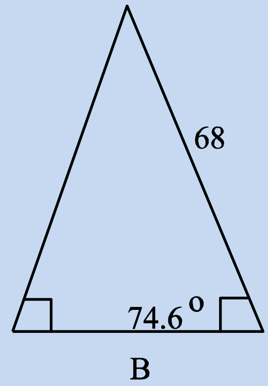
Exercise 1,5

1. Find the lengths marked with letters. Give all your answers correct to 2s.f. All lengths are in metres.

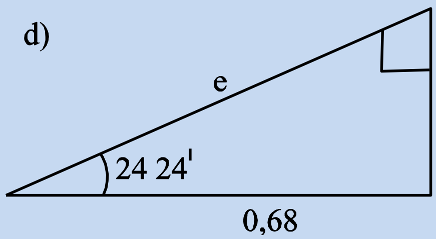
a)



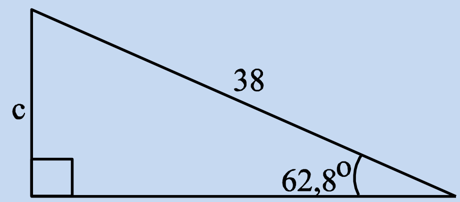
b)



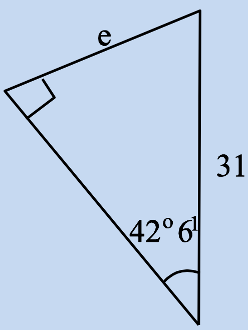
d)



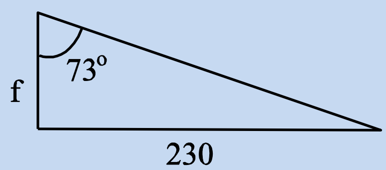
c)



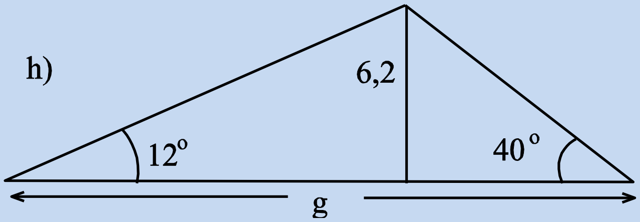
f)



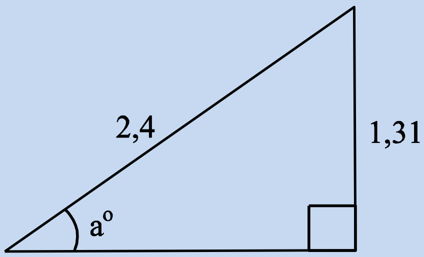
g)



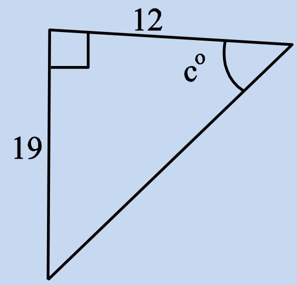
h)

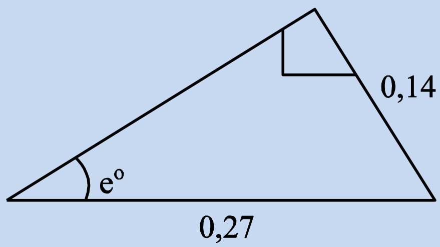


2. Find the angles marked with letters in the diagrams below:



b)





3) A man standing some distance from the foot of a tree notices that the angle of elevation of the top of the tree is 18^0. If the height of the tree is 15m, how far is the man from the foot of the tree.

4) A helicopter hovers vertically above a boat which is 750m from the store. The angle of elevation of the helicopter from the store is 25^O. Calculate the height of the helicopter.

5) From a cliff 80m high Peter looks down on a boat at sea. His angle depression is 9^o. How far is the boat from the cliff.

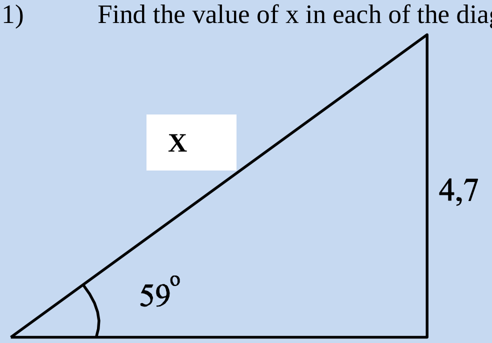
6. A window cleaner leans his ladder against the wall at an angle of 70^o to the horizontal. If it reaches 6,5m up the wall, how long is the ladder.

7. An expedition vehicle carries some stout planks 4,5m long which can be placed to form a ramp to enable the vehicle to surmount obstacles. If the vehicle climb a 12^0 slope. What is the highest obstacle it can surmount.

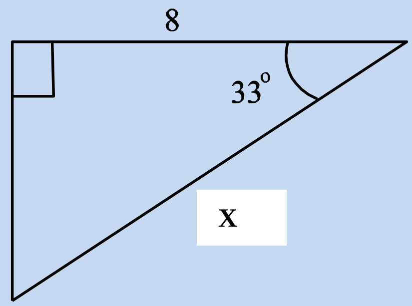
8. A stove rolls 200m down a slope. As it falls it drops 100m vertically. Calculate the angle of the slope.

EXAMINATION QUESTIONS

1) Find the value of x in each of the diagram below



b)



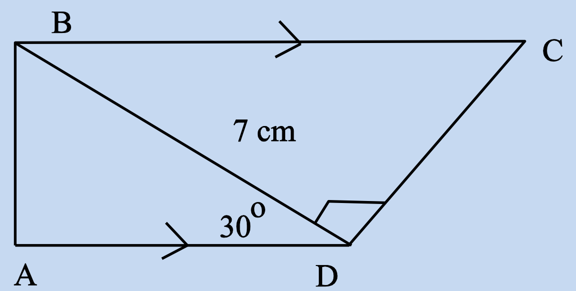
2) In the diagram below, AD is parallel to BC, B AD = BDC=90^o, BDA=30^0 and BD =7cm. Using as much of the information given below as is necessary, calculate i) AD ii) CD

Sin 30^o=0,5 Cos 30^o , Cos 30^o = 0,8660,tan 30 =0,5774

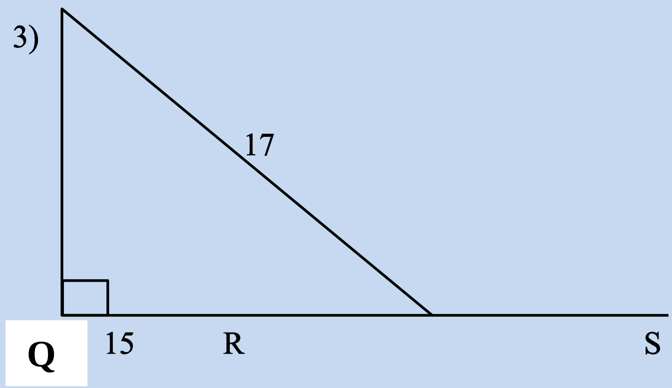
(Cambridge 1988)

REVISIT

2)



3)



In the diagram PQR = 90°, PR=17cm, QR=15cm and QRS is a straight line.

a) Calculate PQ

b) Find, giving the answer as a common fraction

i) cos PRQ

ii) tan PRS

(ZIMEC NOV 2004)

CHAPTER 8

The sine rule and Cosine rule

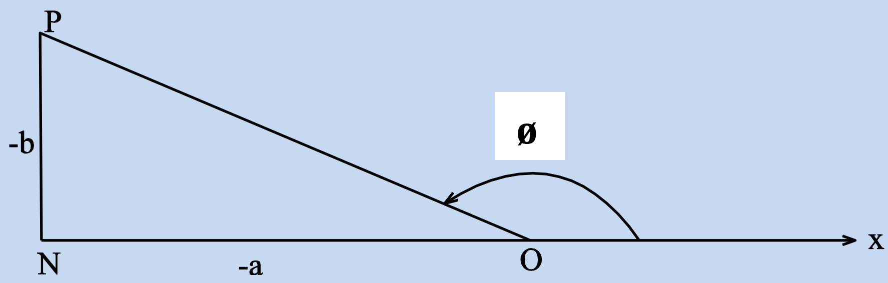
To find the lengths and angles of triangles which are not right angled we use the sine rule and the cosine rule.

Syllabus Objectives

Learner should be able to
a) Find the sine and cosine of obtuse angles.
b) State the sine rule and use it to solve problems
c) State the cosines rule and use it to solve problems
d) Use sine rule and, or cosine rule to solve problems involving bearing.

Obtuse Angles

An obtuse angle is an angle greater than 90^{O} but less than 180^{o}. It cannot occur in a right angles triangle.



In the diagram above OP make an obtuse angle Ø with the positive x -axis. PN is perpendicular to ON. Let PN =+b and ON =-a. OP is taken as positive. Then define

$$\text{Sin } Ø = \frac{NP}{OP} = \frac{+b}{OP} \qquad \text{so Sin Ø is positive}$$

$$\text{Cos } Ø = \frac{ON}{OP} = \frac{-a}{OP} \qquad \text{so Cos Ø is negative}$$

* Cos Ø (obtuse angle) is negative.

NOW NOP = 180 – Ø from the diagram

Sin NOP = Sin (180- Ø) = $\frac{b}{OP}$ = Sin Ø

Also Cos NOP = (180- Ø) = $\frac{a}{OP}$ = -cos Ø

Thus, if Ø is obtuse
Sin Ø = Sin (180^o- Ø)
Cos Ø = Cos (180^o- Ø)
Tan Ø = -tan (180- Ø)

Example 1
Calculate the following

i) Sin 100^o ii) Cos 120^o iii) Tan 147^o 31^1

i) When Ø obtuse
Sin Ø = (180- Ø)
Sin 100 = Sin (180-100^o)
= Sin 80
= 0,9848

ii) When Ø is obtuse
Cos Ø = - Cos (180^o- Ø)
= -Cos (180^o-120)
= -Cos 60^o
= -0,5

iii) When Ø is obtuse
tan Ø =-tan (180^o- Ø)
tan 147^o 31^1 = tan (180^o-147^o 31^1)
= -tan 32^0 29^1
= -0,6369

Working
180^o- 00+60
147^{+1}
32 21^1

Example 2
Find Ø if a) Sin Ø= 0,7660 b) Cos Ø =0,5000
c) Cos = -0,7071 c) Tan Ø = -0,6367

a) Sin Ø = 0,7660
Ø = 50^O
Ø = 50^o – or 130^o (Ø could be either obtuse or acute)

b) Cos Ø = 0,5000 Since value of Cos is positive it means Ø is not obtuse
Ø= 60^o

c) Cos = -0,7071 (Since the valve of Cos Ø is negative it means Ø is obtuse
ignoring the negative
Cos Ø = 0,7071
Ø = 45^o
but Ø must be obtuse
Ø = 180-45^o
= 135^o

e) Tan Ø = -0,6367
Tan Ø = 0,6367
Ø=32 29^1
But Ø must be obtuse
= Ø- 180^o-32^o 29^1
=147^o 3^1

Brief Summary

To find Sin or Cos of on obtuse angle.

- i) Subtract the angle from 180^0
- ii) Find the cosine or Sin of the result
- iii) Make the cosine negative

<u>Exercise 1.1</u>

With side of length a as the base

Area = ½ ab Sin C

Equality the Areas

½ bc Sin A = ½ ac Sin B = ½ ab sin C

Dividing by ½ abc

$$\frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{c}$$

We therefore have this sine formula

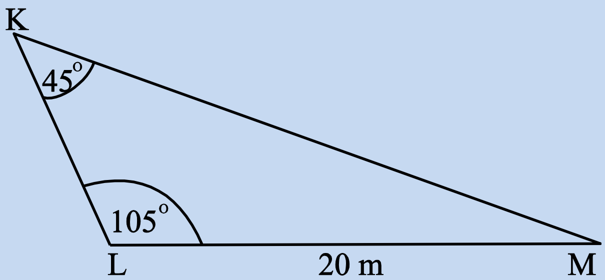
$$\boxed{\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}}$$

$$\boxed{\frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{c}}$$

An alternative form is

Example 3

In triangle KLM below, LKM=45°, KLM=10^0 and LM=20^0. Find KM.



According to sine Rule

$$\frac{LM}{SinLKM} = \frac{KL}{SinLMK} = \frac{KM}{SinKLM}$$

But to Solve you need two ratios

$$\frac{20}{Sin45^\circ} = \frac{KM}{Sin105^\circ}$$

KM Sin 45º = 20 Sin 105º

$$KM = \frac{20Sin105^0}{Sin45^\circ}$$

$$= \quad 20\frac{Sin75^\circ}{Sin45^\circ}$$

working

N0	Log
19,318	1,2860
0,7071	$\bar{1}$,8495
27,45	1,4395

$$= \quad \frac{20\ (0,9659)}{0,7071}$$

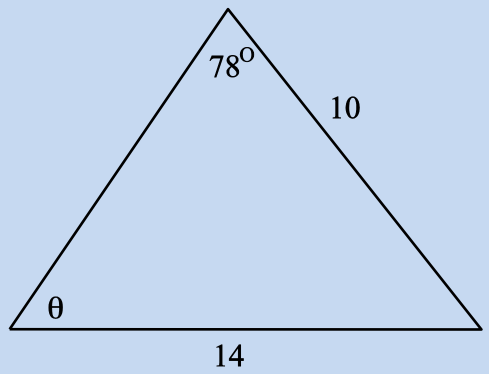
$$= \quad \frac{19,3180}{0,7071}$$

= 27,45m

KM = 27,45m

Example 4

Calculate the angle marked x on the diagram below.



$$\frac{Sin\varnothing}{10} = \frac{Sin78^\circ}{14}$$

$$Sin\varnothing = \frac{10Sin78^\circ}{14}$$

Working

$$= \frac{10(0,9781)}{14}$$

$$= \frac{9,781}{14}$$

No	Log
9,781	0,9903
14	-1,1461
0,6979	1,8442

$$= 0,6979$$

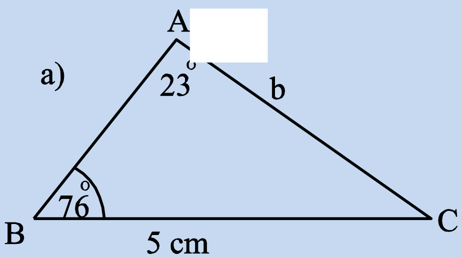
$$Sin\varnothing = 0,6979$$

$$\varnothing = 44^{\circ}15^{1}$$

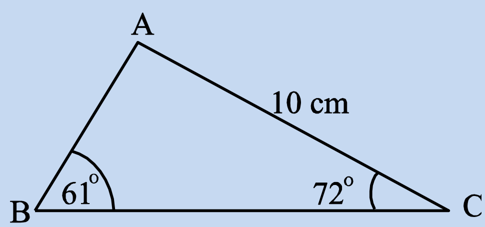
Exercise 1,2

1. Calculate the unknown length or angle in the given diagrams.

a)



b)



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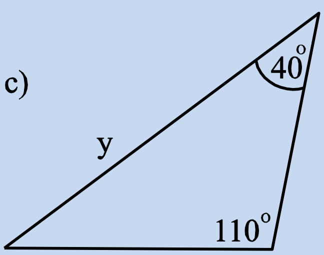
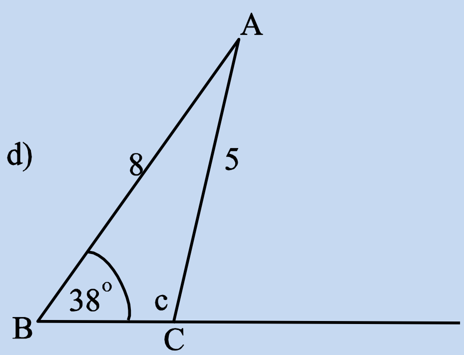
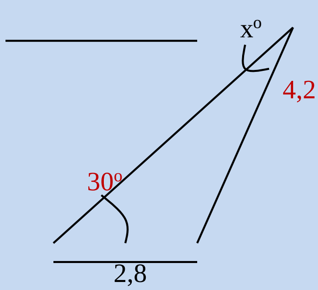


d)

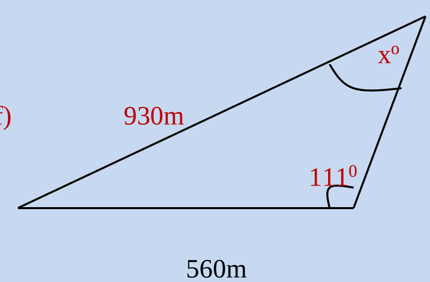


REVISIT

e)



f)



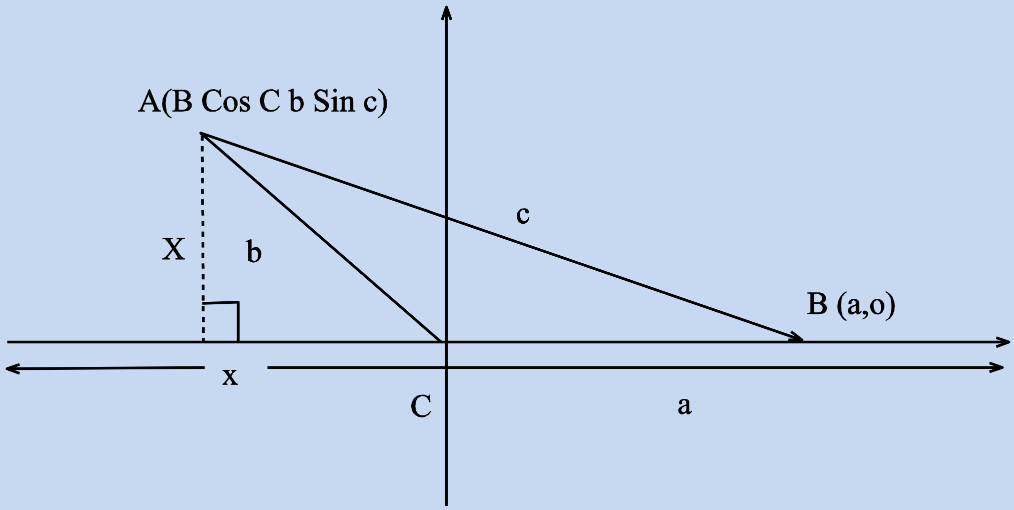
2. In triangle ABC , angle A= $54^0\ 12^1$, angle B= $71^0\ 30,^1$ a = 12, 4cm

Find b

3. In triangle ABC, angle C= $92,2^0$, b=11,2cm, c=39,4cm
Solve the triangle completely

The Cosine Formula

Consider any triangle say ABC. Place the triangle on a pair of co-ordinate axis as shown on the diagram with angle C being obtuse.



The side opposite vertex A has length a units
The side opposite vertex B has length b units
The side opposite vertex c has length c units
The coordinate of B are (a:o). Since angle C is in standard position and A is b units from the origin, the coordinates of A in trigonometric form are (b cos C, b sin C), c is the length of AB.

Now using the distance formula or Pythagoras's theorem

$d^2 = (x^2 - x_1)\ 2 + (y^2 - y)^2$

Let A represent (x_2, y_2) and B represent (x_1: y_1) by subtraction

$C^2 = (b \text{ Cos } c - a)^2 + (b \sin C - 0)^2$

$= b^2 \text{ Cos }^2 - 2ab \text{ Cos } C + a^2 + b^2 \text{ Sin }^2 C$

$= b^2 (\text{Cos }^2 C + \text{Sin } C) + a^2 - 2ab \text{ Cos}$

$C^2 = a^2 + b^2 - 2ab \text{ Cos } C$ $\qquad$ $(\text{Cos }^2 c + \text{Sin}^2 C = 1)$

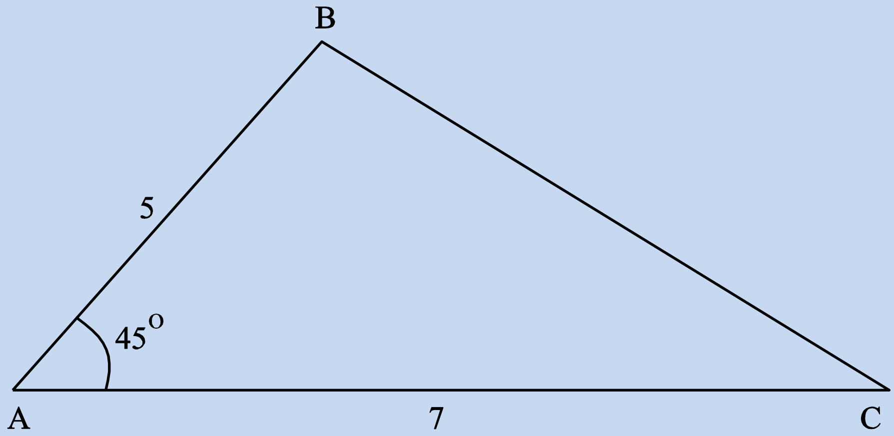
Since the triangle could have been positioned with any of its sides along the x-axis there are two other similar formulas known as the cosine formula given below.

$$\boxed{a^2 = b^2 + c^2 - 2bc \text{ Cos } A}$$

By making different unknowns , the subject other formulars can be derived.

Example 5

In the diagram triangle ABC has side AB=5m, AC=7m and BAC=45°. Find BC.

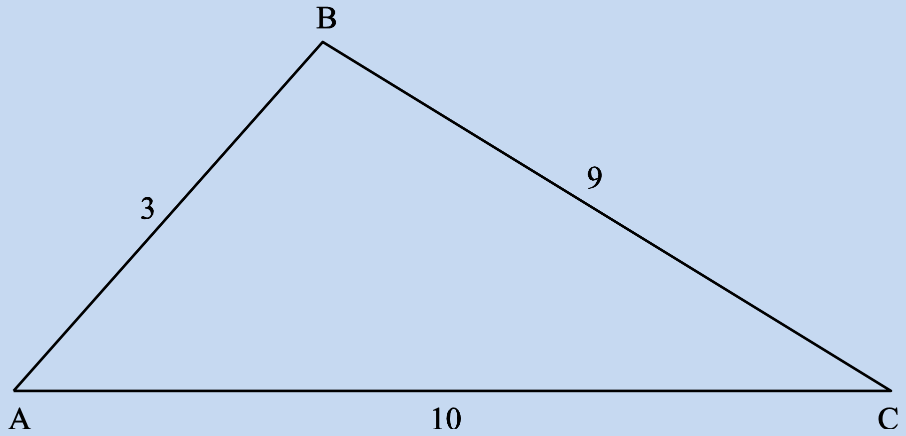


$a^2 = b^2 + c^2 - 2bc \cos A$

$BC^2 = 7^2+5^2 - 2(5)(7) \cos 45$
$= 49 + 25 - 70 \cos 45$
$= 74 - 70(0{,}7071)$
$= 74 - 49{,}497$
$BC^2 = 24{,}503$
$BC = \sqrt{24{,}503}$
$= 5{,}0$ to the nearest tenth

Example 6

In triangle ABC, BC=9m, AC=10m. Find, to the nearest degree, the size of the greatest angle.



From the cosine formula

$$Cos = \frac{a^2 + c^2 - b2}{2ac}$$

$$= \frac{9^2 + 3^2 - 10^2}{2(9)(3)}$$

$$= \frac{81 + 9 - 100}{54}$$

$$= -0,185$$

Negative sign means the angle is an obtuse angle

Cos B = -0,185
B = $180^o - 79^o$
= 101^0

Brief Summary

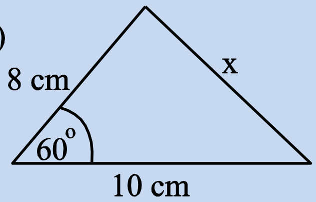
Cosine Rule
$a^2 = b^2 + c^{2-}$ 2bc Cos A

a is the side opposite the angle A: b and C are the sides forming the angle A

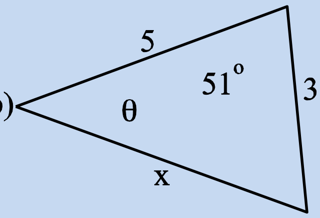
Exercise 1,3

i) In the diagram below calculate the marked side and or angle.

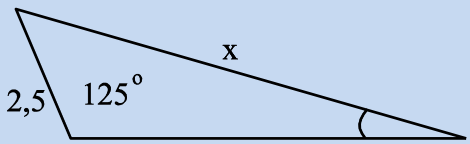
a)



b)



c)



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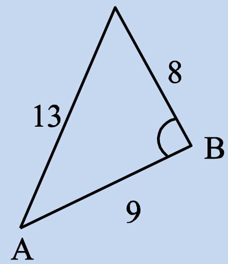

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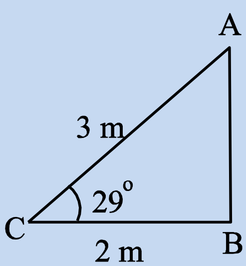

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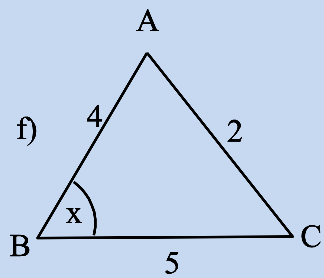
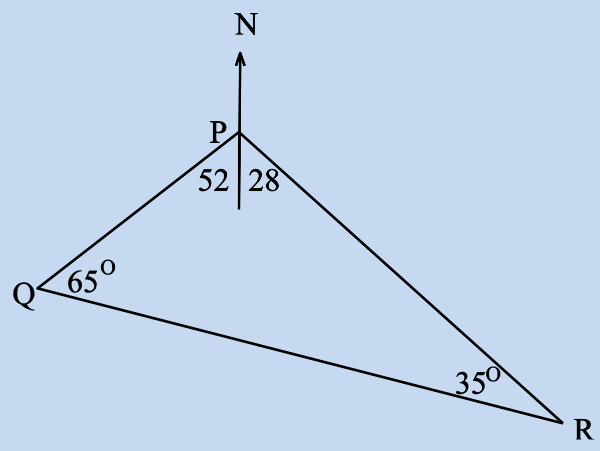


<u>Bearing</u>
Remember bearing is a clockwise measure from the north. It is stated relative to a given fixed point. From the fixed point, the initial reference is the North-south line.

Example 7
The diagram below shows three triangular points with angles as indicated. Calculate the bearing of.

Revisit

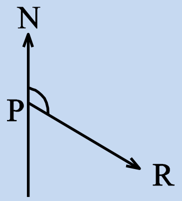


i) R from P

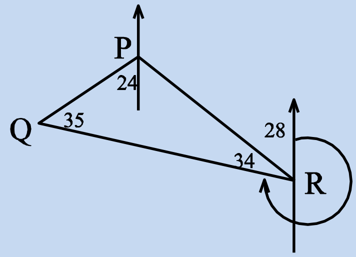
ii) Q from R

iii) R from Q

i) Bearing of R from P

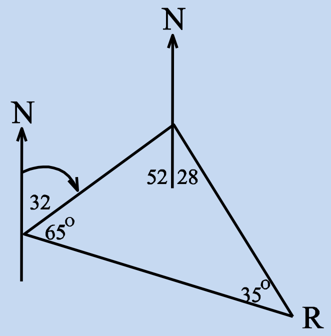


Bearing R from P=180^0-28^0
=152^0



The two north poles form parallel line. We get z angles

:-Bearing of Q from R= 180^0-(35^0 +28^0)
= 117^0



ii)

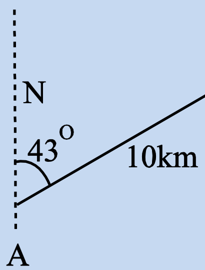
:-Bearing of R from Q= 52^0+ 6

= 117^0

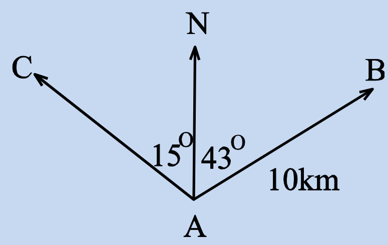
Example 8

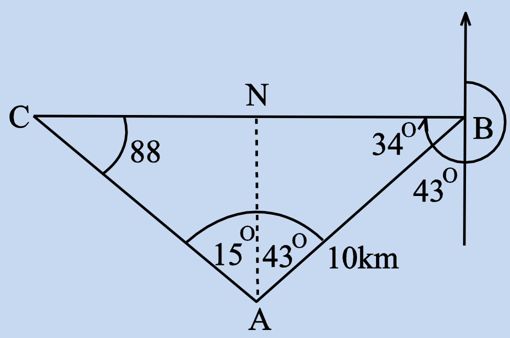
Two boys leave point A. The first moves on a bearing of 043^0 for 10km to point B and the second boy moves on a bearing of 345^o until he reaches c and his bearing from point B is 257^o. Find a) the distance between the two boys at this instant   b) the distance of point C from A.

Step 1



Step 2





Using Sine Rule

$$\frac{CB}{Sin58} = \frac{10}{Sin88}$$

$$CB = \frac{10Sin58}{Sin88}$$

$$= \frac{10(0,8480)}{0,9994}$$

$$= \frac{8,480}{0,9994}$$

Working

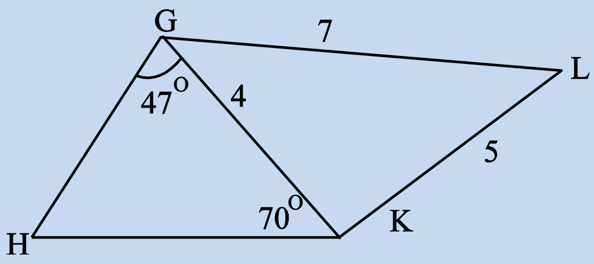
No	Log
8,480	0,9284
0,9994	1,9998
8, 46	0,9286

CB = 8,46m

Distance between the boys is 8,46m

b) finding CA use sine rule or cosine rule

EXAMINATION QUESTIONS



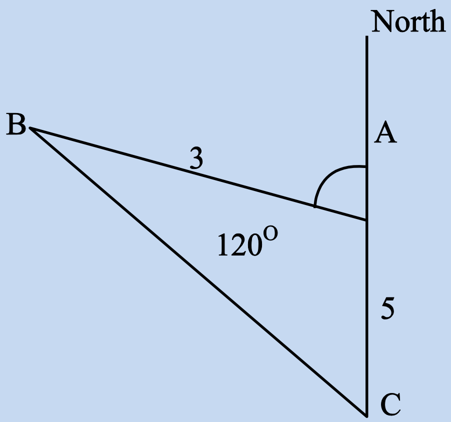
In the diagram above GHK and L are four points on level ground GK=4km, KL=5m, GL=7m, HGK=47^0 and GKH =70^o

i) Calculate a) HK b) GKL

ii) A vertical pole whose top is T, is erected at the midpoint of GL. Given that the height of the pole is 2,87m, Calculate TGL

(CAMBRIDGE 1983)

2) Revisit



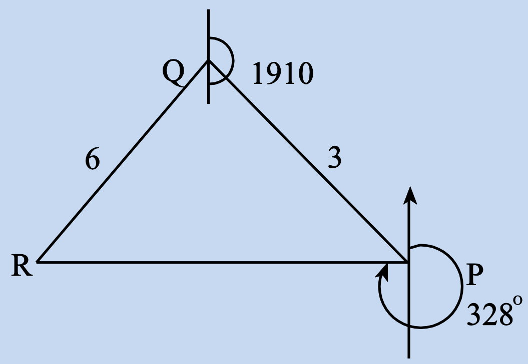
CAMBRIDGE 1983

In the diagram A is 5km due North of C, AB=3km and CAB =120^o, Calculate

i) The bearing of B and A

ii) The distance that is east of B

iii) The distance BC

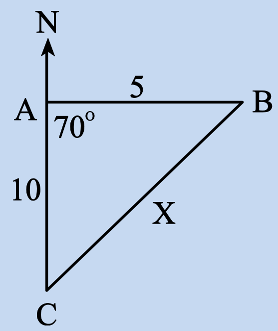


The diagram show three villages P, Q and R. The bearing of Q from P is 328o and the bearing of R from Q is 191°, PQ =3km and QR=km

a) **Calculate**
   i) PQR
   ii) the distance Q is north of P
   iii) PR

b) Find the bearing of R from P, giving your answer correct to the nearest degree

(Zimsec June 2005)



In the diagram, A is 10m due to north of C; AB=5m, BC=x and CAB =70°

(i) Calculate the bearing of A to B
(ii) Calculate the value of x^2

CHAPTER 9

Variation

Variation is concerned with the ways in which one variable depends on one or more variables
There are 4 types of variation namely
1) Direct Variation
2) Inverse variation
3) Joint variation
4) Partial variation

Syllabus objectives
Leaner should be able to
a) Identify direct variation either graphically. Or by calculation
b) Calculate variables in direct variation
c) Identify inverse variation either graphically, or by calculation
d) Calculate the values of variables in inverse variation
e) Calculate the values of variables in joint variation and partial variation.

Direct variation

Occurs when two variables x and y are related in such a way that the ratio of the correspondence values represented by y/x is constant for all pairs of variables x and y.
When x increases the corresponding y value also increases for example the distance and time.

Time (minutes) (t)	0	4	6	8
Distance (km) (D)	0	12	18	24

Ratios $\frac{D}{T} = \frac{12}{4}$ $\qquad \frac{18}{6}$ $\qquad \frac{24}{8}$

$= 3$ $\qquad 3$ $\qquad 3$

Since the ratio $\frac{\text{Distance (D)}}{\text{Time (t)}}$ stay a constant 3, it means Distance is directly proportional to time

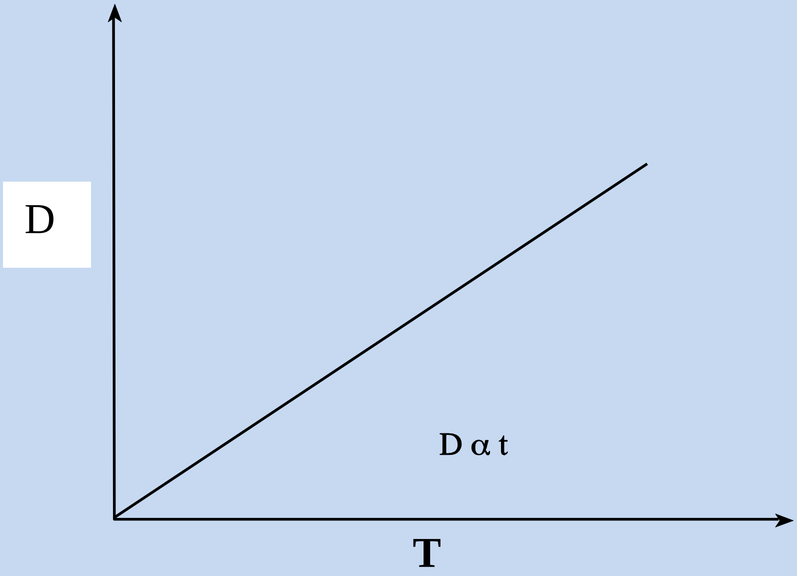
The symbol ∝ is used and means 'Varies with" distance is shortly proportional to time. or is proportional to.
The statement is therefore written as

$D \propto t$

If $D \propto t$, when the ratio $^D/_T$ is the same for all values of D and t , that is, $^D/_t = K$, a constant which is equal to 3 in our case

So, D= kt but k=3
D= 3t

Also the graph of D against is a straight line through the origin.



Other examples of direct variation
i) A stretched string and its tension
ii) The circumference of a circle and radius

Example 1

If y ∝X and y = 8 when x =10

i) Find the equation that connects y and x

ii) Find y when $x = 7\frac{1}{2}$

Y ∝ x

Y= k x

Y = 8 when x =10

8 = 10K

$K = \frac{8}{10}$

$K = \frac{4}{5}$

Equation $y= \frac{4x}{5}$

ii) When x = $7\frac{1}{2}$

$y= \frac{4x}{5}$

= $4(7\frac{1}{2})$

= $\frac{4}{5} \times \frac{15}{12}$

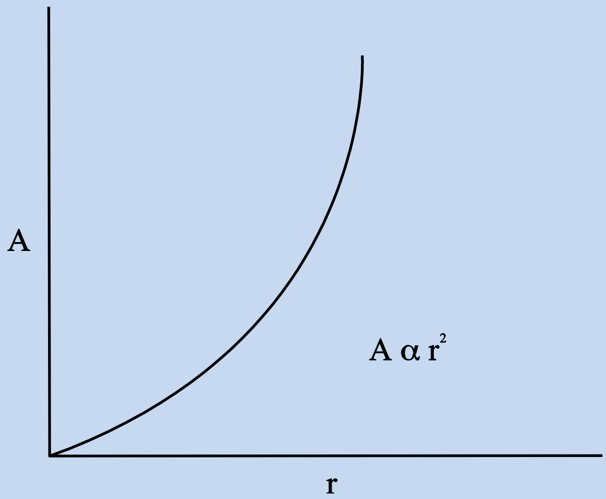
= y = 6

Direct variation between non linear quantities

Some quantities which vary directly are not in linear form

For example the area (A) of a circle varies directly as the square of the radius (r) , $A \propto r^2$

$A = \pi r^2$ (π is a constant). showing the relationship graphically.



Example 2

If $V \propto \sqrt{Q}$ and v=6 when Q = 25. Find the relationship between V and Q ii) Find V when Q = 150

i) $V \propto \sqrt{Q}$

$V = K\sqrt{Q}$

$6 = K\sqrt{25}$

$K = \frac{6}{5}$

$V = 6/5\ \sqrt{Q}$

ii) When Q = 150

$v = \frac{6\sqrt{150}}{5}$

$= \frac{6\sqrt{25} \times \sqrt{6}}{5}$

$= 6\sqrt{6}$

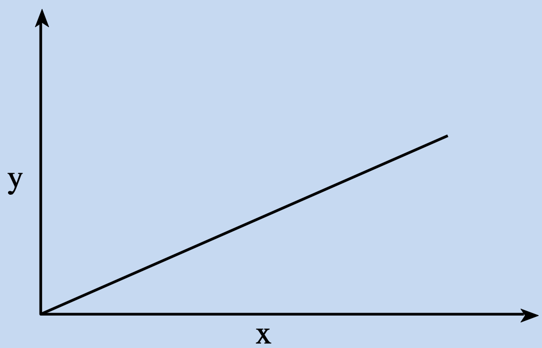
Exercise 1,1

1) If V $\propto$ Q and V=15 when Q=6, find the formula connecting V and Q

2. If D $\propto$ and t and D=400 when t=8, find formula connecting D and t
   Find t when D= 310

3 A vegetable vendor is selling three oranges for 12 cents
   i) Find the cost of 7 oranges (ii) 17 oranges
   ii) Write down a formula for the cost in cents, C, and of number of oranges , n.

4. What is the relation illustrated by the sketch graph below



5) Sketch graph of m $\propto$ n

6) The exchange rate of the US $ to the rand (R) on a particular day was as shown below.

US $	2	3	4	5
R	12,60	18,90	25,20	31,50

a) Show that US $ $\propto$ R
   Find the law connecting US $ and R
b) Find the value of R100,80 in US $
c) Find the value of US $ 12 in Rands
7. The table below shows the cost of paraffin in dollars and the number of gallons at Mr. Dube Stores

Number of	Gallons (n)	2	7	11	13
Cost in	Dollars (c)	1,18	4,13	6,49	7,67

a) Show that c $\propto$ n
b) Find the law connecting c and n
c) Find c when n = 27
d) Find n when c = $10,03

8) The voltage (V) in volts across the resistor varies directly as the current, I, in amperes (A) flowing through the resistor. A current of 0,15A flows when the voltage across the resistor is 2,5v, what current will flow when the voltage is 4,7v.

9. Given that $q \propto p^2$, p= 5 when q=50

i) Find the relationship between p and q

ii) Find q when p =4

iii) Find p when q=100

10. $V \propto h^3$ and v=40 when h=2

i) State the formula for V in terms of h.

ii) Find v when h=3

11. Sketch the following curves, showing both positive and negative values of

a) $m \propto n^2$ b) $x \propto y^3$ c) $c \propto \sqrt{d}$

Inverse variation

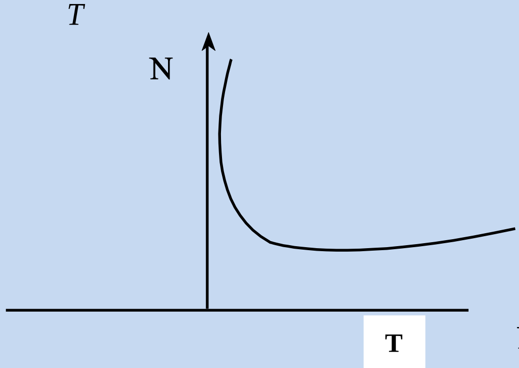
If variables are inversely proportional, when one variable increase the corresponding variables decreases and vice versa. For example, the number of men (N) doing task and the time taken (t) to complete the task.

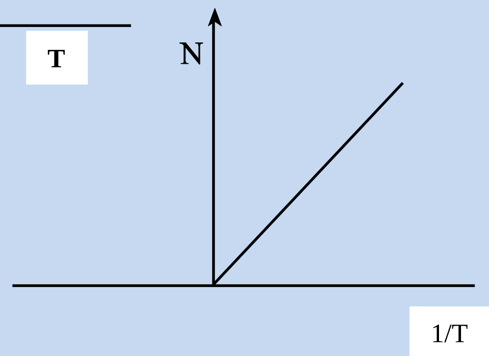
Number of men (N)	1	2	3
Time (in hours) (T)	4	2	1

It can be seen on the table that as the number of manpower is increased the time taken to complete the task is reduced.

We say that N is inversely proportional to T written

Written $N \propto \frac{I}{T}$





$N = \frac{k}{T}$

In general, if $y \propto \frac{1}{x}$ then

i) $Y = \frac{k}{x}$

ii) The graph of y against $^1/x$ is a straight line through the origin

Similarity, if $y \propto \frac{1}{x^2}$ (if y varies inversely as the square of x) then $y = \frac{k}{x^2}$ also the graph of y against $\frac{1}{x^2}$ is a straight line through the origin.

Example 3

If $x \propto \frac{1}{y}$ and $x = 9$ when $y=4$

i) Find the equation connecting x and y
ii) Find the value of x when y =3
iii) Find the value of y when $x = 6$

Example 4

If $x \propto \frac{1}{y}$ and x=9 when y=4

iv) Find the equation connecting x and y
v) Find the value of x when y =3
vi) Find the value of y when $x = 6$

i) $x \propto \frac{1}{y}$

$$x = \frac{k}{y}$$

$$\frac{9}{1} = \frac{k}{4}$$

:-k=36

:- $x = \frac{36}{y}$

ii) When y =3

$$x = \frac{36}{6}$$

y = 12

iii) When x =6

$6 = \frac{36}{y}$

$\frac{6y}{6} = \frac{36}{6}$

Y = 6

Example 5

If $V \propto \frac{1}{W^2}$ and v=18 when w=2. Find the equation connecting V and W

ii) Find W when v =2

i) $V \propto \frac{1}{W^2}$

$V = \frac{K}{W^2}$

$18 = \frac{K}{2^2}$

$K = 18 \times 4$
$= 72$

$V = \frac{72}{W^2}$

ii) When v=2

$v = \frac{72}{W^2}$

$2 = \frac{72}{W^2}$

$\frac{2}{2} W^2 = 72$

$w^2 = 36$
$w = \sqrt{36}$

W = +6

Exercise 1,2

1. Given that f varies inversely as q
   a) Find the equation of p and q in terms of K.
   b) Calculate the value of K when p=2 ½ and q=10
   c) Find q when p=3

2. If $f \propto 1/w$, f=15 when w=5

i) Write down on equation connecting f and W

ii) Find W when P = 10,5

3. If $y \propto \frac{1}{x}$ show graphically the following

i) Graph of y against x

ii) Graph of y against $\frac{1}{x}$

4. If $y \propto 1/x^2$, show graphically the following
   i) Graph of y against x^2
   ii) Graph of y against $1/x^2$

5. If $y \propto 1/x^2$, y =18 when x =2. Find
   i) The value of K
   ii) x when y=8

6. Given that p is inversely proportional to q^2 and that p=100, when q=2 Find the value of p when q=5

7. The height of a cone, of constant volume, varies inversely as the square of its radius. A cone of radius 4cm has a height of 9cm. Find the radius of a cone of equal volume whose height is 4cm.

8. If $y \propto \frac{1}{x+5}$, y=21 when x =3
   i) Calculate the value of k
   ii) Hence, calculate the value of x when y =11

9. When a given mass of gas is kept at a constant temperature, its pressure, P Pascal's, varies inversely as its volume, vcm^3, p=200 when v=1,2. Find p when v=2,2 Find p when V= 2,2 and find v when p = 40.

Joint variation

Occurs when one variable depends on a combination of other variables linked by a multiplication sign. For example the volume of cylinder is $v=\pi r^2h$. The formula shows that the volume of a cylinder depends on the radius and the height. The volume varies as the height and the square of the radius (r)

$$V \propto r^2 h$$
$$V=kr^2 h$$

Example 5

If Z varies directly as x and inversely as y, z=7 ½ when x=3 and y=5

i) Find relationship between x ,y and z

ii) Find Z when $x = {}^{14}/_{5}$ and y=4 ½

$z \propto {}^{x}/_{y}$

$$z = \frac{kx}{y}$$

$$7\frac{1}{2} = \frac{3}{5}k$$

$$\frac{15}{2} = \frac{3k}{5}$$

$$6k = 75$$

$$k = \frac{75}{6}$$

$$k = \frac{25}{2}$$

$$z = \frac{25x}{2y}$$

ii) When $x = \frac{14}{5}$ and y = $4\frac{1}{2}$

$$z = \frac{25x}{y}$$

$$= 25\frac{(\frac{14}{5})}{4\frac{1}{2}}$$

$$= \frac{70}{\frac{9}{2}}$$

$$z = \frac{140}{9}$$

:-

Exercise 1,3

1) If p varies directly as t and inversely as w^2 and if p = $3\frac{1}{3}$, when t = $2\frac{1}{2}$, and w=3. Find the value of p when t=$1\frac{1}{3}$ and w =4.

2) The mass of a sphere varies jointly as the cube of the radius and the specific gravity of the material. A sphere of radius $2\frac{1}{2}$ cm and the specific gravity of 11 7/9 kg/cm^3, has the mass of $6\frac{2}{3}$ $_3$ kg. Find the mass of a sphere of radius $1\frac{2}{3}$ cm and of specific gravity of $2\frac{1}{4}$ kg/cm^3

3. If p varies directly as the square, m, and inversely as the square root of n, p=3 when m=$1\frac{1}{2}$ and n=16. Find p if m= $2\frac{1}{2}$ and n= $6\frac{1}{2}$

4. The curved surface area, A of a cylinder varies directly as the radius, r and the height h, when r=3cm, h=3,5cm and the area is 60 cm^2

i) Find the equation connecting A,h and r

ii) Find the area when h=5cm and r=4,5cm

5. Write equations with the necessary constants for these statements

i) The volume (v) of a sphere varies as the cube of its radius (r)

ii) The curved surface are (A) of a cylinder varies directly as the radius and height (h)

iii) The kinetic energy (E) of a body varies jointly as its mass (m) and the square of its velocity (V).

6. The volume of a cone varies directly as the height, and inversely as the square of the radius r. When V=20m^3, h=2cm and r=2cm

i) Find the formula connecting v, h and r

ii) Find the volume when the height and the radius are increased by 50%

Partial Variation

This type of variation is usually referred to as the sum of two parts

A function of a variable may consist of two more term. This is best illustrated by considering the formula.

$$y = ax + bx^2$$

Where a and b are constants.

Y is the sum of two terms, one which varies directly as x and the other which varies directly as x^2. As a result y varies as x and partly x^2.

Example 6

Y is partly constant and partly varies as the square of x, when y=11, x =3 and when y=12 1/2 , $x = 2\frac{1}{2}$, Find y when x =3.

$$y = a + bx^2$$

When y = 11, x =2

$11 = a + 2^2b$

$11 = a + 4b$ (1)

When y = 12 ½ , x = 2 ½

$y = a + b\ x^2$

$12\frac{1}{2} = a + b(2\,{}^1/_2)^2$

$$\frac{25}{2} = a + \frac{25b}{4}$$

50 = 4a+25 (2) Multiplying every term by L.C.M (2)

Solving the simultaneous equation.

11 = a + 4b (1) x 4 (1)

50 = 4a + 25b (2) x 1 20

44 = 4a + 16b

<u>50 = 4a + 25b</u>

-6= 9b

$$b = \frac{6}{9}$$

$$b = \frac{2}{3}$$

Substituting for b in (1)

11 = a + 4b

11 = a + 4 (2/3)

11 = a+ $^{8}/_{3}$

a = 11 – $^{8}/_{3}$

$= \frac{33 - 8}{3}$

$a = \frac{25}{3}$

= 8 $^{1}/_{3}$

Exercise 1,4

1. y is partly constant and partly varies as the square of x. When y=107, x = -5 and when y=71, x=4
   i) Find the relationship between x and y.
   ii) Find y when x = 11
2. V is partly constant and partly varies with q
   When V=40 q=150, and when v =54, q=192

a) Find the formula connecting V and q

b) Find q when V =65

3. The cost of making a suite is partly constant and party varies as the time taken to make the suite. When the cost is $ 20 , the time taken is 2 hours. When the cost is $ 35 the time taken is 5 hours.
   i) Find the formula connecting the cost (c) and the time (t)
   ii) Find the cost when time taken is 2 ½ hours

4. The cost of household electricity is partly constant and partly varies as the number of units consumed. It cost $ 14,50 to consume 10 units of electricity. 4 units of electricity cost $ 7.
   i) Find formula connecting the cost (c) and the number of units consumed.

5. The cost of feeding delegates at a training centre is partly constant and partly varies at the number of delegates. It costs $ 90 to feed 8 delegates.

   i) Find the relationship between the cost and the number of delegates
   ii) Find the cost of feeding 50 delegates

EXAMINATION QUESTIONS

1. Given that y is directly proportional to x^2 and that y=100 when x =5, find
   i) the value of y when x =1
   ii) the positive value of x when y=36

2. The cost of producing a radio component is partly constant and partly varies inversely with the number made per day. If 100 are made per day the cost is $ 2 per article, if 200 are made per day the cost is reduced to $1,50. What would be the cost of a component if 500 were made per day?

3. It is given that F varies jointly as the square of P and the inverse of Q
   i) Write down the equation connecting f,p,q and a constant k
   ii) Find the value of K when F=3, P=2 and Q=6
   iii) Hence, find value of P when F= 12,1 and Q=5

4. The cost, c dollars, of feeding people at a conference is partly constant and partly varies as the number of people present x

i) Write down the equation connecting c, x and constants h and k

5. The cost of feeding 5 people is $ 4 300 while the cost of feeding 3 people is $ 2 800.

i) Calculate

a) the values of h and k

b) the cost of feeding 9 people

c) the number of people who were fed if $ 12 550 was charged for feeding them.

CHAPTER 10

Distance-Time Graphs

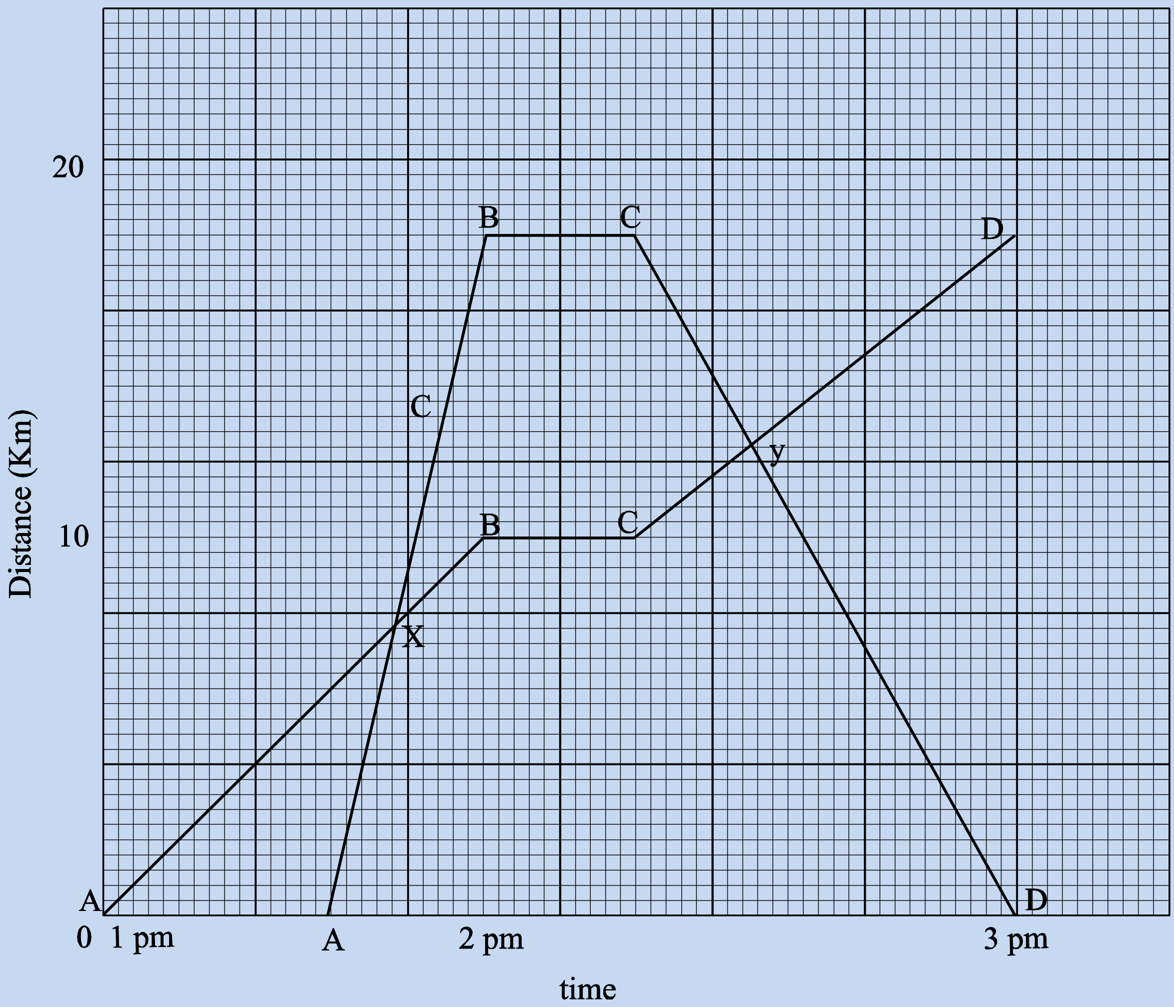
Example 1

Distance is a quantity with magnitude (size) only. It is a scalar quantity

Syllabus objectives
Learner should be able to
a) Describe different stages of a distance –time graph
b) Calculate the distance traveled, time taken, speed etc from the distance time graph
c) Compare two different graphs
d) Draw a distance – time graph

Distance time graphs

Example 1 REVIST GRAPH



The diagram above is a graph representing the journeys of a pedestrian P and a cyclist C. P walks steadily towards the village store 18km away. C cycles to the store.

i) Describe the journey of the pedestrian, P using stages AB, BC and CD

ii) Describe the journey of the cyclist. C using stages AB, BC and CD

iii) Explain what is happening in points x and part of y of the graph.

i) In part AB the pedestrian leaves for his journey at 1pm at a steady speed and by 2pm had covered a distance of 10km

In part BC the pedestrian rested from 2pm to 2:20pm in part CD the pedestrian proceeds in his journey at a steady speed and arrives at 3pm having covered total distance of 18km.

ii) In part AB the cyclist leaves for his journey at 1:30pm at a steady speed by 2pm had arrived covered a distance of 18km

In part BC the cyclist rested from 2pm to 2:20pmin part CD the cyclist returns from the stores and arrives at 3pm

iii) At point X the cyclist passes the pedestrian on his way to the store at about 1:40pm

At point y the cyclist passes the pedestrian on his way back from the store at about 2:32pm

Example 2

Using the diagram in example 1 to answer the following.

i) At what speed did the cyclist travel to the store

ii) What was the pedestrian's average walking

iii) How far from the store was the pedestrian when he met the cyclist at point y

iv) How far did the pedestrian walk between the two points that the cyclist passed him.

i. Speed is given by the gradient of distance time graph

Speed along AB = $\dfrac{\text{Change in distance}}{\text{Change in time}}$

$$= \frac{18-0}{\frac{1}{2}hr}$$

(2:00pm -130pm) 30 minutes

$$= 18 \times \frac{2}{1}$$

$$= 36\text{km/h}$$

ii) Average speed $= \dfrac{\text{total distance traveled}}{\text{total time taken}}$

pedestrian average speed = $\dfrac{18\text{km}}{2\text{h}}$

= 9km/h

iii) At point y the pedestrian had traveled about 12,4km

Distance from the store = 18-12,4km

= 5,6km

iv) Distance walked by the pedestrian = Distance moved from x to B + distance moved from C to y

= 3 + 2,5

= 5,5 km

Further notes on solving distance line graphs

i) Understand the scale being used in each of the axis

ii) In distance time graphs, time is always given in the horizontal axis

iii) Some answers may not be exact but approximated

iv) On calculating the average speed the stoppage time is also included in the total time
v) The speed is given by the gradient of the graph in a distance

Exercise 1,1

1. Fig 1 is a graph representing the journeys of a cyclist C and motorist M. C cycles steadily towards town D. Y drives to town and returns.

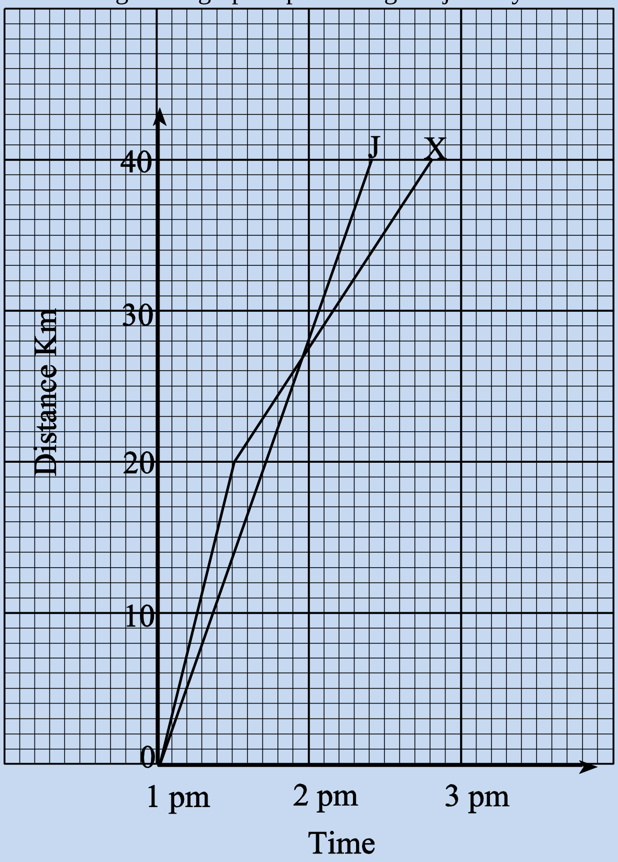
GRAPH

a) i) At what time did the motorist leave for town D
ii) How many minutes did the motorist stay in town D
iv) Calculate the average speed of the motorist
v) Calculate the speed of the motorist on his way to town B

b) i) How long did it take the cyclist to get to town D.
(ii) Calculate the average speed of the cyclist.
iii) Calculate the speed of the cyclist after rest

c) i) At what time did the cyclist and the motorist meet for the first time
ii) What was the distance between the cyclist and the motorist at 11am
iii) How far from town D was the cyclist when the motorist arrived back home
iv) How far did the cyclist travel between the two points that the motorist passed x and y

2. Fig 2 is a graph representing the journey of two cars,



1a) Calculate the speed of car Y.
iii) Calculate the initial speed of car X.
iv) Find the average speed of car X.
v) At what time did the car X and car Y meet.

vi) How far were the two cars from their destination when they meet.
vii) How far apart were the two cars at 2:24pm
viii) How many minutes latter did car X arrive after car Y
ix) At what time did car X change speed?
x) Which car was ahead for a longer time
xi) How far ahead was car X of car when it changed its speed

3. Fig 3 shows the outcome of a 100km car race between car A and Car B. Car A stopped momentarily to refuel.

GRAPH

a) Calculate car B's speed
b) What was car A's speed before it stopped
c) How far ahead of car B was car A when it stopped
d) For how long did car A to overtake car B from
e) How long did it take car A to overtake car B from the time it was overtaken.
f) Which car won the race?
g) By how many minutes did it win the race?
h) What was the distance from the finishing when car A overtaken car B.

Drawing Distance time Graphs

The following points should be followed when drawing graphs

a) Choose a suitable scale. A larger scale increases accuracy
b) Draw the graph by stages. Use the formula Distance = speed x time, to find the unknown.

EXERCISE 1,2

1. A motorist began a journey at 10:45pm. He traveled a distance of 200km for 2hours 15 minutes
   i) At what time did he arrive
   ii) Calculate his speed for the whole journey

2. A man leaves home for a village 20km away at 8 am. He walks steadily at 5km/h after every 45 minutes until he arrived. Draw a travel graph and hence, find the time when he complete his journey

3. A motorist set out a journey of 100km. He travels at a steady speed of 120km/h for 60km then stops to refuel for 15 minutes. He then proceeds with his journey at a speed of 100km/h until he arrives to his destination. Draw a travel graph for his journey and hence find the time when he completes his journey.
   ii) Calculate the average speed for the journey

4. A lorry set out for a journey 20km away at 10:30am. He travels at a steady speed of 30km/h until he gets to his destination. He loads his lorry for the 30 minutes and, then returns at a steady speed of 25km/h. Draw a travel graph for the journey.
   i) Calculate the total time taken for the whole journey
   ii) Calculate the average speed for the journey.

EXAMINATION QUESTION

1. A bus leaves Bulawayo at 9:36am and arrives at Beitbridge bus terminus at 11:27pm
   a) Express 9:36am as a time on a 24 hour clock
   b) How long does the journey take?
   c) Given that the bus was traveling at an average speed of 80km/h. Calculate in km the distance traveled during the journey.

2.

Harare				
141	Kadoma			
275	134	Gweru		
439	x	164	Bulawayo	
379	738	604	440	Victoria Falls

The figure above shows a distance chart for five towns. The distances in a kilometres. From the chart the distance between Gweru and Victoria Falls is 604 kilometres.

a)i) Write down the distances between Harare and Victoria Falls.

ii) Calculate, x, the distances between Kadoma and Bulawayo

b) A motorist travels from Bulawayo to Victoria Falls at an average speed of 80km/h

Calculate

i) the time the motorist takes to complete the journey

ii) the fuel the motorist needs if the car uses 6,5 litres for every 100 kilometres traveled.

(ZIMSEC JUNE 2006)

3) Ten people want to get from A to B, 60km apart. They hire a taxi but this can only take 5 of them at a time. The taxi starts from A at 0900, with 5 of the people and travels towards B at an average speed of 50km/h, while the remaining 5 people start to walk towards B and immediately the taxi returns to meet the remaining 5 people and take them to B.

Show these journeys on a travel graph, ignoring any time spent other than travelling.
Find approximately the time between the two sets of passengers arriving at B

4) Express 12:44am as time on a 24-hour clock.

b) Fungai embarks on a 30km journey. He runs the first 20km in 1 ¼ hours and takes a rest for 30minutes. He then walks the remaining distance in 2 hours. Calculate his average speed for the whole journey.

CHAPTER 11

SPEED-TIME GRAPHS AND AREAS UNDER CURVES

Speed, unlike distance is a vector quality. It has both magnitude (size) and direction.

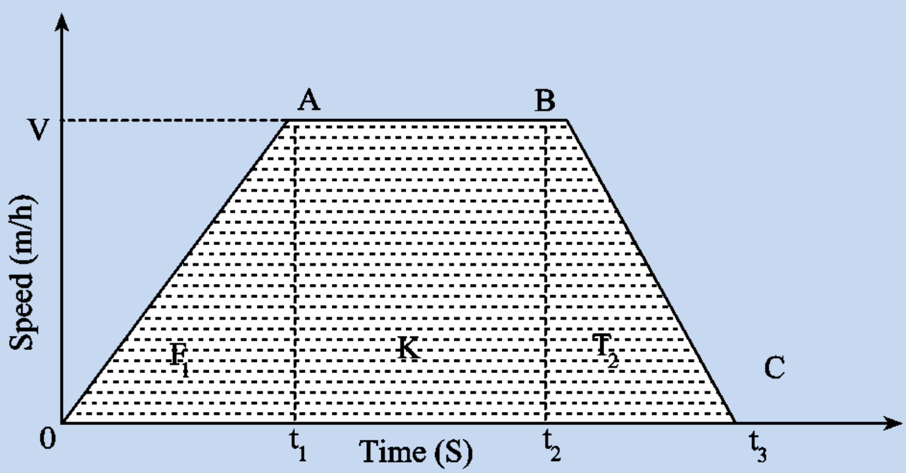
Syllabus objectives

Learner should be able

a) Describe the different stages of speed-time graphs
b) Draw a speed-time graph from given information
c) Calculate the distance, acceleration, time etc from speed time graph
d) Estimate the area under the curve using the trapezium rule.

Describing different stages of a speed-time Graph

Example 1
Fig 1 below shows the speed time graph of a car. Describe the stages OA, AB and BC of the graph. How do you calculate the distance travelled by the car.



Stage OA
In this stage the car is said to be accelerating. Acceleration is the rate of change of speed with time. Since OA describes a straight it means the car was accelerating at a uniform speed. Its speed started from the rest at O and changes by the same quantity every second until it reached a speed of Vm/s after t_1, seconds. The gradient of OA, therefore, gives the acceleration.

$$\text{Acceleration} = \frac{\text{change in speed}}{\text{Change in time}}$$

The units of acceleration are m/s^2

Stage AB
In stage AB of the graph the car is moving at a constant speed Vm/s. It means its speed is no longer changing, between t_1 and t_2, hence, the acceleration is zero. (no acceleration) Practically, the driver is just controlling the steering without applying the accelerator.

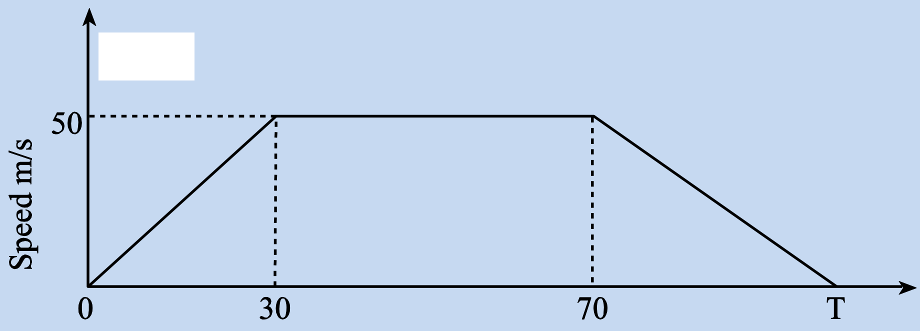
Stage BC
In stage BC of the graph there is deceleration or retardation. The driver is slowing down from a speed of vm/s until the car was stopped at t_3 (i.e. speed is equal to zero). Practically, between t_2 and t_3 the driver applies the brakes to stop the car. The gradient of BC is negative since there is negative acceleration, or deceleration.

Distance
The area under a speed graph gives the total distance traveled. In the diagram it can be found by recognizing the trapezium shape formed and using its area.

Area of a trapezium= ½ (sum of parallel sides) x height
½ (AB + OC) V

The area can also be found by dividing the whole shape into triangle T, rectangle and another triangle T_2 and adding together all the three areas to get the distance.

Example 2
The speed-time graph below represents the movement of a particle between two points.



a) Calculate the acceleration during the first 30 seconds
b) Calculate the distance the car travels from rest before it begins to decelerate
c) Given that the car decelerates at 2 ½ m/s^2, calculate the total time for the journey.

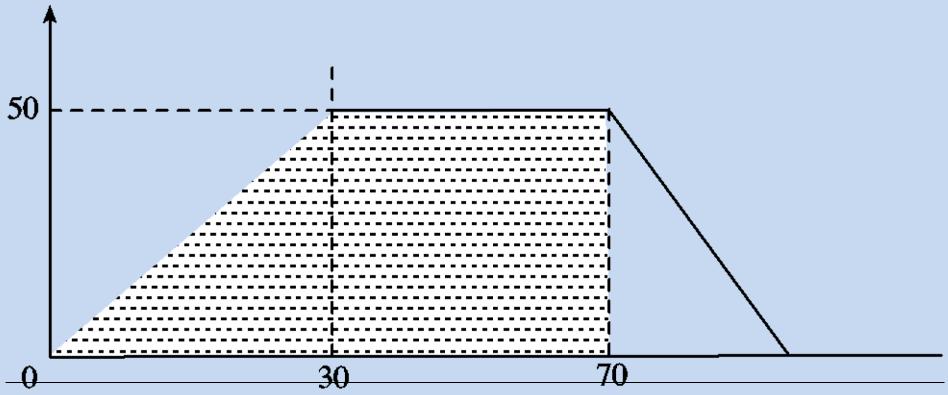
a) acceleration = gradient of a speed time graph

$$= \frac{\text{change in speed}}{\text{Change in time}}$$

$$= \frac{50-0}{30-0}$$

$$= \frac{50}{30}$$

= <u>1,67/sm s^2</u>

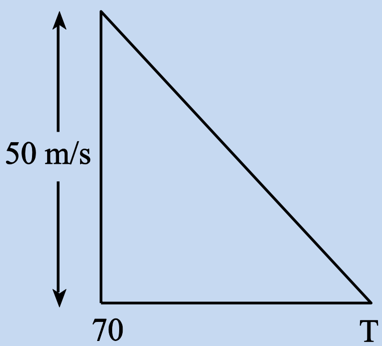


Since area under a speed time graph is equal to the distance covered. The shaded area represents the distance.

Distance = ½ (70 + 40) 50.

= ½ (110) 50
= 2 750m

c)



The gradient of last stage represent the retardation acceleration = gradient
Since deceleration is negative acceleration . remember retardation and deceleration can be used interchangeably.

$$-2\frac{1}{2} = \frac{\text{Change in speed}}{\text{Change in time}}$$

$$-\frac{5}{2} = \frac{50-0}{70-T}$$

$$-\frac{5}{2} = \frac{50-0}{70-T}$$

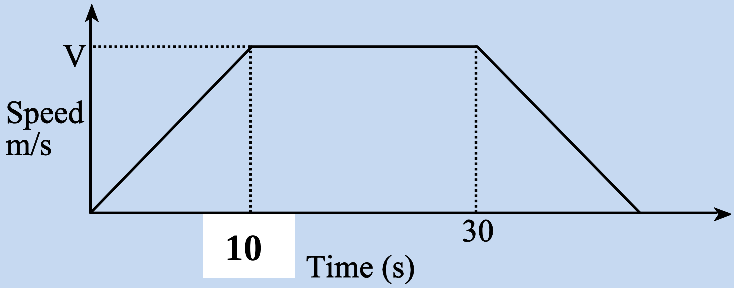
- 5 (70- T) = 50 x 2

- 350 + 5T = 100

$$\frac{5T}{5} \qquad \frac{450}{5}$$

T = 90s
Total time for the journey is 90 seconds

Example 3
The speed time below represents the movement of a different particle between two points.



Given that the distance travelled from the rest before retardation is 250m, find the speed V

Distance = area under the graph
2 50 = ½ (sum of parallel side) x height
2 50 = ½ (20 + 30) V
2 50 = ½ (50) V

$$\frac{250}{25} = \frac{25V}{25}$$

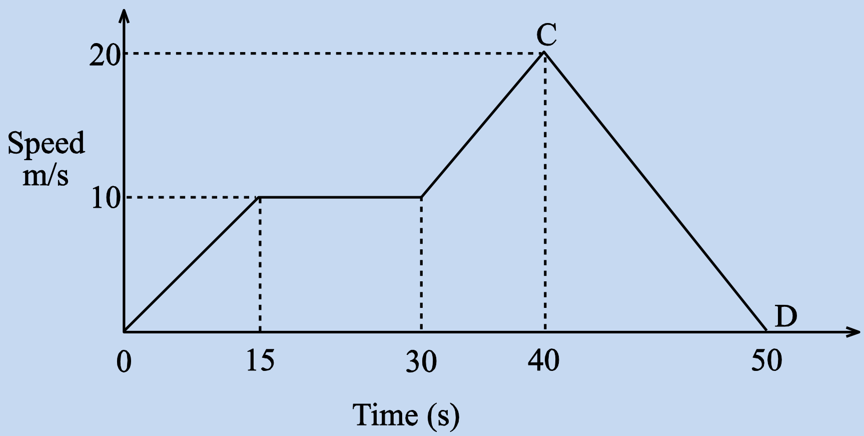
V = 10 m/s

Brief Summary

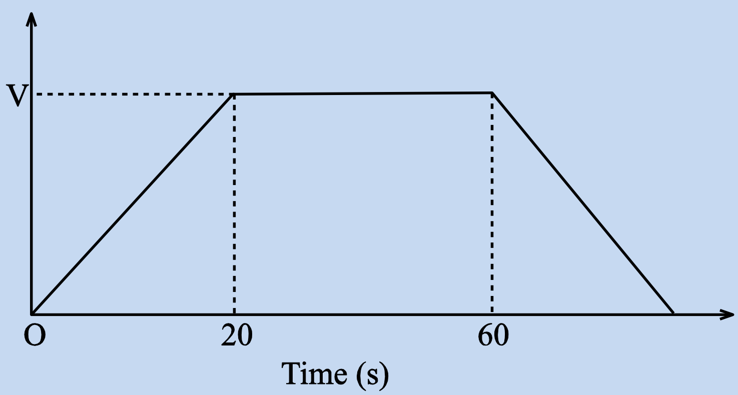
1. The gradient of a speed time graph gives the acceleration
2. When a body is moving at a constant speed, its acceleration is zero. This is shown by a horizontal line on the graph
3. Deceleration or retardation indicate negative acceleration
4. The area under a speed-time graph give the distance travelled by the object
5 Velocity is speed in a given direction. The two can be used interchangeably

Exercise 1,1
Describe the following stages of the journey shown in the speed-time graph below.

i) OA ii) AB iii) BC iv) CD



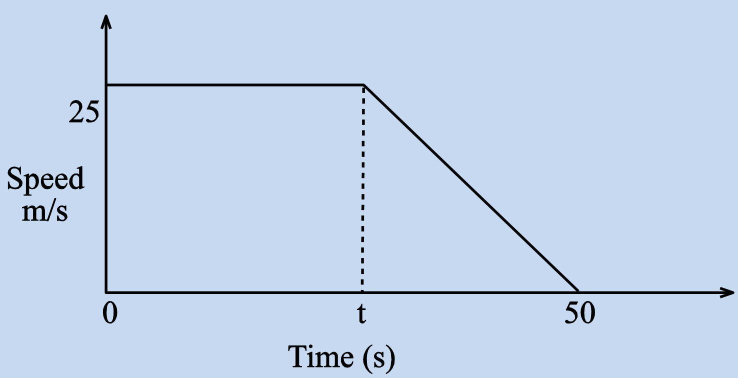
2) The diagram below is the speed-time graph of car.



a) If the acceleration of the car during the first 20 seconds is 1 ½ m/s^2, find the speed V

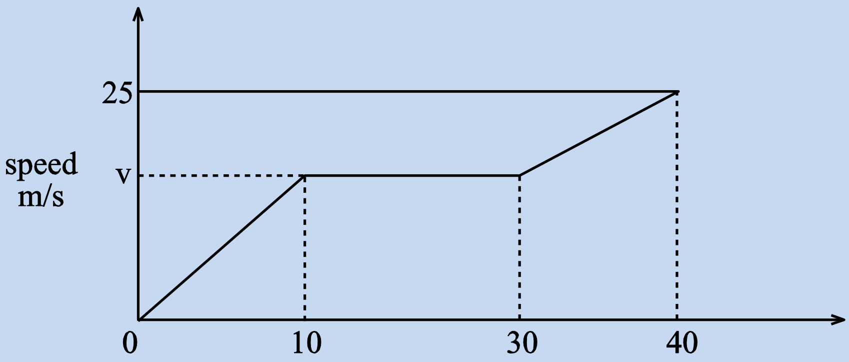
b) Hence, calculate the total time taken for the journey if a distance of 1,4km was travelled
Hint: Change kilometers to metres

3. The diagram below is the speed-time graph of car. The car is traveling at a constant speed of 25m/5 for ts, then brakes are applied until it stops.



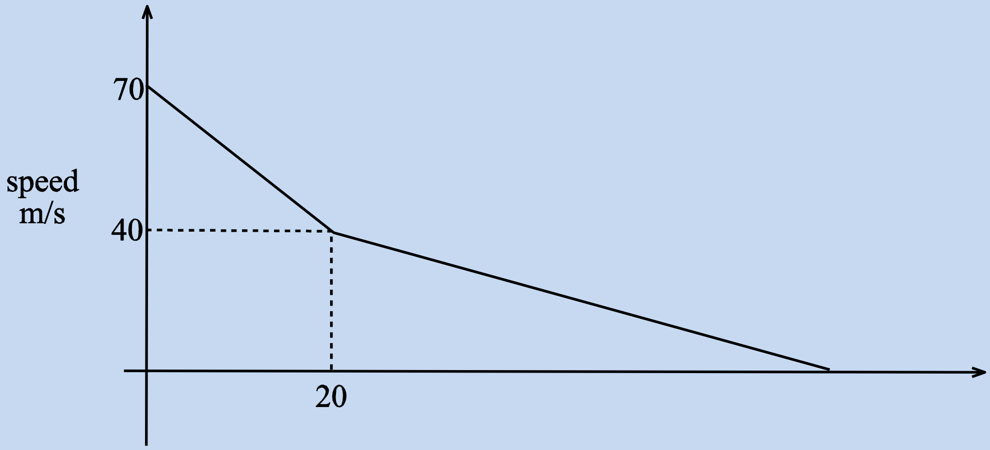
a) If the distance travelled from a speed of 25m/s to rest was 150m. Find the value of t.
b) Calculate the retardation of the car.
c) Calculate the stopping distance of the car.

4. The diagram below show the speed time and graph of a goods train



a) Given that the initial acceleration of the train was 1,5m/s^2, find the value of the speed V
b) Hence, find the distance travelled during the first 15 seconds
c) Calculate the acceleration after 30 seconds.

5) The diagram below shows the speed-time graph of a train which decelerates uniformly from a speed of 70m/s to a speed of 40m/s in 20 seconds. The train further decelerates at 0,4 m/s until it comes to rest.



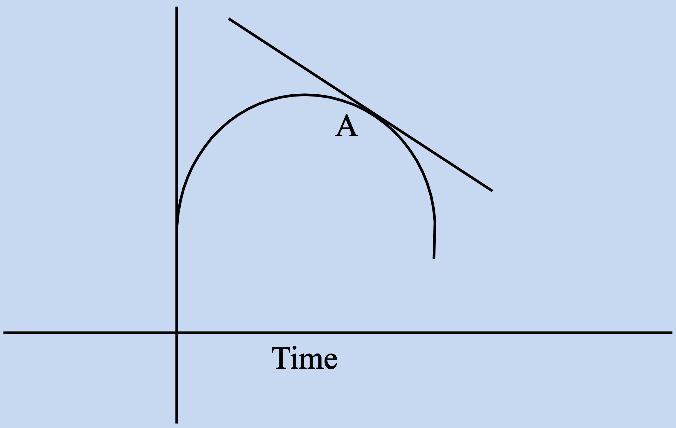
Calculate

i) the deceleration during the first 20 seconds
ii) the total time that the train takes to come to rest
iii) Hence, the distance travelled after 20 seconds

Speed-time curves

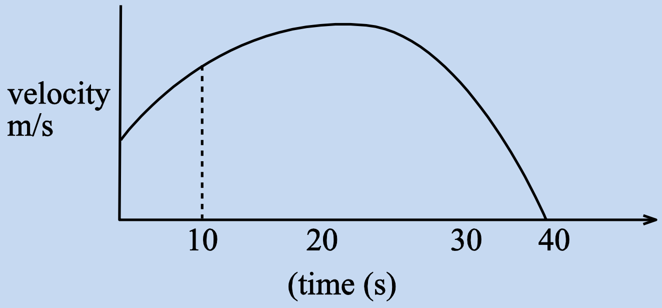
Velocity speed-time curves are commonly used to show the relationship between time and the velocity of moving objects.

Remember that on the work on gradient it was pointed that the gradient changes from point to point along the curve. In speed time graph the gradient represents the acceleration of the object. The fact that the acceleration keeps changing show that it is not uniform hence, it describes a curve. The gradient at any point along the curve is found by drawing a tangent to the curve.



Tangent at point A show the gradient at that point
Hence the acceleration**.** At the turning point the acceleration is Zero

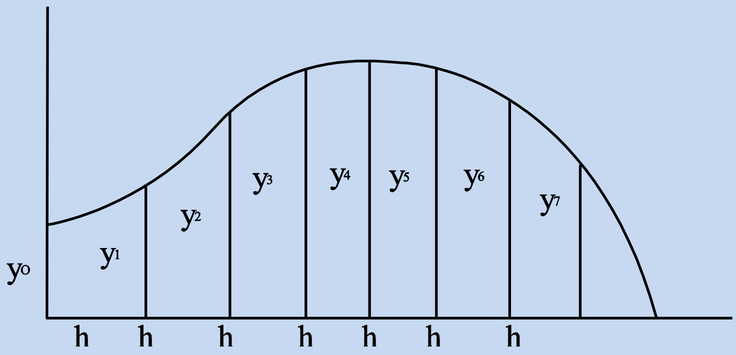
Area under velocity-time curves
The distance under the velocity time graph gives the distance travelled.



Since the area under the curve is not regular and cannot be divided into distinct shapes whose area can be calculated and added, its area is estimated using <u>the trapezium rule.</u>

The area between the boundaries is divided into strips and using the formula for the area of a trapezium to estimate the area of each strip. The more strips taken the better the estimate of the area.

REVIST



In the diagram the heights of each trapezium have been labeled h.

Area of trapezium = ½ (Sum of parallel side) x height
= ½ $h(y_0+y_1)$ + ½ $h(y_1+y_2)$ + ½ (y_2+y_3) +
½ h (y_3+y_4) + ½ $h(y_4+y_5)$ + ½ h (y_5+y_6) +
½ h $(y_6 + y_7)$

Example 4

The table below shows the velocity of a particle which is given below by $v= 3+2t-t_1^2$ where t is time in seconds

T	0	0,5	1	1,5	2	2,5	3
V	3	3,75	a	3,75	3	B	0

a) Calculate the values of a and b

b) Draw a graph to show the relationship

c) From your graph determine

i) the acceleration of the particle. When t=1,5

ii) the distance travelled by the particle in the interval t=1 to t=2,5

a) $v = 3 + 2t – t^2$ $V= 3+2t-t^2$

When t=1 when t=2,5

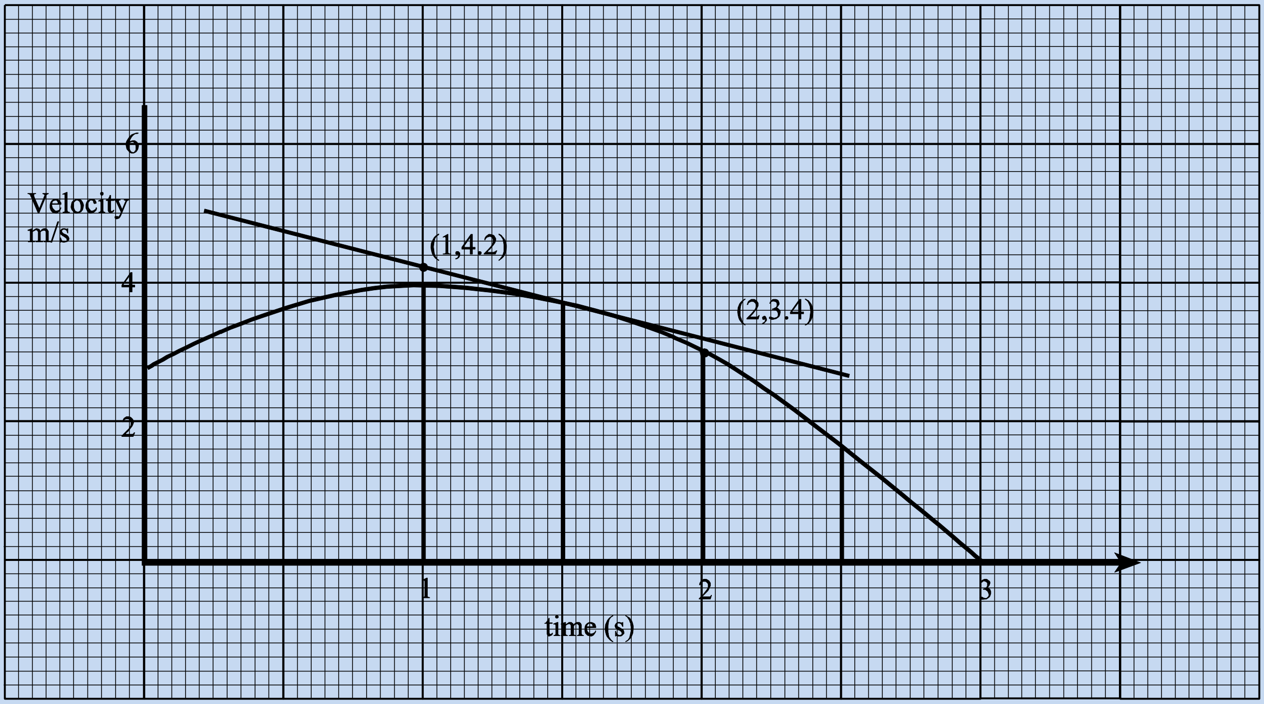
$V=3+ 2 (1) – (1)^{2-}$ $v=3 + 2(2,5) – (2,5)^2$

V = 4 = 3+ 5,0—6,25

a = 4 v = 1,75.

Graph

REVISIT THE GRAPH



i) Acceleration =gradient

$$= \frac{4{,}2-3{,}4}{1-2}$$

$$= -0{,}8$$

ii) Distance=Area under the curve

$$= \frac{1}{2}\left(\frac{1}{2}\right)(4+3,75)+$$

$$\frac{1}{2}\left(\frac{1}{2}\right)(3,78+3)+$$

$$\frac{1}{2}\left(\frac{1}{2}\right)(3+1,75)$$

$$=1,9375+1,6875$$

$$+1,1875$$

$$=4,81m$$

Examination Questions

1. A particle moves along a straight line AB so that after t seconds, the velocity Vm/s in the direction AB is given by $V=2\,t^2-9t+5$.
Corresponding values of t and V are given below

t	0	1	2	3	4	5	6	7
V	5		-5	-4	1	10	23	

Calculate the value of V when t=1 and the value of V when t=7
Taking 2cm to represent 1 second on the horizontal axis, and 2cm to represent 5m/s on the vertical axis, draw the graph of $v= 2t^2 -9t + 5$ for the range $0 \leq t \leq 7$

Use your graph to estimate

i) the values of t when the velocity is zero
ii) the time at which the acceleration is Zero
iii) The acceleration after 6 seconds
iv) Find the distance travelled between t=1 and t=4

2) A particle P travels in s straight line from a fixed point O so that its velocity m/s is given by $V=10+3t - t^2$ where t is the time in seconds after leaving O. Draw a graph for values of t from t=0 to t=5

Calculate

a) Its velocity when t=0
b) the time t when the particle is instantaneously at rest
c) the range of values of t when the acceleration is negative
d) Estimate the distance covered from t=1 to t=3
e) the acceleration when t=3,5 seconds

3. Answer the whole of this question on a sheet of graph paper.

A particle moves along a straight line so that after t seconds its velocity is given by the formula

$V = 5t+7t-2t^2$

A particle moves along a straight line so that after t seconds its velocity, vm/s is given by the formula

$V=5+7t-2t^2$

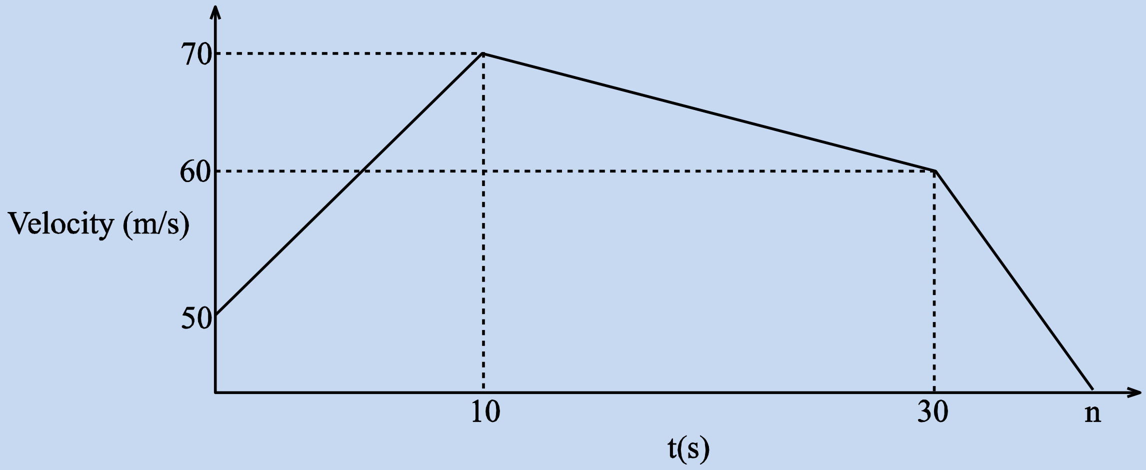
Corresponding values of t and v are given in the table below.

tcs	0	1	2	3	4	5
V(m(s)	5	10	11	8	P	-10

Taking 2cm to represent 1 second on the horizontal axis and 2cm to represent 4m/s on the vertical axis, draw the graph for $v=5+ 7t-2t^2$ for $o \leq t < 5$.

c) use the graph to estimate
i) the maximum velocity of the particle
ii) the value of t when the particle is stationary
iii) the acceleration of the particle when t=5
iv) the distance the particle between t=2 and t=4

(ZIMSEC JUNE 2006)



The diagram is the velocity-time graph of the motion of a particle.

a) Write down the acceleration of the particle during the first 10 seconds

b) Find the distance the particle travelled during the first 30 seconds

c) Given that the distance the particle travelled from t=30 to t=n is 27cm, find the value of n (ZIMSEC 2004)

CHAPTER 12

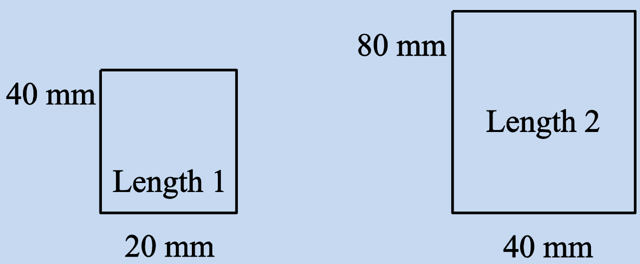
Similarity

Shapes are said to be similar when everything about them is the same except the size. For example an enlarged photograph is similar to the similar original one. Both plane shapes and solid shapes can be similar.

Syllabus Objectives
Learner should be able to
a) Identify similar shapes
b) Calculate the scale factor
c) Calculate the area factor
d) Find the volume factor
e) Use similarity to solve problems

Similar plane shapes
Two plane shapes are similar if one is a copy of the other but on a different scale (i.e. size)
One shape is an enlargement of the other. The ratio of corresponding sides is the scale factor of the enlargement. For example consider similar rectangle and similar triangles.

The shapes below are similar rectangles one is 20mm x 40mm, and the other 40mm x 80mm



Since the two rectangles are similar, the ratio of the corresponding sides should be the same and give the scale factor.

Thus $\frac{Width2}{Width1} = \frac{40mm}{20mm} \quad \frac{length2}{length1} = \frac{80mm}{40mm}$

$= 2 \qquad = 2$

2 is the scale factor. The smaller rectangle has been enlarged two times to form the bigger rectangle.

Area is defined as the space occupied by an object. By counting the small squares in each shape we can get the area of each shape in square units.

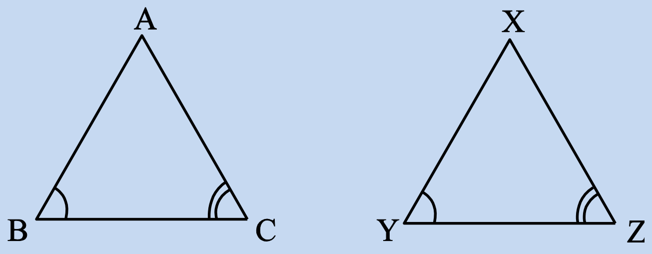
$\frac{\text{Area of bigger rectangle}}{\text{Area of smaller rectangle}} = \frac{800 \text{ units}^2}{200 \text{ units}^2}$
$= 4$
The ratio of the two areas give the area factor
The area factor of the two rectangles is 4

It should be noted that by squaring the scale (i.e 2^2) we get the area factor 4.

The ratio of the two areas give the area factor.
The area factor of the two rectangles is 4

* In general, the ratio for the areas of the similar figures is the square of the scale factor of the two figures.

Similar triangles



Triangles are similar if they are equiangular, that is if the corresponding angles are equal.

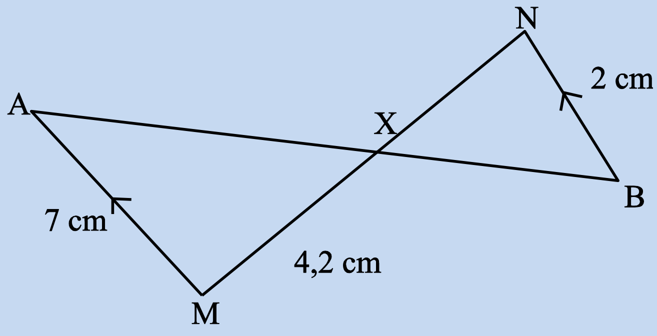
In fig 1, the triangles ABC and XYZ are similar because angle ABC = angle XYZ, angle BCA = angle YZX The third pair of angle must also be equal as the sum of angles in a triangle add up to 180, so two pairs of equal angles are enough to state the similarity of two angles.

It also follows that the ratios of the corresponding sides are equal and gives us the scale factor..

Thus $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$ = the state factor

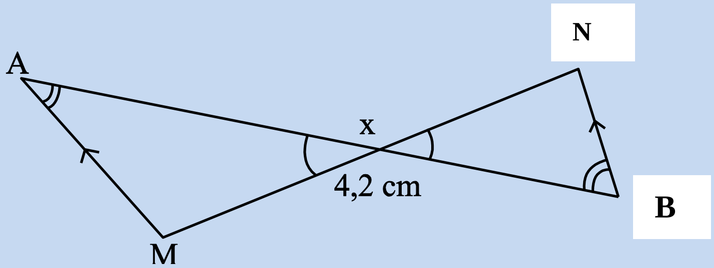
Also if the scale factor of two similar triangles is k, then the ratio of their areas = k^2

Example 1
In the diagram below MA is parallel to BN, AXB and MXN are straight lines. AM=7cm, MX=4,2cm and BN=2cm



a) Name a triangle similar to XAM giving reasons
b) Calculate the length of XN
c) Find the area ratios of two similar triangles
d) If the area of the larger triangle is 21cm^2, find the area of the bigger one.

Solution



a) Triangle XAM is similar to triangle XBN
Reasons angle XAM = angle XBN (angles form Z angles with parallel lines)
angle MXA = angle NXB (vertically apposite angles are equal)

$$\frac{XA}{XB} = \frac{AM}{BN} = \frac{XM}{XN}$$

Using $\frac{AM =}{BN =} \frac{XM}{XN}$

$$\frac{7}{2} = \frac{4,2}{XN}$$

7 xN = 4,2 x 2

$$\frac{7}{7} \times N = \frac{84}{7}$$

XN =1,2 cm

c) Scale factor = 7/2

Area factor = $\left(\frac{7}{2}\right)^2$

= $\frac{49}{4}$

d) Area factor = $\frac{49}{4}$

= 49 : 4

= 49 : 4
21 : less

$= \frac{21}{\cancel{49}\ 7} x\ 4$

$$= \frac{12}{7}$$

$=1,714cm^2$

Area of the smaller one = $1,71cm^2$

Example 2
A map of Bulawayo is drawn to a scale of 1:50 000. On the map the Trade Fair covers on area of $8cm^2$. Find the true area of the Trade Fair in hectares (1 ha=10 000 m^2.

Scale factor = $\frac{50000}{1}$

= $(50\ 000)^2$

Area factor = $(50\ 000)^2$

= 2 500 000 000

Area of Trade Fair = $\frac{2500000000 \times 8}{10000 \times 10000}$

= 200 hectares

Brief Summary
1) The area factor is found by squaring the scale factor.
2) When finding the bigger area or actual area given the smaller are or area on the map, multiply the given area by the area factor.
3) When finding the small area or area on the map, given the larger area or actual area divide by the area factor.
4) Take note of the following conversions

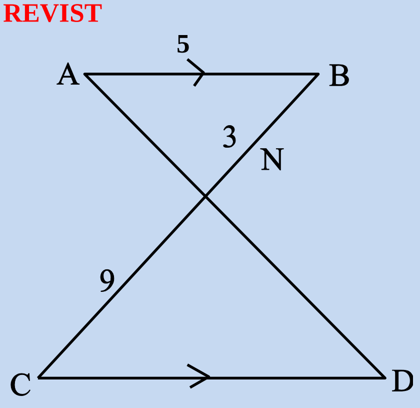
a) 1 ha = 1 000 m^2

b) 1m = 100cm

$1m^2 = (100cm)^2$

$1m^2 = (100\ x100)cm^2$
$1m^2 = 10\ 000\ cm^2$

Exercise 1,1
REVIST



a) State the triangle which is similar to BAN
b) State the scale factor of similar triangles
c) Calculate the length of CD
d) Find the area factor
e) Given that the area of triangle BAN is 2,7cm ^{2} , find the area of the triangle similar to it.

2 Two similar rectangles have corresponding sides in the ratio 5:2. Find the ratio of their areas.

3 Two similar triangles have corresponding sides of length 5cm and 12cm. Find ratio of their own areas.

4. Triangle XYZ is similar to triangle PQR. Given that XY =5cm and PQ =2cm and area of triangle PQR is 12cm^2, find the area of triangle XYZ

5. Two circles which are similar have area of 81cm^2 and 16m^2 respectively
   a) Calculate the ratio of corresponding radii
   b) Given that the radius of the smaller circle is 8cm, find the radius of the bigger circle

6. A map of a city is drawn to scale 1:50 000 on the map the sports stadium covers an area of 12m^2. Find the actual area of the stadium in hectares.

7. A map of school is drawn to a scale of 1:500 , on the map the school covers an area of 8cm^2. Find the actual area of the school in square metres.

Volume of similar solids

Solids refer to three dimensional shapes like cylinders, cubes, spheres, cone etc. They are also similar if all their corresponding linear dimensions are in the same ration.

For example, if the ratio of the corresponding widths, lengths and height of two cuboid are equal then the two solid shapes are asaid to be similar. Again, the ratio of the volumes is the cube of the scale factor of the two solids.

Example 3

Two cuboids are similar. The ratio of their lengths is 3:2 given that the volume of the smaller cuboid is 64cm^3. Calculate the volume of the bigger cuboid.

Scale factor = $\frac{3}{2}$

Volume factor = $(^3/_2)$ 3

$= \frac{3^3}{2^3}$

$= \frac{27}{8}$

Volume of the larger cuboid $= \frac{27}{\cancel{8}} \times \cancel{64}$

$= 216 \text{ cm}^3$

Example 4

The cylindrical containers of maize meal are similar and contain 27kg and 125kg of maize meal respectively. If the radius of the bigger container is 25cm, find the radius of the smaller one.

Volume ratio = Mass ratio

$= \frac{125}{127}$

$= (^5/_3)^3$

Scale factor $= \frac{5}{3}$

Radius of smaller cylinder $= 25 \div \frac{5}{3}$

$= 25 \times {}^3/_5$

$= 15$ cm

Further points
The masses and capacities are equal to the volume factor.

Exercise 1,2

1 Two cylinders are similar. The ratio of their radii is 4:9. The volume of the bigger cylinder is $27{,}9cm^3$. What is the volume of the smaller one.

2. Two similar bricks masses of 729, and 1 342g respectively. What is the ratio of their lengths when the length of the smaller brick is 18,9cm

3. Two similar bricks have corresponding edges of length 10cm and 20cm Find the ratio of their masses.

4. Two similar cylindrical cooking oil container bottles hold 8 litres and 27 litres of cooking oil respectively. If the radius of the bigger container is 15cm, find the radius of the smaller container.

5. The height of a model coca-cola container bottle is 30cm, while the height of the actual bottle is 10cm. If the capacity of the model is 36ml, find the capacity of the actual bottle in litres.

6. A pyramid is 4cm high and has a volume of $45cm^3$. What is the height of a similar pyramid whose volume is 2 880 cm^3

7. Two solids which are similar have volumes of 1 $343cm^3$ and $532cm^3$ respectively
   a) Find the ratio of their sides
   b) Given that it costs \$ 1,80 to paint the surface of the smaller solid. Find how much it would cost to paint the larger solid using the same type of point. Hint surface area = (scale factor)^2

EXAMINATION QUESTIONS

1. Refer to originally Two key boats are geometrically similar and one is 2 ½ times as long as the other.
2. Given that the height of the mast of the smaller boat is 14cm, calculate the height of the mast of the larger boat. ii) Write down the ratio of the surface area of the smaller boat to that of the larger boat, expressing your answer as a fraction.

(Cambridge 1982)

3. A geographical globe has a diameter of 48cm. A miniature model of the globe has a diameter of 8cm.

i) Calculate the surface area of the model

ii) On the globe, the map of Zimbabwe occupies an area of 23, $04cm^2$. Calculate the corresponding area on the model.

c) If the globes are similar, calculate the volume of the model.

$$\left\{ \begin{array}{l} \text{Volume of sphere} = 4/3\ \pi r^3 \\ \text{Surface area of a sphere} = 4\pi r^2 \\ \text{Take } \pi \text{ to be } 3{,}142 \end{array} \right\}$$

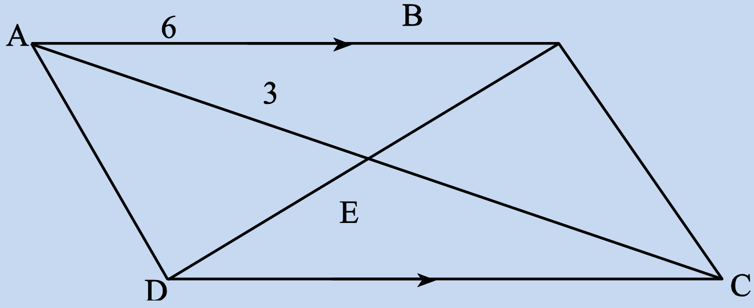
3) The plan of a building is drawn to scale of 1:250

a) Find the length, in metres of a wall which is represented by a line 11,7cm on the plan

b) The area of the floor of a room is $25m^2$. Find in square centimeters, the area on the plan which represents this floor.

REVISIT

4)



In the diagram, ABCD is a quadrilateral with AB parallel to DC. Diagonals AC and BD meet E. AB=6cm, BE=3cm and DC =15cm.

a) Name, in the correct order, the triangle that is similar to triangle ABE

b) Calculate DE

c) If the area of triangle BEC is $22{,}5cm^2$ calculate

i) the area of ΔDEC

ii) the ratio of area of $\dfrac{\Delta ABE}{\Delta ADC}$ in its simplest form

CHAPTER 13

Quadratic Equations

A equation of the form a x^2 + b x + c = 0 where a,b and c are real numbers and a ≠0, is a quadratic equation.

Syllabus Objectives
Learner should be able to
a) Solve quadratic equation by the method of factorization
b) Solve quadratic equations by the method of completing the square
c) Solve quadratic equations using the formula
d) Solving quadratic equations using the graphical method.

Method of factorization
This method involve expressing the quadratic expression as a product of its factors.

Example 1
Solve the following quadratic equations

i) $(3x+7)(4x-1) = 0$
ii) $x^2 - x - 6 = 0$
iii) $2m^2-5m+3=0$

i) $(3x+7)(4x-1)=0$
either $3x+7=0$ or $4x-1=0$
$3x = -7$ or $4x = 1$
$x = \frac{-7}{3}$ or $x = \frac{1}{4}$

x= -2 ½ or ¼

ii) $x^2 - x -6 =0$
$x^2-3x + 2x -6=0$
$x(x-3)+2(x-3)=0$
$(x+2)(x-3)=0$
Either $x+2 =0$ or $x-3=0$
$x = -2$ or $x =+3$
$x = -2$ or 3

ii) $2m^2-5m + 3 = 0$
$2m^2-3m-2m+3=0$
$m(2m-3) – 1(2m-3)=0$
$(m-1)(2m-3) =0$

Either $m-1=0$ or $2m -3 =0$
$m=1$ or $m=3/2$

Exercise 1

1. Solve the following quadratic equations

a)	$(x+1)(x-1)=0$	b)	(a-3) (a+5) =0
c)	$x(x-3) = 0$	d)	y(y-1)(y+2)=0
e)	$x^2(x+2)(x-2)=0$	f)	(5-n)(4+n)=0
g)	$x^2(x+2)(x-2) = 0$	h)	a(2-a)(1+a) = 0
i)	(2y+1) (y+4)=0	j)	(4h-1) (2h-3) = 0

2. Solve the following equations

a)	$x^2-2x-3=0$	b)	$a^2 + 2a - 15 = 0$
c)	$6y^2-13y-6$	d)	$x^2 - 9 = 0$
e)	$y^2 - 4 = 0$	f)	$y^2-5y+4=0$
g)	$U^2 + 2U-35 = 0$	h)	$n^2 + n = 90$
i)	$16x^2 - 1 = 0$	j)	$4-9m^2 = 0$
k)	$6x^2 - 13x - 5=0$	l)	$2x^2-11x+12=0$
m)	$15-31x+14x^2=0$	n)	$9y^2-12y+4=0$

COMPLETING IN THE SQUARE

This method involves manipulating the equation so as to generate a perfect square on one side of the equation.

Given the general quadratic equation.

$ax^2 + bx + c=0$

The following steps should be followed to complete the square.

1) make the coefficient of x^2 unity i.e) by dividing throughout the equation by a

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

2) Take the constant term to the right hand side leaving the terms in x^2 and x on the left.

$$x^2 + \frac{bx}{a} = \frac{-c}{a}$$

3) Complete the square by adding(1/2 co-efficient of $x)^2$ to both sides leaving the terms in x^2 and x on the left.

$$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$$

4) Factorize the left hand side.

$$\left(x+\frac{b}{2a}\right)^2=\frac{-c}{a}+\left(\frac{b}{2a}\right)$$

5) Take the square roots both sides

$$x+\frac{b}{2a}=\sqrt{\frac{-c}{a}+\left(\frac{b}{2a}\right)^2}$$

6) Obtain the values of x

Example 2

Solve the following equation by completing the square.

i) x^2-6 x + 3 =0   ii) 5 x^2-3 x -3=0

i) x^2 – 6 x + 3 = 0
x^2 – 6 x = 0-3

$$x^2-6x+\frac{(6^2)}{2}=-3+\left(\frac{6}{2}\right)^2$$

x^2-6 x +3^2 = -3 + 3^2
(x - 3) ^{2} = 6
(x - 3)^2 = 6

x - 3 = $\pm$ √6

x =√6+3 or -√6+3

ii) $5x^2-3x-3=0$

$x^2 - \frac{3}{5}x - \frac{3}{5} = 0$

$$x^2-\frac{3x}{5}+\left(\frac{3}{10}\right)^2=\frac{3}{5}+\left(\frac{3}{10}\right)^2$$

$$\left(x-\frac{3}{10}\right)^2=\frac{3}{5}+\frac{9}{100}$$

$$x - \frac{3}{10} = \pm\sqrt{\frac{69}{100}}$$

$$x = \frac{\sqrt{69}}{10} + \frac{3}{10} \; or \; \frac{\sqrt{69}}{10} - \frac{3}{10}$$

$$x = \frac{\sqrt{69}}{10} +$$

complete

EXERCISE 1,2

Solve the following by completing the square

a) x^2-2 x -5=0 b) y^2+6y3=0

c) c^2-4c-2=0 d) x^2+4 x +4=0

e) z^2-5z+6=0 f) h^2-5h+4=0

i) 3 x^2+6 x -2=0 j) 5 x^2-3 x -3=0

k) 2 x^2-6 x -1=0 l) 5 x^2-2 x -4=0

m) 15-31 x +14 x^2 =0 n) $9y^2$-12y+4=0

THE FORMULA FOR SOLVING QUADRATIC EQUATIONS

The general form of a quadratic equation is a x^2+ b x +c=0. The roots of this are found by complete the square,

a x^2+b x + c = 0

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = \frac{-c}{a}$$

$$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x=-b\pm\sqrt{\frac{b^2-4ac}{2a}}$$

Example 3
Solve the following equation. Give your answer correct to 2 decimal places.

$2x^2 -3x-4=0$

a= 2 b = -3 c=-4

$$x=\frac{-b\underline{+}\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-3)\underline{+}\sqrt{(-3)^2-4(2)(-4)}}{2(2)}$$

$$=\frac{3\underline{+}\sqrt{9+32}}{4}$$

$$=\frac{-3+\sqrt{41}}{4}$$

$$=\frac{-3\sqrt{41}}{4}\;or\;\frac{-3-\sqrt{41}}{4}$$

$$=\frac{3,403}{4}\;or\;\frac{9,403}{4}$$

=0,850 or -2,350

X = 0,85 or 2,35 to 2.d.p

<u>TAKE NOTE</u>
Sometimes a quadratic equation does not have any real roots. When this happened there is a negative number under the square root sign in the formula. We then say roots are imaginary or there are no real solutions.

Exercise 1,3

a) $x^2 + x$ ̱56 = 0 b) $5x^2 - 13x - 6 = 0$

c) $6x^2 - 5x + 4 = 0$ d) $3x^2 -8x -4=0$

e) $4x^2 - 2x - 1 = 0$ f) $5x^2 - 8x + 2 = 0$

g) $2x^2 + 3x - 4 = 0$ h) $2x^2 + 7x - 3 = 0$

i) $4x^2 + 7x - 2 = 0$ j) $3x^2 - 8x + 2 = 0$

Graphical solution for Quadratic

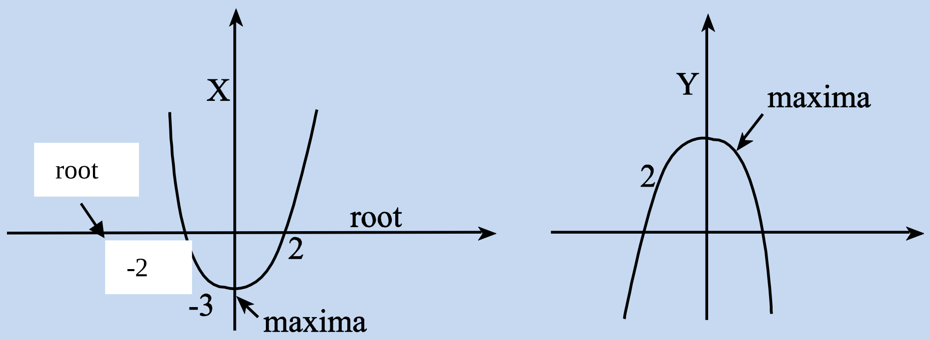
Graphs of quadratic equations are parabola. When the coefficient of x is positive the parabola faces upwards and when it is negative the parabola faces downwards. A parabola facing upwards form a maxima at the turning point while that facing downwards forms a maxima. At all turning points the gradient is zero.

The roots of the quadratic equation are found were the graph crosses the axis

A line of symmetry divides the curve into two equal parts.

When a straight line graph is drawn across the curve, the meeting points of the two graphs forms the solution to the two equations.

Revisit



root of equation x = -2or 2
minima occurs at y = -3
Line of symmetry x = 0

Drawing a quadratic curve

1) Find the scale that will fill the whole graph
2) Draw a smooth curve using very sharp pencil

Example 5

The variables x and y are connected by the equation $y = 1 + 2x - x^2$ and some corresponding values are given in the following table.

X	-3	-2	-1	0	1	2	3	4	5
Y	-14	-7	a	1	b	1	-2	-7	-14

a) Calculate the values of a and b

b) Using the scale of 2cm to represent 1 unit on the x -axis and 2cm to represent 5 units on the y axis draw the graph of $y=1+2x - x^2$ for $-3\leq 5.\ x<5$.

c) Use the graph to write down

i) the coordinates of the maximum turning point of the curve

ii) the roots of the equation $1 + 2x - x^2 = 0$

d) The equation of the line of symmetry

i) By drawing a suitable straight line on the same axes, solve the equation.

$1 + 2x + x^2 = -5$

f) On the graph shade the area by the curve $y=1+2x-x^2$ and the lines y=0 and x=1

a) $y= 1+2x - x^2$

when x=-1

$y= 1+ 2(-1) -(-1)^2$

$= 1-2-1$

$y=-2$

:- a=-2

$y=1+2- x^2$

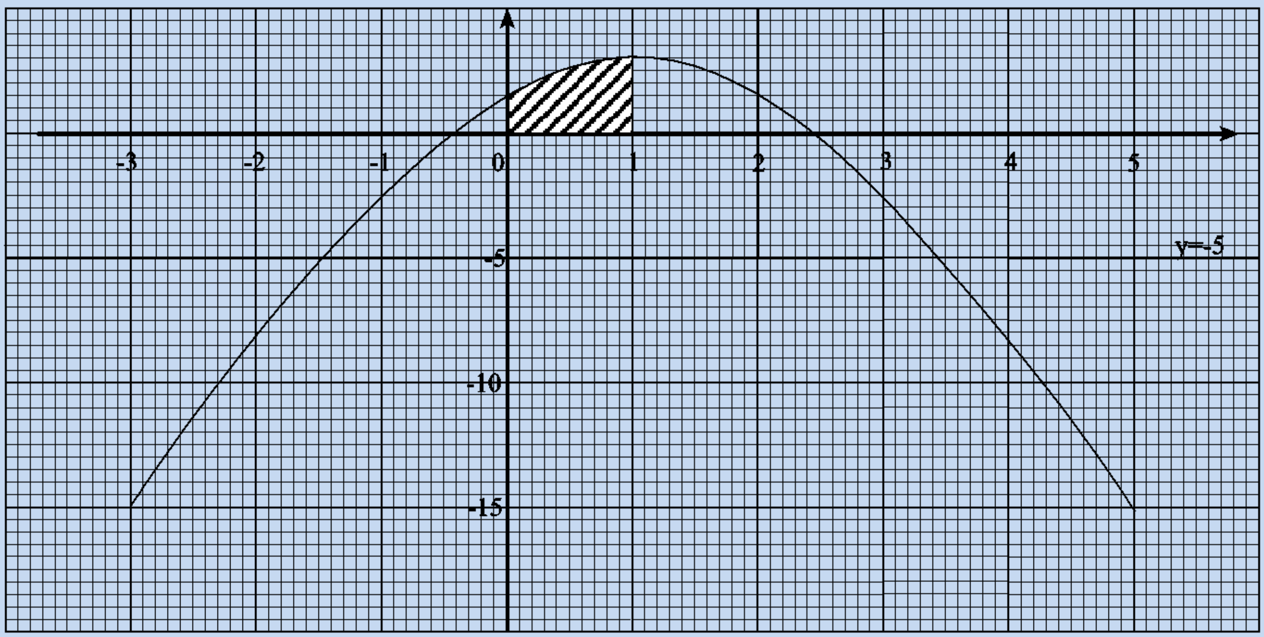
when $x=1$

$y=1+2(1) -(1)$

$= 1+2-1$

$=2$

:- b=2



ci) Maximum turning point (1;2)

ii) X=-0,4 or 2,3

d) Line of symmetry x =1

ii) x=-1,6 or 2,6

Exercise 1,3

1. Draw the graph of the equation $y= x^2- x -5$ for $3\leq x \leq 4$

b) Solve the equation $x^2- x -5=0$

c) Write down the equation of the line of summitry

2) Draw the graph of the equation $y= x^2+ x -2$ for $-4 \leq x \leq 3$

b) Find the maximum coordinates of the graph

c) Solve the equation $x^2+ x -2=0$

d) By drawing a suitable straight line on the some axes, solve the equation $x^2+ x -2$

Examination questions

1. A cyclist made a distance of 50km at an average speed of xkm/h. Write down on expression for the time in hours that he took for the journey. He returned by the some route but his average speed was 3km/h less. Write down an expression of the time, in hours that he took for the return journey.

   Given that the difference between his two times is 1 ½ hours, form an equation in x and show that it reduces to x^2-3x-100=0

   Solve this equation giving your answer correct to 1 decimal place. Hence find the time, correct to the nearest 5 minutes, for the return journey.

2. Answer the whole of this question on a sheet of graph paper

The following is incomplete table of values for y=3- x -2 x^2

x	-2	-1	- ½	0	½	1	1 ½
y	-3	2	m	3	2	0	-3

a) Calculate the value of m

b) Using a scale of 2cm to represent 1/2 unit on the x axis and 2cm to represent 1 unit on the y axis, draw the graph of y=3- x -2 x^2 for $-2 \leq x \leq 1\frac{1}{2}$

c) Use the graph to estimate
   i) the maximum value of y
   ii) the gradient of the curve at x = ½
   iii) the range of values of x for which y is positive

d) By drawing a suitable straight line on the same axes, solve the equation.
   3- x -2 x^2=-1

3. Solve the equation 2 x^2, +3 x -1=0, giving the answer to two decimal places

4. Factorise complete
   a) 3(x -2) + 4y (x -2)
   b) 6 x^2- x -12

5. Solve the equation $3x^2-5x-9=0$, giving answers correct to two significant figures.

(ZIMSEC NOV 2005)

CHAPTER 14

Matrices

A Matrix is a rectangular array of numbers arranged in row and column. Matrices are useful for storing information.

Syllabus objectives
Learner should be able to

a) Interpret a matrix as a store of information
b) State the order of a matrix
c) Add and subtract matrices
d) Multiply matrices with each other and or with a scalar
e) Find the determinant of a 2×2 matrix
f) Find the inverse of a 2×2 matrix
g) Use the matrix method of solving simultaneous equations

Systematical storage of information

ABC limited is a large company that supplies households furniture to its customers X,Y and Z respectively. The company supplies wardrobes, beds and sofa. To keep check on its factory and accounts, there is need to arrange this information systematically.

Month of May

	Beds	Sofas	Wardrobes
Orders from x	50	30	40
Orders from y	40	40	30
Orders from Z	30	0	40

This information in matrix can written as follows.

$$\begin{pmatrix} 50 & 30 & 40 \\ 40 & 40 & 30 \\ 30 & 0 & 40 \end{pmatrix}$$

Each horizontal line of quantities represent all the orders by one customer e.g. (50 40 40) and it is called the row of matrix. So the above matrix has 3 row.

Each vertical line of quantities represented the quantities of each item ordered by each customer and it is referred to as a column of a matrix

- The "number of rows by the number of columns" gives the order of a matrix in the above matrix the order is 3 by 3 (or 3×3). Generally a matrix with arrows and columns is said to have order mxn.

Special matrices

a) Row matrices

These are matrices made up of a single raw e.g (2 3 4)

b) Column matrices

These are matrices made up of a single column e.g $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ etc

c) Square matrices

These are arte matrices made up of the same number of rows and column

e.g $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}; \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & 3 \\ 4 & 1 & 5 \end{pmatrix}; (4)$

Exercise 1

The information below shows the soccer log standing after the first 5 matches

Team	Won	Drew	Lost
A	4	1	0
B	2	2	1
C	0	2	3

a) Write information in matrix form?
b) How many rows form the matrix?
c) How many columns form the matrix?
d) State the order for the matrix

2. Give the order of the following matrices

a) $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ b) $(1 \quad 4 \quad 3)$ c) $\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ & & \end{pmatrix}$ e) (5) f) $\begin{pmatrix} 1 & -2 & 2 \\ 3 & 2 & 8 \\ 1 & 4 & 3 \end{pmatrix}$

3) Give an example of a matrix of the following order

a) 2×2 b) 2×3 c) 1×4
d) 1×1

4. Given the matrix $\begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ & & \end{pmatrix}$

State the element on each of the positions below

a) First row, third column
b) First row, first column
c) Second row, second column
d) third row, last column

Equal matrices

Two matrices are equal if there are of the same order and the correspondence elements of one matrix are equal those of the other.

Example 1

Say whether the following matrices are equal, and if equal find the values of the unknown.

i) $\begin{pmatrix} x & 3 \\ -2 & y \end{pmatrix}$ and $\begin{pmatrix} -1 & 3 \\ -2 & 1 \end{pmatrix}$

ii) (1 2 3) and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

i) The two matrices are equal

$$\begin{pmatrix} x & 3 \\ -2 & y \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ -2 & 1 \end{pmatrix}$$

x= -1 y=1

ii) The two matrices are not equal as they are not of the same order.

Addition and subtraction of matrices

Matrices can be added and or subtracted only if there are of the same order, Corresponding elements are added (or subtracted) to obtain the resulting matrice which will be of the same order as the original matrices.

$$\begin{pmatrix} a & b & c \\ d & e & f \\ & & \end{pmatrix} + \begin{pmatrix} g & h & i \\ j & k & l \\ & & \end{pmatrix} = \begin{pmatrix} a+g & b+h & c+i \\ d+j & e+k & f+l \\ & & \end{pmatrix}$$

Example 1

Given A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ B = , $\begin{pmatrix} 6 & 2 \\ 1 & 3 \end{pmatrix}$ C = $\begin{pmatrix} 1 & 3 & 4 \\ 5 & 6 & -3 \\ & & \end{pmatrix}$

Find a) A + B b) A + C c) B-A

a) A + B $= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 6 & -2 \\ 1 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 1+6 & 2+(-2) \\ 3+1 & 4+1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 0 \\ 4 & 5 \end{pmatrix}$$

b) A + C = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 4 \\ 5 & 6 & -3 \\ & & \end{pmatrix}$

Addition is not possible since some elements of the second matrix have no partners to combine with. Remember addition and subtraction is possible only when the matrices are of the same order.

c)

$$B - A = \begin{pmatrix} 6 & -2 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 6-1 & -2-2 \\ 3-3 & 3-4 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -4 \\ 0 & -1 \end{pmatrix}$$

<u>Exercise 1.2</u>

1. Evaluate the following, indicating where it is not possible.

a) $\begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

b) $\begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & 3 \\ 4 & -1 & 3 \end{pmatrix} + \begin{pmatrix} 8 & 2 & 7 \\ -6 & 3 & 0 \\ 2 & 1 & 4 \end{pmatrix}$

c) $\begin{pmatrix} -8 & 4 \\ -2 & -3 \end{pmatrix} - \begin{pmatrix} -1 & -3 \\ -1 & 4 \end{pmatrix}$

d) $(1.3.4) + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

e) $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -8 \\ 4 \end{pmatrix}$ **f)** $\begin{pmatrix} 3 & -1 & 1 \\ 2 & -4 & 2 \\ 1 & 0 & 3 \end{pmatrix}$

2) Given that A = $\begin{pmatrix} 1 & -3 \\ 5 & 4 \end{pmatrix}$ B = $\begin{pmatrix} -1 & 2 \\ -4 & 3 \end{pmatrix}$ c = $\begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix}$

Simplify

a) A + A b) A – B c) A + C

d) A + (B-C) e) (A+B) – (B+C)

3a) $\begin{pmatrix} x & 3 \\ y & 4 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$, find the value of x and y

b) if $\begin{pmatrix} -2 & x \\ y & 3 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ -5 & 4 \end{pmatrix}$

value of x and y

MATRIX MULTIPLICATION

Multiplication by a scalar

A matrix can be multiplied by a scalar k, (i.e. a number) where the scalar multiplies every element in the matrix

For example

$$K = \begin{pmatrix} V & W \\ X & Y \end{pmatrix} = \begin{pmatrix} K \times V & K \times W \\ k \times X & K \times Y \end{pmatrix}$$

$$= \begin{pmatrix} KV & KW \\ KX & KY \end{pmatrix}$$

Example 2

Solve i) $2\begin{pmatrix} -3 & 4 \\ 5 & -8 \end{pmatrix}$ ii) $\frac{-3}{2}\begin{pmatrix} 2 & 3 \\ -2 & \frac{1}{2} \end{pmatrix}$

i) $$2\begin{pmatrix} 3 & 4 \\ 5 & -8 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 10 & -16 \end{pmatrix}$$

ii) $$\frac{-3}{2}\begin{pmatrix} 2 & 3 \\ -2 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -3 & -4\frac{1}{2} \\ 3 & -\frac{3}{4} \end{pmatrix}$$

MULTIPLICATION OF TWO MATRICES

Matrices can only multiply each other if the number of columns in the first matrix is equal to the number of rows in the second matrix. The easier way of determining whether multiplication is possible is by first finding the order of the matrices. Then what? Suppose first

m x n   n x x

Since these are the same it means multiplication is possible.

The outside combination of m x x predicts the order of the answer after multiplication.

To be specific

order of 1st matrix   Order of second matrix

3×2 2×1

Multiplication is possible

The answer will be of order 3×1

2×3 2×1

Multiplication

Multiplication is done by multiplying the row by the column. The first row in the matrix multiplies all the columns in the second matrix and other row follows.

Thus $\begin{pmatrix} a & b & c \\ d & e & f \\ & & \end{pmatrix}$ $\begin{pmatrix} f & i & j \\ k & l & m \\ n & o & p \end{pmatrix}$

$$\begin{pmatrix} ((a\times f)+(b\times k)+(c\times n)(a\times i)+(b\times l)+c\times o) & (a\times j)+(b\times m)+(c\times p) \\ (d\times f)+(e\times k)+(f\times n)(d\times i)(e\times l)+(f\times o) & (d\times j)+e\times m)+(f\times p) \end{pmatrix}$$

2×3 3×3

Possible $= \begin{pmatrix} af+ba+cn & ai+bl+co & aj+bm+cp \\ df+ek+fn & di+el+fo & dj+em+fp \\ & & \end{pmatrix}$

Example 2
Evaluate

i) $\begin{pmatrix} 3 & 2 & 1 \\ 0 & 2 & 4 \\ & & \end{pmatrix}\begin{pmatrix} 6 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 1 \end{pmatrix}$ ii) $\begin{pmatrix} 5 & 1 \\ 2 & 3 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 6 & 6 \\ 3 & 4 \end{pmatrix}$

i) $\begin{pmatrix} 3 & 2 & 1 \\ 0 & 2 & 4 \\ & & \end{pmatrix}\begin{pmatrix} 6 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 1 \end{pmatrix}\begin{pmatrix} 3\times6+0+1\times1 & 2\times2+1\times5 & (3\times1)+2\times3)+1 \\ 0+0+4\times1 & 0+2\times3+4\times1 & 0+2\times3+4\times1 \\ & & \end{pmatrix}$

2 × 3 3 × 3

Possible

$= \begin{pmatrix} 19 & 9 & 10 \\ 4 & 24 & 10 \end{pmatrix}$

Result
2 × 3

ii) $\begin{pmatrix} 5 & 1 \\ 2 & 3 \\ 4 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 \\ 6 & 6 \\ 3 & 4 \end{pmatrix}$

3 × 2 3 × 2

Multiplication not possible

Further notes
In multiplication, matrices are not communicative, that is to say Ax B ≠ B x A

Exercise 1,3

1) evaluate where possible

a) $\begin{pmatrix} 2 & -3 & 1 \\ -1 & 4 & -3 \\ & & \end{pmatrix}\begin{pmatrix} 2 & -3 & 1 \\ -1 & 4 & -3 \\ & & \end{pmatrix}$ b) $\begin{pmatrix} 2 & -3 & 1 \\ -1 & 4 & -3 \\ & & \end{pmatrix}\begin{pmatrix} 4 & -1 & 0 \\ 3 & -2 & 0 \\ & & \end{pmatrix}$

c) $\begin{pmatrix} 2 & 0 \\ -3 & 2 \\ -1 & -4 \end{pmatrix}\begin{pmatrix} -1 & 2 & -2 \\ 3 & -1 & 4 \end{pmatrix}$ d) $(1.4)\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

-

e) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (1 3) f) (1 3 4) $\begin{pmatrix} -2 & 5 \\ 0 & -3 \end{pmatrix}$

g) $\begin{pmatrix} -3 & 1 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 4 & 1 \end{pmatrix}$ h) (1 3 4) $\begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix}$

2. Given that A = $\begin{pmatrix} 2 & -4 \\ 1 & 3 \end{pmatrix}$ B, = $\begin{pmatrix} -1 & -7 \\ 5 & 6 \end{pmatrix}$

And c = $\begin{pmatrix} 2 & 0 \\ 10 & -1 \end{pmatrix}$

a) -2 A b) AB c) ½ C

d) 3B

3. Find x if $\begin{pmatrix} 2 & 0 \\ x & 3 \end{pmatrix}\begin{pmatrix} 0 & 4 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ 6 & 9 \end{pmatrix}$

4) Find y if $\begin{pmatrix} 3 & 2 \\ 4 & -5 \end{pmatrix}\begin{pmatrix} y & 1 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ 6 & 9 \end{pmatrix}$

5) If L = $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ M= $\begin{pmatrix} 5 & 0 \\ 2 & 6 \end{pmatrix}$ And N= $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

Find

a) L (MN) b) is it true that (LM) N = L (MN)

Identify Matrix

This is a matrix which behaves like one in ordinary multiplication. This matrix is called the identity matrix.

In ordinary multiplication One x any matrix = that matrix

i.e. $1 \times K = K$

In matrice multiplication Identify matrix x any matrix = that matrix

i.e. $$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The identify matrix, I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Determinant

Given any 2 x 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinate of the matrix is defined as ad-bc. Its symbol is det

For example Give n $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

Det= (2 x 4) – (3x1)
= 8-3
= 5

Inverse

In ordinary multiplication a number $k \times$ it , K
K (1/K) = 1

Also, in matrix multiplication, matrix A x its inverse, A^{-1}= identify, I

$A \times A^{-1} = I$

How do we find the inverse of a matrix

Given $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

1) Find first the determinant
Det = ad – bc

When determinant is zero**, stop**. It means the matrix has no inverse.

2) Interchange the elements in the first diagonal $\begin{pmatrix} d & \\ & a \end{pmatrix}$

3) Change the signs of the other two elements without changing their positions.

$$\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

4) Multiply the result of the step 3 by $\dfrac{I}{ad-bc}$ (i.e. $\dfrac{I}{\det}$ Remember when determinant is Zero there is no inverse.

$$\frac{I}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Generally if A $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$A^{-1} = \frac{I}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

When ad-bc = 0, then there is no inverse

A matrix with no inverse is called a singular matrix.
Remember $AA^{-1} = I$

Example 3

Find the inverse of the following matrices

i) $\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}$ ii) $\begin{pmatrix} -4 & 3 \\ 2 & -3 \end{pmatrix}$

i) $\begin{pmatrix} 2 & -3 \\ 1 & 3 \end{pmatrix}$

Det = $(2 \times 3) - 1 \times -3)$
= $6 - (-3)$
= 9

Inverse = $\dfrac{1}{9} \begin{pmatrix} 3 & 3 \\ -1 & 2 \end{pmatrix}$

ii) $\begin{pmatrix} -4 & 3 \\ 2 & -3 \end{pmatrix}$

Det = (-4 $\times$ -3) – (1$\times$ -3)
= 12 – (-3)
= 15

Inverse $\frac{1}{15}$ $\begin{pmatrix} -3 & -3 \\ -2 & -4 \end{pmatrix}$

Summary

i) A singular matrix has no inverse

ii) The identify matrix, I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

iii) Matrix, A x its inverse A $^{-1}$= Identify matrix , I

cii) Inverse of $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Where ad-bc $\neq$ 0

ciii) $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+g \end{pmatrix}$

civ) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$

Exercise 1,4
State matrices which are singular (i.e. det = 0

a) $\begin{pmatrix} 4 & 11 \\ 1 & 3 \end{pmatrix}$ b) $\begin{pmatrix} 6 & 2 \\ 3 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

d) $\begin{pmatrix} -2 & 3 \\ -3 & 5 \end{pmatrix}$ e) $\begin{pmatrix} 5 & -8 \\ -2 & -8 \end{pmatrix}$ f) $\begin{pmatrix} -4 & 2 \\ -6 & 3 \end{pmatrix}$

2) Find the inverses of the following

a) $\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}$ b) $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$ c) $\begin{pmatrix} -2 & 3 \\ -4 & 1 \end{pmatrix}$

d) $\begin{pmatrix} -6 & 4 \\ 4 & -2 \end{pmatrix}$ e) $\begin{pmatrix} -4 & -5 \\ -2 & -3 \end{pmatrix}$ d) $\begin{pmatrix} -4 & -3 \\ 5 & 3 \end{pmatrix}$

3.

$\begin{pmatrix} x & 2 \\ -5 & 2 \end{pmatrix}$ has determinant zero, find x

The two possible values of k

4. Given that $\begin{pmatrix} k-7 & 2 \\ 4 & k \end{pmatrix}$ has determinant -4, find the two possible values of K

b) Hence, write down the inverse of the matrix using one of the values

5. Find the value of y which the matrix $\begin{pmatrix} 2 & k+1 \\ 2 & 5 \end{pmatrix}$ does not have an inverse

EXAMINATION QUESTIONS

1. A = $\begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}$ B = $\begin{pmatrix} \frac{1}{4} & k \\ O & \frac{1}{3} \end{pmatrix}$ and c = $\begin{pmatrix} 12 & 4 \\ -19 & m \end{pmatrix}$

a) Evaluate A^2
b) Evaluate the value of k which makes AB the identity matrix
b) Find the value of m which makes the determinant of A equal to the determinant of C

(CAMBRIDGE)

2) Given

$\begin{pmatrix} k+3 & 6 \\ 2 & k \end{pmatrix}$ if the determinant of the matrix is -14, find 2 possible values of K

3) Express as single matrices

i) $\begin{bmatrix} 1 & 4 & -3 \end{bmatrix}$ $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ ii) $\begin{pmatrix} 1 & 3 \\ 0 & 5 \\ -2 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & -2 \\ 4 & 1 \end{pmatrix}$

4a) Given that M = $\begin{pmatrix} 4 & -9 \\ -2 & 5 \end{pmatrix}$ N = $\begin{pmatrix} 1 & 3 \\ 0 & -3 \end{pmatrix}$ and

L = $\begin{pmatrix} 2d & 4 \\ 1 & 3 \end{pmatrix}$ find

i) M + 2N
ii) MN
iii) the value of d which makes L singular

[ZIMSEC NOV 2006]

5) Given that P= $\begin{pmatrix} -3 & 2 \\ 0 & -5 \end{pmatrix}$ and Q $\begin{pmatrix} 2 & 6 \\ 1 & -4 \end{pmatrix}$

i) Find P + 2Q

ii) Calculate the values of x and y if P $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 2y \end{pmatrix}$

[ZIMSEC NOV 2004]

CHAPTER 15

Vector

A vector is any quantity with magnitude (size) and direction. Displacement (or translation, velocity force and acceleration are all examples of vectors.

Syllabus objectives
Learner should be able to
a) Write a vector as a representation of translation
b) Use the appropriate vector notation, $\begin{pmatrix} x \\ y \end{pmatrix}$ x ; $\overrightarrow{AB}$ or a
c) Add and subtract vectors
d) Carryout scalar multiplication of vectors
e) Find the magnitude of a vector
f) Use vectors to prove and discover properties of shapes

Vectors as a representation of a translation or displacement REVISIT

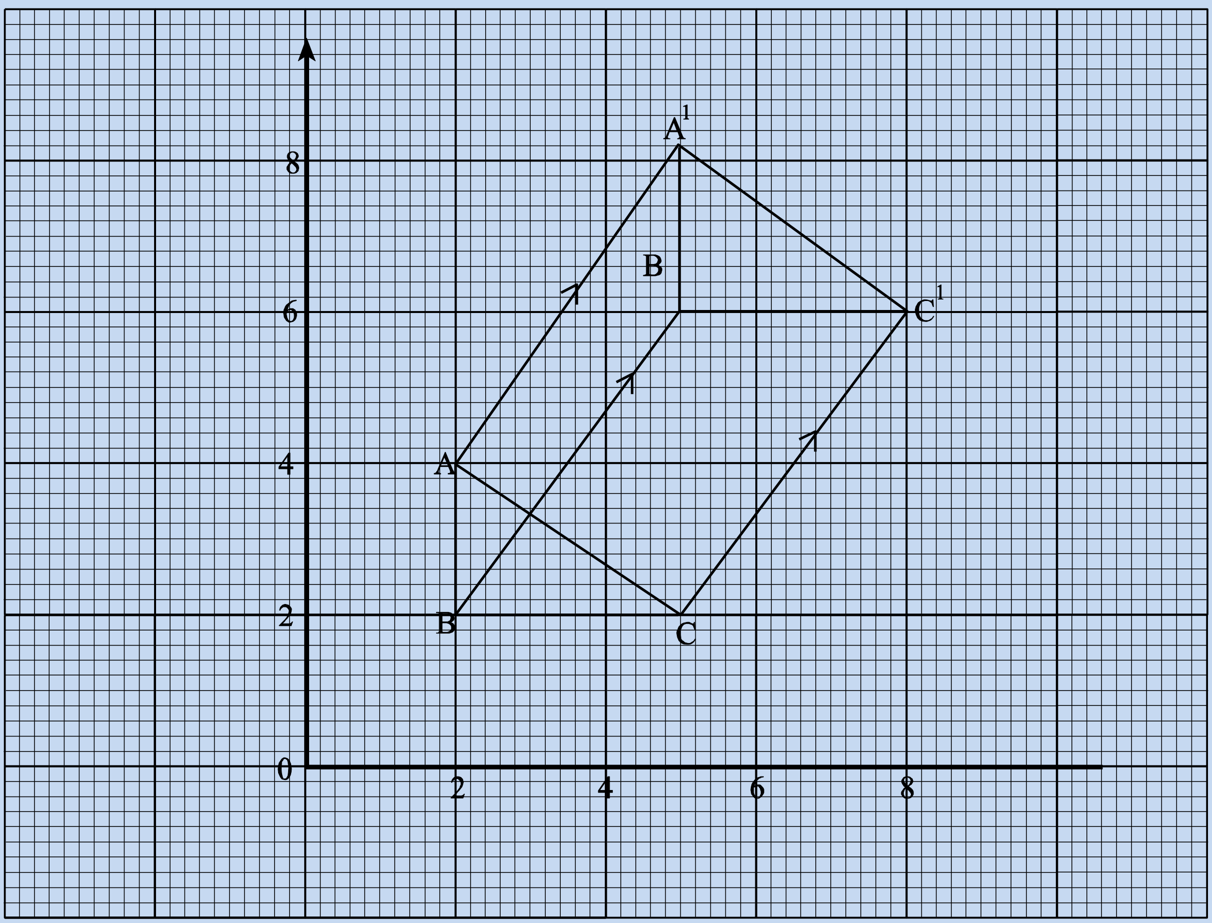


Diagram above shows $\triangle$ ABC being moved upwards without turning to new positions A^1B^1 and C^1. $\triangle$ABC has been translated or displaced to $\triangle A^1B^1C^1$. Lines AA^1, BB^1 an CC^1 represent translation vectors.
To move from A to A 3 units were moved to the right (the x direction) and 4 units upwards (the y direction, we can write $AA^1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ This is called a column vector.

Column Vector

Notice that $\overrightarrow{BB^1}$ and $\overrightarrow{CC^1}$ are also $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\overrightarrow{AA^1}$, $\overrightarrow{BB^1}$ and $\overrightarrow{CC^1}$ are equivalent. It means $AA^1=BB^1 = CC^1$

In general, any translation of the Cartesian plane can be written as a column vector $\begin{pmatrix} x \\ y \end{pmatrix}$ where x represents a movement parallel to the x axis and y represents movement parallel to the y axis
Movements to the right and movement upwards are positive. Movements to the left and movement downwards are negative.

Vector rotation

A vector represents a translation or a displacement which has direction and magnitude.
A vector can be drawn as an arroued line segment pointing in a specific direction. The length of the line segment represents the magnitude of the vector.

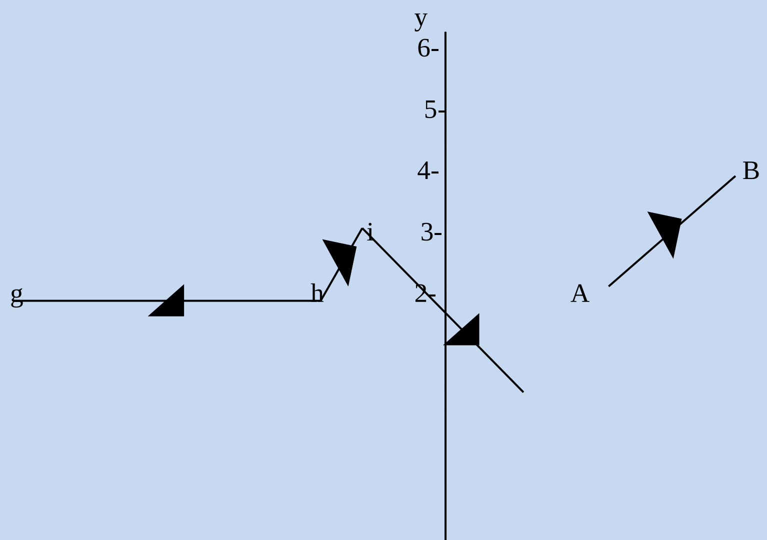
A vector can be named in the form $\begin{pmatrix} x \\ y \end{pmatrix}$; a or $\overrightarrow{AB}$.

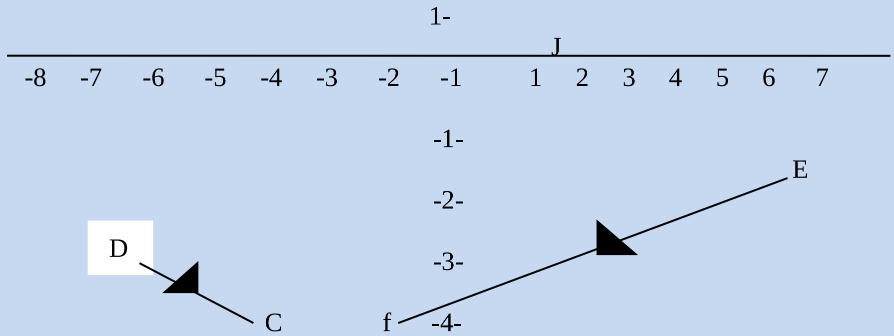
Note that $\overrightarrow{AB}$ is not the same as $\overrightarrow{BA}$

Example 1

In the diagram below the line segments represent vectors AB, CD, ef, gh hi and ij Write these in the form $\begin{pmatrix} x \\ y \end{pmatrix}$

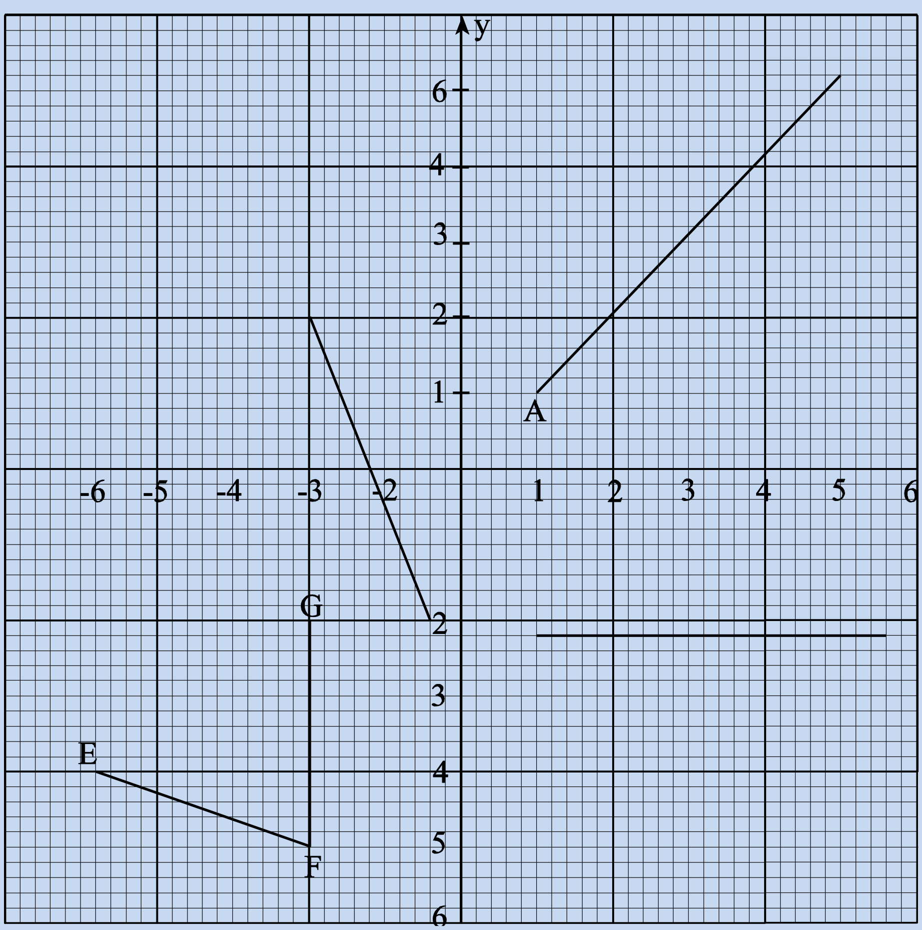
GRAPH





$\vec{AB} = \begin{pmatrix}3\\5\end{pmatrix}$ $\vec{CD} = \begin{pmatrix}3\\2\end{pmatrix}$ $\vec{EF} = \begin{pmatrix}8\\2\end{pmatrix}$ $\vec{GH} = \begin{pmatrix}5\\0\end{pmatrix}$ $\vec{HI} = \begin{pmatrix}1\\1\end{pmatrix}$ $\vec{IJ} = \begin{pmatrix}3\\-3\end{pmatrix}$

Exercise 1,1 **REVISIT**



In the diagram below the line segments represent vectors $\overrightarrow{AB}$, $\overrightarrow{CD}$,$\overrightarrow{EF}$, $\overrightarrow{GH}$ and $\overrightarrow{Hi}$. Write these vectors in column form

2 Using the diagram in Question 1, write the column vector that represent the following.

a) $\overrightarrow{BA}$ b) $\overrightarrow{DC}$ c) $\overrightarrow{FE}$ d) $\overrightarrow{iH}$

3. Using a scale of 1cm to 1 unit both axis in the graph paper. Draw line segment to represent these line segments.

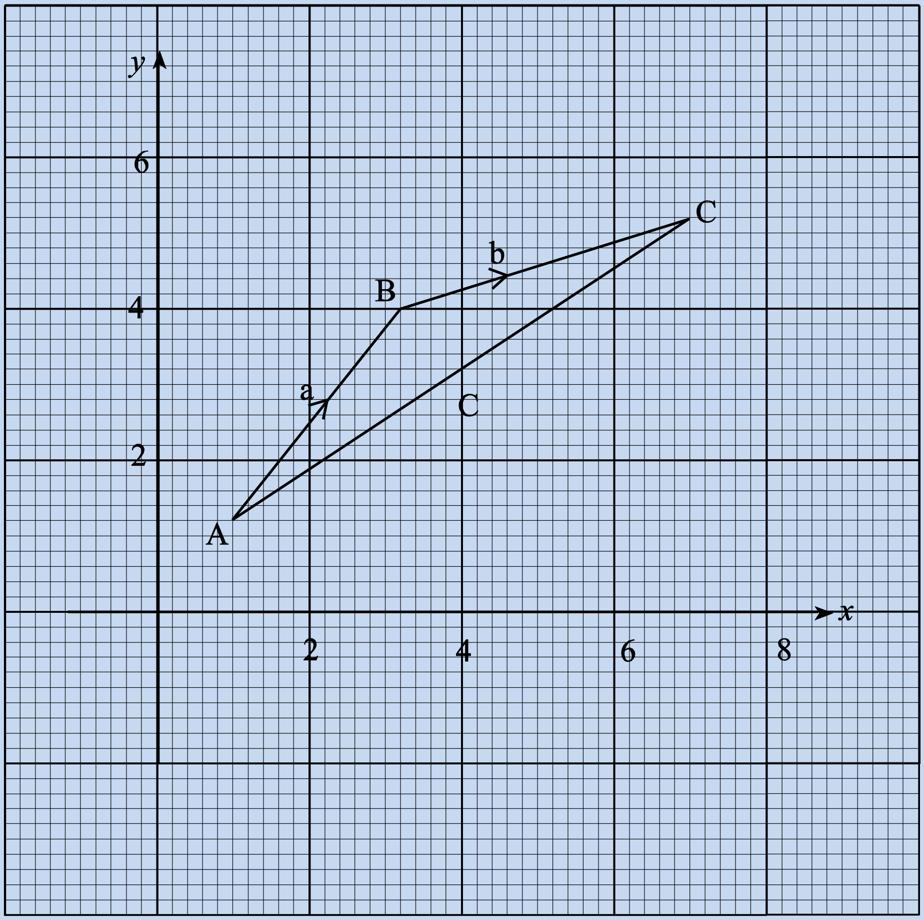
a) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ b) $\begin{pmatrix} -8 \\ 5 \end{pmatrix}$ c) $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ d) $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

e) $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$ f) $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ g) $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ h) $\begin{pmatrix} -5 \\ -5 \end{pmatrix}$

Addition of vectors

revisit

Addition of two vectors is a translation followed by another translation



From the diagram above, a translation of $\overrightarrow{AB}$ followed by a translation BC is equivalent to the single translation $\overrightarrow{AC}$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

Or

a + b = c

By counting squares

a = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ b= $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and c= $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$

a + b = c

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+5 \\ 3+1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ \overline{4} \end{pmatrix}$$

= c

In general if a = $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and b= $\begin{matrix} x_2 \\ y_2 \end{matrix}$

the a+b = $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

$$= \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$

<u>Vectors in opposite direction</u>

If a = $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

REVISIT

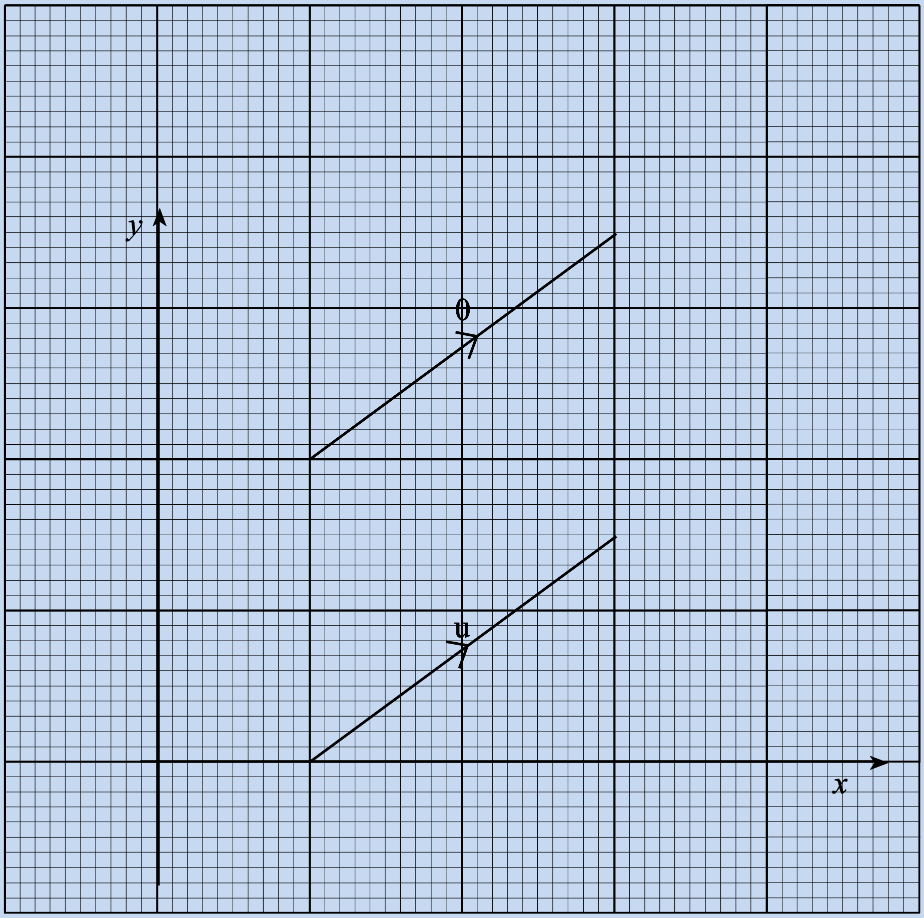


Diagram illustrates vector a and another vector V, with the same magnitude as a and parallel to a but in the opposite direction.

By counting the square

V= $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ a = $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Notice that V is a vector which has the same magnitude as a but which in the opposite direction we say that V = -a

In general, if a = $\begin{pmatrix} x \\ y \end{pmatrix}$ then –a = $\begin{pmatrix} -x \\ -y \end{pmatrix}$

Basically, negating a vector implied changing its original direction.

Subtraction

To subtract vector b from vector a, we add –b to a

Thus a-b = a+ (-b)

Example 2

Given a = $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and b = $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$ find

a) a + b

b) a-b

a) a + b = $\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$$= \begin{pmatrix} 3 + -1 \\ 4 - +2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 - 1 \\ 4 - 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

b) a-b = $\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 3 - & -1 \\ 4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 + 4 \\ 4 + 2 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 6 \end{pmatrix}$$

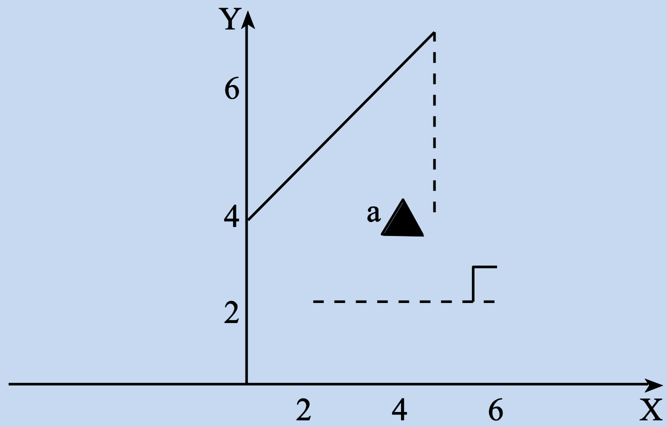
Multiplication by a scalar

A scalar is simply a multiplication. A vector multiplied by a scalar increases its size 3 time e.g 3 $\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$

If a vector is multiplied by a scalar K, the result is a new vector ka which is in the same direction as a but which is k times as big.

Magnitude of a vector or modules of a vector
Magnitude refers to the size of a vector.

revisit



In the diagram above the size of a vector a can be calculated by the Pythagoras theorem

Thus magnitude of a written IaI $= \sqrt{x^2 + y^2}$
Where x is the number of units moved horizontally, and y is the number of units vertically.

$$|a| = \sqrt{5^2 + 4^2}$$
$$= \sqrt{25 + 16}$$
$$= \sqrt{41}$$

This modules of vector a, $|a| = \sqrt{x^2 + y^2}$
Example 3

Given that a = $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$, and b = $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$ find

i) $|b|$b ii) $|a-b|$

i) $|b| = \sqrt{(-1)^2+(-2)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

ii) a –b = $\begin{pmatrix}3\\5\end{pmatrix} - \begin{pmatrix}-1\\-2\end{pmatrix}$

$= \begin{pmatrix}4\\7\end{pmatrix}$

$|a\text{-}b| = \sqrt{4^2+7^2}$

$= \sqrt{65}$

Exercise 1,2

1 Express in the form $\begin{pmatrix}x\\y\end{pmatrix}$

a) $\begin{pmatrix}2\\1\end{pmatrix}+\begin{pmatrix}2\\6\end{pmatrix}$ b) $\begin{pmatrix}-1\\2\end{pmatrix}+\begin{pmatrix}3\\3\end{pmatrix}$ c) $\begin{pmatrix}6\\8\end{pmatrix}-\begin{pmatrix}4\\5\end{pmatrix}$

d) $\begin{pmatrix}4\\1\end{pmatrix}-\begin{pmatrix}3\\2\end{pmatrix}$ e) $\begin{pmatrix}7\\-3\end{pmatrix}-\begin{pmatrix}-1\\0\end{pmatrix}$ f) $\begin{pmatrix}2\\1\end{pmatrix}+\begin{pmatrix}2\\6\end{pmatrix}$

2) Find the magnitude of the following vectors

a) $\begin{pmatrix}3\\0\end{pmatrix}$ b) $\begin{pmatrix}-4\\-5\end{pmatrix}$ c) $\begin{pmatrix}-8\\-6\end{pmatrix}$

d) $\begin{pmatrix}0\\-2\end{pmatrix}$ e) $\begin{pmatrix}15\\-8\end{pmatrix}$

3) Given that a = $\begin{pmatrix}4\\-5\end{pmatrix}$ and b= $\begin{pmatrix}-6\\2\end{pmatrix}$ and c = $\begin{pmatrix}0\\-4\end{pmatrix}$

Find

a) $a - b$ b) $a + b$ c) $a+b-c$

d) $a + c - b$ e) $a+b$

4) If $\overrightarrow{BC} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, find BC

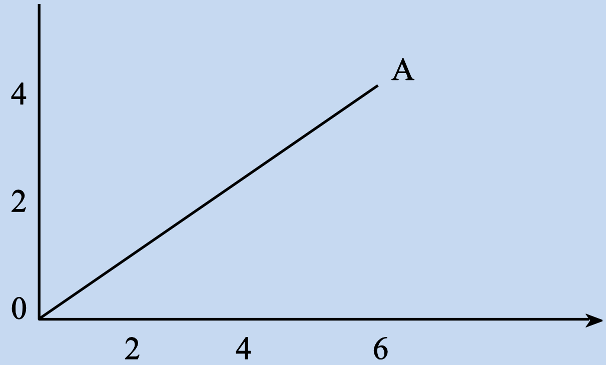
5) Find vector a such that

$$\begin{pmatrix} -1 \\ 4 \end{pmatrix} + a = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Hint : let $a = \begin{pmatrix} x \\ y \end{pmatrix}$

Position Vectors

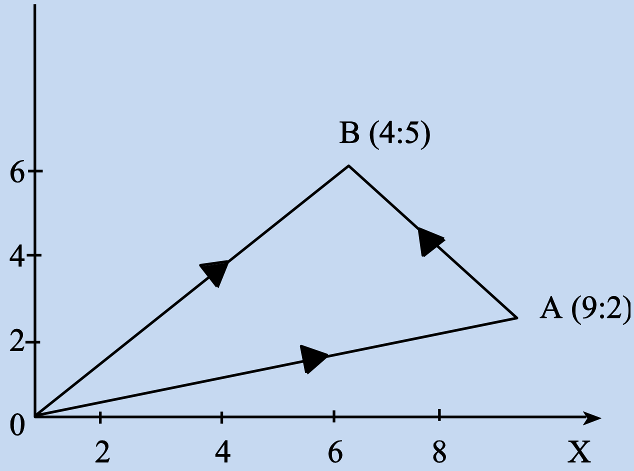
Inse:rt GRAPH



In the diagram above, position vector in the x-y place is the displacement vector from the origin O to A. OP = $\begin{pmatrix} x \\ y \end{pmatrix}$ is called the position vector .

It is called the position vector of A relative to O.

Position vectors can be used to find displacements between points.



In the diagram, to move from B to A you have to move from B to O, then from OA

Thus $\overrightarrow{BA}$ = $\overrightarrow{BO}$ = -$\overrightarrow{OA}$

but BO = - OB

so BA - $\overrightarrow{OB}$ + $\overrightarrow{OA}$

= $\overrightarrow{OA}$ – $\overrightarrow{OB}$

$$= \begin{pmatrix} 9 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

By counting units $\overrightarrow{BA}$ = $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

For any point with coordinates, A (x, y) and B(x_1, y_1)

$\overrightarrow{AB}$ = $\overrightarrow{OB}$ – $\overrightarrow{OA}$

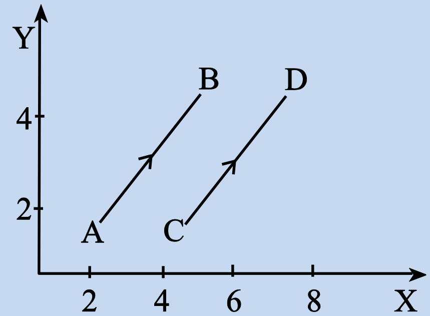
$$= \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} -x \\ y \end{pmatrix}$$

$|\overrightarrow{AB}| = \sqrt{(x_1-x)^2 + (y_1 - y)^2}$

Parallel lines

If $\overrightarrow{AB}$ = K $\overrightarrow{CD}$, then the lines AB and CD are parallel and the length of AB is K times, the length of CD.

revisit



$$\overrightarrow{CD} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$\overrightarrow{AB} = 2CD$

$$\overrightarrow{AB} = 2 \begin{pmatrix} 2 \\ 3 \\ 4 \\ 4 \end{pmatrix} \qquad \overrightarrow{CD} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$|\overrightarrow{AB}| = \sqrt{4^2 + 4^2} \qquad |\overrightarrow{CD}| = \sqrt{2^2 + 2^2}$$

$$= \sqrt{32} \qquad = \sqrt{8}$$

$$= 4\sqrt{2} \qquad = 2\sqrt{2}$$

It can be seen that the size of $\overrightarrow{AB}$ is twice that of CD

In conclusion for parallel vectors

If $\vec{a} = k\vec{b}$, then, IaI = k IBI and $\vec{a}$ is parallel to $\vec{b}$

Example 4

Given point A(6;7) and B(2;-1), find

a) $\overrightarrow{AB}$ b) $\overrightarrow{BP}$ c) $|\overrightarrow{PB}|$

a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OP}$

$$= \begin{pmatrix} 6 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ -8 \end{pmatrix}$$

b) $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$$= \begin{pmatrix} 6 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$

c) $PB = \sqrt{(2-6)^2 + (-1-7)^2}$

$= \sqrt{(-4)^2 + (-8)^2}$

$= \sqrt{16+64}$

$= \sqrt{80}$

$= 4\sqrt{5}$

Example 5
Which of the following are parallel

$a = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $b = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ $c = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $d = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$e = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Vector a is parallel to vector b b= 3a

Vector d is parallel to vector e = ½ d

Exercise 1,3

1) Given $\overrightarrow{OA} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

Find i) $\overrightarrow{AB}$ ii) $\overrightarrow{BA}$ iii) $\overrightarrow{2OA}$

2) If $a = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, which of the following are parallel to

i) $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ ii) $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$ iii) $\begin{pmatrix} -6 \\ -8 \end{pmatrix}$ iv) $\begin{pmatrix} -6 \\ 8 \end{pmatrix}$

v) $\begin{pmatrix} 1 \\ 1\frac{1}{4} \end{pmatrix}$

3. Given point P(5:8) and Q (2;-1), find
a) PQ b) QP

4. Give the following points. Express each of the following as a single column vector.

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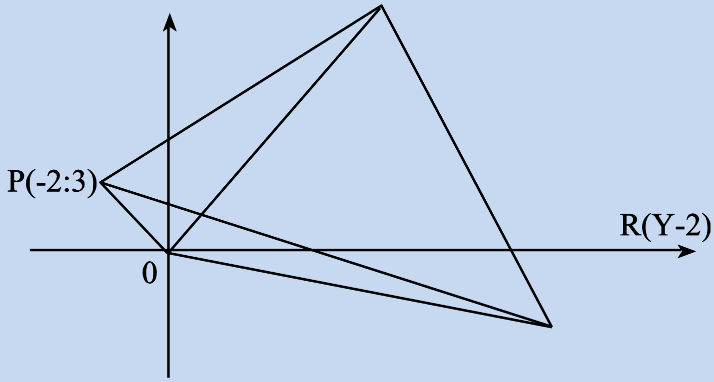


i) $\overrightarrow{OP}$

ii) $\overrightarrow{OR}$

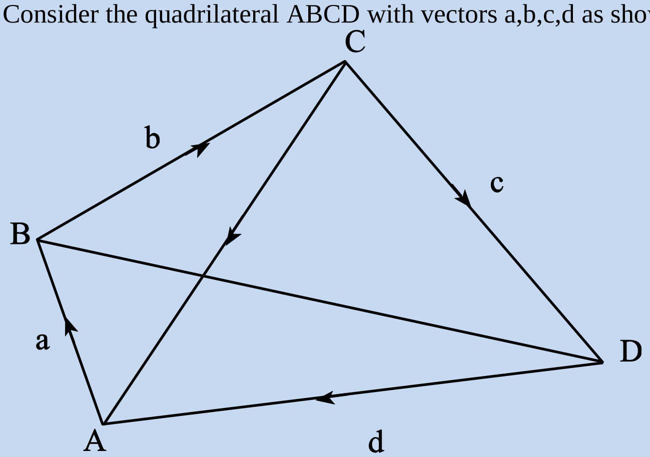
iii) $\overrightarrow{OP} + \overrightarrow{PQ}$

iv) $\overrightarrow{OR} + \overrightarrow{RQ}$

v) $\overrightarrow{OQ} + \overrightarrow{QR}$

The application of vectors is not only restricted for use in the cartesion plane. They can also be used in geometry.

Consider the quadrilateral ABCD with vectors a,b,c,d as shown



To move A to C one can move to B first then from B to C. Also, one can move to D first then to C.

Thus $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ or $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

= a + b   but AD= – DA   DC = -CD

$\overrightarrow{AC}$ = DA   - CD
= -d-c

It should also be noted that the total sum of vector of a closed polygon is Zero.

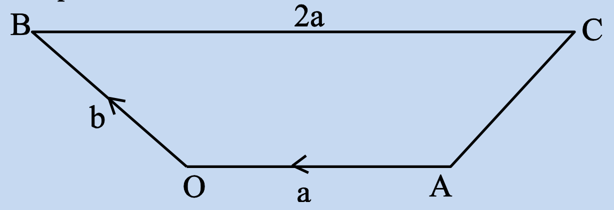
Proof
AC = AB + BC
= a + b

Since $\overrightarrow{AC}$ = - CA
AC + CA = 0

a + b + c+ d = 0 shown

- The total final displacement from the starting point A is zero when the vectors from the sides of a closed polygon

Examples 6



In fig 1 OACB is a trapezium with $\overrightarrow{OA}$ parallel $\overrightarrow{BC}$ and $\overrightarrow{BC}$ = 2a. If $\overrightarrow{OA}$ = a and $\overrightarrow{OB}$=b express, in terms of a and b.

i) $\overrightarrow{BC}$ ii) $\overrightarrow{OC}$ iii) $\overrightarrow{AB}$ $\overrightarrow{AC}$

i) $\overrightarrow{BC}$ = 2$\overrightarrow{OA}$
but OA = a

$\overrightarrow{BC}$ = 2a

ii) $\overrightarrow{OC}$ = $\overrightarrow{OB}$ + $\overrightarrow{BC}$
= b + 2a

iii) AB = AO + OB
but AO = -OA

= -a + b
= b-a

iv) AC = AO + OC
but AO = - OA

= -a+(b+2a)
= a+ b+2a
= a + b

Example 7

In Fig 2 ABCD is a parallelogram. M is the midpoint of $\overrightarrow{BC}$ and L is a point on $\overrightarrow{AD}$ such that 3AL=$\overrightarrow{LD}$. Given that $\overrightarrow{AB}$ =p and $\overrightarrow{BM}$=2q, express as simply as possible, in terms of p and or q i) $\overrightarrow{MD}$. ii) $\overrightarrow{DL}$ iii) $\overrightarrow{LM}$

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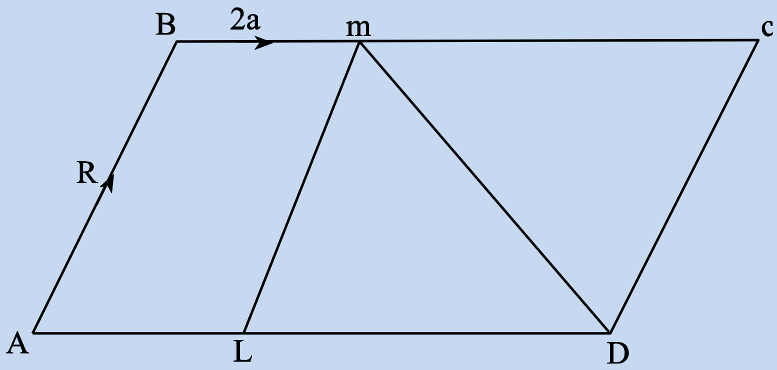

Considering side BC. If m is the midpoint, then the ration of BM:MC = 1:1

i) $\overrightarrow{MD} = \overrightarrow{MC} + \overrightarrow{CD}$

Since $\overrightarrow{BM}$=2q, then $\overrightarrow{MC}$ =2q

DC = $\overrightarrow{AB}$=p (opposite $\overrightarrow{\text{sides}}$ of a parallelogram are equal

ii) $\overrightarrow{MD}$ = 2q + (-(-$\overrightarrow{DC}$)

= 2q-p

ii) $\overrightarrow{DL}$ = ?

$\overrightarrow{AD} = \overrightarrow{BC}$ (opposite sides of a parallelogram are equal.

but $\overrightarrow{BC} = \overrightarrow{BM} + \overrightarrow{MC}$

= 2q +2q

= 4q

= $\overrightarrow{AD}$ = 4q

Since DL lies in the $\overrightarrow{AD}$

Considering AD, if 3AL = $\overrightarrow{LD}$ then ratio of AC: LD = 1:3

A 1 L 3 D

$$LD = \frac{3}{4}\overrightarrow{AD}$$

$$= \frac{3}{4}(4q)$$

$$= 3q$$

$\overrightarrow{DL}$ = - $\overrightarrow{LD}$

= -3q

Example 8

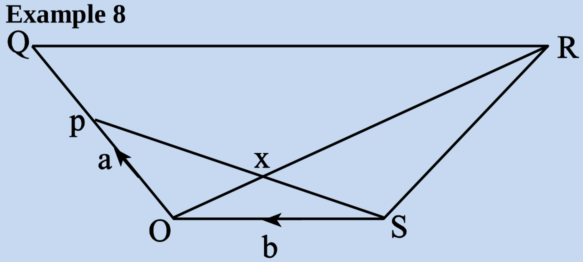

Fig 3 OP = a and OS =b

i) Express SP in terms of a and b

ii) Given that SX = hSP, show that $\vec{OX}$ = ha + (1-h) b

iii) Given that $\vec{OQ}$=3a and QR =2b, write down an expression for $\vec{OR}$ in terms of a and b

iv) Given that OX = k OR use the results of points (ii) and (iii) to find the values of h and k

v) Find the value of the ratio $\frac{PX}{XS}$

i) SP = -$\vec{OS}$ + OP
= -b + a
= a-b

ii) $\vec{SX}$ = h $\vec{SP}$
but $\vec{SP}$ = a-b
SX = h(a-b)
OX= OS + SS
OX = b + h(a-b)
= b + ha – hb

= ha-hb+b

= ha+b-hb

= ha+b-hb

= ha + b(1-h)

ox = ha+ (I-h)b Shown (ii)

iii) $\vec{OR}$ = $\vec{OQ}$+$\vec{OR}$
= 3a +2b

iv) OX = k(3a+2b)
= 3ak + 2bk

Equating OX
3ak + 2bk= ha+ (1-h)b
Equating those with b and a

3k = h (i)
2k =1-h (ii)

Substituting 3k for h

2k = 1-3k
5k= 1
k= $\frac{1}{5}$

h=3k
= 3 (1/5)
h= $\frac{3}{5}$

v) $\frac{PX}{XS}$

$\overrightarrow{PX} = -\overrightarrow{OP} + \overrightarrow{OX}$

$= -a + ha + (1-h)b$

$= -a + \frac{3a}{5} + (1-\frac{3}{5})b$

$= -a + \frac{3a}{5} + \frac{2b}{5}$

$= \frac{-2a}{5} + \frac{2b}{5}$

$= \frac{-2}{5}(a-b)$

$\overrightarrow{XS} = -\overrightarrow{OX} + \overrightarrow{OS}$

$= OS - OX$

$= b - [ha + (1-h)b]$

$= b - [\frac{3}{5}a + (1-\frac{3}{5})b]$

$= b - [\frac{3}{5}a + \frac{2}{5}) b]$

$= b - [\frac{3}{5}a + \frac{2}{5} b]$

$= b - \frac{3a}{5} - \frac{3b}{5}$

$= \frac{-3}{5}(a-b)$

$$\frac{PX}{XS} = \frac{\frac{-2}{5}(a-b)}{\frac{-3}{5}(a-b)}$$

$= \frac{2}{3}$

EXAMINATION QUESTIONS

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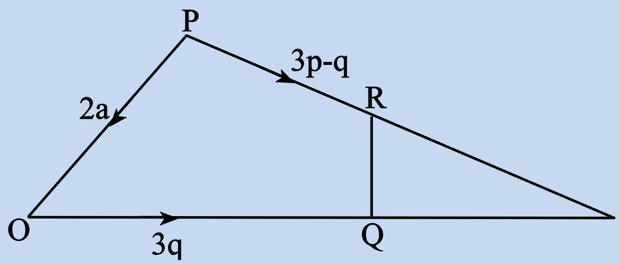

In the diagram, PRT and OQT are straight lines OP=2p, OQ=3q and $\overrightarrow{PR}$ =3p-q

i) Express $\overrightarrow{RQ}$ as simply as possible in terms of p and/or q

ii) Given that $\overrightarrow{PT}$=m, PR, express $\overrightarrow{PT}$ in terms of p, q and m

iii) Given also that OT=n OQ form an equation connecting p,q, m and n. Hence the values of m and the value of n.

(ZIMSEC NOV 2006)

2) O is the origin and A and B are the points (5;12) and (1;4) respectively. Find

a) $\overrightarrow{AB}$

b) Given that BP = $\overrightarrow{OA}$ ++20B, find the co-ordinates of P

3) In the diagram below point X lies on a straight line AB and AX:AX= 1:3 OA = 4p and OB = 4q.

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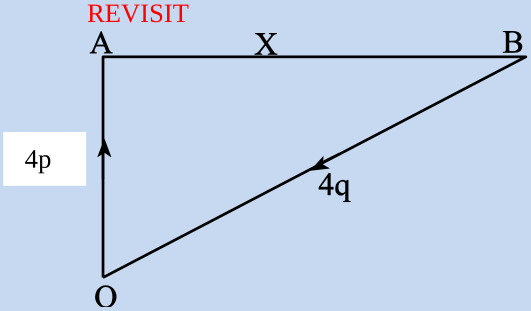
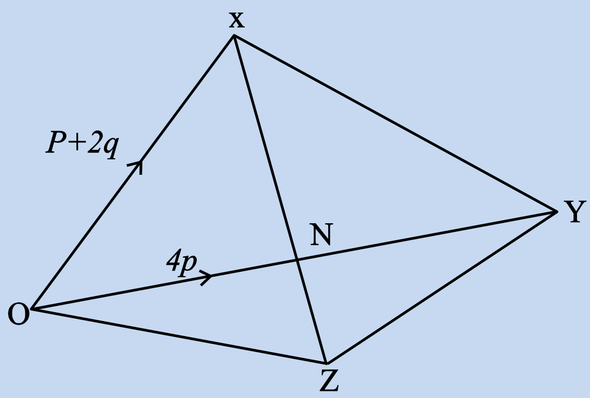


Express in terms of p and/or q

ii) AB ` ii) AX iii) OX



In the diagram, OXYZ is a quadrilateral N is a point on XZ such that XN: NZ =3:1 $\overrightarrow{OX}$= p+2q and $\overrightarrow{ON}$ =4p

a) Express as simply as possible in terms of point

i) $\overrightarrow{XN}$ ii) $\overrightarrow{XZ}$ iii) $\overrightarrow{OZ}$

b) Given that ZY is parallel to OX, express OY in terms of p,q and a constant K

c) If OY = $\frac{4}{3}$ ON

i) Find the value of K

ii) Express ZY in terms of p and q

a) Find the ration $\frac{\text{area of } \triangle \text{ YZN}}{\text{area of } \triangle \text{ OXN}}$ (hint scale factor)

(ZIMSEC NOV 2004)

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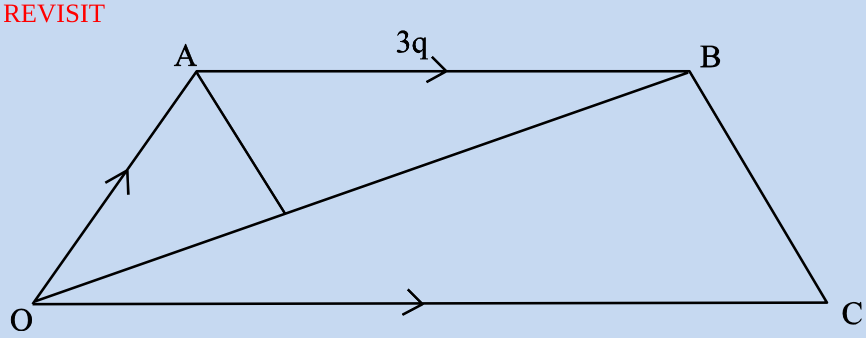


Fig 1

In fig 1, OAABC is a trapezium in which AB is parallel to OC.

a) Given that OA=3p, AB=3q and m is a point on OB such that OM:MB =1:3, express the following vectors in terms of p and q.

i) OB ii) OM iii)AM

b) Given that OC=h AB where h is a scalar, express OC in terms of h and q

ii) Given that BC is =KAM, where k is a scalar, show that OC = (3-2k)p + (3+k)q

iii) Use of the two expression for OC to find the numerical values of h and k

c) AM produced meets OC at N, find in terms of p and q

i) AN ii) MN

(Cambridge 1989)

CHAPTER 16

Ratio, Rate, Proportion

Syllabus objectives
Learner should be able to
a) Solve the problems involving ratio
b) Solve problems involving rate
c) Calculate using the concept of proportion

Ration

A ratio involves comparison of two or more qualities which are in the same units. It can be written as a fraction or with dots (:) separating the quantity. For example, Peter has 50c and John has $1,50. What is the ratio of their money.

Ratio Peter to John = 50c :150c
1 : 3

Ratio John Peter = 150c: 50c
3 1

Example 1

Share $ 60 in the ratio 3:4:8
First divide it into 15 parts (i.e. 3+4+8=15)

Sharing $\frac{3}{\cancel{15}} \times \overset{4}{\cancel{60}}$ $\frac{4}{\cancel{15}} \times \overset{4}{\cancel{60}}$ $\frac{8}{\cancel{15}} \times \overset{4}{\cancel{60}}$

$= \$12$ $= \$16$ $= \$32$

Take note $ 12 + $ 16 + $32 = $ 60

Example 2

(a) Increase 40m in the ratio 3:5
(b) Decrease 40m in the ratio 5:8

a) New length = $\frac{8}{\cancel{5}} \times \overset{8}{\cancel{40}}$

$= 64m$

b) New Length = $\frac{5}{\cancel{8}} \times \overset{5}{\cancel{40}}$

$= 25m$

Brief Summary

a) The quantities in the ratio should be in the same units and should be simplified

b) A ratio x:y can be written as a fraction $\frac{x}{y}$

Exercise 1,1

1) Express the following ratios in their simplest form.

a)	3:9	b)	4 to 12	c)	5kg:15kg
d)	10m:40m	e)	25cm :1m	f)	30cm:0,9m
e)	50c : $2.25	f)	50min:1 ¼ hrs	g)	¼ : ½
h)	4 ½ : 1 ½	i)	$25mm^2:2,5cm^2$	j)	0,96:1,2g
k)	0,24m:9,6m	l)	1,6m : 880cm		

2) Find the result of increasing or decreasing the following quantities

a) Increasing 10cm in the ratio 5:4

b) Decreasing 3,5m in the ratio 5:7

c) Increasing 0,05m in the ratio 6:5

d) Decreasing 24cm in the ratio 3:4

e) Increasing $\frac{3}{4}$ in the ratio 5:3

f) Decreasing 2 ½ days in the ratio 2:5

3) Express the following scales in the form 1:n

a) 1mm represents 10cm

b) 1cm represents 100m

c) 10m represents 2km

d) 7cm represents 630m

4) Find the value of x if

i) x :15 = 3:5

ii) x :2,5=3:5

iii) 7: x =0,56:0,72

5) A man shares $2,40 among his children in the ratio 1:4:5. How does each child get

6) On a sales promotion the trader decreased the price of his shoes by a ratio of 4:5. If the original price was $75,50. What was the new price.

7) Mr. Sibanda business recorded a net profit of $ 10 750 in 2007. The net profit recorded in 2008 was $ 9 250. Express the ratio of the previous profit to the 2008 profit.

8) A map is drawn on a scale of 1cm to 10km

a) Find the scale of the map in the form 1:n

b) If the actual distance between two towns is 50km. Find in centimeters, their distances on the map.

a) A photo graph in the ratio 1:3. What is the new size of a building 0,27cm high.

PROPORTION

It is important to recognize that proportion can be direct proportion or inverse proportion

In direct proportion an increase in one quantity results in a increase in another vice versa.

In inverse proportion an increase in one quantity results in a decrease in another vice verse.

Example 1

1 a) 6 eggs cost 54c. What is the cost of 15eggs

b) It takes 2 hours for 8 men to complete a certain task. How long will it take 3 men to complete the same task.

a) 6 eggs : 54c (since its direct proportion
15 egg : more 15 eggs will cost more)

If it costs more multiply 54c by an improper fraction to get more. (than 54c)

$$x = \frac{15}{6} \; x \; 54$$

= 153c
= $ 1,35

b) Comparing

2 hours : 5 men (since it inverse proportion
Less : 8 men 8 men will take less time to do the job

If they take less time multiply 2 hours by a proper fraction to get less hours (than 2 hours)

$$x = 2 \times \frac{5}{8}$$

$$= \frac{10}{8}$$

$$= \frac{5}{4}$$

= 1 ¼

Take Note

a) Multiplication by an improper fraction increases the quantity, hence multiply by it when its "more"

b) Multiplication by a proper fraction decreases the quantity, hence multiply by it when its "less"

Exercise 1,2

1) A vendor sells three oranges at 12c

a) Find the cost of i) 7 oranges ii) 17 oranges

b) How many oranges could be bought for
i) 36c ii) $1,12

2) A car uses 9,5 liters of petrol in travelling 100 kilometres
How much petrol will it use in travelling

a) 300km b) 270km

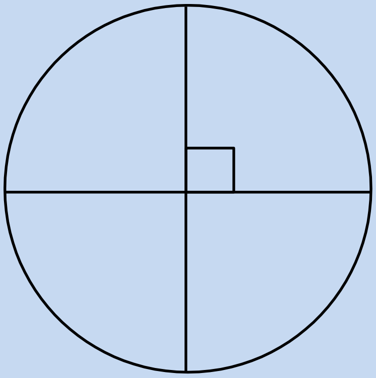
3) A box can carry 450 books of thickness of 1,5cm each. Find;

i) the number of books it can carry of a thickness of double the size.

ii) the thickness of each book in the box when 75 books are carried.

4. A car travelling at a speed of 100km/h takes 1hour 10 minutes to travel a certain distance. Find.
i) the time it will take to travel the same distance when travelling at a speed of 80km/h.
ii) the speed at which it will be travelling when it takes 50 minutes.

5) A circle is divided into equal parts by drawing its lines of symmetry and measuring the corresponding angles formed at the centre. If 2 lines of symmetry form angle 90º each at the centre.



Find
i) the number of lines which form angles of 30º each
ii) the sizes of angles by 8 lines of symmetry

6) A spring stretches when a load is applied. It is thought that the extension of the spring is proportional to the load applied.
a) How much will the spring stretch for loads i) 60N ii) 15N
b) What would the load, if the string stretched i) 64mm ii) 72mm

Rate

This compares quantities of different kind for example the exchange rate of two different quantities, wage rate per day, speed etc.
As a result of comparing two quantities of different units, rate is expressed in two units eg km/h , $/h etc

Example 3

a) A car travels a distance of 100km in 1 ¼ hours. What is its average speed in
a) km/h b) km/min

100km : 1 ¼
Less : 1 hour

$100 \times \frac{1}{\frac{5}{4}}$

$100 \times \frac{4}{5}$

= 80km/h 80km/h means 80km is travelled in one hour

b) 100km : 1 ¼ hours
100km: 75 minutes
Less : 1 minute

$$100 \times \frac{1}{75}$$

1,33km/min 1,33km/min a distance of 1,33km was covered in every minute.

Example 4
A man is paid $ 150 for a week. Calculate his hourly rate of pay.

40hrs : $ 150
1hr : less
3,75

$$\cancel{150} \times \frac{1}{\cancel{40}}$$

$ 3,75/h

Exercise 1,3

1) A train takes 66 minutes to travel 91km. What is its average speed in
a) km/min b) km/h c) m/s

2. A bank exchanged US$ 70 for P500. What was the exchange rate
a) In Pula per US$ b) In US$ per pula

3). Refer If 150 litres of petrol cost $ 300, how much the cost
4) A city has an area of 36km^2 and a population of 60 000 people. Calculate the population density of the city per km^2 correct to 2.s.f.

5) Workers at a factory are paid $ 98 for a 40hour working week. Find their weekly rate of pay

6) A steel beam 5,2m long has a mass of 137,8kg. Find its mass in kg/m

7) The exchange rate of 2 June 2009 was $ US to ZAR8,5. How many US can be bought for ZAR 765

8) A man is paid at a rate of $4,74 per hour. If he works for 4 hours every week. Calculate his monthly per pay.

9) The density of a liquid is 0,12g/mc3. What is the mass of the liquid when the volume is 43,1cm^3.

10) A shop reduces the price of its stock of clothes to clear at a rate of 10c for every $. Calculate the new price for a suite which was marked $ 42,50.

Examination questions

1. Sand, cement and water were mixed in the following proportions by mass to make mortar for building bricks

Sand	112kg
Cement	64kg
Water	24kg

i) Express in its simplest form, the ratio sand: cement to water

ii)a) Calculate the percentage of cement in the mortar

b) Calculate the mass of cement in a freshly moulded brick of mass 12,5g

2. At a certain school the ratio of girls to boys is 6:5. If there are 105boys. Calculate

i) the number of girls in the school

ii) the total student population in the school

3.a) A map of a town is draw to a scale of 1cm: 5km

i) A road on the map is 8cm long. Calculate the actual length of the road, giving the answer in kilometres.

ii) The actual area of the town is 150km$^{2.}$ Calculate in square centimeters, the area of the town on the map

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4) A bank exchanges South African rands for Zimbabwean dollars at the rate of R1 to $6,25

Calculate

i) the amount received in rands for $ 2 250,00

ii) the amount received in dollars for R 150

5) Tapiwa and Netsai share some money in the ratio 2:5. Given that Tapiwa's share is $ 620 000, Calculate Netsai's share

(ZIMSEC NOV 2008)

CHAPTER 17

Mensuration of plane shapes

Plane shapes are two dimensional shapes like triangle, square, rectangle etc. Mensuration involves looking at their perimeter

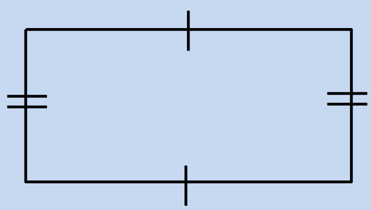
Syllabus objectives
Learner should be able to
a) Define the term perimeter
b) Find the perimeter of plane shapes
c) Define the term area
d) Calculate the are of the plane shapes

Perimeter

Perimeter is the distance around the shape. It is the outside boundary that defines the stage.

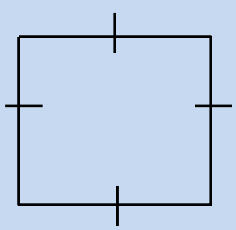
The following are the perimeters of some plane shapes

Rectangle



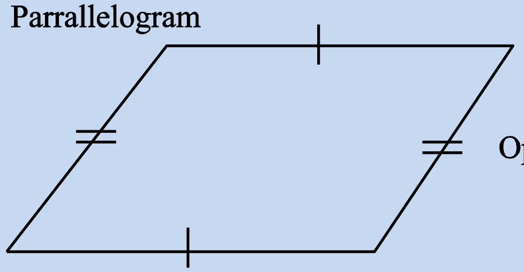
Opposite side a rectangle are equal

Square



All the sides of a square are equal

Parrallelogram



Opposite side a parrallelogram are equal

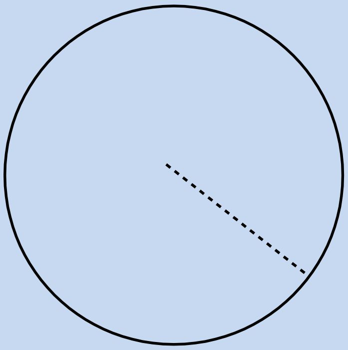
= b+b+l+l

$= 2b+2l$
$= 2(l+b)$

Perimeter $= L+L+L+L$
$= 4L$

Parallelogram

Perimeter $= a + a+b+b$
$= 2a+ 2b$
$= 2 (a+ b)$



Circle

Circumference $= 2\pi r$ or πd

Exercise 1,1 (use $\pi = \frac{22}{7}$)

1) Calculate the perimeter of a rectangle with the following l =length b=breadth
a) l= 4cm b=3cm b) l=5,2 cm b=3,4
c) l=0,5m b= 0,3m d) l=7cm b=4cm

2) Find the length of a rectangle with an area of $63cm^3$ and breadth of 7cm

3) A length of wire is bent to make a square of side 4,7cm. What is the length of wire in the square?. If the wire is to be painted at cost of 23c per centimeter. How would be the total cost.

4) A parallelogram has sides of length 13cm and 7cm. What is the perimeter of the parallelogram

5) Find the circumference of a circle of radius
i) 7cm ii) 3,5cm iii) 3,5cm

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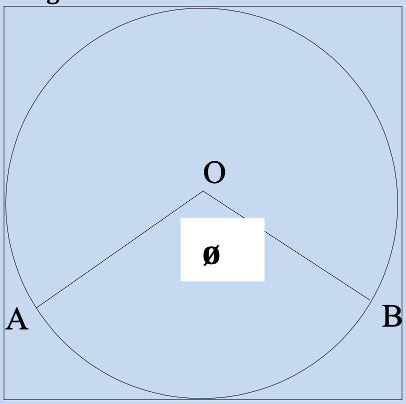
Length of arc



The diagram show a minor arc AB being subtended by the angle øat the centre O.

Since the angles at the centre O add up to 360^{O}, the angle ø covered an arc which $\frac{ø}{360}$ of the whole circumference of a circle.

Thus length of arc AB = $\frac{ø}{360} \times 2\pi r$

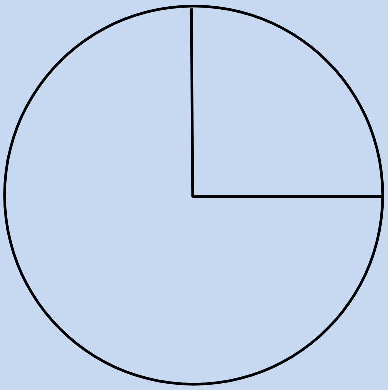
Length of arc= $\frac{ø x 2\pi r}{360}$

$$\boxed{\text{Length of arc} = \frac{ø x}{360} 2\pi r}$$

Example 1

a) Calculate the length of the arc of circle radius 7 subtended by an angle of 90^{0}

b) Find the perimeter of the sector



a) Length of arc = $\frac{\text{Ø}}{360} \times 2\pi$

$$= \frac{\cancel{90}^{\,1}}{\cancel{360}} \times \cancel{2} \times \frac{22}{\cancel{7}} \times 7$$

$$= 11\text{cm}$$

b) the shaded part represents the sector

Perimeter = 11cm + 7cm + 7cm
= 25cm

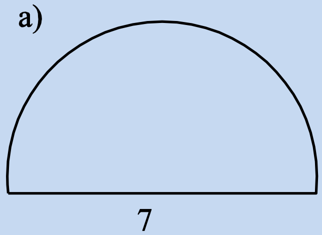
Exercise 1,2
Calculate the missing item to complete the table

1) Calculate the missing item to correct the table

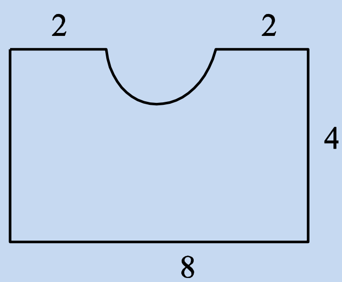
	Radius	Angle	Length of arc
a)	5cm	100^o	
b)	7cm	75^o	
c)		110^o	12cm
d)		108^o	132^o
e)	19cm		21cm

2. Find the perimeter of the shapes given below. All measurements are in cm. Use $\pi = \frac{22}{7}$

a)



b)



3) The arc of a circle of radius 30cm subtends an angle of 110^O at the centre. Use the value of 3,142 for π to calculate the length of the arc.

4) What angle does on arc of 12cm subtend all the centre of a circle of radius 12cm

Area
Area is defined as the space occupied
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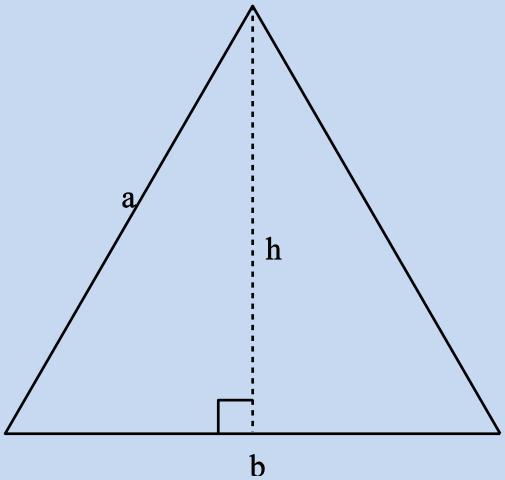
The area of rectangle above can be found by counting the total number of square that make up the rectangle thus Area = 18 units

Alternatively area can be easily found by multiplying the number of squares making up the length and the number of squares making up the width.

Thus Area = length x breadth
= 3 x 6
= 18 units^2

Area of a rectangle = lb

The triangle



Basically when the perpendicular height is given.

Area of triangle = ½ base x height
= ½ bh

Diagram in Fig show a triangle with side a and base b. The perpendicular height can be dropped as shown by trigonometry.

Diagram in Fig. 1 show a triangle with side a and base b. The perpendicular height can be dropped as shown by trigonometry. $\text{Sin}\theta = \frac{h}{a}$

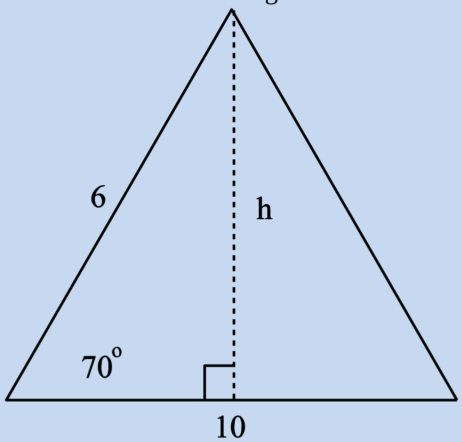
$h = a \text{ Sin } \theta$

Substituting into formula

Area of a triangle = ½ b (a sin θ)

Example 2

Find the area of the triangle below



Area = ½ b x h

= ½ (10)h

But h= 6 Sin 70^0

$= \frac{1}{2}\cancel{6} \times 10 \text{ Sin } 70^o$

$= 30 \text{ Sin } 70^o$

= 30 (0,9397)

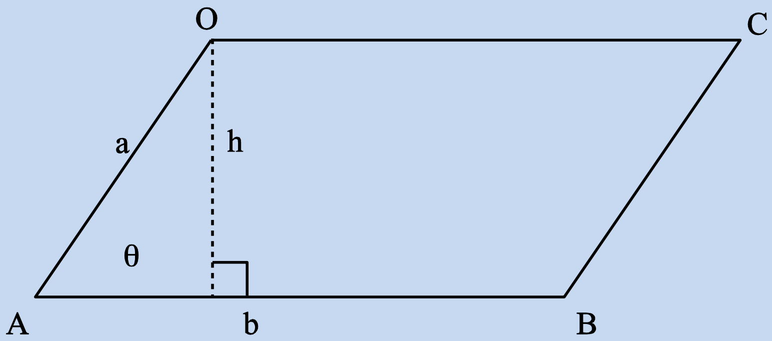
=28,20 to 2.d.p

Area of a parallelogram.

A parallelogram is a special type of a rectangle. Since the area of a rectangle = length x breadth, then in general the area of parallelogram = base x height

$= b \times h$

When the height is not given on a triangle trigonometry is used to calculate the height.

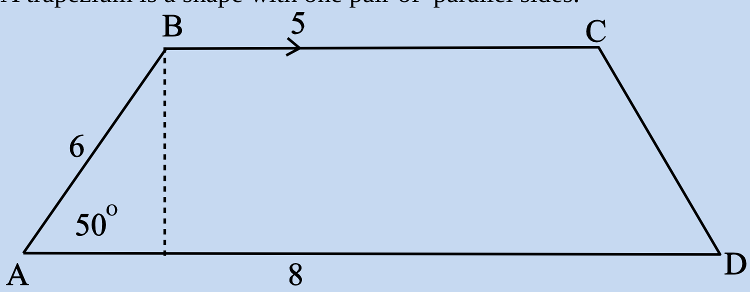


Area of parallelogram = base $\times$ height
= b $\times$ h

But h = a sin θ

In general area of parallelogram without the given height = ab Sin θ. Where b is the base and a is the side forming the angle with the base.

Area of a trapezium
A trapezium is a shape with one pair of parallel sides.



Area of a trapezium= ½ $\times$ (sum of parallel sides x height)
= ½ (8+5) h
= ½ 13 x h

But h= 6 sin 50^0
= ½ $\times$ 13$\times$6 sin 50^0
=29,87
=29,9cm^2 to 3.S.F

Exercise 1,3
1. Calculate the area of each of the shapes. All dimensions are in cm. Use 3,14 for π

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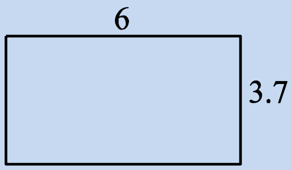
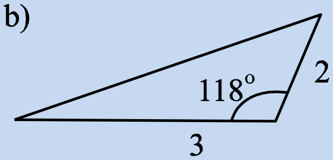
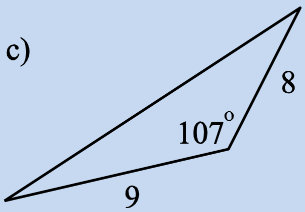
a)



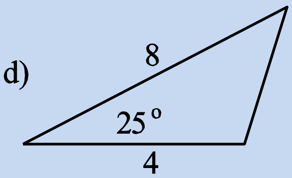
b)



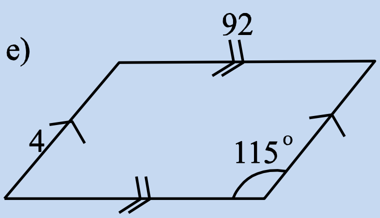
c)



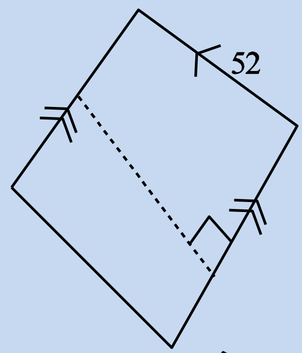
d)



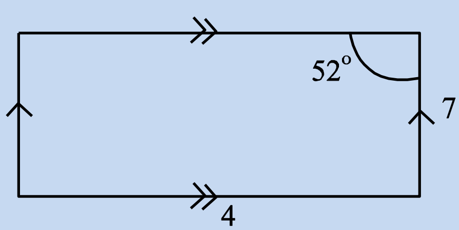
e)



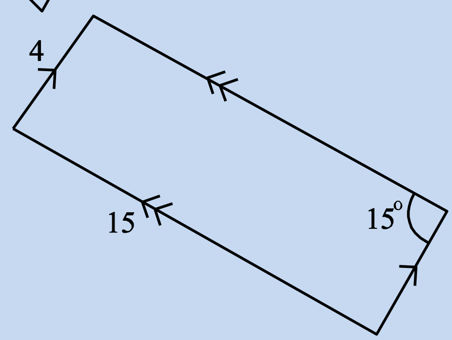
f)



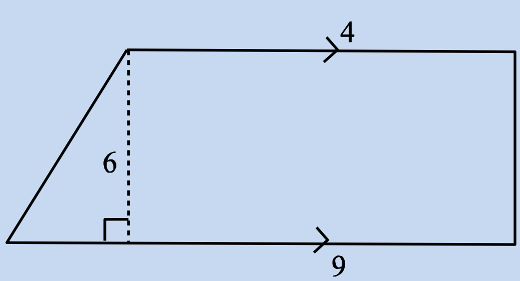
a)



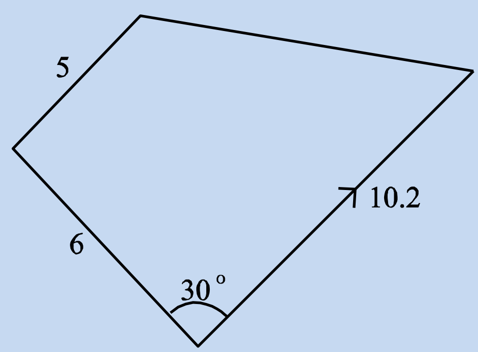
b)



i)



ii)



2) The area of a trapezium is $52cm^2$. Its parallel sides are of length 11cm and 5cm. Calculate the height

3) The width of rectangle is half its length. If the area of the rectangle is $72mc^2$ Calculate its
i) Width ii) its length

4) The area of a trapezium is $71{,}5cm^2$. If the parallel sides are 9, 25 and 3,75cm long. Find the perpendicular distance between them.

5) If the area of a triangle ABC is 12cm. If BA=6cm and ABC=34^o. Find the base BC

Area of Circle

The area of a circle of radius , r, is πr^2.

Area of a sector

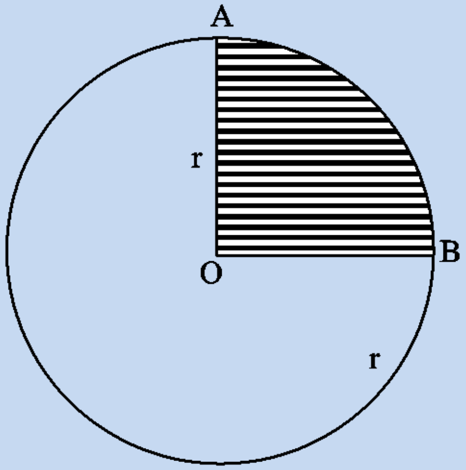


Diagram show a circle with sector AOB formed by angle, $\ominus$, at the centre

Like the perimeter of an arc, the area of the sector is proportional to the angle at the centre,

Since the area of the whole circle is πr^2.

Area of sector AOB = $\frac{\theta}{360} \times \pi r^2$

Example 4

A sector 70^o is shaded on a circle of radius 3cm. Calculate its area.

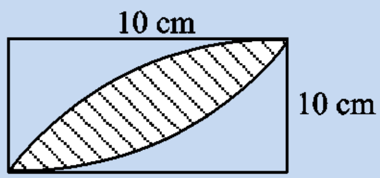
Area of a sector = $\frac{\theta}{360} \times \pi r^2$

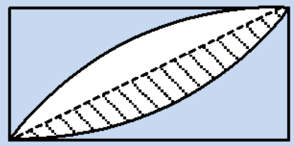
$= \frac{\cancel{70}}{\cancel{360}} \times \overset{0,78}{\cancel{3,12}} \times 3 \times 3$

$= 5{,}46cm^2$

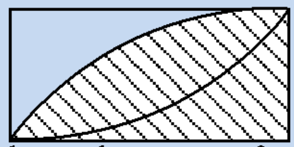
Example 5

Calculate the shaded part. Given that all arc are circular and dimensions are in cm.

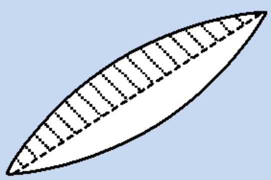




shaded area shows area of a triangle



shaded area shows area of a sector



shaded area represents area of a segment

Area of square = 10×10
= 100 mc^2

Area of one triangle = $\frac{1}{2} \times 100\text{cm}^2$
= 50cm^2

Area of sector = $\frac{90}{360} \times 3{,}12 \times 10 \times 10$
= 78cm^2

Area of one segment = Area of Sector – Area of triangle

= $78\text{m}^2 - 50\text{cm}^2$
= 28cm^2

Shaded area represents area of a segment

The shaded part in the original share represents the sum of the two segments.
Total area of the shaded part = 28+28
=\ $56cm^2$

Exercise 1,4 use $\pi \frac{22}{7}$

1a) Complete the table below for areas of sectors of circles show working in each case.

	Radius	Angle of sector	Area of Sector
a)	7,5cm	81^o	
b)	4,7cm	143^0	
c)	13,7	31^o	
d)		35^o	$8{,}3cm^2$
e	47cm		$7{,}71cm^2$

2. In the diagram, the area of the sector and the area the square is equal. Find the angle of the sector

3) Calculate shaded in parts below. All dimensions are in cm all are circular

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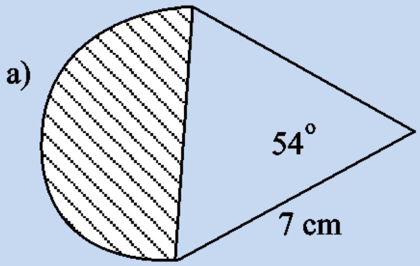
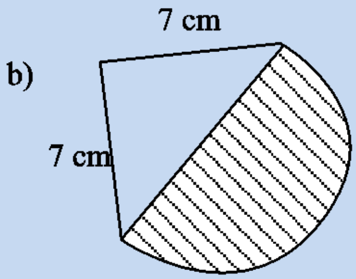




4. The area of sector formed by an angle of 60^o is $3{,}62cm^2$. Calculate the area of the circle.

5) The minute hand of radius 1,5cm long moves 10 minutes
i) Calculate the angle it forms in 10 minutes
ii) Hence, calculate the area of the sector if forms

6)

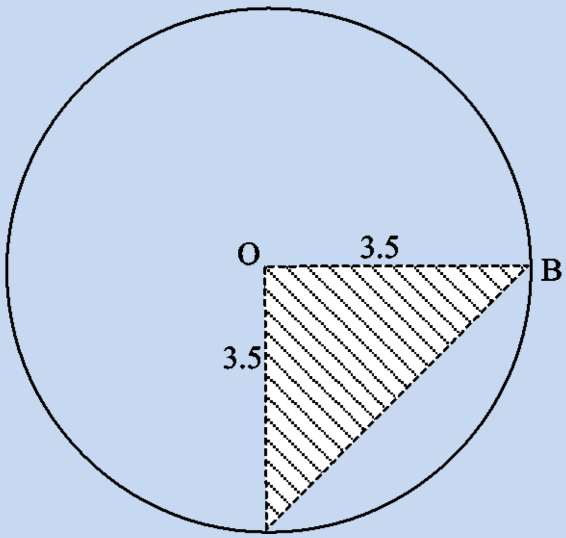


Fig 1

In Fig 1 calculate the area of

h) the sector bounded by arc AB, radius OB and OA

ii) triangle AOB

iii) the un shaded segment

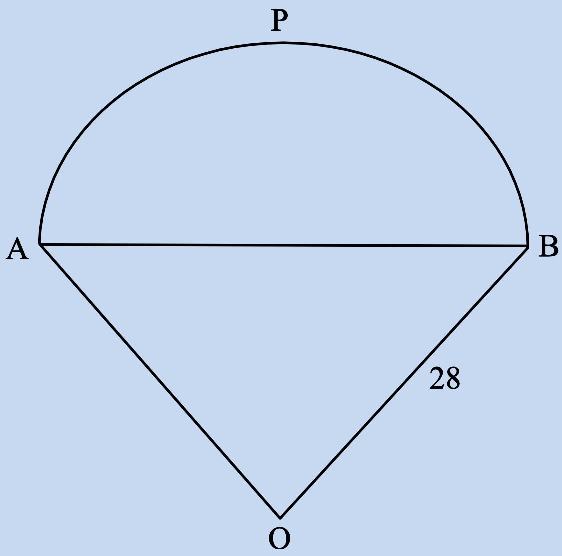
EXAMINATION QUESTIONS

1) The circumference of a circle , centre O, is 99cm

a) Taking π to be 22/7, calculate the diameter of the circle

b) Given that A and B are points on the circumference of the circle such that angle AOB=80^0, calculate the length of the minor arc AB.

2) The trapezium ABCD has AB=20cm, BC=13cm and DC=15cm. The angle DAB =90^o and AB is parallel to DC. Calculate a) the length AD b) The area of the trapezium ABCD.



Take π to be $\frac{22}{}$

7

In the diagram, OAPB is a sector of a circle centre O and radius 28mm, AB=43mm

Calculate

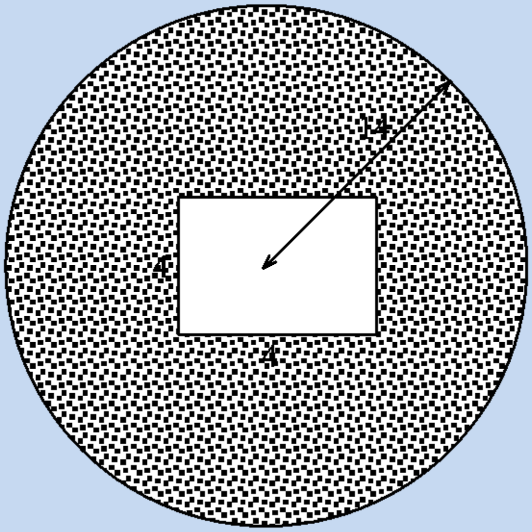
a) the angle AOB

b) The area of

i) Sector OAPB

ii) triangle AOB

iii) the segment APB



The diagram above shows the cross-section of a circular metal disc of radius 14mm. The disc has a control hole which is a square of side 4mm

i) Calculate a) the circumference of the disc

b) the shaded area in mm^2

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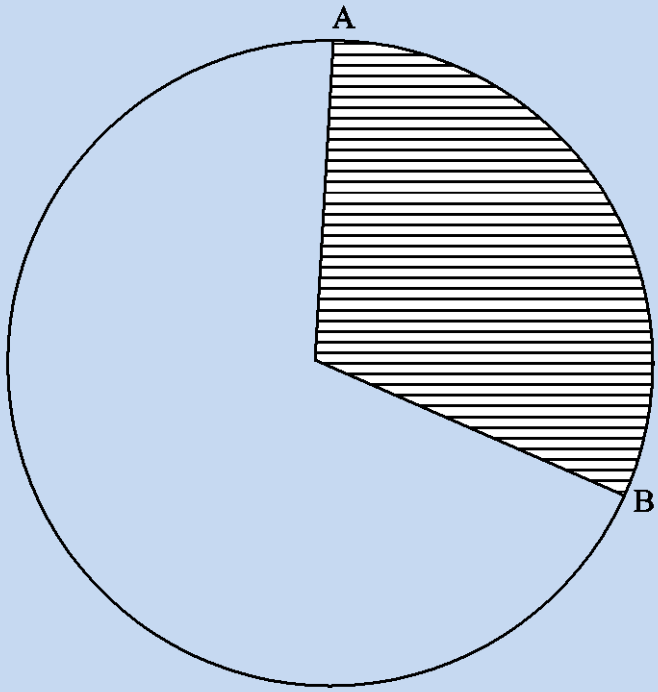

a) In the diagram, the shaded sector AOB is 7/15 of the circle centre O
Calculate AOB

b) Calculate the radius of a circle whose area is $154cm^2$

(Take π to $\frac{22)}{7}$

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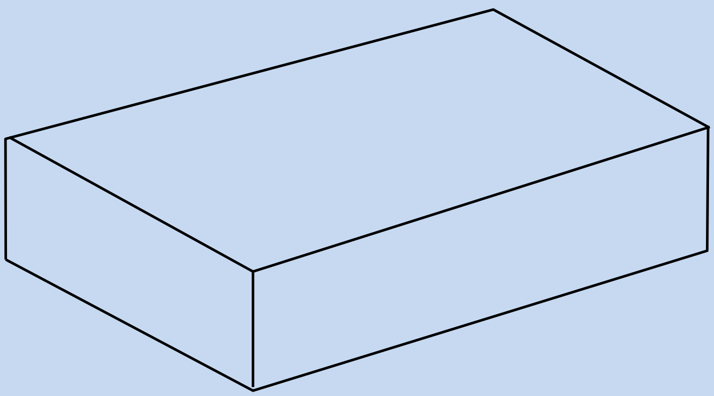
CHAPTER 18

MENSURATION OF SOLID SHAPE

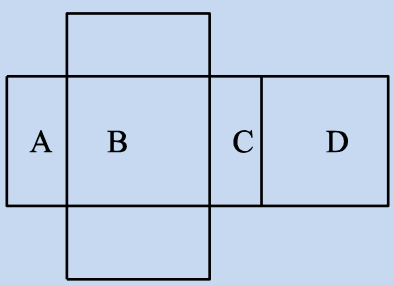
Solid shapes are three dimensional shapes like cylinder, cuboids, cone sphere etc

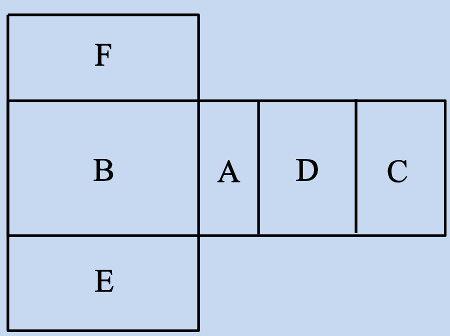
SYLLABUS OBJECTIVES
Learner should be able to
a) Calculate the surface area of the shape
b) Calculate the volume of solid shapes using their formula
c) Apply volume to find solutions to given problems

SURFACE AREA

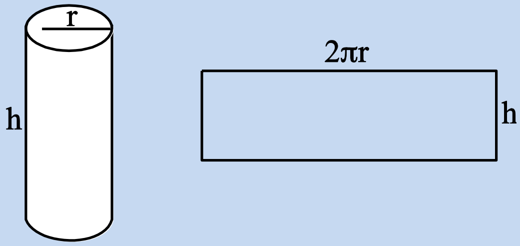


The diagram shows a box of matches which has a shape of a cuboid. The shape is made up of a cuboid. The shape is made up of rectangular plane shapes.
When the box is cut opening the following plane shape would be formed.





On folding back either (a) or (b) we get back to the original box
The total surface area of the matchbox can be found by adding all the areas of rectangular faces A to f
The Total surface area = Area of A + Area of B + Area C + Area of D + Area of E + Area of F.
For a cylinder, the curved surface can be opened out to give a rectangle of height h and length 2 πr
(2is the circumference of the circular top)



Thus curved surface area = $2\pi r \times h$

$= 2\pi rh$

The total surface area of solid shape is basically the sum of the areas of plane shapes (or faces) making up that solid shape.

Surface Area and volume of solids

Below are the formulae of surface area and volume of common solid shapes.

Cuboid

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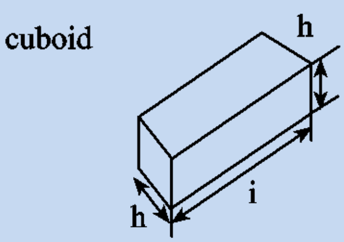
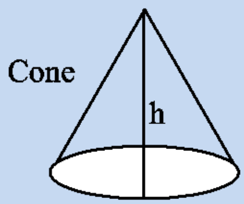


Volume= (1bh)

Surface area= 2 (1b+ 1h +bh)

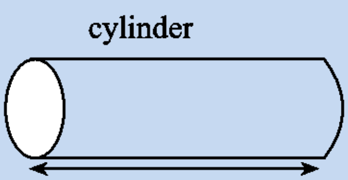


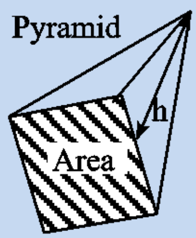
Volume =1/3 $\pi r^2 h$

Curved surface area= πrl

Total surface area=

$\pi nrl + \pi r^2$





Volume = $^1/_3$ x base area x height

= $^1/_3$ Ah

Volume = $\pi r^2 h$

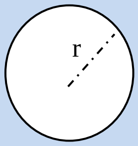
Curved Surface area = $2\pi rh + 2\pi r^2$

$= 2\pi r(h+r)$

Cylinder

Pyramid

sphere



Volume = $^4/_3\ \pi r^3$
Surface area= $4\pi r^2$

Exercise 1,1

1) A cylinder has a height of 9m and a diameter of 6m. Find.
i) its volume
ii) its curved surface area

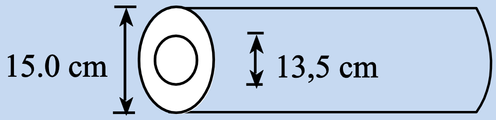
2) Calculate the volumes of cones
a) of base radius 1,7 and height 5,5cm
b) of base radius 3,4 height 11,0cm
c) What is the ratio of the volumes of the two cones

3) A cone has a volume of 39,7cm. Given that its base radius is 3,2cm. Find its height
ii) Calculate its curved surface area.

4) The base of a pyramid is a square of side 8,3cm
The other edges are of length 9,7cm
Calculate
a) The area of a triangular face'
b) The height of the pyramid
c) The volume of the pyramid

Example 1

A metal drainage has external diameter of 15,0cm and internal diameter of 13,5cm. Calculate the volume of metal in a piece of pipe 7,2cm long. Use $\pi = 3{,}142$



Volume $= \pi r^2 h$

External volume= $3,142 \times (15/2)^2 \times 7,2$

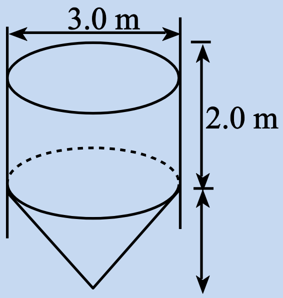
$= 3,142 \times 225 \times 1,8$
$= 1272,51\text{cm}^3$

Internal volume = $3,142 \times (13,5/2)^2 \times 7,2$
$= 3,142 \times (13,5 :- 2)^2 \times 7,2$
$1030,73\text{cm}^3$

Volume of metal = $1\ 272,51\ \text{cm}^3 - 1\ 030,73\text{cm}^3$
$= 241,78\ \text{cm}^3$

Example 2
A metal storage bin, with a lid, is shaped as shown in the diagram below and is made of sheet metal.

i) Calculate the area of sheet metal needed
ii) Calculate the capacity of the bin.



Area of sheet metal needed = curved surface area of a Cylinder + curved surface area of cone

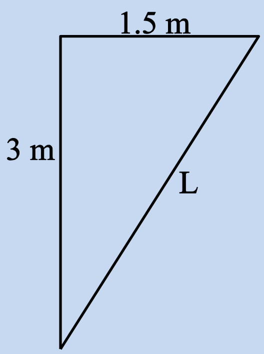
$=2\pi r\ (h+r) + \pi rl$

$=\pi r\ (2\ (h+r) + l)$

$=3{,}142\ (1{,}5)\ (2)\ (2 + 1{,}5) + L)$

$=4{,}\ 713\ (7+L)$

From cone



$L^2 = 1{,}5^2 + 3^2$

$= 2{,}25 + 9$

$= 11{,}25$

$= 11{,}25$

$= 3{,}35$

Substituting L

Area $= 4{,}713\ (7+3{,}35)$

$= 4{,}713\ (10{,}25)$

$= 48{,}31^2$

ii) Capacity = volume

= volume of cylinder + volume of cone

$=\pi r^2 h_{cy} + \underline{1}\ \pi r^2 h_{co}$ where h_{cy} = height of cylinder

3 $\qquad\qquad$ h_{co}= height of cone

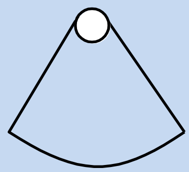
$=\pi r^2 (h_{cy} + \frac{1}{3} h_{co})$

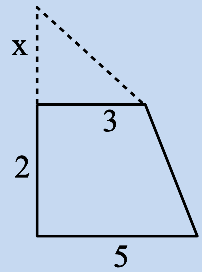
$=3{,}142\,(1{,}5)^2\,(2 + 3)$

$=35{,}35m^3$

Example 3

Diagram show frustum of a cone, top and bottom diameters being 6cm and 10cm respectively and depth is 2cm. Find the volume of the frustum.





$$\frac{x+2}{x}=\frac{5}{3}$$

3x + 6 = 5x

6 = 2x

x =3

Volume of frustrum $\qquad$ = Volume larger cone – volume of smaller cone

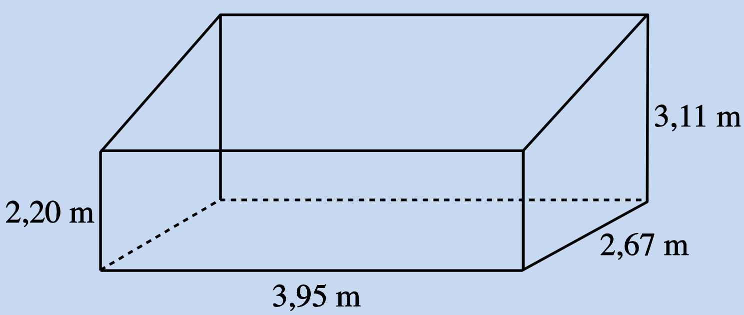
$= 1/3\ \pi\ (5)^2 \times 5 - 1/3\ \pi\ (2)^2 \times 3$

=1/3 Iπ (125-12)

=1/3 π (113)

=37,33π cm3

EXERCISE 1,2



2,

The diagram show a shade of height 3,11m at the back and 2,20m at the front. Its length is 3,9m and its depth from the front to back is 2,67m. Calculate

a) the area of a side wall

b) the volume of the shed

Hint: shade formed by a cuboid and a triangular prism

2) A swimming pool is 25,0m long and 15,0 wide. The bottom slopes steadily and at the shallow end the pool is 1,00m deep and at the deep end its depth is 2,75m. How many litres of water does the pool hold. ($1m^3$ = 1 Litre)

3) A wooden box, without a lid has a length of 48cm, width of 40cm and height of 20cm. If the cardboard making the box is 1cm thick. Calculate

a) the volume of the box

b) the volume of cardboard used to make the box

4) A water level in a swimming pool 25m long and 9,15m wide is 12,5cm below maximum. If water is being supplied at a rate of $116m^3$ per minute, how long will it take to fill.

5) A petrol storage tank is a cylinder of a diameter 7,3m and 4,1m. Calculate the mass of petrol that it will hold. Density of petrol that it will hold

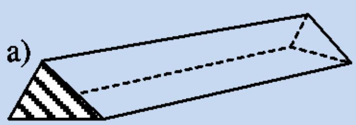
Density of petrol = 895kg/m3

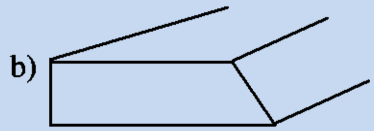
Hint : Density = $\frac{\text{mass}}{\text{Volume}}$

Volume of a prism

A prism is a solid which has a uniform cross-section

A cross-section refers to the end view from any direction.





In the diagram (a) and (b) the shaded parts represent their cross sections

The volume of prism is found by multiplying the cross sectional area by the height (i.e the distance to the other opposite similar face)

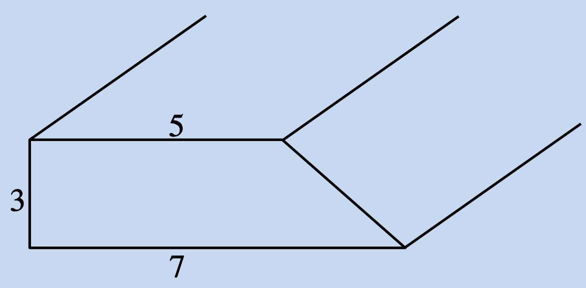
Thus, volume of a prism = cross sectional area x height

When the prism is horizontal like on the above example

Volume of a prism = cross –sectional area x length

Example 4

Find the volume of the diagram below. Given that its length is 9m



Volume = Cross sectional Area x height

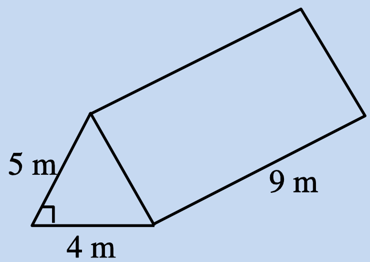
$= \frac{1}{2}(5+7)\, 3 \times 9$ Cross section is a trapezium)

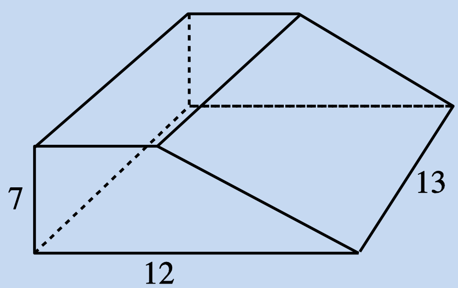
$= 18 \times 9$

$= 162\text{cm}^2$

Examination Questions

1) Find the volume of the solids





2)a) Cylinder A has base radius 3,5cm and height 10cm. Calculate the volume of A. Take π to be $\frac{22}{7}$

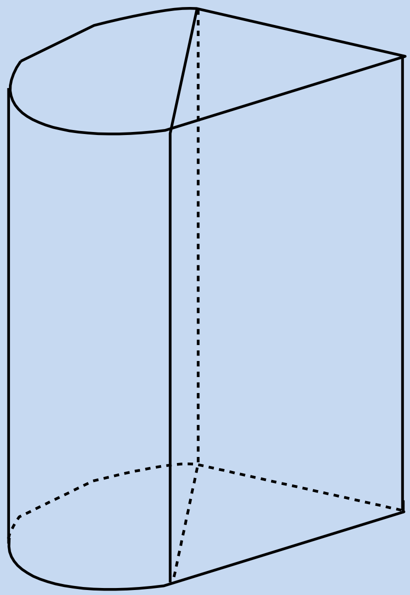
b) A solid metal cylinder B has volume 216cm^3.

i) Calculate the volume of a cylinder which has the same height as B but base radius twice that of B

ii) Given that the cylinder B is melted down and made into a cube, calculate the length of an edge of the cube

C.S.C

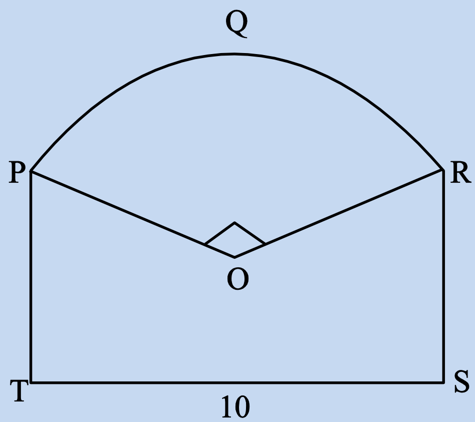
3. In this question take to be 3,142



The diagram represents a wooden block in the shape of a prism. The faces ABCD and EFGH are horizontal and all the others are vertical. AC and EG are the diameters of the semi-circles and EFG respectively.
Given that EH=GH=10cm, and HD=20cm, EG=16cm Calculate

i) The area of the semi-circle EFG
ii) The area of the triangle
iii) The volume of the block
iv) The mass of the block in kilograms, given that the density of the wood is0,8g
v) The area of the curved surface

Cambridge, November 1990



The diagram above represents the cross section of a barn. The width is 10m, the vertical sides PT and RS are each 15m and the roof PQR is an arc of a circle, centre O, with POR=90°

a) Calculate
i) the radius of the sector PQRO
ii) the area of the cross-section

b) Given that the barn is 30m long, calculate
i) the volume of the barn
ii) the cost of painting the outside of the roof correct to the nearest $10 at $250 per square metre

CHAPTER 19

Angle Properties

Syllabus Objectives
Leaner should be able to

a) state all the required properties of angles
b) State the properties of polygons

Angles

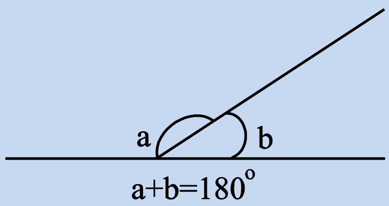
A angle is a turn measured from a reference point
A full turn makes an angle of 360^o. A straight line or half turn makes an angle of 180^0

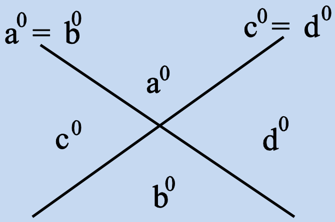
Types of angles

- Acute angles – These are angles less than 90^o
- Obtuse angles – angles greater than 90^0 but less than 180^0
- Reflex angle – angles greater than 180^o and less than 360^0

Properties of angles

Angles on a straight line add up to
180^0



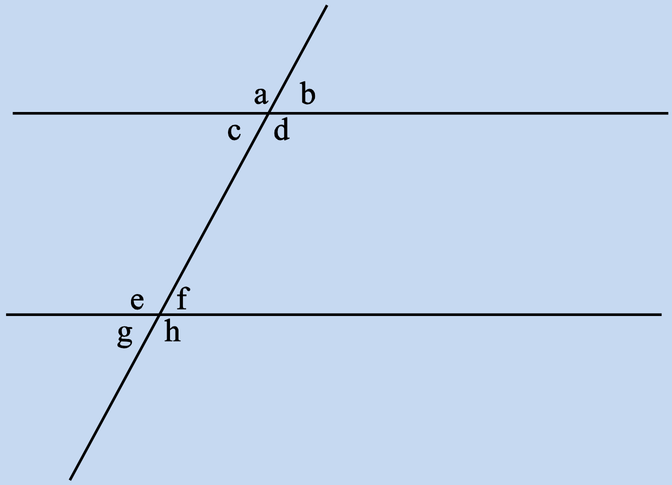


When two straight lines cut each other vertically opposite angles are formed. Vertically opposite angles are equal

Also angle a^0,b^0c^0 and d^0 angles at a point, they add up to 360^0
Angles at a point add up to 360^0

Angles in parallel lines

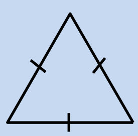
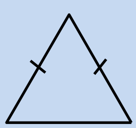
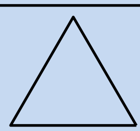
When two parallel line are cut by a third line called the transversal, 8 angles are formed



a) Allied/co-interior
(add up to 180^0 c+d =180^o, f+d=18^0
b) Alternate angles (Z angles)
e=d, c=f
g   h
c) Corresponding angles
a=e, b=f, h=d, c=g

Angle properties of triangles

Triangles are 3 sided shapes whose angles add up to 180^o

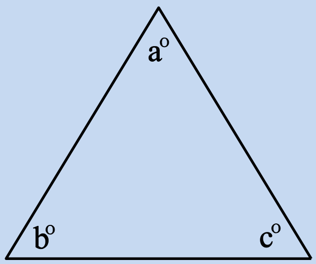
Type of triangles	Properties
 Equilateral triangles	a) All three sides are equal b) **All three angles are equal**
 Isoceles triangles	a) Two sides are equal b) Base angles are equal
 scalene triangles	a) All three sides are different

Additional types of triangles
A right angled triangle has one right
An acute-angled triangle has each angle less than 90^0
An obtuse-angled triangle has one angle greater than 90^0

Further proportions

$a^0 + b^0 + c^0 = 180^0$

1. The sum of the interior angles are equal



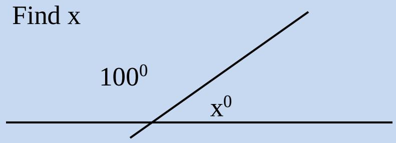
If one side is produced (extended, the exterior angle formed is equal to the sum of the opposite angles. That is, $e^0 = a^0 + b^0$

Exercise 1,1

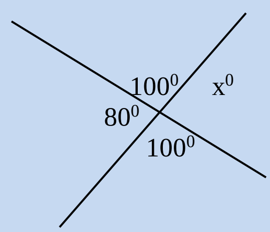
1) Name three types of angles with examples

2) Find x

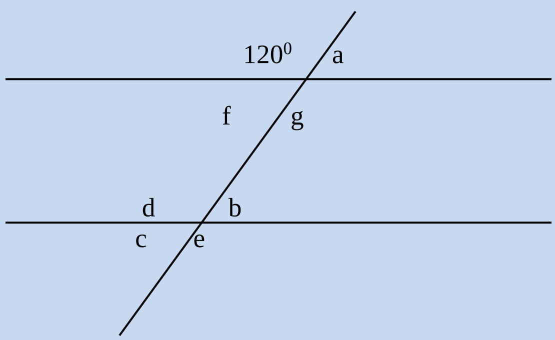
a)



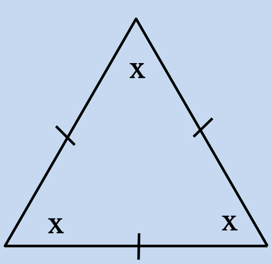
b)

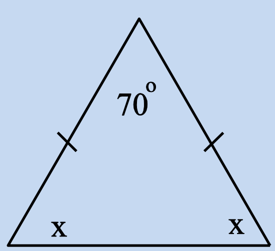


3) Find the values of the unknown letters with reasons

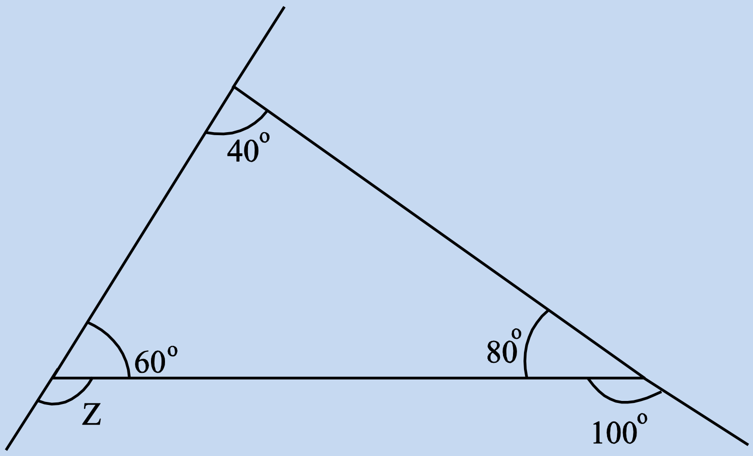


4. Calculate the value of x





6) Find the value of the letters

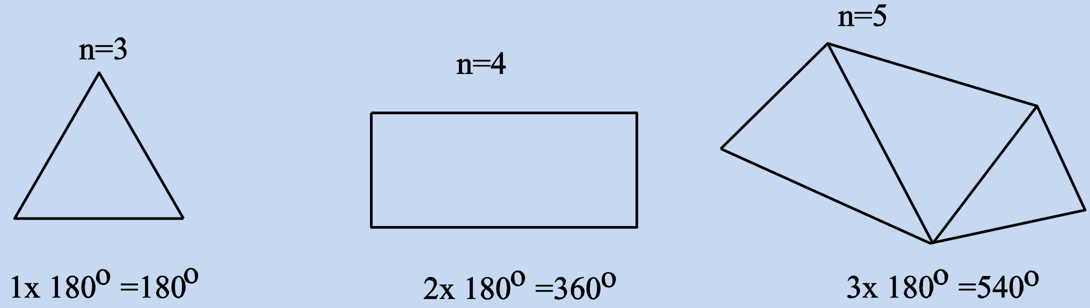


Angle properties of polygons

A polygon is any closed plane shape with straight sides e.g. a triangle, quadrilateral etc

Different types of polygons

Type of Polugon	Number
Triangles	3 sides
Quadrilateral	4 sides
Pentagon	5 sides
Hexagon	6 sides
Heptagon	7 sides
Octagon	8 sides
Nonagon	9 sides
Decagon	10 sides



Interior angles of polygons

Consider the polygons below with increasing number

Each polygon has been divided into possible number of triangles to aid in finding them some of interior angles.

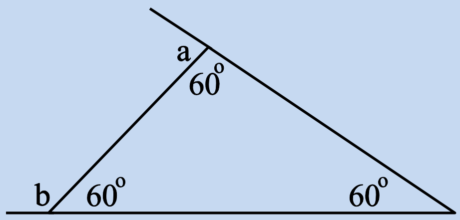
It emerges from the pattern that to find the sum of interior angles of an n-sided polygon =(n-2)180
or =(2n-4)90

In a rectangular polygon the sides are equal in length, the angles are also equal.

Size of each angle = $\frac{(n-2)\ 180^o}{n}$

Exterior angles polygons

Consider on equilateral triangle with interior angles of size 60^o. Each is produced to from exterior angles as shown



$a^0 = 60^0 + 60^0$ b= 60^o+60^o
$c^0= 60^o + 60^0$
$a^0+ b+c= 60^o + 60^o + 60^o + 60^o+60^o$
$=360^0$
Since a+b+c = 360, it is shown that the sum of exterior angles is $360^{0.}$

The sum of the exterior angles of any polygon is 360^o.

Example 1
Calculate the sum of interior angles of a regular octagon (8 sides)

Sum of interior angles = $(n=2)\ 180^o$
But n=8
$= (8-2)\ 18^0$
$= 6 \times 180^o$
$= 1\ 080^o$

Example 2
How many sides has a rectangular polygon if each interior is 108^0

Size of rectangle = $\frac{(n-2)\ 180^0}{n}$

$\frac{108}{1} = \frac{(n-2)\ 108}{n}$

108n =180 (n-2)
108n = 180n-360
360=180n-108n
$\frac{360}{72} = \frac{72n}{72}$

n =5 sides
the polygon has 5 sides

19.1.1 Exercise 1,2

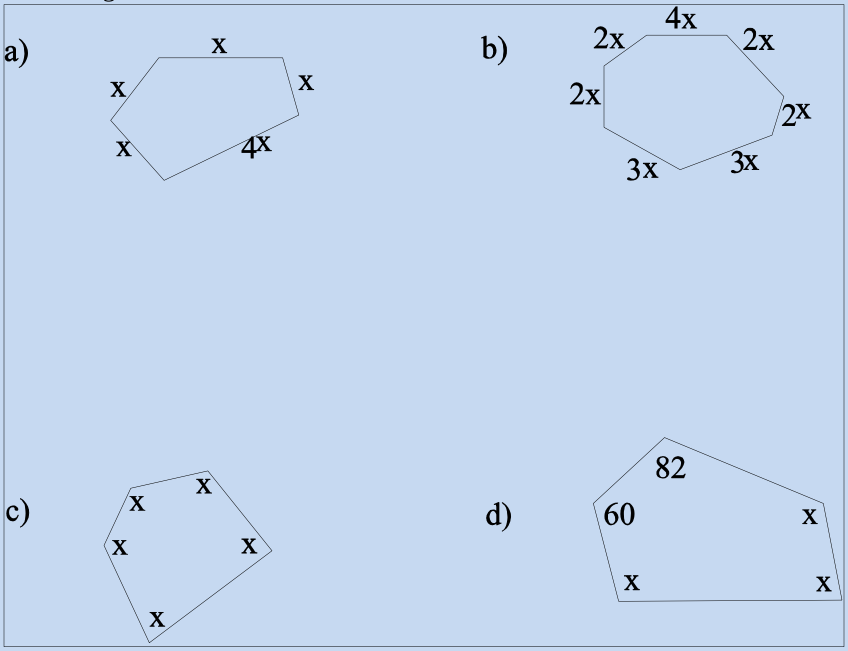
1) Calculate the interior of regular polygons with

a) 6 sides   b) 10 sides   c) 15 sides

2) Find the number of sides that a regular polygon has if its interior angles are

a) 108^0 b) 156^0 c) 120^0 d) 150^0

3) In the diagrams below, calculate x



4) Refer to question 3. Find the values of each of the indicated angles in each diagram

EXAMINATION QUESTIONS

1) ABCDE is a pentagon. The angles at A,B,C,D,E are x^o, $4x^o$,$5x^o$ respectively. Find the value of x and prove that AB is parallel to ED

2) The interior angle of a regular polygon is 162^o. Find the number of sides of the polygon.

3) A polygon has 8 equal sides

a) State the special name of the polygon

b) Calculate the special name of the polygon

4) The interior of a polygon is four times as big as the exterior angle Calculate the size of interior angle.

CHAPTER 20

PERCENTAGES

A percentage is a fraction with a denominator of 100 e.g. 13% is $^{13}/_{100}$

Syllabus objectives
Learner should be able to:
a) Change percentage to fractions or decimals, vice versa
b) Express one quantity as a percentage of the other
c) Calculate percentage increase and percentage decreases.
d) Calculate the required information in buying and selling
e) Solve problems involving simple interest

Changing percentage to fractions or decimals

To change a percentage to a fraction divide by 100

Example 1

Change 30% to i) a fraction ii) a decimal

i) $30\% = \frac{30}{100}$

$= \frac{3}{100}$

ii) $30\% = \frac{30}{100}$

$= 0{,}3$

Changing fraction or decimals to percentages
To change a fraction or decimal to a percentage you multiply it by 100

Example 2
Change the following to percentages

i) $\frac{3}{5}$ ii) 0,45

i) $\frac{3}{5} \times 100$

$= 60\%$

ii) $0{,}45 \times 100$

45%

Expressing one quantity as a percentage of another

Example 3

Express \$ 50 as a percentage of \$ 200

$\frac{\$\,50}{200} \times 100$

= 25%

Exercise 1,1

1) Change the following percentages
i) to fractions in their lowest terms
ii) to decimals

a)	20%	b)	30%	c)	38%	d)	80%
e)	75%	f)	5%	g)	$33\frac{1}{3}$%	e)	

2) Change The following to percentages

a) $\frac{3}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) 0,5 e) 0,33 f) 0,22

g) 0,275 h) $\frac{1}{20}$

3) Express the first quantity as a percentage of the second

a) \$ 9 : \$ 100 b) 75: \$200 c) 30:\$150 d) 300m:1,5km
e) 1,2 litres : 2 litres
h) 3mm:1cm
f) 70:\$2

4) Calculate

a)	5% of 20c	b)	20% of \$1
c)	30% of \$ 1,50	d)	75% of 250g
e)	94% of 1 000	f)	33 1/3 of \$ 21
g)	$66\,^2/_3$ % of 24		

5) The pass mark in an examination is 40%
If the examination is marked out of 120 what is the pass mark.

6) Kudzai scores 35 marks out of possible 50 marks in a test. What percentage did she get.

7) A piece of elastic 48cm long is stretched to 60cm. What percentage of the original length is the increase

8) A trader allows a 20% deposit on a stove that cost \$500. How much is the deposit

Percentage increase and decrease

Example
a) Increase \$50 by 10%
b) Decrease \$ 20 by 5 %

a) The new price is 110% of the original price

110% of \$50 $= \frac{11\cancel{0}}{1\cancel{00}} \times \underline{\$\,5\cancel{0}}$

$= \$\,55$

b) The new price is 95% of the original price

95% of \$20 $= \frac{95}{1\cancel{00}\ 5} \times \underline{20}$

$= \$\,19$

BUYING AND SELLING

Commonly used terms

Cost price - This is the price paid for an article selling price

Selling price -This the price at which the article is sold.

Profit - The excess of selling price over the cost price (i.e selling price- cost price)

Loss -The excess of cost price over the selling price (i.e cost price-selling price)

The profit or loss is often expressed as a percentage of the cost price.

Percentage profit (or loss) = $\frac{profit}{\cos t}$ (or loss) x $\frac{100}{1}$

Example 5

A retailer buy a suit from a wholesaler at \$ 20. He sells the suit at \$ 25. Calculate .

i) The profit made

ii) the percentage profit

i) Profit = selling price – cost price

= \$ 25 - \$ 20

= \$ 5

ii) % profit = $\frac{\text{Profit}}{\text{cost price}} \times 100$

$= \frac{5}{2\cancel{0}} \times \overset{5}{1\cancel{00}}$

= 25%

Example 6

A trader make a 10% profit by selling a two plate stove \$ 60. What is the cost price of the stove

110% : \$60 use proportion

100% : less

$$\frac{100}{110} \times 6\cancel{0}$$

$$= \frac{600}{11}$$

= $ 54,54

OTHER TERMS USED

Hire purchase method of buying

Exercise 1,3

1)	$ 75 by 5%	b)	60c by 2%	c)	$ 1,50 by 10%
d)	0,25cm by 4%	e)	65cm by 75%	f)	$ 38 by 61%

2) Decrease the following quantities by the given percentage

a)	$ 50 by 4%	b)	20c by 75%	c)	$14 by 12%
d)	7m by 5%	f)	0,08 by 2%	g)	$25 by 4%

3. The price is $ 60 after an increase of 10%. Find the original price

4) The price is $ 150 after an increase of 25%. Find the original price.

5) The price is $42 after a decrease of 5%. Find the original price

7) An article is bought $ 1, 50 and sold at a 10% profit. How much was the selling price?

8) A stove is bought at $ 230 and sold at a less of 6%. What is the selling price?

9) Find the cost price of an article which is sold for $ 21,25 at a profit of 5%

10) Find the cost price an article which is sold for $ 15,25 a less of 3%

11) A man's weekly wage is increased from $ 125 to $ 140. Calculate the percentage increase

12) A retailer bought some hats from $1, 90 each and sold them at 30% profit. What was the selling price of each that?

13) By selling a bicycle for $ 78, a shopkeeper makes a profit of 30% of his cost price. At what price should he sell it to make a profit of 25% of his cost price?

14) In a batch of 150 articles 6% were defective. Calculate the number which were not defective

15) A vendor buys 150 oranges for $3, 75 and sells them all at 3 cents. What percentage profit did the make?

SIMPLE INTEREST

INTEREST

This is the price paid for using borrowed money similarly, when for instance a person saves his/her money with the bank, he/she receives payments of the interest proportional to his or her savings.

Principal

This refers to the money borrowed or saved. The interest is usually a percentage of the principal for each year of the loan.

If the interest is withdrawn (or paid) each year, the principal remains the same and this is called simple interest. Per annum means per year.
If a principal of $P is deposited for T years at a rate of R% per annum, the simple interest, $1 is given by the formula.

$$I = \frac{P \times R \times T}{100}$$

The formula can be re-arranged to change the subject of the formula to find the principal (P), or the ratio R, or the time (T).

- It is advisable to calculate using the time in years.

Example 7
Calculate the simple interest an $ 100 for 2 ½ years at 5% p.a

$$I = \frac{PRT}{100}$$

$$= \frac{100 \times 5 \times \frac{5}{2}}{100}$$

=$ 12,50

Example 5
At what rate per annum is simple interest paid when $ 720 yields an interest of $ 216 in 6 years.

$$I = \frac{PRT}{100}$$

$$216 = \frac{720 \times R \times 6}{100}$$

$$216 = \frac{4320R}{10}$$

Make R the subject of the formular

2 160 = 432R
540
R = ~~2 160~~
~~432 1~~
108
135
= ~~540~~
~~108~~
27
45
= ~~35~~
~~27~~
9

R= 5%

Exercise

1) Calculate the simple interest on the following
a) $ 50 for 1 year at 5% pa.
b) $ 75 for 1 ½ years at 5% p.a
c) $ 110 for 10 months at 6%
d) $120for 1 year 6 months at 3 ½ p.a

2) Calculate the principal that will earn an interest of

a) $54 in 1 year 6 months at 8% p.a
b) $ 252 in 5 years at 9% p.a
c) $ 374 in 4 years at 11 % p.a
d) $12,96 in 3 years at 9% p.a

3) Find the time which
a) $ 240 will earn $ 120 at 5% p.a
b) $ 200 will earn $ 75 at 8% p.a
c) $ 245 will earn $137,20 at 14% p.a

4) Calculate the rate per cent, per annum of which
a) $ 162 will earn $ 43,20 in 4 years
b) $ 70 will earn $216 in years
c) $ 225 will earn $ 54 in 4 years
5) A bank lends $ 6000 to a company for 8 months. At the end of that period $ 6 048 is repaired. Calculate the annual rate of interest which was charged.

6) Find the amount after 2 years in which $ 840 will earn $ 157, 50 at 12 ½ % per annum, simple interest.

7) Calculate the simple interest on $ 480 invested for 5 years at 9% per annum

8) A bank charges $ 28 simple interest on a sum of money which is borrowed for four months. Given that the rate of interest is 15% per annum, Calculate the sum of money.

EXAMINATION QUESTIONS

1) A man brought a picture for $325 and sold it at a profit of 12%. Calculate the selling price
b) A dealer made a profit of 20% by selling a car for $ 630. Calculate the price he paid for the car.
Cambridge
2) A shop marks an article so as to make a profit of 30% on the cost price. In a sale discount of 10% was allowed off the marked price. If the article was sold in the sale, state the actual percentage profit made by the shop.

3) Calculate the principal that earns $ 1 520 simple interest at 12% per annum.

4)a) During a drought, a farmer lost 20% of his herd of cattle and was left with 40 only. Calculate his original herd.

b)

Special Offer !! 40% OFF ON ALL MARKED PRICES

A shop displayed the above advertisement on its windows

Calculate

i) The selling price of an article marked $ 2 500 on this shape

ii) The amount by buying the article in this shop

5) A agriculture class keeps 100 broiler chickens at a time. The basic costs of inputs are shown in the table below.

Cost of 100 Chickens	Cost of 50kg broiler starter mash	Cost of 50kg Finisher mash
$ 4 800	$ 2 180	$ 2 118

The class needs 20kg of boiler starter mash and 100kg of finisher mash to feed the chickens for six weeks.

a) Calculate the basic cost of buying and keeping 100 chickens for six weeks.

b) After six weeks all the chickens are sold at $300 each. Calculate the profit made

c) A second batch of chickens was bought when the price of feeds had gone up by 15%

i) Calculate the capital needed to buy and raise this batch

ii) After six weeks 55 of the chickens were stolen and the rest were sold at $ 375 each

Calculate the percentage profit made.

CHAPTER 21

Probability

Probability is a chance that an event will occur. It is a fraction which lies between 0 and 1, which gives us an idea of likelihood of an event happening or not happening.
If the probability of an event is one (1) it means the event will certainly happen. If its zero (0), then it will certainly not happen.

Syllabus Objectives
Leaner should be able to
a) Explain experimental probability and calculate the probability
b) Explain theoretical probability and make related contributions
c) Explain mutually exclusive events in probability and make related calculation
d) Explain independent events in probability and solve related problems
e) Solve probability using outcome tables and tree diagrams

Experimental probability

This is a form of probability that uses record of past events to predict the future. For example, If Peter managed to pass 4 out of the five tests that he has written, then based on that record we can predict that Peter will pass the sixth test.

Practically, experiential probability is used in many situations like predicting the likely rainfall pattern of the coming season, to predict outcomes based on previous survey results, to predict the outcome of a match based on the previous results of the two teams that are playing.

Basic Formular for probability
Probability measures the likelihood of a required outcome. It is usually given as a fraction

$$\text{Probability} = \frac{\text{number of required outcomes}}{\text{Number of possible outcome}}$$

Using the previous example of Peter where he passed 4 out of the 5 test written so far.
The probability of Peter passing the next test,
Written as

$$P\text{ (passing)} = \frac{\text{number of required outcomes (i.e number tests he has passed)}}{\text{Number of possible outcomes are i.e the number tests written)}}$$

$$= \frac{4}{5}$$

$$P\text{ (failing)} = \frac{\text{number of required outcomes (i.e the u\textbackslash number tests failed)}}{\text{Number of possible outcomes (i.e the number of tests written)}}$$

$$= \frac{1}{5}$$

Taking P (failing) as P (not passing)

P (Passing) + P (not passing) = 4/5 + 1/5

=1

P (Passing) = 1- P (not passing)

Also
P (not passing) = 1-P (Passing)
Generally if the probability of something happening is x, then the probability of it not happening is 1-x
The sum of the probability of something happening and the probability of that something not happening is equal to 1.

Example 1
In a school, it is known that 20 students out of every 50 advanced level students, make it to university. Find
i) the probability of students who will go to university next year
ii) the probability of students who will not go to university
iii) The number of students who will go to university if 75 students sit for their advanced level examinations

i) P (students going to university) = $\frac{\text{number of required}}{\text{number of possible outcome}}$

$$= \frac{20}{50}$$

$$= \frac{2}{5}$$

ii) P (Student not going to university) = 1 – P (students who will go).

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

iii) Number of students = P (students going to university) x 75 going to university.

$$= \frac{2}{5} \times 75$$

= <u>30 students</u>

Example 2
There are six slices of bread in a plate. Four slices are spread with jam while two slices are spread with margarine.
i) What is the probability that a slice chosen at random is spread with jam
ii) What is the probability that a slice chosen of random is spread margarine.

i) P (Jam) $= \frac{4}{6}$ $= \frac{2}{3}$

ii) P (margarine) $= \frac{2}{6}$ $= \frac{1}{3}$

At random means without choosing carefully!

Exercise 1,1
1) A box contains seven red and three blue ball-point pens. Mary randomly pick a pen at randomly from the box, find
i) P (red) ii) P (blue) iii)P (neither blue nor red)
Hint – neither, nor means none of the above mentioned

2) There are twenty girls and fifteen boys in form four A. A pupil is chosen at random from the classes. Find.
   i) P (girl) ii) P (boy)
3) On a hospital two in every 100 babies born die. Find the probability that
   a) a born baby will die
   b) a born baby will not die
   c) if 75 babies are born, how may would you expect to
      i) die ii) live
4) Midday temperature during a week were 21^o, 21^o, 20^o, 19^o, 18^o. What is the probability that the midday temperature in the next day will be

a) 21^o b) 20^o c) 30^o

5) Team A and Team B have played each other 10 times. Team A has 5 games. Team B has won 3 times and they have drawn twice. What is the probability that on the next match

a) Team A will win b) Team B will win c) The two teams will draw

d) which team is likely to win the match

6) One in every sixty bulb in a shop is faulty. If a customer picks one from the shelf what is the probability that

i) it is faulty ii) it is not faulty

7) Table below gives result of a survey on the number of customer that come to their shop by gender. The table show the total number of customers in the two different times of day and the number of those that were male and female.

	Total number of customers	**Number of males**	**Number of females**
Morning	50	15	35
Afternoon	25	10	15

a) For the whole day, find the total number of i) males that enter the shop
ii) Females that enter the shop
b) Find the percentage of the customer that were males
c) Find the percentage of the customers that were females
d) Find the profitability that the next customer to enter the shop is likely to be female
e) Of the next 10 customers how many are likely to be females

Theoretical probability

These refers to probability that can be found without any prior experiment or record of past event. For example the fact that a coin has two sides, a head and a tail, the p (tail) = ½ and p (head) = ½. It is based on the fact that the probability of each outcome is equally likely due inherent fairness.

Example 3

A die is thrown. Find the probability of getting

i) a 3 ii) or even number

i) P (3)= $\frac{\text{number of required outcome}}{\text{Number of possible outcome}}$

$$= \frac{1}{6}$$

ii) P (even number) = $\frac{\text{number of required outcome (i.e. 2, 4, 6)}}{\text{Number of possible outcome (1, 2, 3,4,5,6}}$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Example 4
A card is picked at random from a pack of 52 playing cards. What is the probability that it is a2

P (a2) = $\frac{\text{number of required outcomes}}{\text{Number of possible outcomes}}$ $\frac{\text{(i.e 2 of clubs, 2 of hearts, 2 of spades)}}{\text{2 of diamonds)}}$

$= \frac{4}{52} = \frac{1}{13}$

Exercise 1,2
A fair 6 sided die is thrown. Find the probability of getting

a) a3 b) a 10 c) aO d) a6
e) either a1,2,3,4, 5 or 6
f) odd number
g) a prime number
h) a number divisible by 6.
i) a number greater than 3
ii) a number less than 5

2. A card is picked at random from a pack of 52 playing cards. Find the probability of picking
a) 5 of spades b) the 3 of clubs
c) the j of hearts d) a 4 of diamonds
e) a Queen f) an Acee (A)
g) a diamond h) a black King
j) a black 5 k) either a 3 or a 4

3) Two coins are tossed together. What is the probability of getting?
a) a tail and a head b) two tails c) two heads

4) A letter is chosen at random from the alphabet. Find the probability that is.
a) Q b) a vowel
c) one of the letters of the word TEXTURE
d) one of the letters of the word PROBABILITY
e) either X or Y
f) either A or B

Mutually exclusive events

Two events are said to be mutually exclusive if they cannot both happen at the same time. For example, on throwing a die, the event, "a score of 3 is obtained and the event score of four on a single throw, is mutually exclusive. These two events cannot happen at the same time.
However, the event of getting a score of 3 and the event of getting an odd number of score are not mutually exclusive. **Why**? on a single throw you can get a score of 3 which happens to be an odd number of score.
There are 6 possible score on a die that is 1 or 2 or 3 or 4 or 5 or 6. Since P (score of 3) and P (score of 4) are mutually exclusive. It means if you do not get a 3 there is a chance that you will get a 4.

Since Probability = $\frac{\text{number required outcomes i.e.}}{\text{Number of possible outcomes}}$

So P (3 or 4) $= \frac{2}{6}$

$$= \frac{1}{3}$$

Alternatively

P (3 or 4) $= P(3) + P(4)$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

We said P (3) and P (odd number of score) are not mutually exclusively

P (3 or odd number of score) $= \frac{3}{6}$

$$= \frac{1}{2}$$

Comparing the results

P (3 or odd number of score) = P (3) + P (odd number of score)

$$= \frac{1}{6} + \frac{3}{6}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

On comparison it can be seen that one method gives a probability of ½ while the other gives a probability of $^{2}/_{3}$. It therefore means, for events which are not mutually exclusive.

P (A or B) ≠ ((A) + P(B)

e.g P (3 or odd number of score) ≠ P (3) + P (odd number of score)

But for mutually exclusive events

P (A or B) = (P (A) + P(B) (Sum rule)

e.g P (3 or 4) = P (3) + P (4)

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{1}{3}$$

Independent Events

Events are said to be independent if the occurrence of one does not affect the occurrence of the other.

For example on tossing a coin and throwing a die, the events “tail” “and score 3” are independent.

Thus P (A and B) = P (A) x P (B) (product Rule)

Example 5
A card is chosen at random from a pack of playing cards. What is the probability that is either an Ace or a the Queen of hearts.

Since the two are naturally exclusive
P (Ace or Queen of hearts) = P (Ace) + Queen of hearts

$$= \frac{4}{52} + \frac{1}{52}$$

$$= \frac{5}{52}$$

Example 6
A box contain 4 red pens and 6 blue pens. One is picked at random without replacing the first one, a second pen is picked at random. What is the probability that both pens are red.

P (red and red) = P (red) x of (red)

First pick
4 red + 6 blue P (red) = $^4/_{10}$

Second Pick
Suppose red was picked on th first pick and not replaced
Remains
3 red + 6 blue p (red) = $^3/_9$

P (red and red) $= \frac{4}{10} + \frac{9}{9}$

$$= \frac{12}{90}$$

Did you know this about playing cards
A pack of playing cards contains 52 cards in 4 suits
Clubs, diamonds, hearts and spades
There are 13 cards in each suits A, 2,3,4,5,6,8,9,10 J,Q,K
Clubs and spades are black, diamonds and hearts are red

Exercise 1,3

1 Cards numbered 1 to 12 inclusive are placed in a hat and a card is drawn without looking. What is the probability that the number drawn is

a) divisible by 4
b) even
c) either prime or divisible by 3
d) both even and divisible by 3
e) either 2 or 3

2) A box contains 5 blue and 3 yellow counters. One is taken out and replaced and then a second draw is made. Find the probability of obtaining.

i) two red counters
ii) one of colour
iii) two white counters

3) Seven cards are placed in a box. Three have crosses on them and the others are plain. One cars is drawn out and not replaced. Then a second is drawn out. Find the probability of drawing.

i) two crosses cards   ii) one cross only   iii) cards with no crosses

4) A die is thrown. What is the probability of getting?

a) a two

b) a three or a five

c) a one or a two or a three

d) a square

5) A bag contain 3 yellow balls, 4 black balls and 5 white balls. A ball is picked at random. What is the probability that it is either

a) A yellow or black   b) black or white

c) yellow or white   d) yellow, black or white

6) A card is drawn from a pack of playing card then returned to the pack. A second card is chosen. What is the probability that both cards are red.

7) A coin is tossed and a die is thrown. What is the probability of getting a head and prime number.

8) A bag contains 8 yellow marbles and 7 blue marbles. A marble is chosen at random and it is not replaced. A second marble is then randomly picked. What is the probability that the 2 balls chosen are (a) both blue b) both yellow.

Outcome tables

These are used to summarize the outcome of two or more independent events.

Outcome table of tossing the coins at the same time the same time.

		Coin 2	
		H	T
Coin 1	H	HH	HT
	T	TH	TT

Outcome table of total score obtained after throwing two dice. Complete the table

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4						
5						
6						

Example 7

When the two dice are thrown what is the probability of getting an even number of score?

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9

4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

P (even number of score) = $\frac{18}{36}$

$= {}^{1}/_{2}$

P (even number of score = P (2 or 4 or 6 or 8 or 10 or 12

= P (2) + Q(4) + (6)+ P(8) + P(10) +(P12)

$= {}^{1}/_{30} + {}^{3}/_{36} + {}^{5}/_{36} + {}^{5}/_{36} + {}^{3}/_{36} + {}^{1}/_{36}$

$= \frac{18}{36}$

= ½

Tree diagrams

Example 8

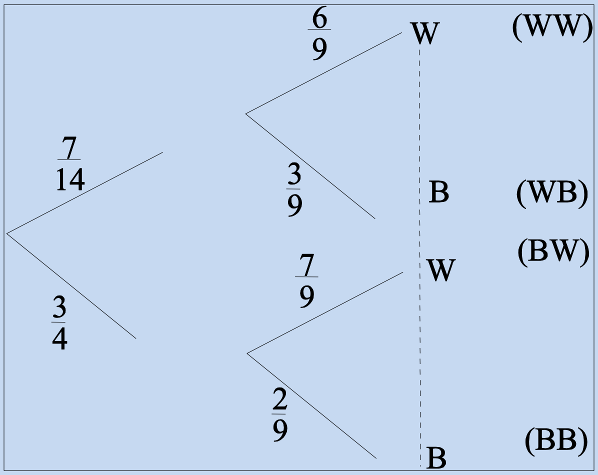
There are seven white beads and three black beads in a box. A bead is selected at random from the box and is not replaced. A second bead is then chosen at random. Find the probability that the two beads are different colours.

If the first bead is white, then six of the remaining nine beads are white and the probabilities of the second bead being white and black are 6/9 and 3/9 respectively.

I the first bead is black, the seven of the remaining nine beads are white and the probabilities of the second bead being white and black are 7/9 and 2/9 respectively

REVISIT

Second draw



P (beads are of different colours) = P (white and Black)

= **P (WB) or P (BW)**

$= ({}^{7}/_{10})\ ({}^{3}/_{9}) + ({}^{3}/_{10})\ ({}^{7}/_{9})$

$= {}^{21}/_{90} + {}^{21}/_{90}$

$= {}^{42}/_{90}$

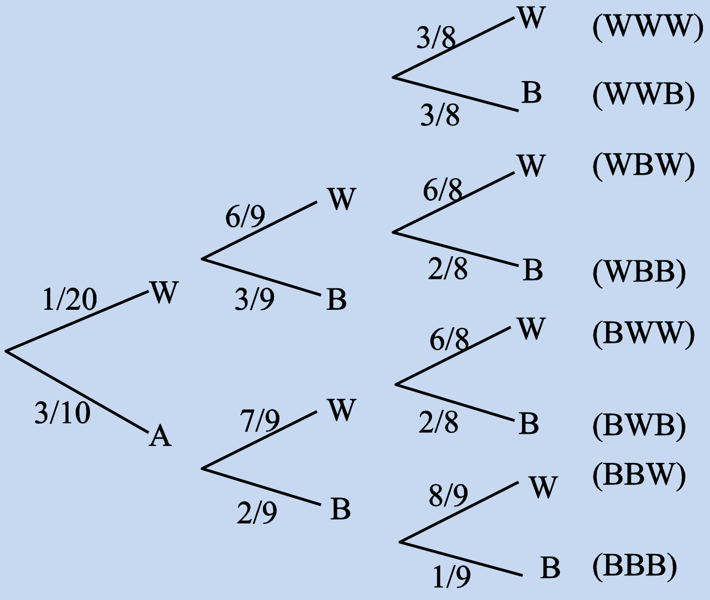
$= {}^{14}/_{30}$

$= {}^{7}/_{15}$

Example 9

In example 8 the second bead chosen was not replaced. A third bead was randomly chosen. Find the probability of
i) Choosing 3 white
ii) At least a black

REVISIT



P (3 white) = P (WnWnW)
$= ({}^{9}/_{10})\ ({}^{6}/_{9})\ ({}^{5}/_{9})$
$= \frac{7}{24}$

P (at least a black) = P (WWB) or P(WBW) or P (BWW)
Or P (BWB) or (BBW) or P (BBB)
$= ({}^{7}/_{10})\ ({}^{6}/_{9})\ ({}^{3}/_{8}) + ({}^{7}/_{10})\ ({}^{3}/_{9})\ ({}^{6}/_{8}) + ({}^{7}/_{10})\ ({}^{3}/_{9})\ ({}^{2}/_{8}) + ({}^{3}/_{10}) + ({}^{7}/_{9})({}^{6}/_{8}) + ({}^{3}/_{10})\ ({}^{7}/_{9})\ ({}^{2}/_{8})$
$+ ({}^{3}/_{8})\ ({}^{2}/_{9})\ ({}^{8}/_{9}) +$ P $({}^{3}/_{10})\ ({}^{2}/_{9})\ ({}^{1}/_{9})$

$$= \frac{126}{720} + \frac{126}{720} + \frac{42}{720} + \frac{126}{720} + \frac{126}{720} + \frac{42}{720} + \frac{48}{720} + \frac{6}{720}$$

$$= \frac{3}{8}$$

P (at least a black) = 1 – P (all white)
= 1 – 7/24

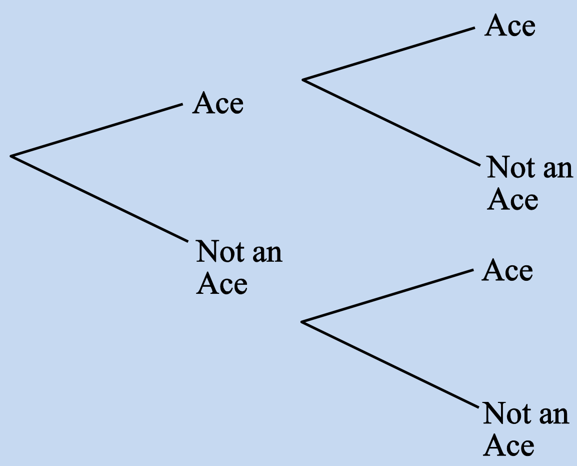
$$= \frac{18}{24}$$

$$= \frac{3}{8}$$

Exercise 1,4

1. Two fair dice are thrown. Find the probability of obtaining i) a score of seven
ii) a score of 6 iii) a score less than 6 iv) a score more than 6
2. A bag contains three red balls and two blue balls. Two balls are randomly chosen without replacement. Draw a tree diagram to represent the outcomes.
b) What is the probability that i) both balls are red ii) both balls are blue
iii) one is white and the other black.
3. A coin is tossed and die is thrown. Represent the set of all possible outcomes in a diagram
b) What is the probability of obtaining
i) a head and an even number ii) a tail and a number less than 5
iii) a head and a multiple of 5
4) There are fourteen girls and sixteen boys in form four A. If three pupils from the class are selected at random, represent the outcome on a tree diagram
b) Find the probabilities that
a) three boys are selected b) two boys and one girl are selected
5. It is found that the probability that a particular brand of fire works will light is $^{1}/_{8}$. John bought two.
a) Draw a tree diagram of “light” and “fail” to represent the possible outcomes
b) Use it to find the probability that both of his fireworks light
6) Ann buys three fireworks. Represent the possible outcomes on a tree diagram and use it to find the probability that at least two of her fireworks light.
7) Draw a tree diagram to show the possible outcomes for tossing a coin three times. Use it to find the probability of getting (a) exactly two heads (b) at least two heads
8) A card is drawn from a pack of 52. It is then replaced and a card is drawn again. Copy and complete the tree diagram below and use it to find the probability
a) the aces b) at least one ace

First Card



9) A class consist of eight girls and seven boys. Three students are randomly selected to represent the class in a clean up campaign. What is the probability that
a) they are all girls b) two of them are boys

Examination Question

1) A ball is dropped at random into one of eight holes, numbered as shown below.

●	●	●	●	●	●	●	●
1	2	1	2	1	2	1	2

The numbered under each hole gives the score obtained when the ball drops into that hole

a) State the probability of scoring 1

b) If the ball has dropped twice, find the probability of scoring i) a total of 6, ii) a total of 4

2. A box contains one hundred transistors of which twenty are defective. Two transistors are selected at random. Find the probability that

a) both are defective b) just one is defective

3) A packet contains 8 red sweets, 5 green sweets and 7 yellow sweets. All the sweets are identical except for colour

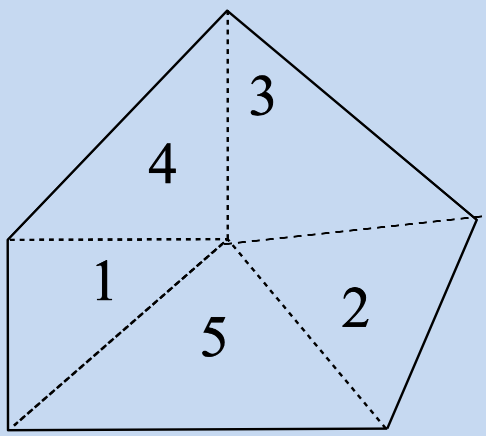
Two sweets are picked from the packet at random, one after the other without replacement. Giving each answer as a common fraction in its lowest terms, find the probability that

a) both sweets are red

b) the two sweets are of the some colour

c) at least one sweet is green

[ZIMSEC NOV 2004]



The diagram shows a regular pentagonal spinner, The side which comes to rest on the table gives the score. Two such spinners are spun together and the result noted. Giving the fraction in its lowest terms, find the probability of that.

a) the product of the score is 16

b) the sum of the two scores is 9

CHAPTER 22

Statistics

Statistics involves collecting and presenting and interpreting information in a manner that is easy to understand.

Syllabus Objectives

Leaner should be able to

a) Calculate the mean, median and made
b) Draw and interpret a bar chart
c) Draw and interpret a pie chart
d) Draw and interpret a histogram
e) Draw and interpret a cumulative frequency curve.

Mean, Median and Mode

Mean

The mean is the sum of all quantities divided by the total frequency. (number of quantity under measure)
For example, given the marks obtained by 10 pupils in a class as follows:

4;4;5;5;5;5;6’7;8;8

Mean = $\frac{\text{Sum of marks}}{\text{Number of pupils}}$

$= \frac{57}{10}$

$= 5{,}7$

The marks can be presented in a frequency distribution table as follows:

Mark (x)	4	5	6	7	8
Frequency (f)	2	4	1	1	2

The frequency represents the number of pupils who got a particular mark

From the frequency distribution table the mean is calculated by the formula, mean = $\frac{\sum fx}{\sum f}$

$= \frac{(4x2) + (5x4) + (6x1) + 7x1) + 8x2)}{2+4+1+1+2}$

$= \frac{57}{10}$

$= 5{,}7$

From the frequency distribution table most pupils (4) get a mark of 5. The mark 5 is therefore called the model of the data
The mode of the data is the quantity with the highest frequency.

The other measure is the median which is middle value when all quantities are arranged in ascending order. If there are even number of quantities the median is the average of the two middle quantities. Using our previous example of works.

4;4;5;5;5;5;6;7;8;8

$$\text{Median} = \frac{5+5}{2}$$

= 5

From the frequency distribution we need to locate the median using its position by the

Median position = ½ (n+1) th where n is the total number of quantities

= ½ (10+1)th

= $5,5^{th}$

Mark (x)	4	5	6	7	8
Frequency (f)	2	4	1	1	2

From the frequency distribution table the position $5,5^{th}$ lie between two marks of 5. Hence the median

$$= \frac{5+5}{2}$$

= 5

The mean can be obtained by $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

Alternatively an assumed mean can be used let assumed mean be5.

Mark (m)	Frequency	Deviation from assumed mean (Mark-assumed Mean)	Frequency X Deviation F (m-5)
4	2	-1	-2
5	4	0	0
6	1	1	1
7	1	2	2
8	2	3	6
	=10		(m-5)=7

Mean Mark = $5 + \frac{7}{10}$

= 5 + 0,7

= 5,7

Example 1

The table shows the number of goals scored in the league football matches

Number of goals x	0	1	2	3	4	5	6	7
Number of matches (f)	4	7	12	11	3	4	1	2

a) Calculate

i) the mean ii) mode iii) median

b) use an assumed mean to find the mean

i) mean = $\frac{\sum fx}{\sum f}$

$$= \frac{(0x4) + (7x1) + (12x2) + (11x3) + (3x4) + (4x5) + 1x6) + (2x7)}{4+7+12+11+3+4+1+2}$$

$$= \frac{116}{44}$$

= 2,6

ii) Two goals is the mode

iii) Median = ½ (n+1)th

= ½ (44+1)

= 22,5th

Median = 2

b) Let the assumed mean be 3

Number of goals (x)	Frequency	Deviation from assumed Mean (x-3)	F(x-3)
0	4	-3	-12
1	7	-2	-14
2	12	-1	-12
3	11	0	0
4	3	1	3
5	4	2	8
6	1	3	3
7	2	4	8
	∑ f=44		∑ f (x-3)= -16

Mean = $3 + \frac{-16}{44}$

= 3-0,37

= 2,6

Exercise 1,1

1) The shoes sizes of a group of 30 students are
4;4;5;5;5;5;7;8;5;7;6;6;8;7;4;6;7;6;8;6;

For the above data

a) Show the information in a frequency distribution table

b) State the modal shoe size

c) Find the median

d) Calculate the mean using
i) the formular
ii) the assumed mean

2) Use the assumed mean to calculate the mean number of children per family from the following distribution if families.

Children in family	1	2	3	4	5	6	7	8	9	10	11	12	13	14
No. of Families	4	5	10	10	9	5	3	2	2	1	1	1	1	1

3) Using the table in question 2 calculate

i) the mean ii) mode iii) median

4) The table shows the distribution of marks in a test

Marks	40	41	42	43	44	45	46
Frequency	2	5	7	6	4	3	3

a) State the mode and median mark

b) Calculate the mean mark

Bar Charts

It is a statistical graph in which bars are drawn such that their lengths or heights are proportional to the quantities they represent.

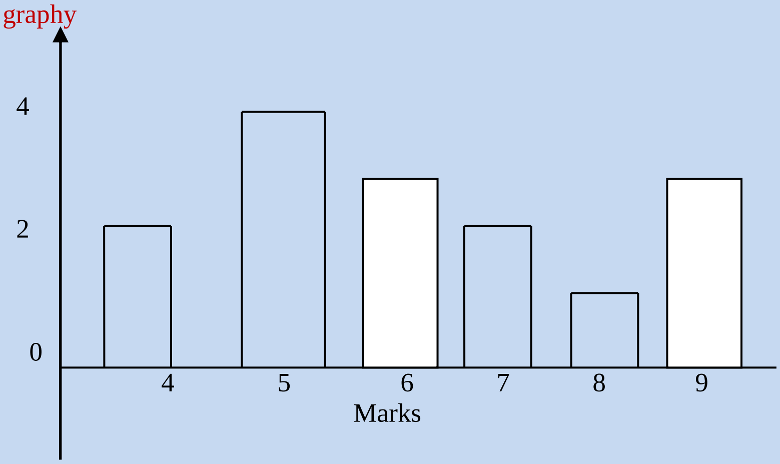
Table shows the distribution of marks on a test.

Mark	4	5	6	7	8	9
Frequency	2	4	3	2	1	3

Draw a bar chart to illustrate the information

REVISIT

graphy



It should be noted that

a) the width of all the bars are of the equal width and the bars do not touch

b) heights of the bars are proportional to the marks

Pie Chart

It is a pictorial representation of the numerical data. The data is represented by angles in the sector of a circle, which are proportional to the given data.

Example 2

A farmer plants five types of flowers on 45 hecters of land as shown on the table.

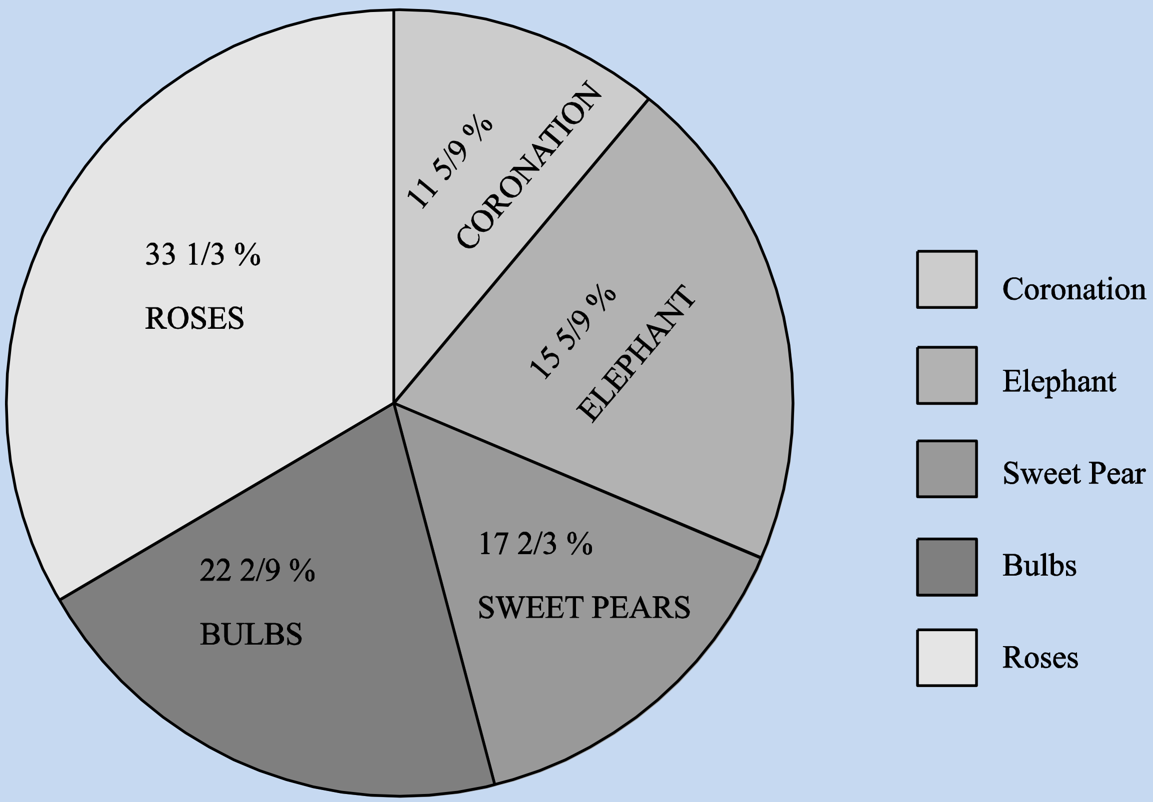
Hectare	Flowers
15	Roses

10	Bulbs
8	Sweet peas
7	Elephant Ear
5	Carnations

Show the information on a pie chart

Hectare	Angle of sector	Each number is represented by a sector of a circle or pie. The angle of each sector is proportional to size of the number
15	$^{15}/_{45}$ x 360^0 = 120^O	
10	$^{10}/_{45}$ x 360^0= 80^0	
8	$^{8}/_{45}$ x 360 = 64^0	
7	$^{7}/_{45}$ x 360^0-56^0	
5	$^{5}/_{45}$ x 36^0 = 40^0	
Total 45	360^0	

<u>Pie Chart</u>



In a pie chart, each number is represented by the area of a sector of a circle and thus by the angle of a circle
Measure the angle with a protractor to give an accurate picture.

- percentages may also be added on the chart
- All the angles should add up to 360^0
- Pie charts are meant to give a quick and clear comparison of the relative magnitude of the various items.

Exercise 1,2

1) Diagram below is a bar chart showing the means of transport of pupils in class.

a) which is the most common mode of transport
b) How many pupils walk to school
c) How many pupils were in a class

REVISIT

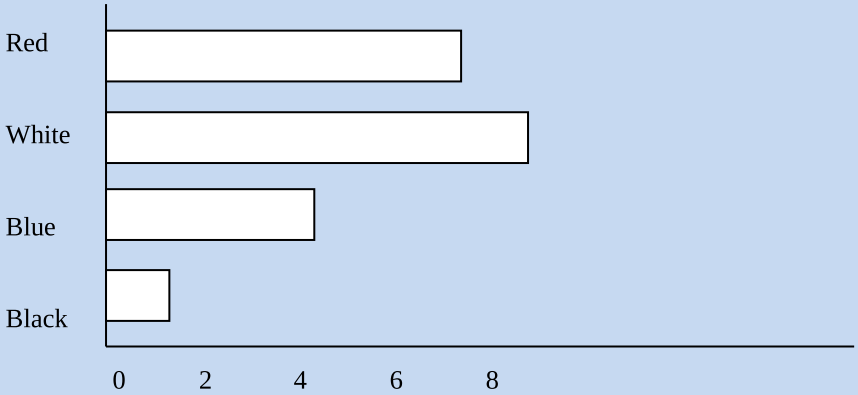
Graphy

d) What fraction of the class use a bike to school

2) Diagram below is a bar chart showing different colours of car passing a certain point.

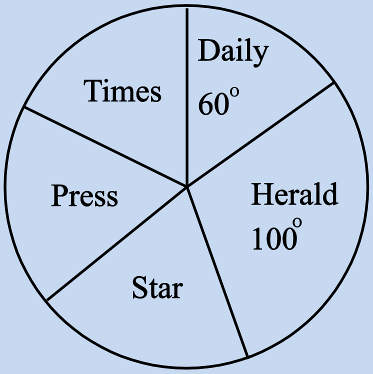
Insert:

REVISIT
Graphy

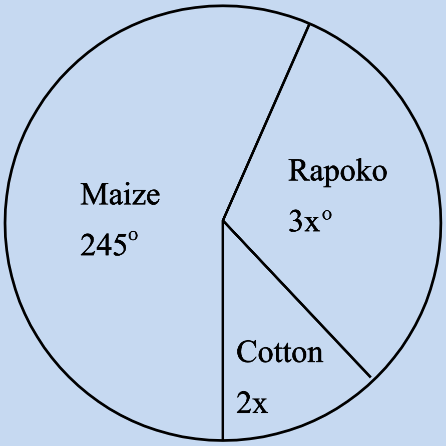


a) How many cars passed the point
b) which colour forms the mode
c) Which car colour forms the median
d) Calculate the mean

3) The pie chat represents the readership of 5 different newspapers on a survey on 200 people.



a) What angle represents one person
b) How many people read the Daily newspaper
c) How many people read the Herald
d) If 30 people read the press what angle does it need
e) What is the size of the angle shared by the Times and the star
4) Pie Chart shows the distribution of a piece of land to grow different crops.



i) Calculate the value if x

ii) Given that 9 hecters of land was used to grow maize calculate the total number of hecters

5) The table shows study time given to each subject during the week.

Subject	English	History	Maths	Commerce
No of hour	4	5	8	3

a) Draw a bar graph to represent this information

b) Draw a pie chart to represent this information

6) The table below shows the contribution of each of their four products to their $ 100 weekly revenues.

Product	W	X	Y	Z
Revenue	20	40	10	50

a) Show this information on a

i) Bar graph

ii) Pie chart

Histogram

Histogram is a form of statical representation of data used for continuous data. The kind of data that we were dealing with in bar graph was discrete data. It involved counting the number of goals, cars, marks, eggs of which a fraction can not be obtained. In continuous data, if we measure for instance the height of a plant we are likely to get a measurement say between 120 and 121 cm (i.e 120,3; 120;8;120;98 etc. In most cases the actual measurement is not exact but rounded to the nearest centimentre or millimeters. A height recorded as 120cm could have been 120,4 or 119,7cm

Grouped data

When a variable has a large number of different values, the values are often placed in groups or class and the data is presented in a histogram

For example, the marks on a test may be shown in a frequency distribution table as followings

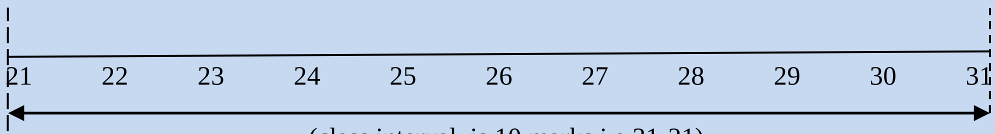
Mark x	21-30	31-40	41-50	51-60	61-70	71-80	81-90
Frequency f	2	4	7	8	3	1	1

Alternatively the information could be shown in inequality form as follows:

Mark x	20<x≤30	30<x- ≤40	30<x≤40	40<x≤50	50<x≤70	60<x≤70	70<x≤80
Frequency f	2	4	7	8	3	1	1

Class boundaries and class interval (class width)
These are prerequisite in the drawing of a histogram
Class interval = Upper class boundary – Lower class boundary
Taking the 21-30 class interval

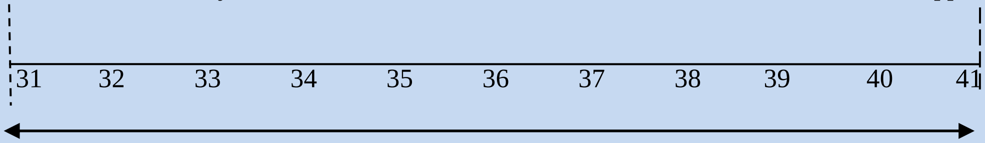
Lower boundary Upper boundary



(class interval is 10 marks i.e 31-21) interval

Also taking the 31-40

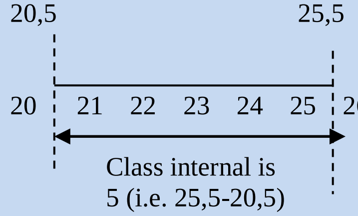
Lower class boundary Upper class boundary



Class interval is 10 marks (i.e 41-30) or cross width

For continues data like the height, age, time etc remember we said a height of 21cm could be on approximation of any value between 20,5 and 21,5cm. Also 25cm could be an approximation of any value between 24,5 and 25,5
Hence for a class of 21-25 (cm)

Lower boundary upper boundary
20,5 25,5



Class internal is
5 (i.e. 25,5-20,5)

Find the Mean
To find the mean the midpoint of the classes is use.
Drawing a histogram with equal class widths

Example 3

Draw a histogram and frequency polygon for the frequency distribution below.

Mass (x)	70-74	75-79	80-84	85-89	90-94	95-99	100-104	105-109
Frequency	3	6	9	13	13	8	6	2

a) Calculate the mean
b) What is the mode/class
Insert:

REVISIT
Graph

Example 4:
Draw a histogram for the frequency distribution below:

REVISIT GRAPH

Height (x)	15-20	21-23	24-26	27-32	33-38
Frequency	8	9	11	11	4

Insert:

REVISIT Graph

To draw a histogram with unequal class width.

1) Find the class width (upper class boundary- lower class)
2) Calculate the frequency density, $= \frac{\text{Frequency}}{\text{Class width}}$
3) Draw the histogram joining the upper class limits.

Exercise 1,3

1) The frequency distribution of marks obtained by so 50 candidates in an examination were grouped as follows:

Marks (k)	0-9	10-9	20-29	30-39	40-49
Frequency (f)	5	9	15	16	5

a) Calculate the mean
b) Draw a Histogram for this distribution

2) A class of 30 pupils were measured and their heights recorded to the nearest cm as follows:

163	152	164	161	153	164	159	171
154	167	155	158	161	157	161	164
166	150	163	154	155	155	153	152

a) Compile a grouped frequency table for these heights using class intervals 150-154; 155-159 etc

b) Calculate an estimate of the mean height of the pupils in the class

c) Show the data on a histogram.

3) The frequency distribution of the heights of 25 boys are measured to the nearest cm and are then grouped as follows

Height (cm)	$150<x \leq 155$	$155 < x \leq 160$	$160< x \leq 165$	$165 < x \leq 170$	$170 <x \leq 175$
Frequency	4	8	7	5	1

a) Calculate the mean height

b) Draw a histogram and a frequency polygon

4) The frequency distributions shows the average yield produced by farmers

Average yield per ha in tones (x)	$0 \leq x < 20$	$20 \leq x < 30$	$30 \leq x40$	$40 \leq x < 50$	$50 \leq x < 80$
N0. of farmers with (f) this yield	20	20	38	30	12

a) Show this data on a histogram

b) Calculate the mean of the distribution

5) Draw a histogram for the frequency distribution below:

Height cm	$15<x \leq 20$	$20<x \leq 23$	$23<x\leq 26$	$26<x \leq 29$	$29 <x \leq 32$	$32<x \leq 40$
Frequency	3	4	6	15	9	3

b) Construct a frequency polygon

c) Calculate the mean.

CUMULATIVE FREQUENCY

Example 5

The height of 56 plants, grown under experimental conditions, are given in the following table.

Height (cm) (x)	$x\leq 10$	$10<x\leq 20$	$20<x\leq 30$	$30<x\leq 40$	$40<x\leq 50$	$50<x\leq 60$	$60<x\leq 70$
N0 of plants (f)	1	2	4	6	13	22	8

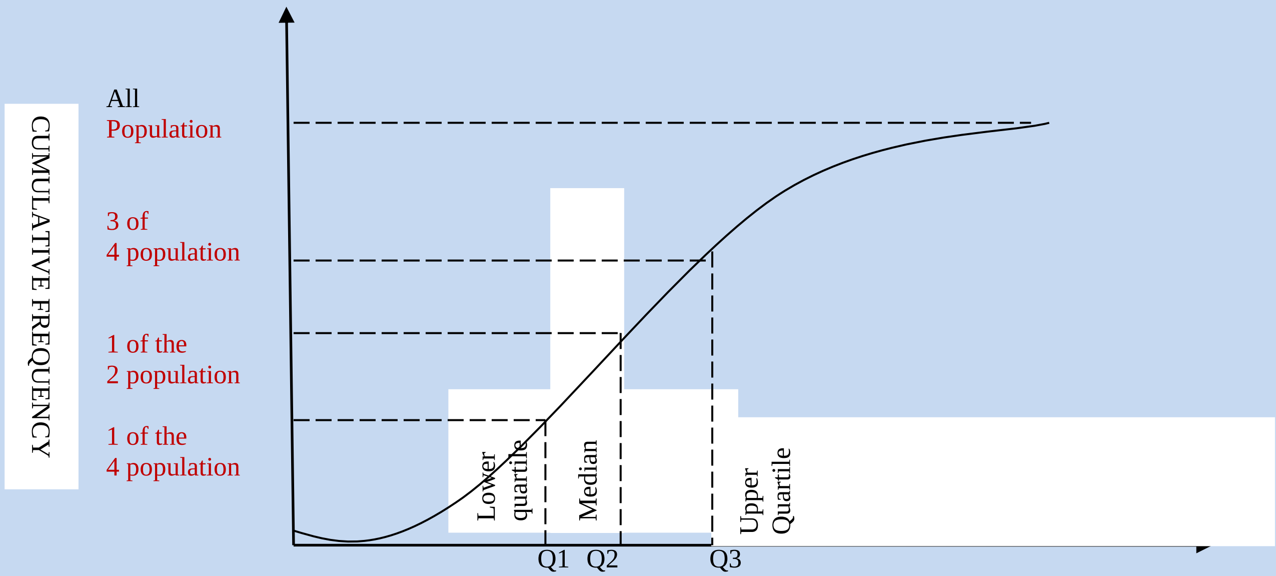
i) Draw a smooth curve cumulative curve for these results.

ii) Showing your method clearly use your graph to estimate
a)The median b) The interquartile range, of this distribution.
c)Semi-interquartile range

iii) If one plant is selected at random find, the probability that its height is a) greater than 50cm.

REVISIT

Insert:
Graph



i) Step 1
Make a cumulative frequency table

- Cumulative frequency 'This is the sum of frequencies to produce a running total
- Using our example 5. Each total gives the number of plants less than the upper boundary of the last class added, that is, e.g. (1+2) plants measured less than 20cm, (1+2+3) plants measured less than 30, (1+2+3+6) plants measured less than 40cm etc.

Quartiles

The frequency range is divided into four equal parts called quartiles.

Median (Q_2)

Divides the frequency range into two equal parts.

It position is given by, ½ (n+1) th Q_2=51cm, means ½ the plants measured less 51cm

Lower Quartile (Q_1)

It is ¼ (or 25%) way along the cumulative frequency axis

Its position is given by, ¼ (n+1)th Q=42cm means 25% of the plants measured less than 42cn

upper Quartile (Q_3)

It is ¾ (or 75%) way along the cumulative frequency axis. It position is given by, ¾ (n+1)th. Q_3=57cm, means 75% of the plants measured less 4 than 57xm.

Interquartile range

This is the difference between the values of the variable at the upper and quartile (i.e Q_3-Q_1)

Semi-interquartile range= $\frac{Q_3-Q_1}{2}$

Summary

a) For a population of numbers the cumulative frequency corresponding to n is the number of members of the population less than or equal to.

b) Half of the population is less than the median

c) A quarter of the population is less than the lower quartile. Three quarters of the population are less than the upper quartile.

Exercise 1,4

1. Diagram below shows a cumulative frequency graph for the masses of 60 tomatoes

Insert:

REVISIT

Graphy

a) Write down the median

b) Find the upper quartile and the lower quartile

c) Use the graph to copy and complete the following frequency table

Mass in grams	**Frequency**
More than 15 but not more than 20	3
More than 20 but not more than 25	4
More than 25 but not more than 30	
More than 30 but not more than 35	
More than 35 but not more than 40	
More than 40 but not more than 45	
More than 45 but not more than 50	

2) Table below show the distribution of marks obtained by 200 candidates in an examination which was marked out of 100.

Mark	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	2	5	10	22	43	65	28	20	5

a) Construct a cumulative frequency curve for the data

b) Estimate the median from the graph

c) Calculate the interquartile range

d) Estimate the proportion of candidates who scored 60% or less

3) The table shows the frequency distribution of the marks of 250 students

Mark	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	8	16	21	30	40	55	37	26	12	5

a) Draw a cumulative frequency curve

b) Estimate the median and the inter quartile range

c) The number of student who scored more than 65 marks

d) The pass mark if 70% of the students passed.

4) The table below gives frequencies for the marks obtained by pupils in a test, which was marked out of 20.

Marks	0-2	3-5	6-8	9-11	12-14	15-17	18-20
Frequency	2	4	9	14	19	8	4

a) Draw a cumulative frequency curve for this data

b) Use your graph to find,

i) the median ii) the lower quartile

iii) the upper quartile vi) the interquartile range

Examination Questions

The table below shows the speeds of 100 cars passing through a survey point on a highway.

Speed (v) km/h	$50< v \leq 60$	$60 < v \leq 80$	$80 < v \leq 100$	$90<v\leq 120$	$100< v \leq 120$
Number of Cars (f)	15	12	40	20	x
Frequency density	1,5	0,6	Y	2	y

a) Find the values of x, y and z
b) State the modal class
c) Using a scale of 2cm to represent 10km/h on the horizontal axis and 2cm to represent 0,5 units on the vertical axis, draw a histogram to illustrate the information.
d) Using an assumed mean of 8,5km/h copy an complete the following table and hence estimate the mean speed of the cars.

Speed (v) km/h	Frequency (f)	Deviation D=V-85	Fxd
55	15	-30	-450
70	12		
85	40	0	0
95	20	10	200
110			Total =

Mark (x)	0< x≤ 10	10<x≤20	20<x≤30	30<x≤40	40<x≤60	60<x≤70	70<x≤80	80<x≤90	90<x≤100
Frequency	8	15	P	29	83	20	10	10	5
Cumulative Frequency	8	23	43	72	155	180	q	198	200

Above is a cumulative frequency table for the distribution of marks obtained by 200 students in Geography test.
a) Find the value of p and the value of q
b) Using the scale of 2cm to represent 20 marks on the x axis and 2cm to represent 20 students on the y axis draw the cumulative frequency curve for the distribution.
c) Use the graph to estimate
i) the median mark
ii) the number of students who scored 75 marks or more
d) Calculate an estimate of the mean mark for the top 45 students

3) In an examination taken by 100 candidates, the marks scored were shown in the following table

Marks	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of candidates	6	8	9	10	20	15	11	9	7	5

i) State the modal class
ii) Estimate the median
ii) Estimate the lower quartile .

CHAPTER 23

NUMBER PATTERNS

Number patterns depict a patterns or sequence of events within a compound or structure

Syllabus objectives
Leaner should be able to
a) Solve problems involving number patterns

Patterns can be shown in numbers, shapes etc
They can also be shown in operations such as multiplication

Consider the pattern for the multiples of 11

1^{st}		2^{nd}		3^{rd}
11	$\times$	1	=	11
11	$\times$	11	=	121
11	$\times$	111	=	1221
11	$\times$	1111	=	12 221
11	$\times$	11111	=	122 221

A pattern develops after the first line 11x1, where the two between the two ones are increased by one at each successive step standards.

A closer analysis of the pattern may result with the derivation of the formular which makes it easy to solve the pattern.

Looking at the three column of numbers it can be seen that the 1^{st} column does not change downwards. It remains eleven i.e. 11

On the second column an additional digit of 1 is included in each next step downwards. Also in the same column it can be noted on further analysis that in step 1 there is a single one, step 2, two ones, step 3, 3 ones.
This observation make possible to predict accurately that in for instance step 7 of this column would be 1 111 111 .
Also having observed the pattern for third column we are able to write the step 7 as

$$10 \quad \times 1\ 111\ 111 = 12\ 222\ 221$$

Patterns depicted by shapes etc.
They came also be shown in operation such as multiplication.

Also having observed the pattern for third column we are able to write the step 7

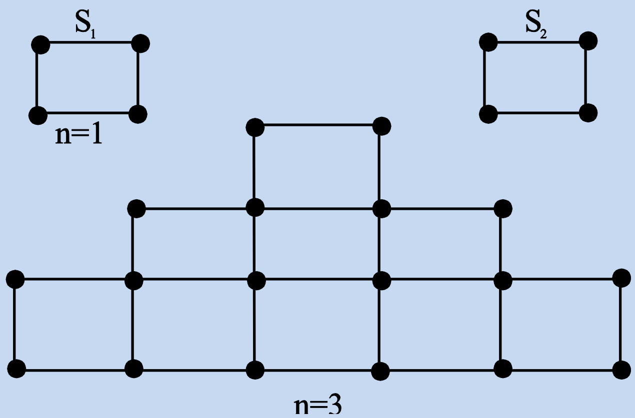
$$11 \times 1\ 111\ 111 = 12\ 222\ 221$$

Patterns depicted by shapes.

S_1 S_2

n=1

n= 1



The letter n represents the number of rows of squares in each shape. The number of squares, S, and the number of dots D, the shape is recorded on the table.

Number of rows (n)	1	2	3	4
Number of squares (s)	1	3	5	7
Number of dots (I)	4	8	12	16

The number of rows increase like the natural counting numbers. The number of squares from odd numbers in their chronological order.

The number of dots are multiplies of 4

Examination Questions

1. Consider the pattern

$1^2 - o^2 = 1$

$2^2 - 1^2 = 3$

$3^2 - 2^2 = 5$

$4^2 - 3^2 = 7$

…………..

…………..

$x^2 - y^2 = 10^1$

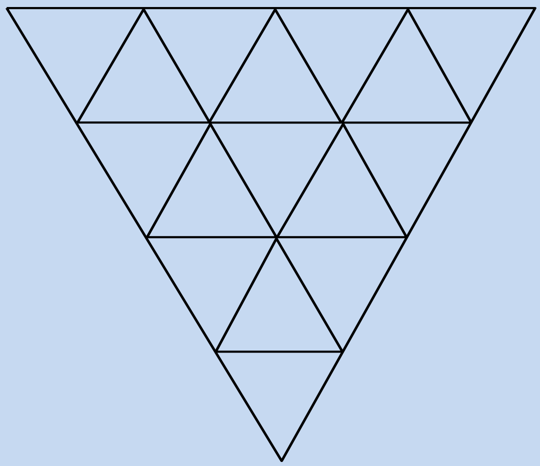
i) Write down the seventh line in the patterns

ii) Find the value of $143^2 - 142^2$.

iii) Find the integer values of x and y which satisfy the equation.

$x^2 - y^2 = 10l$

2. The sticks are arranged to form an equilateral triangles and eventually the shape shown



Row 4, Row 3, Row 2, Row 1

a) Complete the following table that was constructed using the diagram above.

Number of triangle in each row (n)	1	3	5	7
Number of sticks in each row (s)	3	7	11	15
Number of points of which 2 or more sticks meet (p)	3	5	7	9

b) Using the diagram or the table, write down an equation connecting

i) S and n

ii) P and n

iii) S and P

iv) Hence or otherwise, find, S and P when n=21

3) Study the patterns below and answer the questions that follow:

	Column 1	Column 2	Column 3
Row 1	2	2	4
1	3	7	10
2	4	14	18
4	5	23	28
5	6	34	40
6			
.	.	.	.
.	.	.	.
.	.	.	.
17		U	340
.	.	.	.
.	.	.	.

t			W
.	.	.	.
.	.	.	.

i) Write the sixth row of the pattern
ii) Write down the value of u
iii) Express w in terms of t

(ZIMSEC NOV 2005)

4) Study the pattern below

Row

1st	$0 \times 8 + 1 = 1 = 1^2$
2nd	$1 \times 8 + 1 = 9 = 3^2$
3rd	$3 \times 8 + 1 = 25 = 5^2$
4th	$6 \times 8 + 1 = 49 = 7^2$
5th	$10 \times 8 + 1 = 81 = 9^2$
6th	___ _= ___=_____
.	
.	
.	
rth	$p \times 8 + 1 = 441 = 21^2$

a) Complete the 6th row of the pattern
b) Find the values of p and r

(ZIMSEC NOV 2004)

(ZIMSEC 2006)

CHAPTER 24

Consumer Arithmetic

Syllabus Objectives
Learner Should be able to

a) Solve problems involving payments and wages of employees
b) Solve problems involving various types of household bills

Personal Income

A person's pay could be in various form

a) Salary

This refers to a form of payment that comes after a specific period of time, for example, after a month it is a monthly salary, after a year it is an annual salary.

ii) Wage

This refers to a payment that is calculated after a relatively short period of time, for example hourly, daily or weekly wage.

iii) Commission related pay

This form of payment is based on the attainment of set targets. The more the worker surpasses a certain target the more the salary.

A person salary is summarized in a document called a payslip or salary advice.

Items that commonly increase the net salary
a) Allowances e.g. allowances
b) Increments
c) Overtime

Items that commonly reduces the net salary
a) Pension payments
b) P.A.Y.E (Pay as you earn)
d) Bank charges
d) Certain subscriptions and levy etc

PAYE

PAYE tables issued by the tax department are used to calculate the tax to be paid by each employee based on their pay.

MONTHLY RATE 2009

				Rates		
from	-	to	150	multiply by	0%	
from	151	to	500	multiply by	20%	Deduct 28
from	501	to	1,000	multiply by	25%	Deduct 55
from	1,001	to	1,500	multiply by	30%	Deduct 105
from	1,501	to	3,000	multiply by	35%	Deduct 180
from	3,001	to	above	multiplied by	37,5%	Deduct 256

Example

If an employee earns

$ 800 per month

The tax will be calculated thus:

$ 800 x 25%-$55=

$ 145 per month

Example 1
A manager earns $ 1 200 per month. How much income tax does he pay.

Income tax = $\frac{30}{1\cancel{00}} \times 12\cancel{00} - 105$

= $ 255

Exercise 1,1
1) Below

Spec. teach.all	149.00	Transport all	0.00
Housing all	79.00	forex allow	159.00
Pension	73.88	nssa contrib.	100.00
Tchrs un. Zim	6.50	zimta subs	29.88
Zimta Subs	0.75	Fidelity lif	10.00
p.s.m.a.s d	5.00	p.a.y.e	188.33
3% aids levy	5.65		
SAL 747.00 ALL		387.00 DED	322.63

US$ `100 Bank: intermarket head office Acc No. 104441968

Fig 1
Fig 1 above shows an extract of the salary of a civil servant

a) How much income tax was paid by the employee
b) How much money did the employee take home on that month.
c) Show that all deduction amount to $ 322.63
d) Show that all the allowances, excluding the forex allowance amount to $ 387,00
e) How much money was paid towards insurance

2) Use the monthly tables in example 1 to determine the amount of tax paid by each of the following professionals
a) A teacher who earns a monthly salary of $ 480
b) A manager who earns a monthly salary of $ 1 400
c) A chief executive officer who earns a monthly salary of $ 3 250

3) The following is an extract of fortnightly pay as you earn table for February to December 2009

An employee earns $ 480 every fortnight
i) Calculate the tax paid fortnightly
ii) How much does he remain with after paying the tax
iii) Calculate the total tax paid in one month
iv) Hence, determine which payment period results in more tax being paid between monthly pay and fortnightly pay.
b) How much does an employee who earns $ 900 fortnightly pay his tax
ii) How much does she remain with after paying the tax
c) How much does an employee who earns $ 50 pay as tax

4a) An employee in $ 501 to 1000 tax band pays an income tax of $125. Calculate his monthly salary.
b) An employee in the $ 1 501 to $ 3 000 tax band pays an income tax of $ 436. Calculate her monthly salary.

Buying and Selling

Discount
This is a incentive given to the buyer to induce him or her to buy in cash. For example for buying an item costing $ 100 in cash a 25% discount can be given. This results in the buyer paying $ 75 instead of the $ 100.

Hire purchase

This involves allowing buyers to make part payments over a period of time when purchasing costly items. The part payments are called installments. These could be paid monthly over six months, twelve months, twenty four month etc.
In most cases the buyer is asked to first pay a certain percentage as deposit then the remainder is then divided to find the monthly payment. In other cases the buyer does not pay any deposit.

VALUE ADDED TAX (VAT)

This is a form of tax charged on goods. Every stage of production add value to the product hence tax is charged.

Example 2
A stove cost $ 450 cash. It can also be bought by the customer paying 20% and 6 months installments of $ 90.
a) Calculate the deposit paid
b) Calculate the hire purchase price
c) How much more would be paid by hire purchase

a) Deposit $= \frac{2\cancel{0}}{1\cancel{00}} \times 45\cancel{0}$
$= \underline{\$\ 90}$

b) Hire purchase price = ($ 90 × 6) + $ 90
= $ 540 + $ 90
= $ 630

c) Additional amount paid = $ 630-450
= $ 180

Exercise 1,2

1) Find the price if a discount of
a) 15% is given on a cost price of $ 73
b) 8% is given on a cost price of $ 62
c) 7,25% is given on a cost price of $ 48

2) A set of leather sofas have a cash price of $ 1 250. On the purchase, a 20% deposit is paid followed by 12 monthly installments of $ 120.
a) Calculate the deposit paid
b) Calculate the hire purchase price
c) Calculate the percentage increase on paying by hire purchase

3) The selling price of a television set is $ 340. The shop gives a 12 ½% discount for cash. What is the cash price?

4) The hire purchase price of a refrigerator including a deposit of $ 450 is $ 5 730. If the 24 monthly installments are allowed.
i) Calculate the monthly installment paid
ii) If a deposit of 20% the cost price was allowed. Calculate the cost price.
5) A customer pays $ 87,50 cash after get a discount of 12 ½ %. Calculate the original price of the item bought.

6) A new computer cost $ 2 780. 10% discount is given for cash. The hire purchase price of the computer is 20% deposit and 12 monthly installments of $ 185.

a) Calculate the discount price.
b) Calculate the hire purchase price
c) Find the difference between the two prices

7) A retail shop sells at a dozen eggs at $ 1,50 or each at 30c.
a) How much does it cost to buy three dozens in dozens
b) How many eggs using the cost at part (a) can be bought at 30c
c) What would be the savings of buying the eggs in dozens than of unit cost.
8) A pair of jeans cost 32,50 including valve added tax (VAT) charged at 8%. How much tax does the government receive.
9) In electricity value added tax is charged monthly on the total cost incurred on that month at 10%. If the power utility has fixed monthly charge of $ 7,30 and 933 units are used at 6,65c a unit. Calculate the value added tax (VAT).

BILLS

Electricity Bills

Electricity bills have the following components

- Fixed monthly charge
- Energy charges per unit, for the stipulated units and all other units.

To determine the number of units used one has to deduct the previous reading from the present reading

To calculate the total costs one has to multiply the number of units used by the charges as stipulated and add fixed costs and other costs.

Example 1

Standard prices for the supply of electricity to domestic customers.

Fixed monthly charge = $ 330
Energy charges per unit
First 300 units = $ 1,60
All other units = $ 3,50

Below is an extract of Mr. Dumisani Electricity bill for may 2006.

Description	Previous Reading	Present Reading	No. of units	Cost
Energy	66689	67039		
Fixed Monthly charge				300
Sub Total				
VAT 15%				

Using the rates stated above calculate, for the month of May.
a) The number of units of electricity used
b) The cost of the electricity used.
c) the VAT that Mr. Dumisani will have to pay
d) The total amount to be paid

a) Number of units = Present reading – Previous reading
= 6 7039-66689
= 350 units

b) Cost of electricity used = (500 x $ 1,60) + (50 x $ 3,50)
= $ 655,00

c) VAT = 15% x ($655 + $ 330)
= $ 147,75

d) Total Amount = $ 655 + 330 + 147,75
= $ 1 1 22,75

Exercise 1,3

1) Mr. Dlodlo's telephone bill as shown below

Date	**Reading Previous**	**New**	**Amount**
24/12/2004	38739	38351	$ 226 980
	Rent		$ 5 490
	VAT 15%		$ 34 870.50
	Sub Total		$ 267 340.50

a) How many units were used during the month?
b) What is the cost of one unit consumed
c) What is the fixed charge
d) Show that the VAT amounted to $ 34 870,50
e) Hence, show that the total due was $ 267 340

2) Below is the account details for water Meter reading period the month of November 2007 for Mr. Shumba. Amounts are in Z$

Balance brought forward

30062007	0000004658	131992.26 @ 19.00	2061.24

Supplementary				
Vat				(6920.61x0.15)
Water – Fixed Charge				(1x 7984.53)
Water consumed				(490.48.3)
Meter reading	Previous	New	Consumption	
PU10170	21622	21685	63	
** Interim reading **				
Sewage Fixed-VAT				
Sewage HD Domestic				
Meter Reading	Previous	New	Consumption	
PU10170	21622		63	
Solid Waste Management – VAT				(1x4310)

NOW DUE AND PAYABLE

This statement shows payments and before 15/08/2007
Payment is due by 24/08/2007

a) Calculate the cost of 1 unit of water consumption
b) Calculate the total cost of water only for the month of November
c) Calculate the cost of 1 unit sewage Domestic
d) Calculate the total cost of sewage and solid waste of the month of November
e) Hence, or otherwise show that payment date for November is $ 264 929.50

PAST EXAMINATION QUESTIONS

1) The electricity bills of a certain household for the month of December 2004 and January 2005 are shown below.

December 2004

Description	Previous reading	Present reading	Consumption	Rate cents	Total $ c
Balance b/f					3 150,99
Payment					3 151.00 CR
Energy charge	31 565	31 834	269	m	1 894,57
Fixed Monthly Charge					530,49
Value Added Tax (VAT)					363,76
Amount Due					2 788,81
January 2005					
Balance b/f					2 788,81
Payment					5 000.00 CR
Energy Charge	31 834	32 331	n	1909	9 496,84
Fixed monthly Charge					1 067,69
Value Added Tax (VAT)					1 253,00
Amount Due					9

a) Find the values of m, n and q

b) Calculate

i) the rate of which Value Added Tax (VAT) was changed in December

ii) the percentage increase in the monthly fixed charge for the two months.

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2a) In 2004 a motorist filled up his fuel tank with 45 litres of fuel bought of $ 3 450 per litre

i) Calculate the amount he paid for the fuel.

ii) If the price of fuel per litre included 15% Value Added Tax (VAT), calculate the price of fuel per litre

iii) The motorist then used 12,5 litres in travelling a distance of 196 kilometers. Calculate the rate of fuel consumption of the car in kilometers per litre.

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CHAPTER 25

GRAPHS: GRADIENT

Gradient is a measure of the slopeness or steepness. Things that we see everyday like hills, mountains, roads etc have different gradients. The focus on this chapter would be on the gradient of a straight line. Curves will also be covered as the last aspect.

Syllabus objectives

Learner should be able to:

a) Calculate the gradient of a straight line from its given coordinates.
b) Interpret and obtain the equation of a straight line in the form y=m x+c.
c) Identify parallel straight line using gradients
d) Estimate the gradient of a curve by drawing a tangent at a given point.

Gradient of a straight line

Consider the diagram below.

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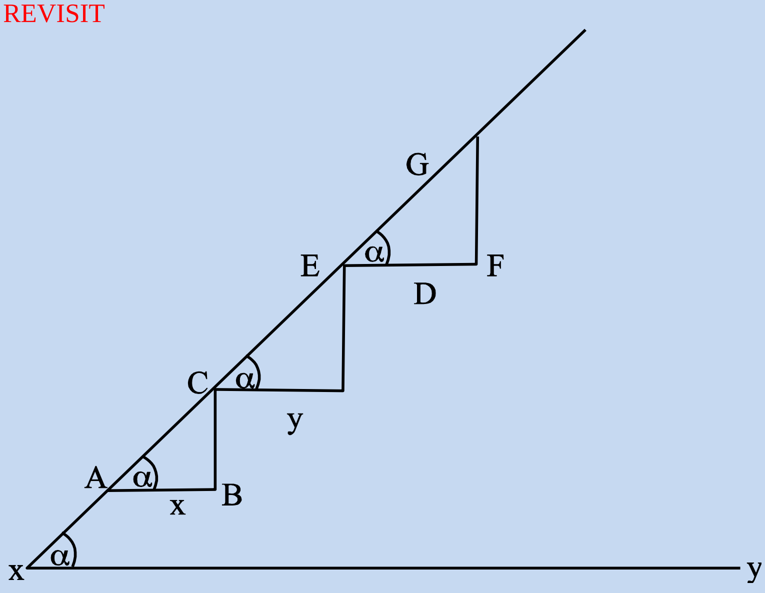


If xy is a horizontal line and makes an angle ∝ with the XG. The triangles ABC, CDE and EFG are similar.

Hence $\frac{BC}{AB} = \frac{DE}{DC} = \frac{GE}{FE}$

Each of these rations are a measure of the gradients of the line XG. The gradient of XG at A is the same as at C and also the same as at E. This therefore, means that the gradient of a straight line is the same at any point on it.

Tan $\propto = \frac{BC}{CB} = \frac{DE}{DC} = \frac{GF}{EF}$, so tan ∝ is also a measure of the gradient.

Taking tan $\propto = \dfrac{BC}{CB}$

BC shows a vertical increase from a point B to a point C. it is an increase in y axis AB shows a horizontal increase from a point. A to a point B. it is an increase in x axis.

Tan $\propto = \dfrac{\text{Increase in y from B to C}}{\text{Increase in x from C to B}}$

But tan x = gradient of AC

:-Gradient of AC $= \dfrac{\text{Increase in y}}{\text{Increase in x.}}$

Example 1:
Find the gradient of the lines A,B,C and D in the diagram below:

Insert:

REVISIT Graphy

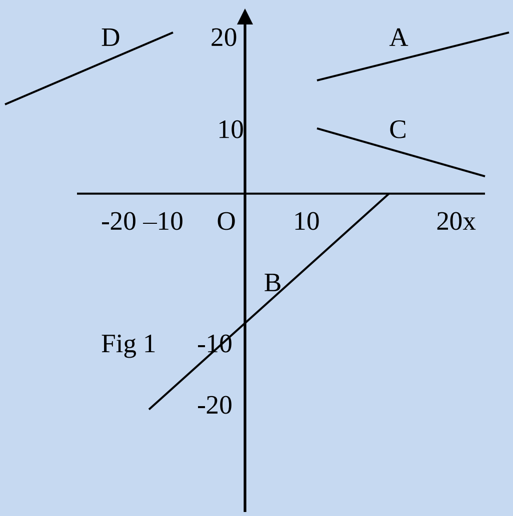


Fig 1

a) Gradient of line A $= \dfrac{\text{Increase in y}}{\text{Increase in x}}$

$= \dfrac{5}{10}$

$= \dfrac{1}{2}$

b) Gradient of line B $= \dfrac{\text{Increase in y}}{\text{Increase in x}}$

$$= \frac{15}{10}$$

$$= 1\frac{1}{2}$$

c) Gradient of line C $= \frac{\text{Increase in y}}{\text{Increase in x}}$

$$= \frac{8}{-12}$$

$$= \frac{-2}{3}$$

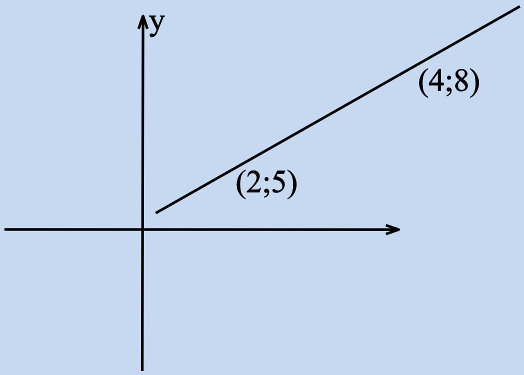
d) Gradient of line (i) $= \frac{\text{Increase in y}}{\text{Increase in x}}$

$$= \frac{5}{15}$$

$$= \frac{1}{3}$$

Take Note
For line C the gradient is negative because as y increase the value of x decrease horizontal. So when one variable increases as the other decreases the gradient is negative.

Calculating Gradients given the points



Consider a line which passes through the points (2;5) and (4;8).

Gradient $= \frac{\text{Increase in y}}{\text{Increase in x}}$

$$= \frac{8-5}{4-2}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

Generally, the gradient of a line passing through a pair of points $(x_1;y_1)$ and $(x_2;\ y;_1)$ is given by

Gradient = <u>Increase in y</u>
Increase in x

= <u>Differences in the y coordinates</u>
Differences in the x coordinates

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

Take note. In the formular either pair of coordinates can take $(x_1;\ y_1)$ or $(x_2\ ;\ y_2)$

<u>Exercise 1.1</u>
<u>Insert:</u>
<u>REVISIT Graph</u>

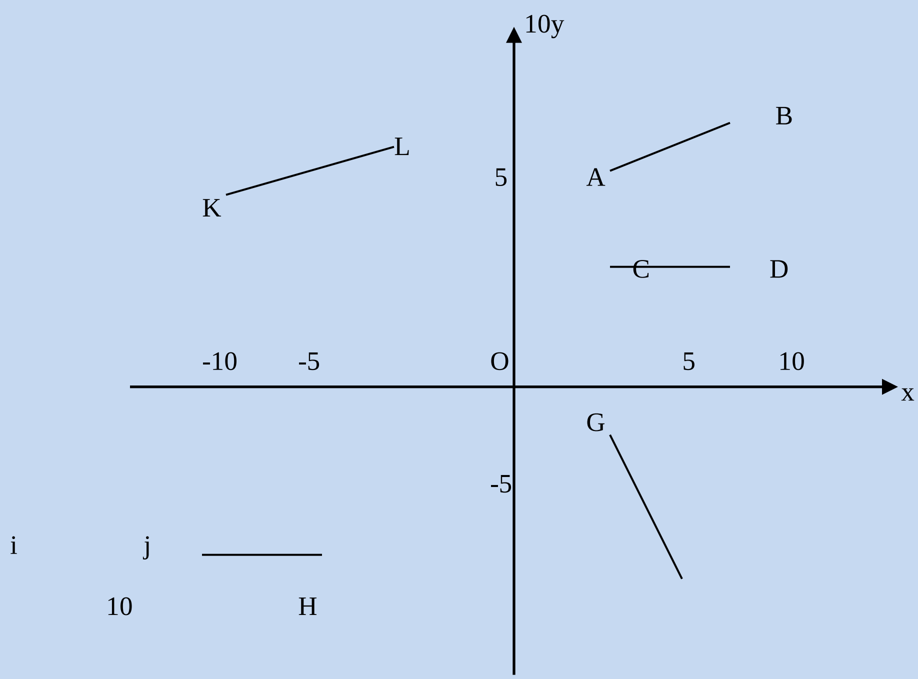


Fig 2

1) In Fig 2 Calculate by taking measurement the gradients of the given lines.
2) Find the gradient of lines joining the following pairs of points.

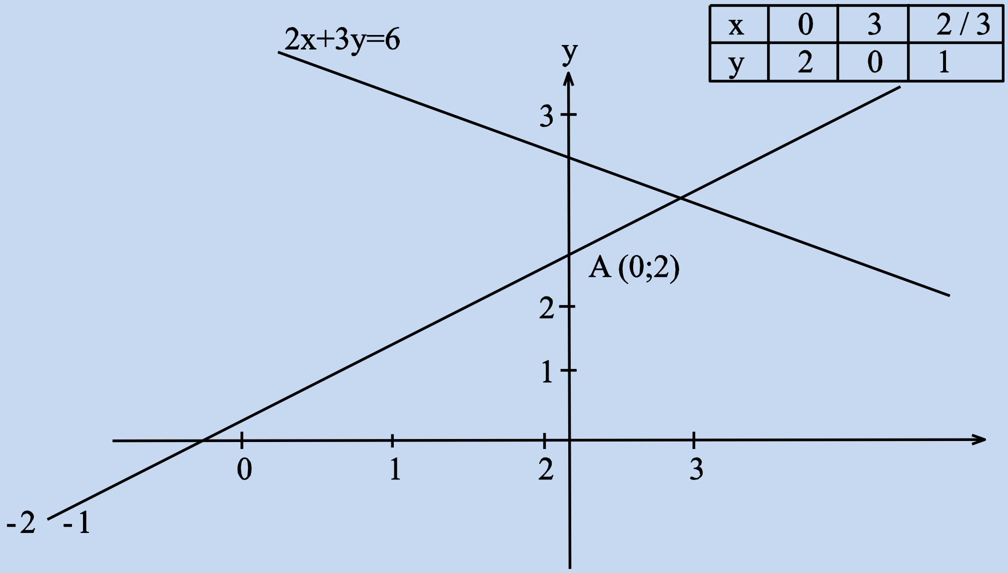
a)	(8;6); (1;4)	b)	(2;5); (4;8)
c)	(5;2), (0,0)	d)	(0,0), (10,8)
e)	(0;5); (-5;0)	f)	(-3,3); (10;8)
g)	(-4;3); (8;-6)	h)	(-3;--3), (-1;5)

<u>Sketching Graphs of Straight Lines</u>

An equation 2 x +3y = 6 can be drawn on the graph as shown below

REVISIT

x	0	3	2 / 3
y	2	0	1



The above diagram show that the graph crosses y axis at point A. where the value of y= 2 when x =0.

Calculating the gradient of the line

$$\text{Gradient} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$= \frac{2-0}{0-3} \qquad \text{Using points A (0,2) and B (3,0)}$$

$$:- \quad \text{Gradient} = \frac{-2}{3}$$

Making y the subject of the formula of the equation ; $2x + 3y = 6$

$$\frac{3y}{3} = \frac{6}{3} - \frac{2x}{3}$$

$$y = 2 - \frac{2}{3}x$$

$$y = \frac{-2}{3} \times +2$$

Comparing the equation with results on the graph shows that:

j) $\frac{-2}{3}$ on the rearranged is equal to the gradient of the line.

ii) +2 on the rearranged equation is equal to line value where the graph crosses the y axis (point A). which is called the y intercept.

Conclusion

When y is the subject of the equation the coefficient of x gives the gradient and the other value gives the y Intercept.

Generally, an equation of a straight line is in the form y = mx +c. where m is the gradient, and c is the y Intercept.

Example 2

Sketch the graph of the line whose equation is 3x +7y =21

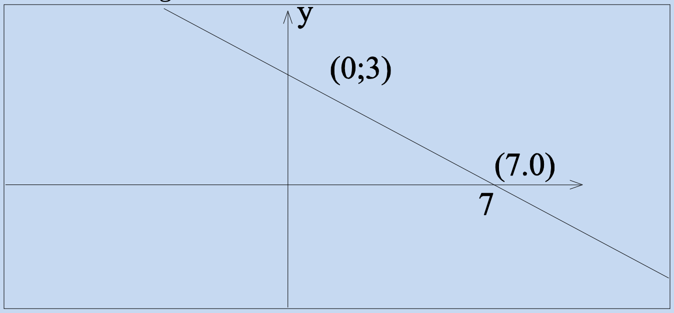
$y = \frac{-3x}{7} + 3$ (Reducing it to the form of an equation of a straight line i.e. y= mx +c.

From equation y intercept is +3. i.e. (0;+3)

Since two pairs of coordinate are needed to sketch.

When y = 0,x = 7

Then sketching **REVISIT**



Brief Summary

- On sketching a straight line two pairs of coordinates are required of which one is provided by the y intercept in the equation of a straight line.

- The other coordinates can be calculated by using any value of x to find the corresponding value of y. it is advisable to find the value of x when y=o (i.e. x intercept).

Exercise 1,2

1) For each of the lines represented by the following equations, write down
i) the gradient
ii) the y intercept
iii) the coordinates of the y intercept.

a) $y = 2x + 4$ b) $y - 7 = 2x + 2$

c) $3y = 21 - 2x$ d) $3x + 2 = -2y$

e) $3x + 7y = 5$ f) $4x + 3y = 2$

2) Sketch the graphs of the following equation.

a) $3x + 2 = -2y$ b) $2x + y = 5$

c) $2x - y = 8$ d) $2-5y-9=0$

e) $5x - 2y = 0$ f) $4x-2y+1=0$

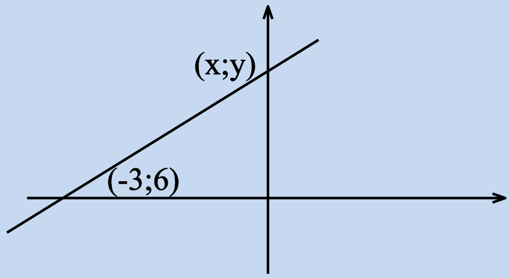
g) $4x - 3y = 5$ h) $7x + 4y - 8 = 0$

Finding the equation of a straight line

a) Given its gradient and a point on the line.

Example 3

Find the equation of straight line of gradient 7 passing through point (-3;6)



Rough Sketch
Other passing point unknown

Gradient $= \dfrac{Y_2 - Y_1}{X_2 - X_1}$

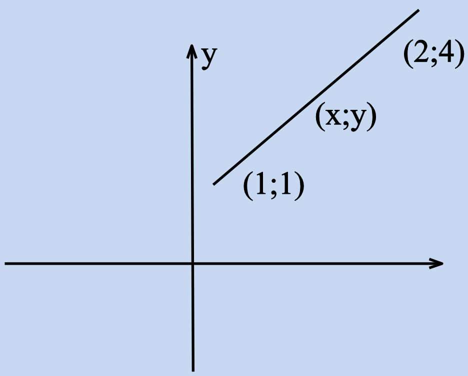
$7 \quad = \quad \dfrac{y-6}{x-(-3)}$

$7 \quad = \quad \dfrac{y-6}{x-3}$

$7(x+3)= y-6$
$7x +21 =y-6$
$7x + 21+6=y$
$7x +27=y$
: - $y=7x +27$
:- Equation of the straight line is $y= 7x +27$

2) **Finding the equation of a straight line**
Given two points on the line

Example 4
Find the equation of a straight line that passes through points (2;4); (1,1)



Find the gradient

$$\text{Gradient} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$= \frac{4-1}{2-1}$$

$$= \frac{3}{1}$$

:- Gradient= 3

Use an unknown other point (x;y) and any of the given points to find equation.

Using (1;1) and (x;y)

Gradient = $\frac{Y_2 - Y_1}{X_2 - X_1}$

$\frac{3}{1} = \frac{x-1}{x-1}$

$3(x-1) =$ y-1
$3x$ -3+1 =y
$3x$ –2=y
:- y= $3x$ -2

Take Note

The gradient on a straight line is the same everywhere on the line.

Parallel Lines

Parallel lines have the same gradient

Exercise 1,3

1) Find the equation of the line which passes through the point
a) (0;0) and has a gradient of 4
b) (1,1) and has a gradient of 2
c) (-2,4) and has a gradient of -1
d) (-1;2) and has a gradient of $2\frac{1}{2}$
e) (0;-5) and has a gradient of –3/4

2) Find the equation of the line, which passes through the points

a) (2;4), (1,1) b) (3;p) (1;1)
c) (0;4); (2p,9) d) (3;9), (7;-1)
e) (-3;3), (10,8) f) (0,0), (-3,5)

3) The line y= $2x$ +k pass through the point (1;2).
i) By substitution x =1 and y =2, find the value of k.
iii) Find c so that y= $3x$ +c passes through (-3,4)

4) Write down an expression for the gradient of a line joining the points (5,2) and (3;k).
Find the value of k if his gradient is $\frac{3}{4}$
Another line is parallel to the one in (a) and passes through (-5,-2). Find its equation.

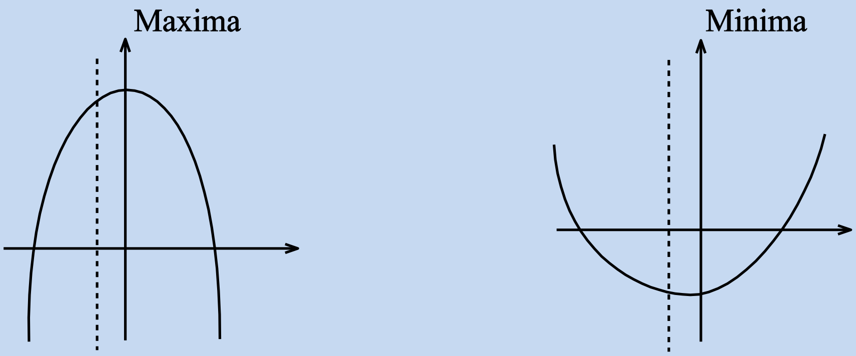
5) Two lines m and n are parallel. If m passes through (0;5) and n has gradient $-\frac{2}{7}$ and passes through (-5;0), find the equations of the two lines.

6) The points (2;3), (5,2) and (3;k) lie on the same line. Find the value of k.

The Gradient of a curve

On a straight line the gradient at every point along the line is the same. With a curve the value of the gradient changes from one point to the other.

To get the gradient at a point, a tangent to the curve is drawn to the point and the gradient of the tangent give the gradient at that point.



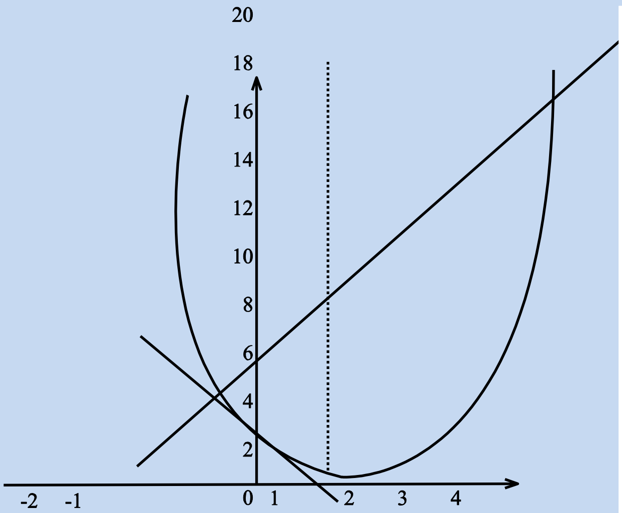
In a curve maxima and minima occur at the turning points where the gradient is Zero. When the graph is facing upwards we have a minima. When facing downwards we have a maxima.
The line that divides the graphs into two equal halves is called the line of symmetry.

Example 5
Draw the graph of y= 2 x^2- 4 x +3. Using a scale at 2cm to 1 unit on the x- axis and 1cm to 2 units on the y- axis. From your graph, find

a) The equation of the line of symmetry of line curve.
b) The gradient of the curve at $x = 0$
c) The minimum value of y

REVIST

x	-2	-1	0	1	2	3	4
y	19	9	3	1	3	9	19



a) Line of symmetry 1 x=1
b) Gradient at x=0
Gradient =Gradient of targent
=y-y
2 1
x2-x
=6-0
-1,1-1
=6-0
-1,1-1
=-6-0
-2,1
=-60
21
=-20
21= -2,86

when plotting use a share pencil to produce a smooth curve.
The tangent should just touch the point.

Exercise 1,4

1) a) Copy and complete the table of values for y= 3 + 5 x - 2 x^2

x	-2	-15	-1	-0,5	0	0,5	1	1,5	2	2,5	3	3,5	4
y	-15	-9			3	5				3			-9

b) Draw the graph of y= 3 + 5x- $2x^2$ using suitable scales

c) Use the graph to find the gradient of curve at

i) x = 0,5 ii) x = 1,3.

2) Copy and complete the table of values for y = 2 x - x^2 +1

x	-3	-2	-1	0	2	3	4	5
y	14	-7			1	-2		-14

b) Draw the graph of y= 2 x - x^2+1

c) Find the gradient of the curve at x =3

Examination Questions

1. Write down an expression for the gradient of the line joining the points (6;k) and (4;1). Find the value of k if this gradient is 3/5.

b) Find the equation of the line through the point (-4;5) with gradient –2 (Cambridge).

2a) Find the equation of the straight line which passes through the points (0;3) and (3;0)

b) Show that the equation of the straight line which passes through (o,c) and (c,o) is x +y = c.

3. The following is an incomplete table of values for the function y= x^3- 6 x^2 +3 x +10.

x	-1,5	-1	0	1	2	3	4	5	5,5
y	-11,4	0	10	P	0	-8	-10	0	1,14

a) Calculate the value of P.

b) Using a scale of 2cm to represent 1 unit on the x- axis and 2cm to represent 5 units on the y-axis, draw the graph of y= x^3-$6x^2$ +3x +10 for $–1,5 \leq x \leq 5,5$.

c) Use the graph to write down

i) The coordinates of the maximum turning point of the curve.

d) Find the gradient of the curve when x = 4

(ZIMSEC NOV 2005)

LINEAR PROGRAMMING

It is a method of solving problems involving two variables that are subject to certain conditions using the graphical representation of inequalities.

Syllabus objectives

a) Derive inequalities from a given situation

b) Show clearly the region, which contains the solution set.

c) Use the graph to solve the problem.

Remember the following points from the previous chapter on Inequalities to show the wanted region.

a) Re-write the inequalities as equations.

b) Establish two pairs of coordinates to plot and draw the appropriate line. (either bold or broken)
c) Test for each inequality to shade the unwanted regions.
d) Make sure the boundaries (lines drawn) cross one another.
e) Shade the whole space to the ends of the graph paper to indicate the wanted region clearly.

Take note

a) All points on solid boundaries are in the wanted region.
b) All point on broken line are not in the wanted region

Exercise 1,1 (Revision)

1) Shade the unwanted region of the following inequalities
a) $y < x$, $x \leq 4$, $y > 2$
b) $x > 1$, $y < x$, $3y > 2x - 15$
c) $x \geq 0$, $y \geq 0$, $x + y \geq 2$
d) $x + y \leq 4$, $x - y \leq -2$; $2x + y \geq 2$

Linear Programming

Example 1
A manager of a warehouse intends to order some chairs. Some of them are lounge chairs and the rest are dining chairs. He requires at least 100 lounge chairs and at least 200 dining chairs but he does not wish to have more than 600 chair altogether. A lounge chair takes up four units of storage space whilst a dining chair occupies one unit of storage space. The maximum storage space available is 1 500 units.

If x is the number of lounge chairs and y is the number of dining chairs, write down the four inequalities which represent restrictions. Draw a graph and clearly indicate the region which represents the possible values of x and y.

The profit on a lounge chair is \$20 and that on a dining chair is \$7. Write down an expression for the profit \$P and find the values of x and y which give the maximum profit.

Solution

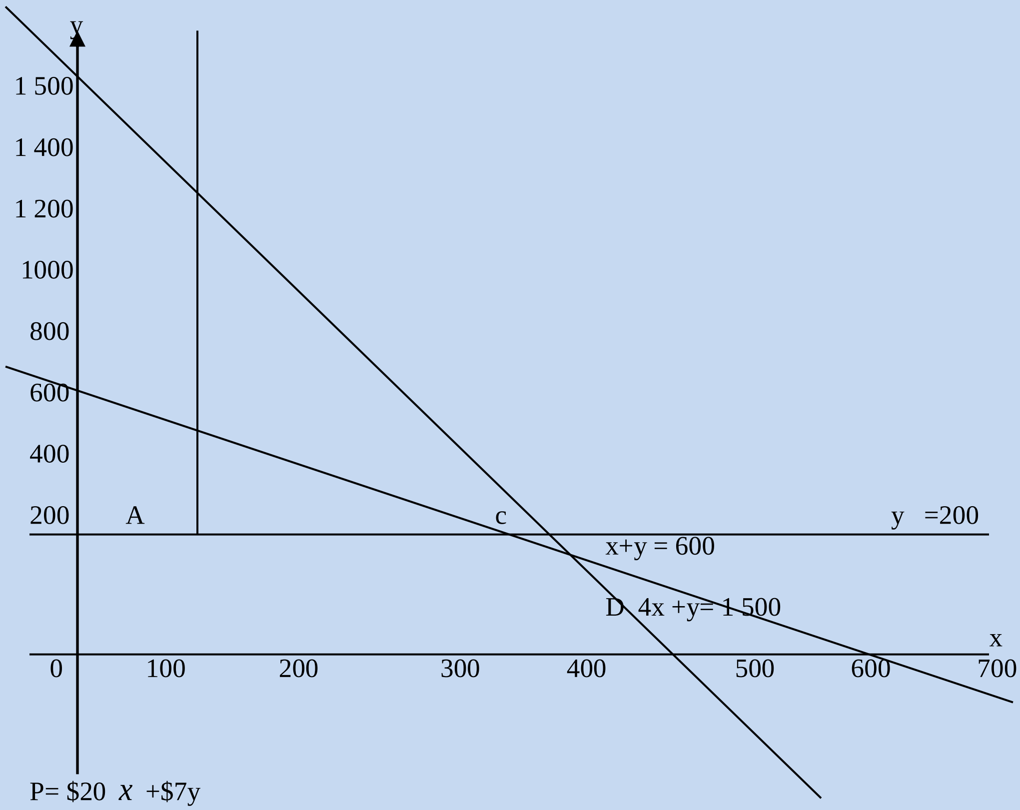
Four inequalities

1) $x \geq 100$ 2) $y \geq 200$

3) $x + y \leq 600$ 4) $4x + y \leq 1\ 500$

Insert:

REVISIT Graph



P= $20 x +$7y

All points in the unshaded region are the possible combinates of lounge (x) and dining chairs.

The combination which gives line highest value for P is required
Profit Margin
$P= $20 x +$7y

The corner points are usually the good points for testing. All the four corner points are

A (100; 200).	P	= 20 (100) + 7 (200)
		= $2 400
B (100, 500)	P	=20 (100) + 7 (500)
		= $2 500
C (370;220)	P	=20 (370) + 7 (220)
		= $8 940
D (375;200)	P	=20 (375+ 7(200)
		= $8 900

:- Point C gives the maximum profit. When 370 lounge chairs and 220 dining chairs.

Brief Summary

Points on the broken line are not part of the solution set.

- Try to use the whole sheet of paper and for each line shade a large area in the unwanted region.
- The corner points on the boundaries usually give the maximum and minimum combinations.

Common terms to look out for in word problems

Word	**Meaning**
at least 5	$x \geq 5$
A maximum of 20	$x \leq 20$
A minimum of 3	$x \geq 3$
More than 4	$x > 4$
Less than 10	$x < 10$
20 at most	$x \leq 20$

Exercise 1,2

1. A canning factory employs x unskilled workers and y skilled workers.

a) Express the following information as inequalities

i) The total number of worker is at least 75

ii) Unskilled workers are paid \$160 an hour, skilled workers are paid \$240 an hour and the total wage bill is less than \$14 400 per hour.

iii) The number of skilled workers is at least one quarter of the number of unskilled workers.

b) Represent these inequalities on a graph

c) Use your graph to find the greatest number of (a) unskilled, ii) skilled workers that would be employed.

2. Mr. Shumba is the manager of an organization in Harare and he has decided to buy some new desks and chairs for his staff.

a) He decides that he needs at least 5 desks and at least 10 chairs but he does not wish to have more than 25 items of furniture altogether.

Taking x to be the number of desks and y the number of chairs, write down three inequalities, other than x > O and y > O, which satisfy these conditions.

b) Each desk will cost him \$120 and each chair will cost him \$80. He has a maximum of \$2 400 to spend altogether. Write down another inequality in x and y which satisfy these conditions and show that it reduces to $3x + 2y \leq 60$.

c) The point (x, y) represents x desks and y chairs. Using 2cm to represent 5 desks on the x - axis and 2cm to represent 5 chairs on the y axis draw x and y axis for $O < x < 30$ and $O < y < 30$.

d) Write down the possible combinations of desks and chairs in which:

i) The number of desks and chairs is equal

ii) The number of chairs is three times the number of desks

Examination Questions

1) A summer gala is being held in a village to raise funds for a school and one lady offers to make cushions and table clothes. One cushion requires 50 minutes preparation time and 75 minutes of machine time. One tablecloth requires 60 minutes of preparation time and 45 minutes of machine time. The lady makes x cushions and y tablecloths.

a) Given that at least 2 ½ hours is spent on preparation and that the machine is available for a maximum of 15 hours show that $5x + 6y \geq 75$ and $5x + 3y \leq 60$.

b) Given also that the total preparation time is less than or equal to the total machine time, show that $y \leq \frac{5}{3}x$

c) The point (x,y) represents x cushions and y tablecloths. Using a scale of 1cm to one cushion on the horizontal and 1cm to one tablecloth on the vertical axis. Construct and indicate clearly by shading the unwanted regions, the regions in which the point (x,y) must lie.

d) The profit from the sale of each cushion is $4 and that from each tablecloth is $2. Use your graph to find the maximum profit which the lady can make. (Cambridge, June 1985).

2) Mr. Hove Manufactures table and chairs using softwood and hardwood.

A table requires 5 metres of softwood and 3m of hardwood.

A chair requires 45m of softwood and 48m of hardwood.

Let x be the number of tables made and y be the number of chairs made.

a) Using the above information, write down two inequalities other than $x > 0$ and $y > 0$ in x and y, which satisfy these conditions.

b) In order for Mr. Hove to make a profit, he should manufacture more than 2 tables and at least 4 chairs. Write down two inequalities, one in x and the other in y, which satisfy these conditions.

c) The point (x;y) represents x tables and y chairs manufactured. Using a scale of 2cm to represent 2 tables on the horizontal axis and 2cm to 2 chairs on the vertical axis. Draw the axes for:
$0 < x < 16$ and $0 < y < 16$.
Indicate clearly by shading the UNWANTED REGION, the region in which (x,y) should lie,

d) Use your graph to write down all possible combinations which give the maximum number of tables and chairs manufactured.

ZIMSEC JUNE 2008

CHAPTER 26

Revisit the whole chapter

CIRCLE GEOMETRY

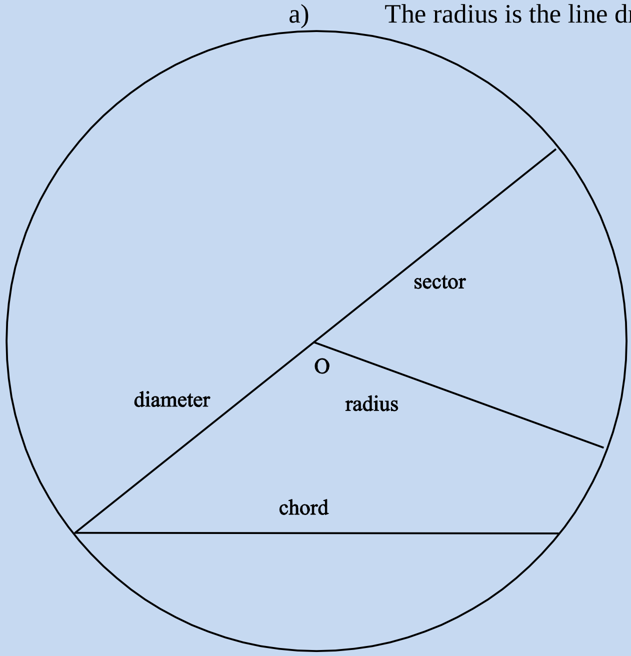
Syllabus Objectives

Leaner should be able to:
a) Define and name the various parts of a circle and its regions.
b) State all the required circle theorems
c) Solve problems using the appropriate theorems

Parts of a circle

Major arc

If O is the center of circle shown.
a) The radius is the line drawn from the circumference to the center



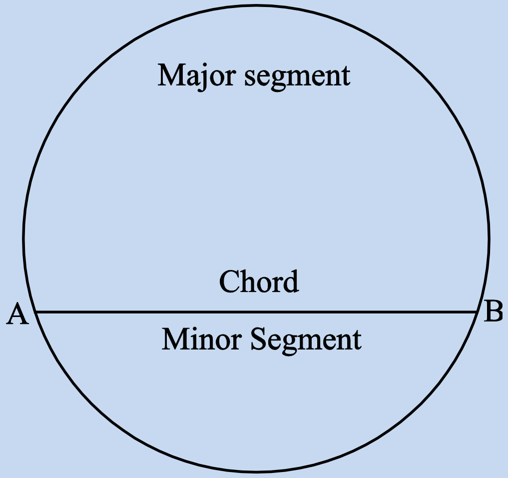
b) The circumference is the outer boundary forming the circle.
c) An arc is any incomplete circumference

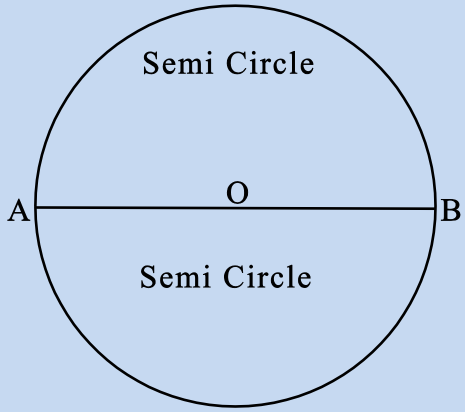
a) The diameter is a straight line drawn from the circumference through the center to the circumference on the other side.
b) The chord is a straight line drawn from the circumference to the other side on the circumference without passing through the center.
f) The sector is a region bounded by two radii and an arc.

Region in a circle

arc

A segment is a region bounded by the chord and the arc
The chord and minor arc AB form a minor Segment.
The chord and the major arc AB form a major segment





If AB is a diameter which divide
the circle into two equal halves.

the two regions formed are called
semi-circles
Semi means half

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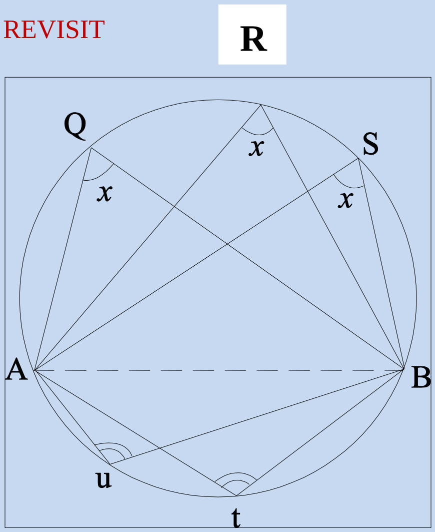


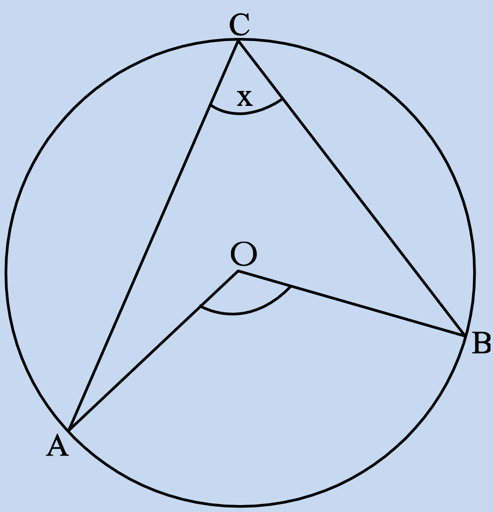
Fig 1

In Fig 1 AB is a chord Q, R, S, T and U are points on the circumference of the circle. Angles AQB, ARB and ASB are in the major segment. They are subtended by minor arc AB Angles AUB and At B are all in the minor segment. They are subtended by the major arc AB.

Angle AQB= ARB= ASB
Angle AUB = AT B

Theorem 1

All angles subtended by the same arc are equal or Angles in the same segment are equal.



in fig 2, if O is the center of the circle. Angle AOB is a a center and is subtended by arc minor arc AB.

Angle ACB is at the circumference and is also subtended by arc AB by arch AB. angle AOB = 2 ACB.

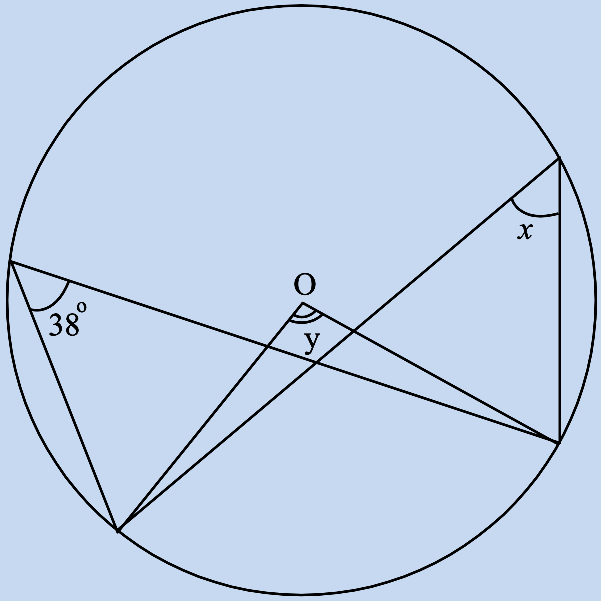
Fig 2

Theorem 2

Angle at the center is twice the size of the angle on the circumference when subtended by the same arc.

Example 1

In the diagram O is the center of the Circle.
Find the value of x and y.



a) $x = 33^0$ (angles subtended by the same arc are equal

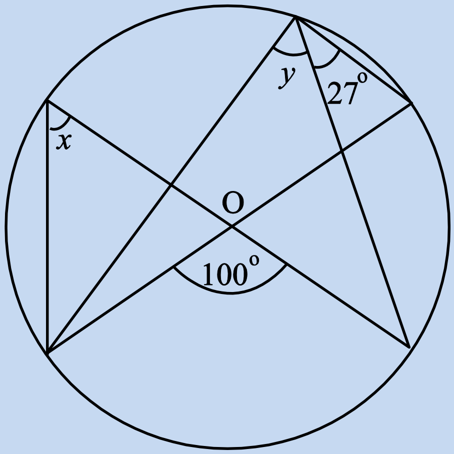
b) $y = 2 \times 33^0$ (angles at the center is twice) at the circumference

c) $=66^0$. When subtended by the same arc.

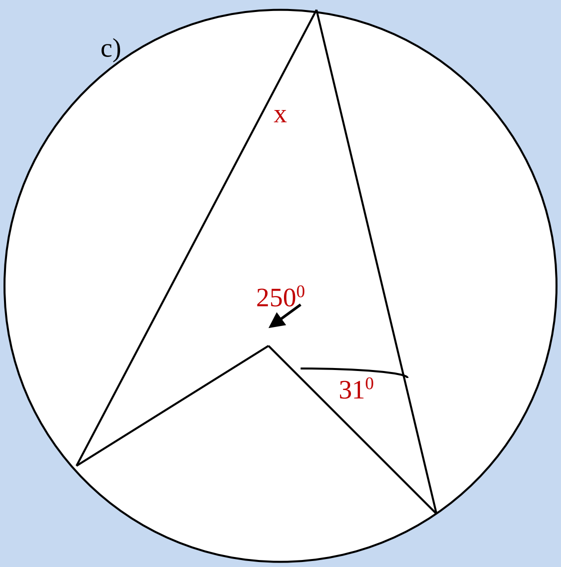
Exercise 1,1

Find the angle marked with letters given that O is the center. State clearly your reasons for each step.

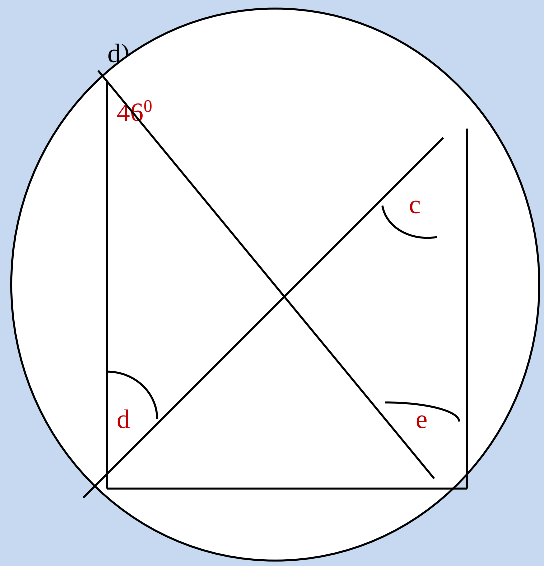
a)



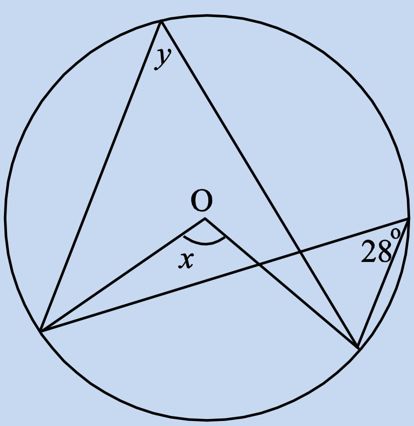
c)

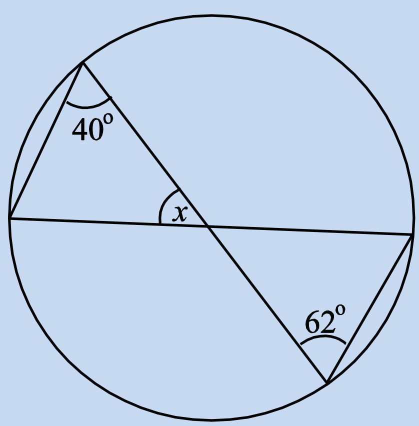


d)

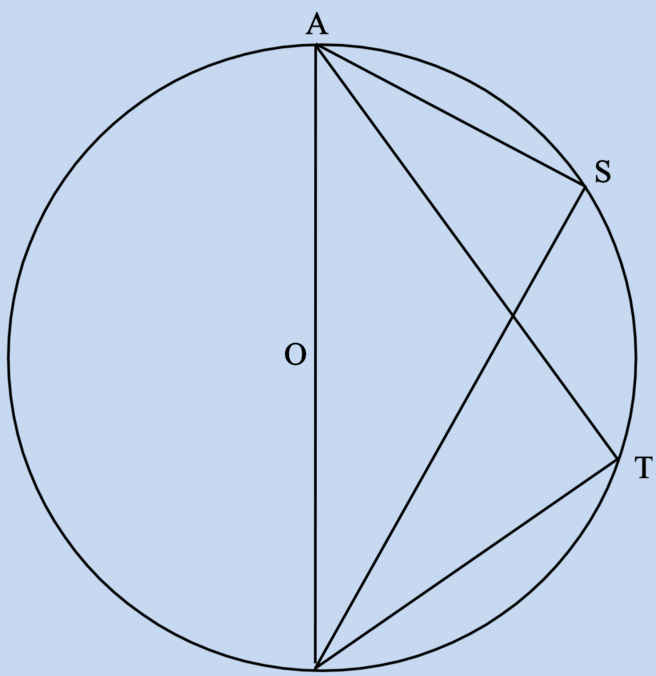


Hint: Make imaginary constructions of radius. Base of isosceles triangle are equal.





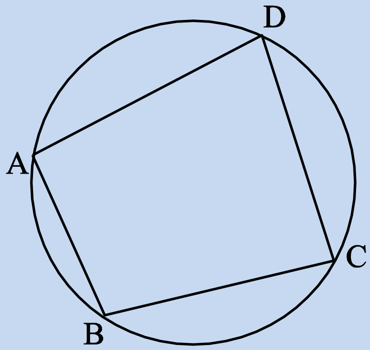
Theorem 3
The angle in a semicircle is a right angle.

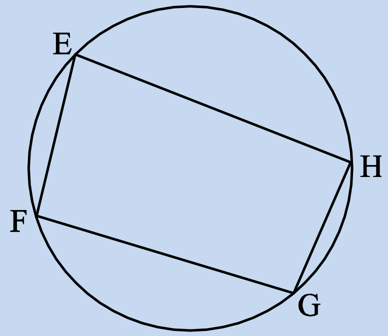


If point O is the centreof the circle. Angles ASB and ATB are in a semi-circle hence are all right angles.

Cyclic Quadrilateral

When all the vertices of a quadrilateral inside a cycle touch the circumference of the cycle a cyclic quadrilateral is formed.

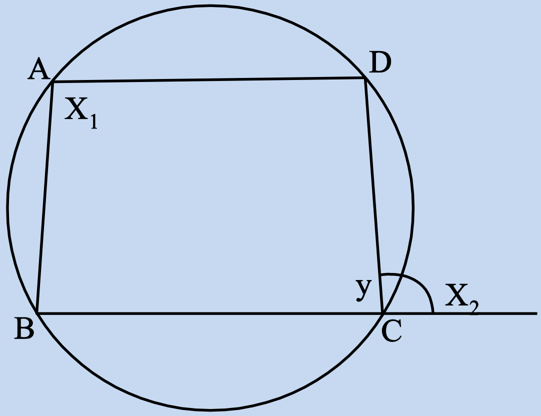




In cyclic quadrilateral ABCD. Angle ADC which is equal to x is in a major segment while ABC which is y is the minor segment.
Angle x and y are opposite angle of a cyclic quadrilateral.

Theorem 4

The opposite angles of a cyclic quadrilateral are supplementary (add up to 180^0)
Angles in opposite segments are supplementary.



If BC is produced in cyclic quadrilateral ABC an exterior angle BCX is formed. If x_2 represents exterior angle and y and x_1 , opposite interior angle then.

(1) $x_1+y = 180$ (opposite angle of a cyclic quadrilateral add up to180
(2) $x_2 = 180-y$ (angles in a straight line add up to 180.

From (1) $y= 180-x_1$ but $x_2 = 180-y$
:- $x_2= 180- (180-x1)$
:- $x_2 = x$

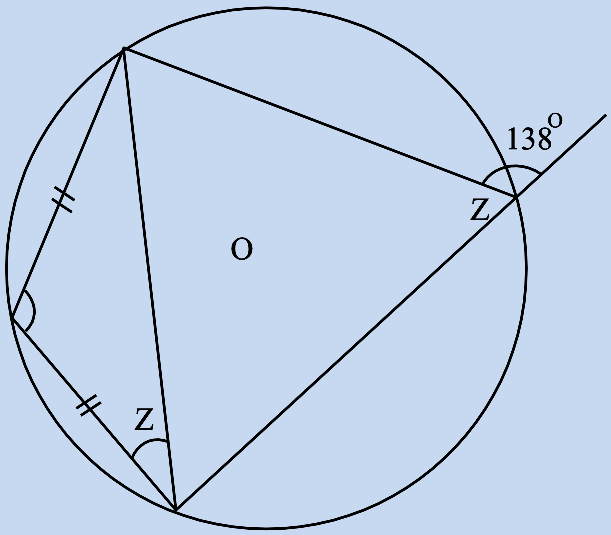
Angle BAD = DCX

Theorem 5

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Example 2

Calculate the marked angles O is the centre of the circle.



i) Z= 18^0- 138^0 (angles is a straight line add up to 180^0)
= 42^0

or

ii) y = 138^0 (Exterior angle is equal to the interior opposite angle).
y +z = 180^0

y+ z= 180^0 (Opposite angles of a cyclic quadrilateral are supplementary.

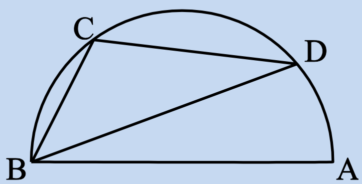
y+ 42 = 180^0
: - y =138^0

$x^0 = \frac{180^0 - 138^0}{2}$ (Base angles of an isosceles triangle are equal)

$= \frac{42^0}{2}$

$= 21^0$

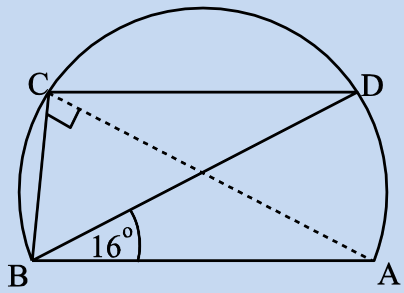
Example 3

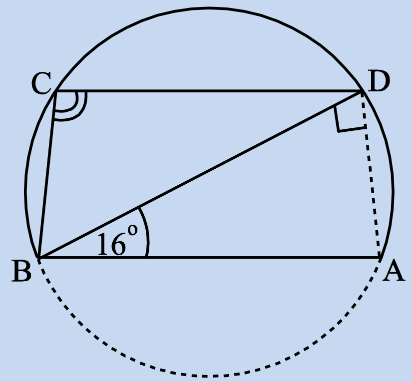


AB is a diameter of Semi Circle ABCD. If angle ABD= 160^0, Calculate angle BCD.

Solution

Make constructions





Join CA.

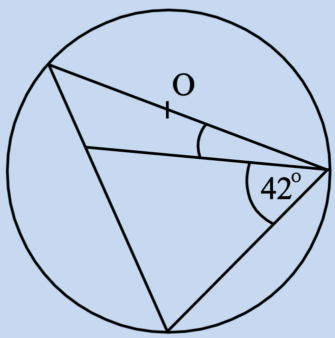
Angle BCA = 90^0 (Angles in a semicircle are right angles)

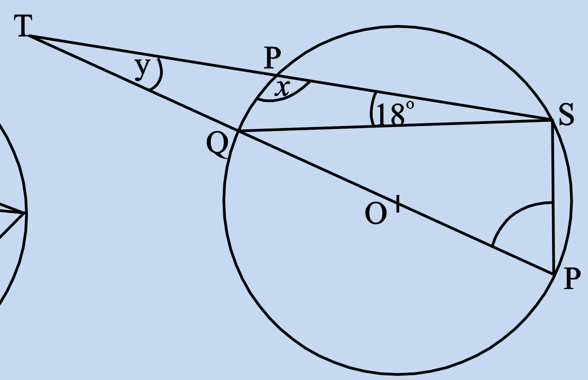
Angle ACD = 16^0 (angle subtended by the same arc are equal)
:- angle BCD = $90^0 + 16^0$
$=106^0$

Join DA

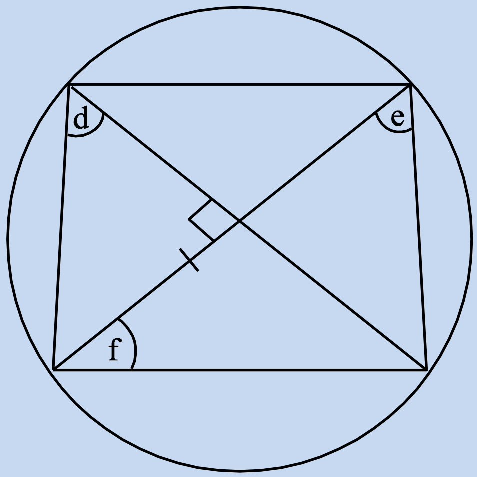
Angle BDA= 90^0 (angle in a semi circle are right angles.
Angle BAD=180- (90+16) (angle in a triangle
= 74^0
Angle BCD = 180^0- 74 (opposite angles)
= 106^o of cyclic quad.

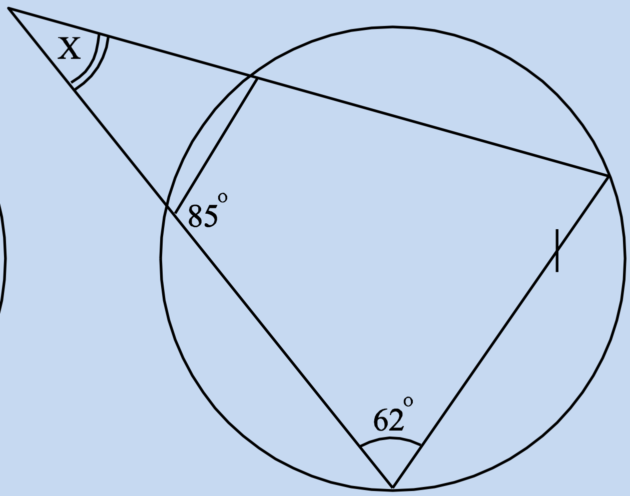
Exercise 1,2
Find the angle marked with letters. Stating clearly your reasons for each step. O indicates the centre of the circle.



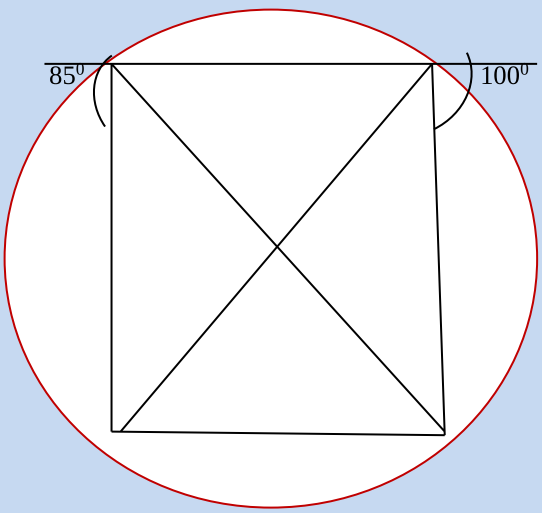


Hint: Make constructions

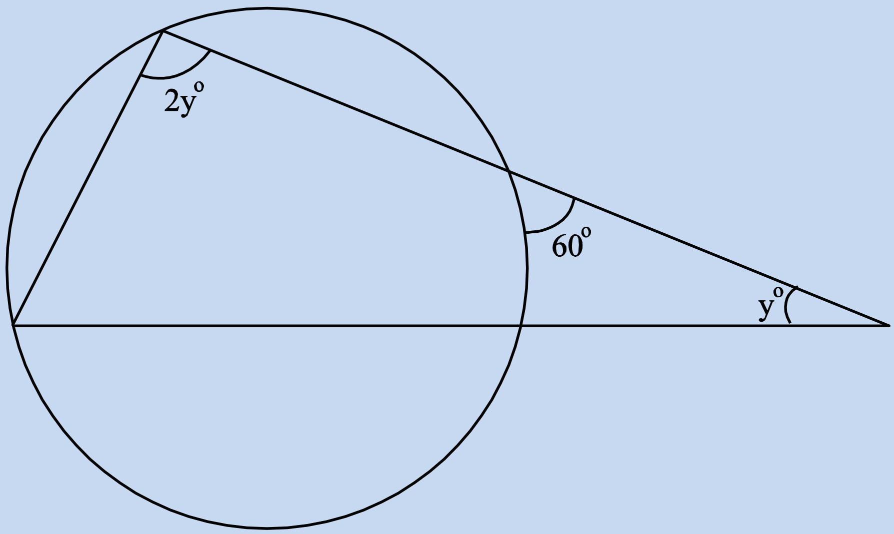




e)



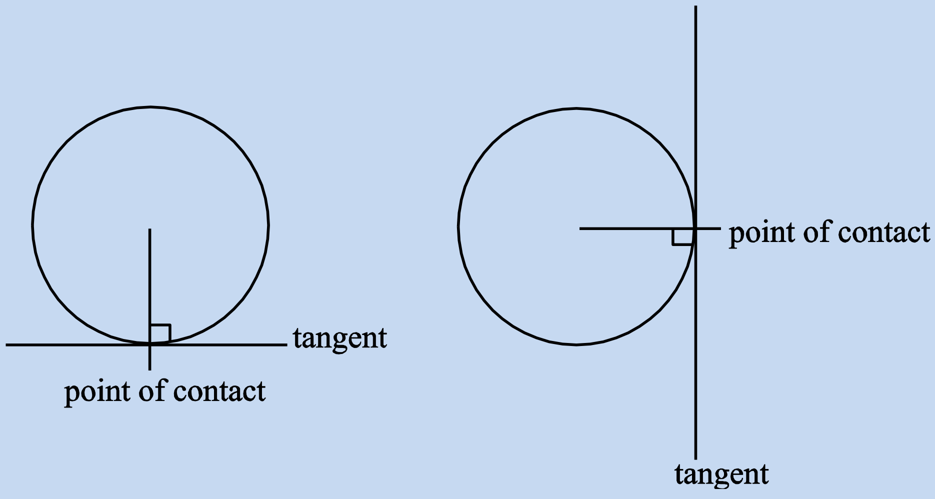
f)



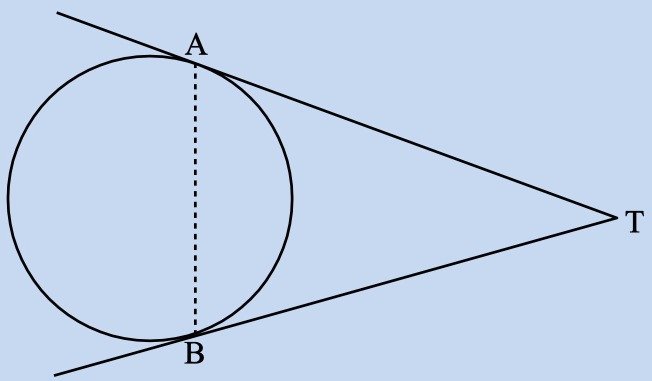
Tangents

A tangent is a line that just touches the circumference of a circle.

1) A tangent is perpendicular to the radius at the point of contact



ii) Two tangents draw from an external point are equal in length.



TA =TB. Hence TAB is an isosceles triangle.

Remember the following point on angles also apply in cyclic quadrilaterals.

a) Angles at a point add up to 360^0

b) Vertically opposite angles are equal

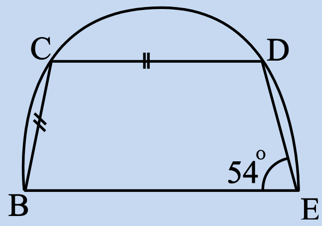
c) Given parallel lines all theorems concerning parallel line apply i.e. Z- angles are equal, corresponding angles add up to 1800 etc.

d) Base angle of isosceles triangles are equal.

e) Similarity and congruency can also be used as an aid on finding solutions

Examination Questions

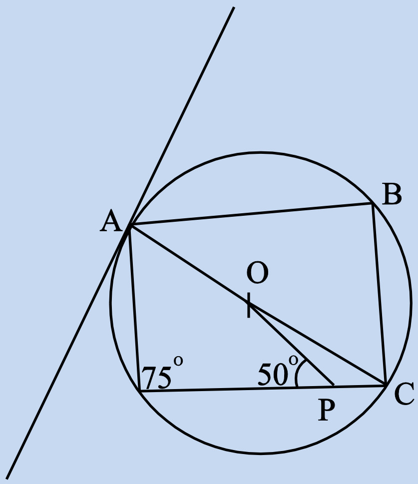
1)



In the diagram B, C, D and E are point on a semi-circle with BE as the diameter. Given that angle BED= 54^0 and BC= CD, Calculate.

a) angle EBD b) angle BCD c) angle DBC

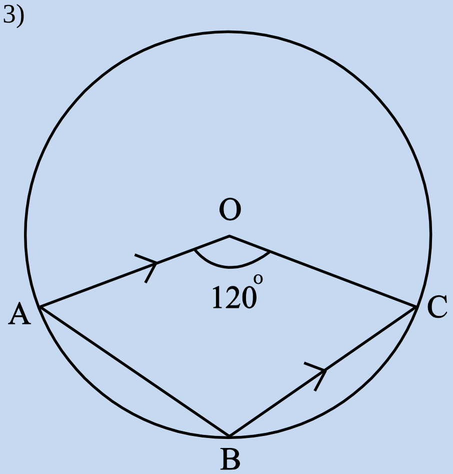
2)



In the diagram, the points A,B,C and D lie on the circumference of circle centre O. The line ST is a tangent at A and the radius AO when produced meet DC at P. Given that angle ADC= 75^0 and angle APD = 50^0 calculate.

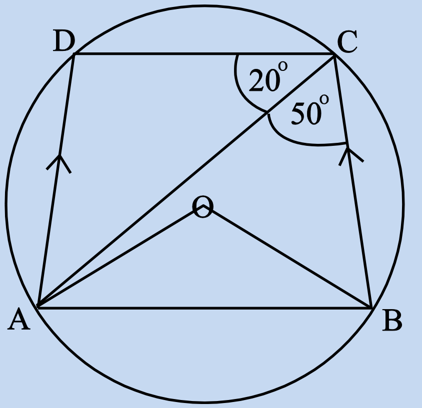
i) angle ABC ii) angle SAD iii) angle POC

3)



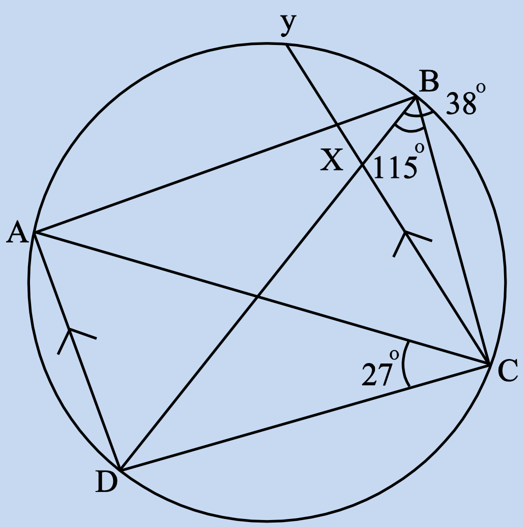
In the diagram O is the centre of the circle. AO is parallel to BC and angle AOC= 120^0.
Calculate the value i) angle ACB ii) angle ABC

4)



In the diagram, above, O is the centre of the circle and AD is parallel to BC. Given that ACB = 50^0 and ACD= 20^0, calculate.

i) angle ADC ii) angle DAB



In the diagram, ABCD is a cyclic quadrilateral. The straight line CXY is parallel to DA angle ACD= 27^0, DBC= 38^0 and BXC = 115^0

a) Calculate the following angles

i) BCY ii) ABD iii) ADB iv) ACY v) CAB

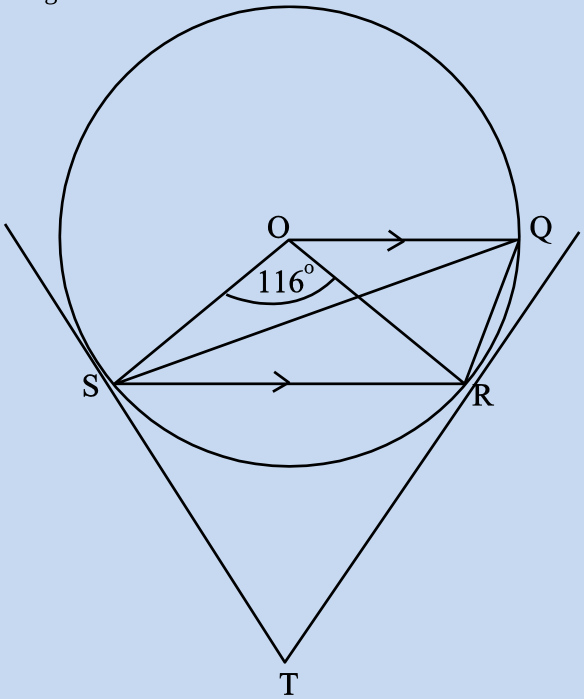
b) State the reason why arc YB is equal to arc AD.

c) Name, in the correct order, the triangle which is similar to triangle BXC

(ZIMSEC NOV 2004)

In the diagram, TR and TS are

Tangents to the circle centre O.



SR is parallel to
OQ and SOR= 116^0 Calculate the angles
i)SQR
ii) RSQ
iii) RTS

(ZIMSEC JUNE 2008)

CHAPTER 27

INDICES, SQUARES, SQUARE, CUBES AND CUBE ROOTS

Syllabus Objectives

Learner should be able to

a) State all the laws of indices and apply them
b) Find the square and square roots of given numbers by various methods
c) Calculate the cube and cube roots of numbers.

Indices
X^2 means XxX. 2 is the power or index of the base x.

Fundamental laws of indices

1) $a^x \text{ x } a^y = a^{(x+y)}$

2) $a^x :- a^y = a^{x-y}$

3) $a^{-x} = \dfrac{1}{a^x}$

4) $a^o = 1$

5) $(a^x)^y = a^{xy}$

6) $a^{1/x} = \sqrt[x]{a}$

7) $a^{x/y} = (\sqrt[x]{a})^y$

Multiplication and division using indices

Law 1 $a^x \text{ x } a^y = a^{x+y}$

when multiplying powers of the same base add indices

Law 2 $a^x :- a^y = a^{x-y}$

Law 5 $(a^x)^y = a^{xxy}$
$= a^{xy}$
Law 4 $a^o = 1$ (every number raised to the power zero is 1).

Proof for law 1 e.g $a^2 \text{ x } a^3$ =(axa) x (axaxa)
= axaxaxaxa
$= a^5$
$a^2 \text{x } a^3 = a^{2+3}$
$= a^5$

Try to prove law 2 and law 5

Example 1

Simplify

a) $3a^2$ x $5a^4$ $18q^6$: - $3q^2$ c) $(3m^4)^2$ d) 100^o

$3a^2$ x $5a^4$= 3x a^2 x 5x a^4

$= 3 \times 5 \times a^2 x\, a^4$ Collecting like terms

$=15 \times a^{2+4}$

$=15 \times a^6$ Law 1

$=15a^6$

b) $18q^6$:- $3q^2$ = $\dfrac{18q^6}{3q^2}$

$= \dfrac{18\times q^{6-2}}{3}$ Law 2

$= 6 \times q^4$

$= 6q^4$

c) $(3m^4)^2 = 3^2 \times (m^4)^2$ Power 2 affects both 3 and m4

$= 9 \times m^{4x2}$ Laws 5

$= 9 \times m^8$

$= 9m^8$

d) $100^0 = 1$ Law 4

Take Note

For Law 5 $(a^x)^y = a^{xy}$

1. The power outside the bracket raises everything inside the bracket to that power.
2. A negative number raised to an odd power is negative $-2^3 = -8$

   A negative number raised to an even power is positive e.g. $(-2)^2 = 4$

Exercise 1,1

Simplify the following

a) $2^4 \times 2^3$ b) $a^5 \times a^4$ c) $4a^2 \times 8a^4$

d) $m^{10} \div m^6$ e) $c^6 \div c$ f) $\dfrac{18x^4}{3x^1}$

g) $\dfrac{24\times 108}{8\times 10^2}$ h) 27^0 i) $10^{3x} \div 10^x$

j) $5^x :- 5^{2y}$ k) $x^3 :- x^{-4}$ l) $8a^{-5} \times 4a^6$

m) $(b^3)^4$ n) $(g^{-3})^5$ o) $(3^{-4})^2$

p) $(4v^3)^2$ q) $-3(a^2)^3$ r) $(-d^5)^4$

s) $(5mn^2)^3$ t) $\dfrac{(-c)^2 \times c^4}{(-c)^5}$ u) $\dfrac{-(x^2)^3}{x^4 \times (-x)}$

4) **Negative Indices**

Law 3 $a^{-x} = \dfrac{1}{a^x}$

Example 2

Simplify

a) 5^{-1} b) $\left(\dfrac{2}{3}\right)^{-2}$

a) $5^{-1} = \dfrac{1}{5^1}$ b) $\left(\dfrac{2}{3}\right)^{-2} = \left(\dfrac{1}{\frac{2}{3}}\right)2$ Law 3

Law 5

$= \dfrac{1}{5}$

$= \dfrac{1}{\frac{4}{9}}$

$= 1 :- \dfrac{4}{9}$

$= 1 \times \dfrac{9}{4}$

$= \dfrac{9}{4}$

$= 2\dfrac{1}{2}$

Fractional Indices

Law 6 $a^{1/x}$ = $\sqrt[x]{a}$ Where a = o

Law 7 $a^{y/x}$ = $(\sqrt[x]{a})^y$

Example 3

SimplifyREFER TO ORIGINALY COPY

a) $27^{1/3}$ b) $16^{-3/4}$ c) $2\times^{1/2}\ (2x^3)^{2/3}$

a) $27^{1/3} = \sqrt[3]{27}$ Law 6

$= 3$

b) $16^{-3/4} = \frac{1}{16^{3/4}}$ Law 3

$= \frac{1}{\sqrt[4]{(16)^3}}$ law 7

$= \frac{1}{2^3}$

$= \frac{1}{8}$

c) $2x^{1/2}\ x\ (2x^3)^{3/2} = 2x^{½}\ x\ 2^2\ x\ 3^{3\ (3/2)}$

$= 2x^{½}\ x4x^{9/4}$

$= 8x^{½}\ x\ x^{9/4}$

$= 8\times^{11/4}$

$= 8(\sqrt[4]{x})^{11}$

Brief Summary

The Square root of a number has a negative and a positive value e.g. $\sqrt{25}$ = 5 or –5. Since 5x5 = 25 and -5x-5=25

Square root. Sign is distributive e.g. REVISIT

$\sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}}$

$= \frac{5}{2}$

$x^{\frac{1}{2}}$ the same as $\sqrt{x}$ e.g. $25^{½} = \sqrt{25}$

$= 5$

Exercise 1,2

Simplify

a) 10^{-3} b) $(-4)^{-3}$ c) 2^{-5} d) $(1/2\ )^{-2}$

e) $(^{-}27)^{1/3}$ f) $64^{\ 4/3}$ g) $\left(\frac{-8}{64}\right)^{-\frac{2}{3}}$ h) $\left(1\frac{9}{16}\right)^{\frac{1}{2}}$

i) $(1/9)^{-1}$ j) $3^{\ ½} \times 3^{\ -3/2}$ k) $0{,}04^{1/2}$

l) $0{,}027^{\ 2/3}$ m) $2a \text{ x } 3a^{-1}$ n) $3^{\ x+1} \text{ x } 3^{x}$

o) $\sqrt{(1252)^{-1/3}}$ p) $(2x)^{\ ½} \text{ x } (2x^{3})^{\ 3/2}$

Squares and Square Roots

To get 25 you need to multiply five by itself twice. I.e. $5 \times 5 = 25$

$3 \times 3 = 9$

$\sqrt{25} = 5$, $\sqrt{9} = 3$.

Means 5 should multiply itself twice to give answer 25 Means 3 should multiply itself twice to give answer 9

$5 \times 5 = 5^{2}$
$= 25.$

Perfect squares five raised to the power two is read as 5 squared. All numbers whose square roots are integer are said to be perfect squares. For example 49, 100 and 144 are perfect squares. Since their square roots are 7, 10 and 12 respectively. 5 is not a perfect square since the $\sqrt{5}$ is 2,236 which is not a whole number but a fraction.

Using Factor to find square roots of perfect squares

Example 4

Find the a) $\sqrt{144}$

2	144
2	72
2	36
2	18
3	9
3	3
1.	

144	$= 2^{2} \times 2^{2} \times 3^{2}$.	Express as product of its prime factors
	$= 2 \times 2 \times 3$	with factors raised to the power two
	$= 12$	-Take all factors raised to power two and multiply

:- $\sqrt{144} = 12$

Exercise 1,3

1a) Which of the following are perfect squares
4, 8, 9, 12, 17, 36, 1 225, 6, 25, 704, 729

2) Find the square roots of the following
a) 729 b) 1089 c) 324 d) 1 225 e) 1 600

3) Find x if i) $x^2 = 9$ ii) $x^2 = 3^2 + 2^2$ iii) $x^2 = 8^2 = + 6^2$

1) Simplify
a) $(1\frac{1}{2})^2$ b) $(1\,{}^4/_5)^2$ c) $\sqrt{2\frac{1}{4}}$ d) $\sqrt{12\frac{1}{4}}$

Hint: reduce to improper fraction first.

2) Evaluate
a) $0{,}3^2$ b) $0{,}02^2$ c) $0{,}013^2$

d) $\sqrt{0{,}036}$ e) $\sqrt{0{,}0009}$ f) $\sqrt{0{,}01}$ g) $\sqrt{0{,}0081}$

6) Calculate $\sqrt{225}$, Hence state the values of $\sqrt{2{,}25}$ and $\sqrt{0{,}0225}$

Cubes and Cube roots

Cube root of a number x is denoted as $\sqrt[3]{x}$

$\sqrt[3]{x}$ means which number can multiply itself three time to give the answer x

for example $3 \times 3 \times 3 = 27$

:- $\sqrt[3]{27} = 3$

Cube of a number x is denoted as x^3. Meaning that a cube of a number is the result of multiply that number by itself three times

$$3^3 = 3\times 3\times 3$$
$$= 27$$

Perfect cubes

These are numbers whose cube roots are integers. For example

$\sqrt[3]{27} = 3$, $\sqrt[3]{64} = 4$ but $\sqrt{49}$ is a fraction:

Note
All cube roots of negative numbers are negative e.g $\sqrt[3]{-27} = -3$

$x^{1/3}$ is the same as $\sqrt[3]{x}$ e.g. $27^{1/3} = \sqrt[3]{27}$
$= 3$

Exercise 1,3
1a) Find the values of:
a) 6^3 b) 3^3 c) 4^3 d) 8^3

2a) Find the values of:

a) $\sqrt[3]{1}$ b) $\sqrt[3]{8}$ c) $\sqrt[3]{27}$ d) $\sqrt[3]{1\,000}$

e) $\sqrt[3]{64}$ f) $\sqrt[3]{125}$ g) $\sqrt[3]{343}$

3) Find x if i) $x^3 = 1000$ ii) $x^3 = 27$ iii) $x^3 = 125$

4) Calculate the exact value of:
a) $0{,}081^{1/3}$ b) $(3\ ^3/_8)^{1/3}$

Examination Questions

1) Simplify:

a) $12x^3 : -2x^2$ b) $27x^{7/2} : -9x^{11/2}$ c) $(2a^{1/2})^4$

2. **Evaluate**
i) $(0{,}2)^2$ ii) $99^2 - 1^2$

3) Find the value of a, given that $2^a = 64$

4) Given that $b^{1/2} = 4$, find the value of b
5) If $c = 6{,}3 \times 10^8$ and $d = 7{,}0 \times 10^3$, find the value of:
$\sqrt{^c/_d}$ giving your answer in standard form

3) Evaluate
a) 4^{-2} b) 16 ¾ c) $5^{2/3} \times 5^{1/3} \times 5$

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CHAPTER 28

SIMULTANEOUS EQUATIONS

The equation y = 1-3 x consist of two unknowns x and y. many pairs of x and y satisfy the equation e.g (0;1), (1,-2), (2,-5) etc.

The equation 2 x +y = 5 also consists of two unknowns, x and y. Many pairs of x and y values also satisfy the equation e.g. (0,5), (1; 3), (3, -1) etc. However, only one pair of x and y values satisfy these two equations that is (-4;-13). When two such equations are true at the same time they are called simultaneous equations.

Syllabus Objectives

Learner should be able to
a) Solve simultaneous equations by the method of substitution
b) Solve simultaneous equations by the method of elimination
c) Solve simultaneous equations graphically
d) Check the correctness of their solutions.

Substitution
It is a method, which involves replacing one unknown with something so that you deal with a single unknown.

Example

Solve the following simultaneous equation
$3x + 5y = 11$
$2x - 3y = 20$

$3x + 5y = 1$	(1)	identify your equations as
$2x - 3y = 20$	(2)	equations (1) and (2)

Taking equation (1) — Take either equation and make any of its unknown the subject

$3x + 5y = 11$
$5y = 11 - 3x$
$y = \frac{11-3x}{5}$
$2x - 3y = 20$

Substitute for y into equation (2)

$2x - 3\frac{(11-3x)}{5} = 20$ (2)

$10x - 3(11 - 3x) = 100$ Multiply by 5
$10x - 33 + 9x = 100$

$10x + 9x = 100 + 33$
$\frac{19x}{19} = \frac{133}{19}$

$x = 7$

Now substitute for x in either (1) or (2)

$2x - 3y = 20$
$2(7) - 3y = 20$
$-3y = 20-14$
$-3y = 6$

$y = -2$

:- $x = 7$ $y = -2$

Checking by substitution into all the equation.

(1) $3x + 5y = 11$
$3(7) + 5(-2) = 11$
$21 - 10 = 11$

(2) $2x - 3y = 20$
$2(7) - 3(-2) = 20$
$14 + 6 = 20$

Take note

The method is useful when the coefficient of one unknown in any of the equations is one.

Exercise 1,1

1. Solve the following simultaneous equations using the method of substitution.

a) $y = 1 - 3x$
$2x + y = 5$

b) $y + 2x = 6$
$2y + x = 3$

c) $2a + b = -3$
$a - 3b = -5$

d) $2y - x = 5$
$3y + 2x = -24$

e) $5x + 3y = 2$
$y + 2x = 0$

f) $m + 3n = 7$
$9m - 2n = 5$

g) $3x - 4y = 8$
$4x + 4y = 27$

h) $r + s = 1$
$4s = 3 + 2r$

I) $3x + y = -5$
$x - 2y = 18$

Method of elimination

This method involves temporarily getting rid of one unknown in the simultaneous equations by addition or subtraction.

Example 2

Solve the following simultaneous equation using the method of elimination. $3x + 5y = 11$
$2x - 3y = 20$

$3x + 5y = 1$ (1)
$2x - 3y = 20$ (2)

$3x + 5y = 11 \times 2$ Multiply (1) by 2 and (2) by 3
$2x - 3y = 20 \times 3$

$6x + 10y = 22$
$6x - 9y = 60$ Subtract the two equation (1)- (2)

10y- (-9y) = 22-60
19y = -38

y = -2

Substituting the value of y in either equation

$3x + 5y = 20$
$3x + 5(-2) = 20$
$3x = 20 + 10$
$\frac{3x}{3} = \frac{30}{3}$

$x = 10$
:- $x = 10$ =y =-2

Take note:
When given the fractions multiply every term by the lowest common denominator to reduce to linear equations.

Exercise 1,2

1. Solve the following simultaneous equations.

a)	$4x+3y=1$ $3x+4y=10$	b)	$3x-4y=8$ $4x+4y=27$	c)	$3x+y=-3$ $x-2y=18$
d)	$3x+2y=7$ $x-y=1.5$	e)	$3x+2y=0$ $2x-3y=20$	f)	$3b+4c=22$ $2b-23=-6c$
g)	$\frac{3a}{4}+\frac{2b}{3}=12$ $\frac{2a}{5}+\frac{b}{3}=12$	h)	$x+2y=9$ $\frac{1}{2}x-3y=1$	I)	$\frac{3y}{2}-\frac{3x}{5}=2$ $\frac{2x}{3}-\frac{5y}{2}=4$
j)	$8p-7q=13$ $3p+2q=28$	k)	$2x+5y+11=0$ $3x+4y+6=0$	l)	$7k+3n=21$ $5k+2n=15$

Graphical method of solving simultaneous equation

The method involves drawing straight-line graphs and where the two lines meet or intersect gives the solution.

Example 3

Solve graphically the following simultaneous equations

$3x + y = -5$
$x - 2y = 18$

Table of values

$3x + y = -5$

x	o	-2	-3
y	-5	-1	4

$x - 2y = 18$

x	18	0	8
y	0	-9	-5

The values of x and y at the point of intersection satisfy both equation, hence the solution is x= 1,1 and y = -8,4
Insert:

INSERT GRAPH

Exercise 1,3

1. Solve the following graphically

a)	$y = 1-3x$ $2x + y = 5$	b)	$3x + y = -5$ $x - 2y = 18$	c)	$y + 2x = 6$ $2y + x = 3$
d)	$4x + 3y = 11$ $3x + 4y = 10$				

Word Problems involving Simultaneous Equations

Example 4

80 saucers and 70mugs cost $940. 80mugs and 70 saucers cost $965. Calculate the cost of a saucer and a mug.
Let S represent saucers
Let M represent mugs

Form equations

80s +70m	=	940 (1)
70s+ 80m	=	965
80s+70m	=	940 x 8
70s + 80m	=	965 x 7
640s + 560m	=	7 520
490s+ 560	=	6 755

Subtracting

150s	=	765
s	=	$5,10

Substituting s

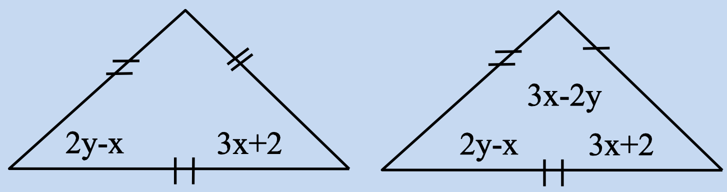
80 (5,10)	+ 70m = 940
40,8 +7m	=94
7m	=94 – 40,8
7m	=63,2
m	= $7,74

:- Cost of Saucer = $5,10 cost of mug is $7,74

Exercise 1,4

1) 3 tomatoes and 5 onions cost 88c. 5 tomatoes and 3 onions cost 72c. Calculate the cost of one tomato and one onion.
2) The sum of two numbers is 7. When the bigger number is subtracted from four times the smaller number the result is 3. Find the value of the two numbers.
3) The diagrams below shows the angles in two equilateral triangles a and b.

a) b)



i) Form two equation using triangles a and b respectively.

ii) Solve the equations to find the value of x and y

4) The sum of Peter and John's ages is 14.
The difference of their ages is 4. Find both the ages of John and Peter.

5) Mary's mother is 30 years older than her. Five years ago, her mother's age was 3 times that of hers. Find their present ages.

6) Anna and Betty each receive some change after buying sweets. The sum of Anna's change and half of Betty's change is \$0,50.
The difference between half of Anna's change and one- sixth of Betty's change is \$1,50. Find both Anna and Betty's change.

Examination Question

1) Solve the Simultaneous equations
$6x + 4y = 3$
$4x + 6y = 5$ ZIMSEC: NOV 2003

2) Solve the simultaneous equation
$4x - 2y = 5$
$x + y = \frac{1}{2}$

3) Solve the following pairs of equations ZIMSEC: NOV 2002
$0{,}5x - 0{,}4y = 0{,}8$
$x - 0{,}9y = 0{,}6$

4) Solve the following simultaneous equations
$2x - \frac{7}{2}y = \frac{23}{2}$

$$1\frac{1}{2}x + \frac{y}{2} = \frac{-3}{4}$$

5) Calculate a and b if $5^{2a-b} = 1$ and $\frac{2^4}{4} = 2^{9a-3b}$

CHAPTER 29

REVISIT THE WHOLE CHAPTER

Inequalities

Inequalities deals with quantities that are not equal. One quantity could be greater than or less than another quantity.

Syllabus Objectives

Leaner should be able to:

a) Use the following inequality signs as required $>$, $<$, $\leq$, $\geq$

b) Solve linear inequalities of the form ax + b > c and or c < a x +b < d where a, b, c, d are rational.

c) Represent inequalities and their solution sets on a number line.

d) Represent inequalities and their solution sets on a Cartesian plane.

The number line

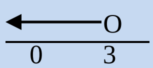
The inequalities in one variable can be illustrated in a number line.

Example 1

Illustrate the following inequalities in a number line.

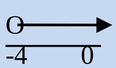
a) $x < 3$ b) $x > -4$ c) $x \leq 2$

a) $x < 3$ x = (2,1,0,-1,-2…y



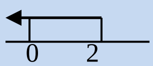
-All number to the left of 3 fall in the set excluding 3.
-Since 3 is excluded the circle is not shaded.

b) $x > -4$ $x = \{ -3,-2, -1, 0,1,2\ldots\}$



-All number to the right of -4 fall in the set.
-Since -4 is not part of the set, the circle is not shaded.

c) $x \leq 2$ $x = \{ 2, 1, 0, -1, -2\ldots\}$



Reads x is less than or equal to 2.
Since two is included in the set the circle is shaded.

Brief summary

a) Number on a number line increase to the right:- 1 > -20

b) The open circle above the number on the number line shows that the number is excluded from the set it applies to $<$ less than and $>$ greater than.

c) The shaded circle above the number on the number line shows that the number is included in the solution set. It applies to $\geq$ and $\leq$.

d) The zero helps to separate positive from negative numbers.

e) Solution set refers to a particular set represented by inequality e.g for $x \leq 2$ the solution set is $x = \{ 2, 1, 0,-1,-2\ldots\}$

Example 2
Illustrate the following inequalities on a number line.

a) $0 < x \leq 4$ b) $-1 \leq x \leq 4$

Insert:

a) $0 < x \leq 4$ $x = (1,2,3,4)$

0 ————0 Since zero is not include it is not shaded
0 4

b) 0 ————0
-1 0 4

Exercise 1,1

1) Give the solution set of the following inequalities

a) $x > 2$ b) $x \geq 4$ c) $x < -3$

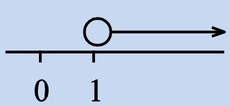
d) $x > e$ e) $-4 < x \leq -2$ f) $0 \leq x \leq 5$

g) $-7 \leq x \leq 10$ h) $3,5 \leq x \leq 7$ i) $-3,7 < x < 6,5$

j) $-2\frac{1}{2} < x < 2$ k) $0,75 < x \leq 4$

2) Represent each solution set in question one on a number line

3) Write the inequalities represented by the diagram.

a)


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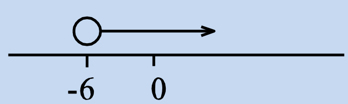

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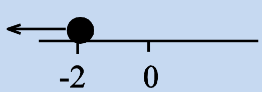

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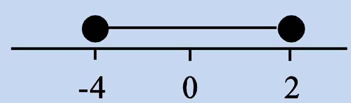

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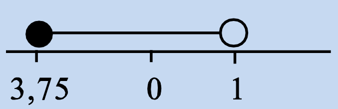

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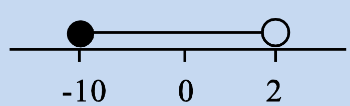

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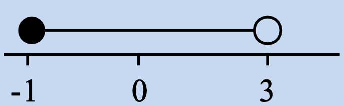

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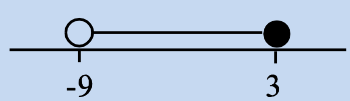


Solving inequalities

Inequalities can be solved the way equations are solved. With inequalities, when dividing by a negative the sign is reversed.

Example 3

Solve and illustrate the solution on a number line.

a) $-3 < 2x + 1$ b) $-2x \le 4$ c) $-1 < \frac{2x-3}{2}$

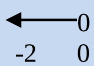
d) $-1 < 2x - 3 \le 5$

a) $-3 < 2x + 1$ Taking 1 to left (change sign)

$-3-1 < 2x$

$-4 < 2x$

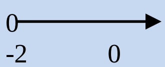
$-2 < x$ Divide by 2



b) $-2x \le 4$ Divide by a negative 2 and change Sign in the process

$x \ge -2$

$x \ge -2$



c) $-1 < \frac{2x-3}{2}$ multiply by 2 both side to remove fraction

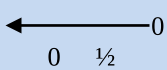
$2(-1) < (\frac{2x-3}{2})^2$

$-2 < 2x - 3$

$-2 + 3 < 2x$

$½ < 2x$

$½ < x$



d) $-1 < 2x - 3 \le 5$

$-1 < 2x - 3$	breaking the	$2x - 3 \le 5$
$-1 + 3 < 2x$	inequality	$2x \le 5 + 3$
$2 < 2x$		$2x \le 8$
$1 < x$		$x \le 4$

Now join the two solution

$1 < x \le 4$

0		0
0	1	4

Remember

Integers are positive and negative numbers including zero…-3;-2,-1,0, 1,2,3…

Exercise 1,2

1) Solve the following inequalities

a) $x - 7 < 2$ b) $3 - x < 9$ c) $4x - 12 \leq 0$

d) $-10x \leq 100$ e) $7x + 1 < 2x + 3$ f) $3(x + 2) < 15$

g) $3x \geq 8 + 5x$ h) $5(7-y) > 3(2-y)$ i) $2/3 > \frac{7}{x+5}$

k) $x + \frac{3}{7} \leq \frac{x}{5}$

2) Represent each of the solution set in question 1 on a number line.

3) List the integer values of x which satisfy the following

a) $\{x : 3{,}75 < x < 10\}$

b) $\{x : -8 \leq x < ½\}$

c) $\{x : x$ is perfect square and $0 < x \leq 40\}$

d) $\{x : x$ is an odd number and $0 < x \leq 10\}$

4) If $-4 \leq x < 2$ and $-1 < x < 4$, x and y being integers find Hint: List the solution set of the two inequalities e.g $x = \{-4;-3,-2,-1,0,1\}$

a) the maximum value of x -y

b) The minimum value of x -y

c) The least value of $2xy$

d) the greatest value of $2xy$.

Hint: maximum occurs when substituted value give the biggest answer. Vice versa for minimum.

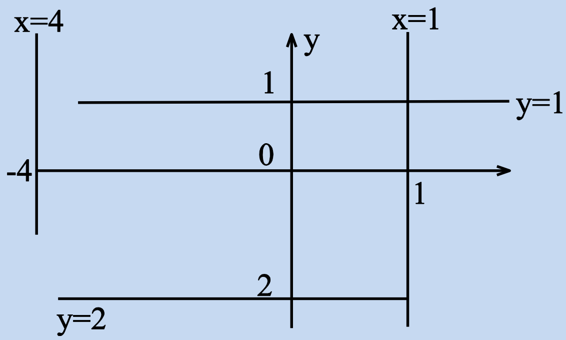
GRAPHINC LINEAR INEQUALITIES (CARTESIAN PLANE)

The Cartesian plane consists of the x axis and the y axis. Along the y axis the value of $x = 0$. Also along the x axis the value of y=0.

In a Cartesian plane vertical lines are x= a, and horizontal lines are y =a.

For example the following lines would show as follows

a) $x = 1$ b) $y = -2$ c) $x = -4$ d) $y = 1$



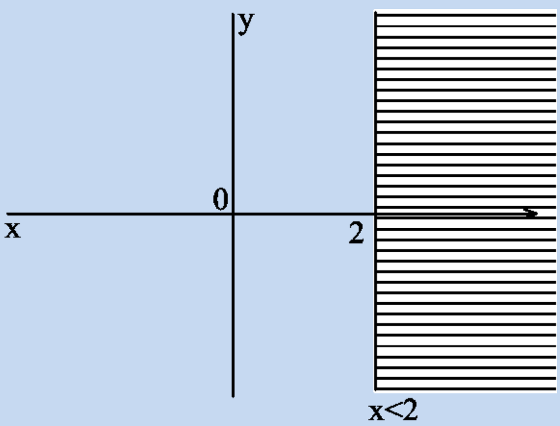
Also inequalities can be represented in the cartesion plane.

Example 4

Show by shading the area represented by the following inequalities

Insert:

a) $x < 2$ b) $x \leq -2$ c) $y \geq 2$

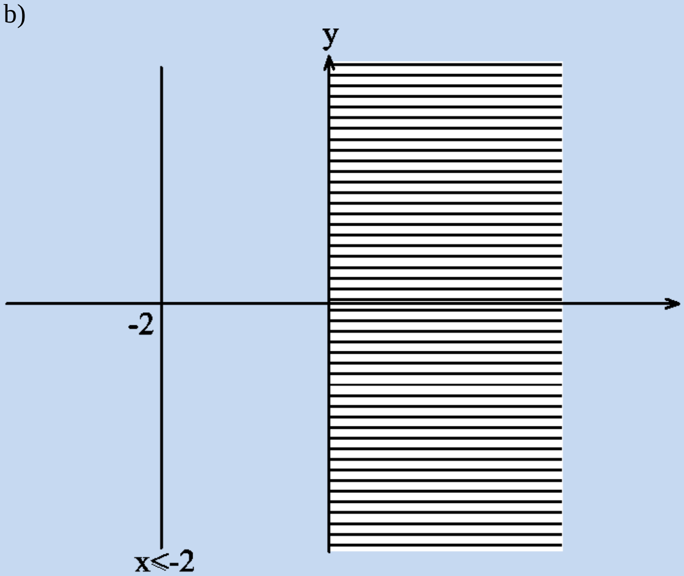


Notes

1. If first draw the vertical line X =2 through 2.
2. Since two is not included in The set draw a broken line
3. Shade the region without the without the solution set. Numbers less than 2 are to the left of it.

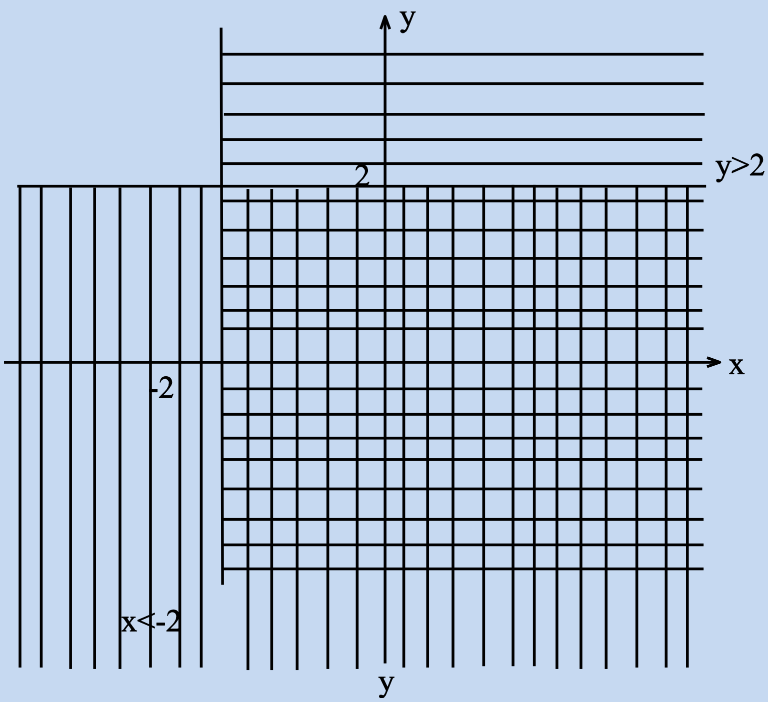
<u>Insert:</u>

b)



Since minus two is included use a bold/continuous line.

c)



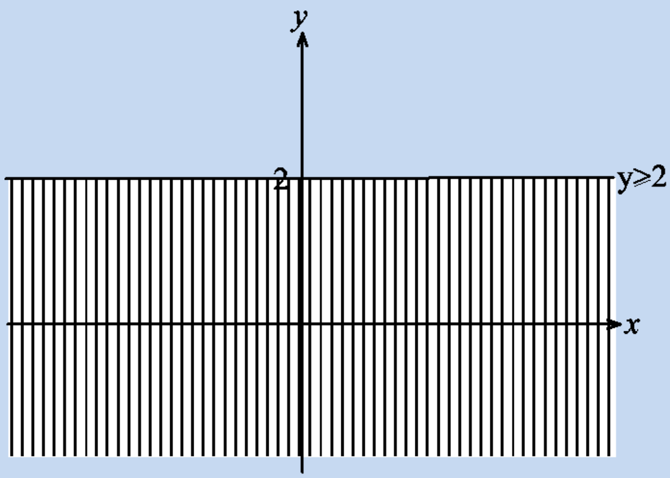
This time the line is horizontal.

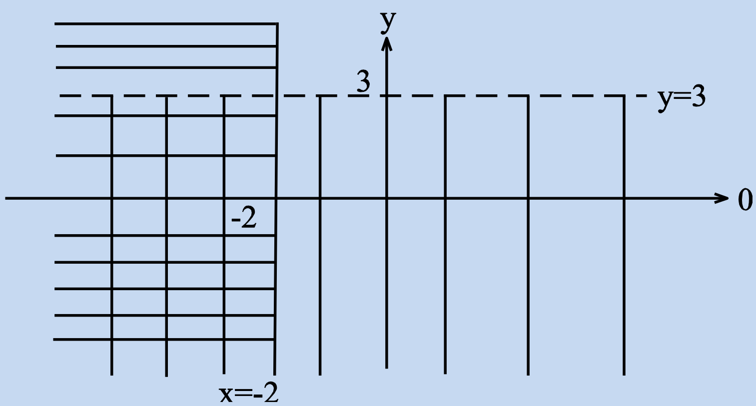
$Y \geq 2$ Numbers greater than 2 are above the line.

Example 5

Show by unshading the following inequalities

a) $\{(x, y): x \leq -2, y \geq 2\}$ b) $\{(x, y): x \geq 2, y > 3y\}$





Notes:
Unshade region represents
y=2 the solution set

y =3 Unshaded region represents the solution set

Brief Summary

When the sign ≥ or ≤, use a bold line.
When the > or <, use a broken line

The remaining unshaded region represents the solution set.

<u>Exercise 1,3</u>

1) On square paper, draw the graphs of the following inequalities by shading the unwanted region.

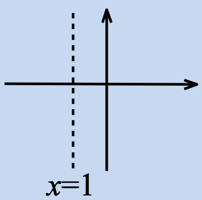
a) $x > 2$ b) y ≤, 3 c) $x < -2$ d) y ≥-2

e) y > 0 f) 1 ≤ x 4 g) -1 < y < 3 h) {(x,y), x > 1,y ≤ 2}

i) {(x,y), x ≤ -1, y > 0} j) {(x,y), x >4, y ≥2} k) {(x,y), x <-1, y ≤ 2}

2) Write down the inequalities represented by the following diagrams drawn by shading the unwanted region.

a)



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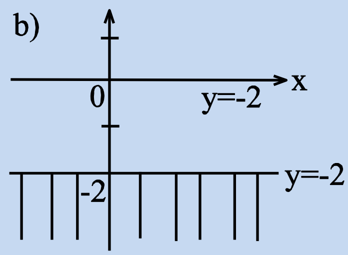
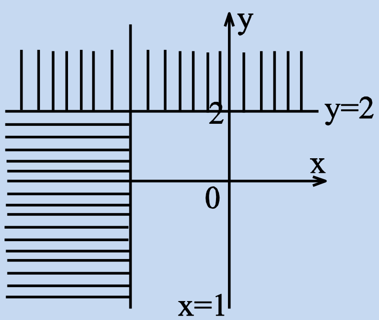


c)



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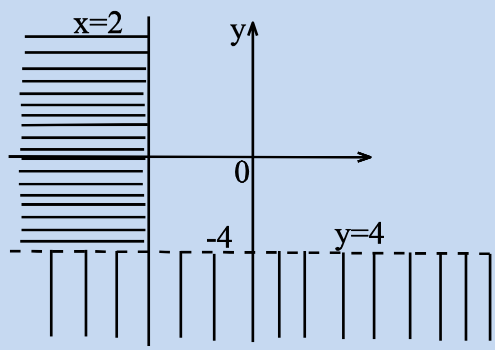
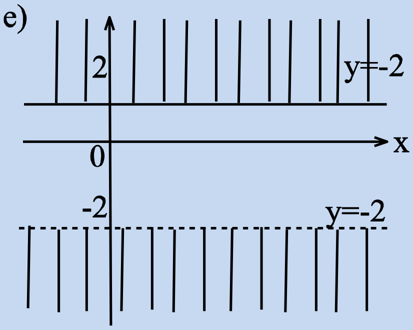
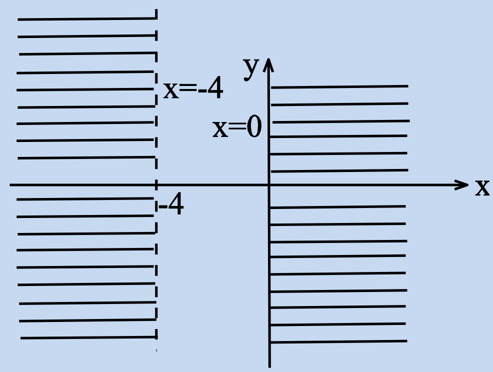


e)



e)



Inequalities in two variables

Linear inequalities in one variable can be shown on the one dimensional number line. Solution of inequalities in two variables lie in a region of two dimensional x,y plane.

Example 6

Shade the region being described by the following inequalities.

a) $3x + 4y < 12$ b) $y \geq 0$, $y < 3x$, $x + y \leq 4$

a) $3x + 4y < 12$

Step 1- Reduce the inequality to the equation $3x + 4y = 12$

Step 2- Determine where the graph crosses the x and y axis

Graph crosses the x axis when y = 0

Insert:

$3x + 4(0) = 12$

$\frac{\cancel{3}x}{3} = \frac{\cancel{12}^{4}}{3}$

$x = 4$

Coordinate (4;0)

Graph cross the y axis when x= 0

$3(0) + 4y = 12$

$\cancel{4}y = 12$

$y = 3$

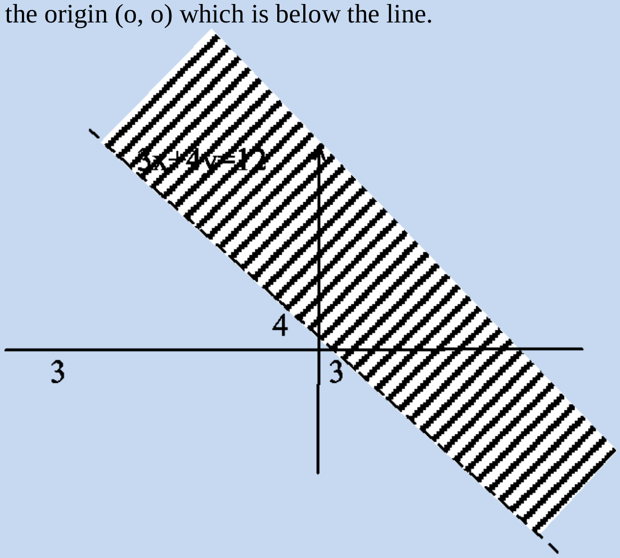
Coordinate (0,3)

Step 3- Draw the line using coordinates

Since the sign is < use broken line.

Step 4 – Test the point above the line or below the line to determine unwanted region

e.g use the origin (o, o) which is below the line.



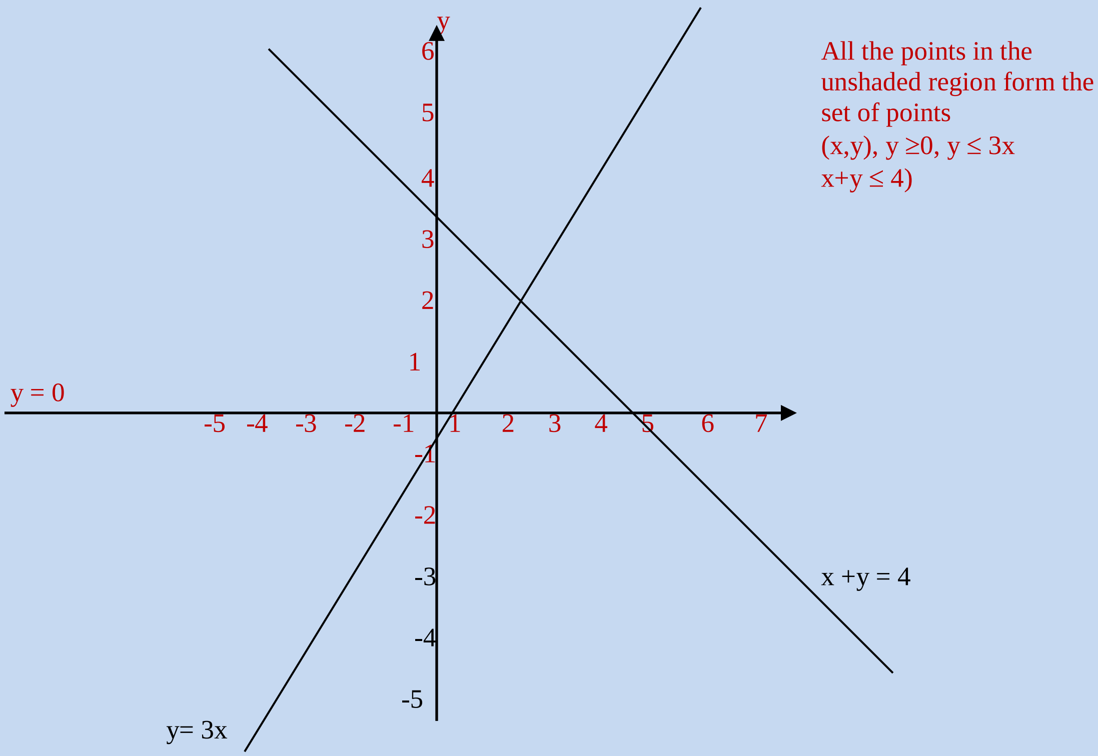
$3x +4y < 12$
$3 (0) +4 (0) < 12$
$0 < 12$ true

:- Point (0;0) is contained in the set of point $3x +4y < 12$.

Also the fact that it is below the line means all point below the line from the set. Hence, all the points above the line are in the unwanted region and are shade.

Step 4: Shade the unwanted region

b) $y \geq 0$, $y \leq 3x$, $x +y \leq 4$
insert:



Take note

Various point below or above the line can be used for testing**. not only point (0,0)**
Points on the bold line form part of the solutions set.
Point along the broken do not form part of the solution set.

Exercise 1,5

1. Draw the regions defined by each of the following. Use graph paper.

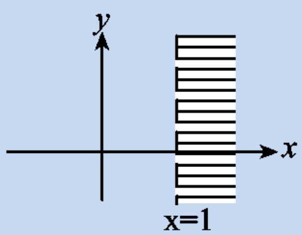
a) $y \geq 0$ b) $x < -4$ c) $x < 0$ d) $x - y \leq 5$

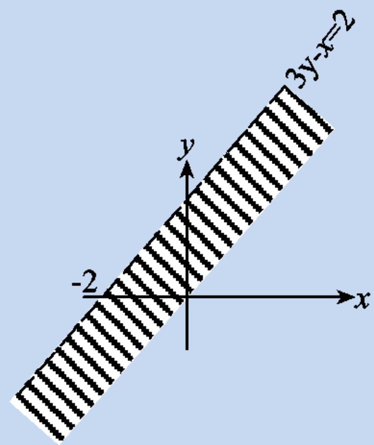
e) $x - 2y > 4$ f) $3x + y \leq -3$ g) $12 \leq 3x + y$; $x + 3y < 6$

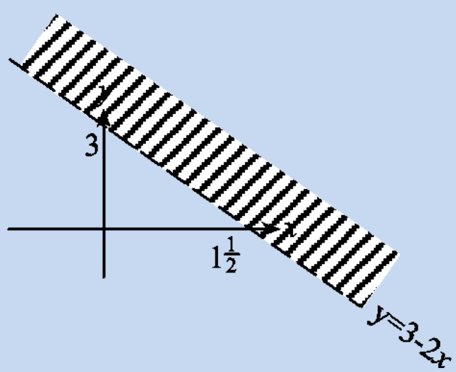
h) $x \geq -4$; $y \leq 2$, $x - y < 2$, i) $y \geq 1$, $y - x < 5$, $2x + y \leq 0$

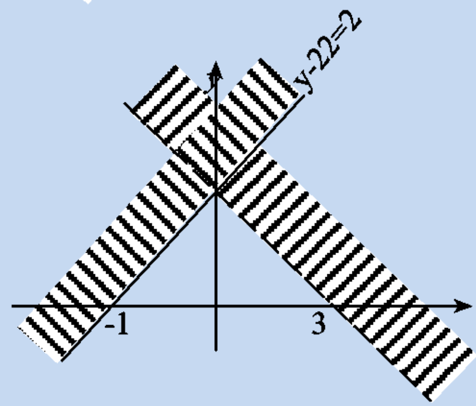
j) $y \leq 4$, $x - y \leq 1$, $2x + 3y > 12$ k) $y \geq 0$, $x - y > 2$, $3x + 4y \leq 12$

2. Using question 1h and 1i state all integral values of x and y.
3. Give the inequalities which define the shade region.









Examination Questions

1) Using a scale of 2cm to represent one unit on each axis. Draw on the same axes the lines $2x + y = 8$ and $3x + 4y = 24$ for those values of x from 0 to 8 for which $y \geq 0$.

i) Shade the region R defined by the inequalities
$2x + y \leq 8$, $3x + 4 \leq 24$, $x > 0$ and $y > 0$

ii) S is the set of points (x,y) such that x and y are integers and $(x,y) \in R$. Find n (S).

iii) Find the greatest value of c for which an element of S lies on the line $4x + y = c$ (Cambridge: 1984)

b) Use the region R to answer the following questions.

k) Write down 3 inequalities other than $y \leq -\frac{2}{5}x + 32$ which define R

(i) State the maximum value of y

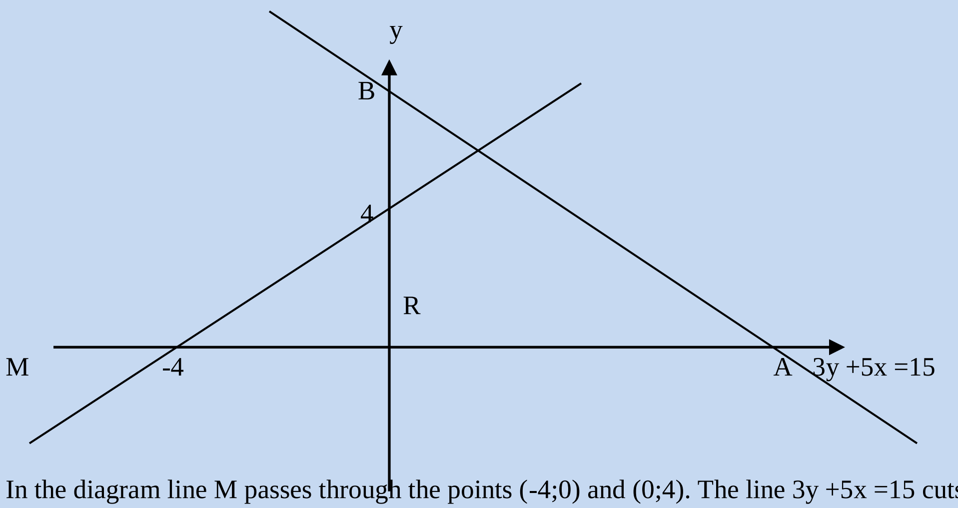
iii) Given that (x;y) is a point inside the region R and that x and y are integers, write down the value of x and the value of y which make $(x + y)$ a maximum.

iv) Find the maximum value of $40x + 20y$

(ZIMSEC NOV 2006)

Insert/ shade

Graph



In the diagram line M passes through the points (-4;0) and (0;4). The line $3y + 5x = 15$ cuts the x- axis and the y axis at A and B.

i) Write down the coordinates of A and the coordinates of B.
ii) Find the equation off line M
iii) Write down two inequalities, other than $y \geq 0$, which define the region R

(ZIMSEC NOV 2008)

CHAPTER 30

Linear equations formulate and substitution

An equation is a statement that two algebraic expressions are equal in value.

Syllabus Objectives

Learner should be able to:

a) Solve equations in various form

b) Change the subject of the formulae

c) Substitute and solve problems involving formulae

Solving linear equations

Solving an equation involves finding the real number value of the unknown, which makes the equation true.

Example 1

Solve

a) 3y +6 =18-y b) 8 (y-2)-5y = 26

c) 3y +6 =18- y Add y both side and also
3y +y= 18-6 subtract 6 both sides

$$\frac{4y}{4}=\frac{12}{4}$$

y =3

b) 8(y- 2)- 5y =26

8y –16 –5y =26 Remove bracket- 5y is not in the bracket Hence not multiplied by 8.

8y –5y = 26 +16 Collecting like term and adding

$\frac{3y}{3} = \frac{42}{3}$ Collecting like term and adding 6 both sides

y = 14

Exercise 1,1

a) Solve the following equations

a) $4x = 48$ b) $4x - 7 = 17$ c) $5x = 4x + 2$

d) 7d = 10 +5d e) 5 = 5c-4 f) $6x + 5 = 13 + 4x$

g) 3w +4 = 16- 2w h) 8-5z = 20-8z I) $5 - 4x + 3 = 8 - x$

j) 5c +9 = 20-4c k) 12- 3y-3= 9-5y

2a) Solve the following equations

a) 2 (2-2q) = (3-q) b) 5a + (3+a) = 21

c) 76- (5b –2) =10 d) $8x - (5n + 13) = 7$

e) $0 = 3 - (4x - 15)$ f) 5-3 (a +3) = -25

g) 4 (3 x +2) = 2(3+ x) h) 7 (5q-4)- 10 (3q-2)=0

i) 2 (5 x -1), = 9 x –3 (x -4) j) 4 = 5 (5p-2)-9 (3p-2)

Equations With Fractions

Examples 2

Solve the equations

a) $\frac{5d}{6}-\frac{3d}{4}=\frac{-2}{3}$ b) $\frac{7}{x}\frac{-7}{3}=0$

c) $\frac{5}{n-1}=\frac{3}{n-4}$ d) $\frac{2x+1}{2}\frac{-x-1}{3}=0$

a) $\frac{5d}{6}-\frac{3d}{4}=\frac{-2}{3}$

$\frac{10d}{d}-\frac{9d}{8}=8$ Clear the fraction by multiplying every term by the lowest common denominator 12

b) $\frac{7}{x}-\frac{7}{3}=0$

$\frac{7}{x}=0+\frac{7}{3}$ Rearrange by adding 7/3 both sides , Cross-multiply and divide by 7

$7x=21$

$x=3$

c) $\frac{5}{n-1}=\frac{3}{n-4}$

3(n-1) = 5 (n-4) Cross multiplying

3n- 3 = 5n- 20 Removing brackets
-3 +20 = 5n –3n Collecting like terms

$\frac{17}{2}=\frac{2n}{2}$

n= 8 ½

d) $\frac{2x+1}{1}-\frac{x-1}{3}=0$

$$\frac{6\ (2x+1)}{2}-\frac{6(x-1)}{3}=0\times 6$$ Multiply every term by the L.C.M

6 x +3 –2 x +2 = 0
4 x = -3-2
4 x = -5

$$x=1\frac{1}{4}$$

Notes
Solution can be checked by substituting the value of the unknown into the original equation.

<u>Exercise 1,2</u>
Solve the following equations

a) $\frac{x}{2}-\frac{x}{5}=6$ b) $\frac{z-}{4}\frac{z}{6}=\frac{1}{2}$ c) $\frac{1}{2}d+1\frac{1}{4}=\frac{1}{4}d$

d) $\frac{5x=}{6}\frac{2}{3}+\frac{3x}{4}$ e) $\frac{1}{3}p-\frac{1}{5}p=3$ f) $\frac{m}{4}=\frac{m}{3}-\frac{1}{6}$

g) $\frac{2x+1}{3}-\frac{3-x}{2}=\frac{x}{4}$ h) $\frac{c+1}{5}=\frac{1}{2}$ i) $\frac{y-2}{5}=0,75$

j) $\frac{5}{x}=4$ k) $\frac{3}{y-1}=\frac{4}{y+1}$ l) $\frac{5x+16}{4}+\frac{x}{2}=\frac{4x+2}{3}$

Word Problems

<u>Examples 3</u>
A worker's weekly wage is calculated by multiplying the daily rate by the number of day worked. A transport allowance of $7 is given on top. If the total amount of money pocketed weekly is $50,find the daily rate of a 5 days working week.

Let the daily rate be x

$$5x+7=50$$
$$5x=50-7$$
$$\frac{\cancel{5x}}{\cancel{5}}=\frac{\cancel{43}8,60}{\cancel{5}}$$

$$x=\$8,60$$

: - The daily rate is $8,60

Brief Summary
When solving word problems follow these steps.

1) When not given choose a letter to represent the unknown quantity.
2) Express the given information in terms of the chosen letter.
3) Form an equation and solve it.
4) Check the correctness of the solution against the given information

Exercise 1,3

1) Peter thinks of a number x. he halves it and then subtracts 3. The answer that he get is 1. Find the value of x.
2) A certain number is subtracted from 7. The result is multiplied by 4 and the answer is 11. Find the number.
3) The perimeter of a rectangle is 30cm. if the width of the rectangle is one-quarter its length. Find i) The length of the rectangle.
   ii) The width of the rectangle

4) The result of taking 4 from a certain number and trebling the result is the same as that of taking 3 and doubling the result.

a) Express this statement as an algebraic equation.
b) Hence find the number.

5) Mr. Shumba and Mr. Hove are farmers who each sold equal number of cattle. Mr. Shumba initially had 10 cattle while. Mr. Hove had 8. After selling, half of Mr. Sibanda’s remaining cattle was equal to three quarter that of Mr. Hove’s remaining cattle. Find the number of cattle that the sold.
6) The result of adding 3 to a certain number and dividing the result by 5 is the same halving than number find the number.
7) One metal rod is 12cm longer than another. One half of the longer rod is equal to three-quarters of the shorter rod. Find the length of the shorter rod.

Formulae
A formula is an equation which contains two or more different unknowns (i.e. letters) and such letters represent measurable things (quantities). For example in finding the volume the formula, V= Lbh.

Change of Subject

In V= Lbh, V is said to be subject of the formula. The equation can be changed to different ways that can make each letter the subject. For example, making L the subject.

$\frac{V}{bh} = \frac{L\cancel{bh}}{\cancel{bh}}$ Diving both side by bh.

$L = \frac{V}{bh}$

Example 4

Make x the subject of the formulae in the following

a) $yx - c = wx$ b) $\frac{x}{a} + \frac{x}{b} = 1$ c) $\frac{w}{c-x} = p$

a) $yx - c = wx$

$yx - wx = c$ Collect like terms take out x since its common

$x(y-w) = c$ Divide by y- w to remain with x on one side

$$x\frac{\cancel{(y-w)}}{\cancel{y-w}} = \frac{c}{y-w}$$

$$x = \frac{c}{y-w}$$

b) $\frac{x}{a} + \frac{x}{b} = 1$b

$$ab\frac{x}{a} + ab\frac{x}{b} = 1(ab)$$ Multiply by L.C.M to remove fraction

$bx + ax = ab$ take out x

$x(b+a) = ab$

$x\frac{\cancel{(b+a)}}{\cancel{(b+a)}} = \frac{ab}{b+a}$ Divide both sides by b +a

$x = \frac{ab}{b+a}$

c) $\frac{w}{c-x} = \frac{p}{l}$

Cross multiply to remove fraction

$p(c-x) = 1\,xw$

$pc - px = w$

$pc - w = px$ Make px positive by adding it both sides

$pc - w = px$ Divide both sides by p

$x = \frac{pc-w}{P}$ Do not bother that x is on the left side

Take Note

1) Fractions are removed by multiplying by the L.C.M or by cross-multiplying.
2) When Multiplying every numerator on the division line should be inside the brackets

Exercise 1,4

Make x the subject in each of the following.

a) $v = wxy$ b) $ax = c$ c) $dx = vy$

d) $x + z = w$ e) $wx - zx = v$ f) $\frac{z}{x} = y$

h) $\frac{v}{x} - y = z$ i) $\frac{x}{c} + \frac{x}{d} = 3$ j) $\frac{w}{x} + \frac{v}{x} = 4$

k) $d(c-x) = yx$

l) $z = \frac{2c+3x}{3c-2x}$

m) $\frac{a}{b-a} = c$

n) $\frac{1}{u} + \frac{1}{x} = \frac{1}{f}$

o) $L = \frac{2a}{E} \; x - r$

p) $t = \frac{3p}{x} + 5$

q) $= \frac{nE}{R + xr}$

r) $\frac{a}{p} - \frac{b}{x} = c$

Problems involving Squares and Square Roots

To remove the √ sign in any algebraic expression you square e.g

$$(\sqrt{x})^2 = x$$

Proof $\sqrt{x} = x^{½}$

$(x^{½})^2 = x^{½ \times 2}$ law of indices $(x^a)^b = x^{ab}$

To remove the square or power 2 in any algebraic expression you find the square root or raise to the power half e.g (x2)

$\sqrt{x^2} = x$

Proof $\sqrt{x^2} = (x^2)^{½}$

$= x^{2 \times ½}$ Law of indices $(x^a)^b = x^{ab}$

$= x$

Exercise 1,4

Make x the subject in each of the following

a) $V = wxy$

b) $ax = c$

c) $dx = vy$

d) $x + z = w$

e) $wx - zx = v$

g) $\frac{z}{x} = y$

h) $\frac{v}{x} - y = z$

i) $\frac{x}{c} + \frac{x}{d} = 3$

j) $\frac{w}{x} + \frac{x}{x} = 4$

k) $d(c-x) = yx$

l) $z = \frac{2c+3x}{3c-2x}$

m) $a = c$ b-a

n) $1 + 1 = 1$ u x f

o) L = 2a E x-r

p) $t = 3p + S$ x

q) $R = \frac{nEu}{R + xr}$

r) $\frac{a}{p} - \frac{b}{x} = c$

Problems involving Square and Square Roots

To remove the √ sign in any algebraic expression you square e.g.

$$(\sqrt{x})^2 = x$$

Proof $\sqrt{x} = x^{½}$

$(x^{1/2})^2 = x^{½ \times 2}$ Law of indices $(x^a)^b = x^{ab}$

To remove the square or power 2 in any algebraic expression you find the square root or raise to the power half. E.g

(x^2) proof $\sqrt{x^2} = x$

Proof $\sqrt{x^2} = (x^2)^{½}$

$= x^1$

$= x$

Example 5

Make the given letter the subject

a) $a^2 + b^2 = c^2$,a b) $T = 2\pi + \sqrt{L/g}$,L

a) $a^2 + b^2 = c^2$

$a^2 = c^2 - b^2$ Subtract b^2 both sides

$a = \sqrt{c^2 - b^2}$ Find the square root both sides

b) $T = 2\pi\sqrt{L/g}$

$\frac{T}{2\pi} = \sqrt{L/g}$ Divide both side by 2π

$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{L/g}\right)^2$ Square both side to remove the square root sign

$\frac{T^2}{4\pi} = \frac{L}{g}$ Multiply both sides by g to make L the subject

$\frac{T^2 g}{4\pi^2} = L$

$L = \frac{T^2 g}{4\pi^2}$

Take Note

When finding the square or square root everything inside the bracket is affected e.g

Exercise 1,5

In each of the given formulae make the each of the given letter(s) the subject.

a) $a = w^2x$ (w) b) $A = 2nr^2$, (r)

c) $a^2 = b^2 + c^2$ (b) d) $x^2 = a$ (x)

e) $\sqrt{x} = a$ (x) f) $\sqrt{2x} = c$ (x)

g) $\sqrt{ax} = b$ (x) h) $\sqrt{y^2 + x^2} = c$ (y)

i) $d\sqrt{x^2 + a^2} = 3a$ (x) j) $H = \frac{M(V^2-U^2)}{2gx}$ (V)

k) $H = \frac{W^2}{2g}(R^2 - r^2)$ (r) L) $V^2 = u^2 + 2as$ (u)

m) $1 = \frac{E}{\sqrt{R^2 + w^2L^2}}$ (R) n) $D = \sqrt{{}^{3h}/_{2}}$ (h)

Substitution

It involves replacing unknowns with specific values in a formulae.

Example 6 (use 22/7 for π)

The volume of a cylinder of radius r and height h is given by $V = \pi r^2 h$

a) Make r the subject of the formula

b) Calculate the radius of the cylinder of Volume $176cm^2$ and height 14cm

a) $V = \pi r^2 h$

$\frac{V}{\pi h} = r^2$

$\sqrt{v/\pi h} = r$

:- $r = \sqrt{v/\pi h}$

b) $r = \sqrt{v/\pi h}$

$V = 176cm^2$ $h = 21$

$r = \sqrt{\frac{176}{22/7 \times 142}}$

$r = \sqrt{4}$

$= 2cm$

Exercise 1,6

i) The surface area of a sphere of radius r is given by $S = 4\pi r^2$

a) Make r the subject of the formulae.

b) Given that $V = 616cm^2$ find r the radius

2) The surface area of a cylinder of radius r and height h is given by $S = 2\pi r(h + r)$

a) Make h the subject of the formulae.

b) Find the height h given that $r = 7cm$ and $v = 440cm^2$

3) The simple interest, \$1, on a sum of money, \$P after T years at R% is given by the formulae

$$I = \frac{PRT}{100}$$

a) Make R the subject of the formulae

b) Given that P= \$2 400, T= 2 years and I= \$440, find the rate, R.

4. The total cost C dollars of running a hotel is made up of a fixed value a dollars and a value which varies as the square of the number of people p. The values are connected by the formulae $C= a + 5p^2$

a) Find the value of C when a = 80 and p = 5

b) The fixed value being \$80, make p the subject of the formulae and find p when C= \$260.

Examination Questions

1. Given that $\frac{ax - 3}{2a - 3x} = 2$, express x in terms of a

2. Given the formulae $S= ut + \frac{1}{2} at^2$

a) Make a the subject of the formulae

b) Given that S= 16m when U= O and t= 2s find a

3. The formula connecting the mass Mg of metal washers of external radius Rmm, and internal radius r mm and a thickness of Tmm, is: given by $M= \frac{T (R+ r) (R- r)}{40}$

a) Find the mass of a washer of external radius of 18mm and 12mm and thickness 1,2mm.

b) 1 000 washers each of radii of 24mm and 16mm are stacked together to form a pile and have a total mass of 12kg. Find the thickness of these washers.

4) Given the formulae $S= \frac{1}{2} n (r + l)$

i) Calculate the value of S when n= 13, r= 7 and l= 11.

ii) Make r the subject of the formulae

5. Given that $Z= \frac{4w}{V-S}$

i) Calculate the value of Z when x= 6 and S= 2,5

ii) Express v in term of Z and W.

6. Given that $Z= r\sqrt{n-1}$

i) Find Z when r= 0,3 and n= 50

ii) Express n in terms of Z and r

ZIMSEC JUNE 2006

CHAPTER 31

Construction

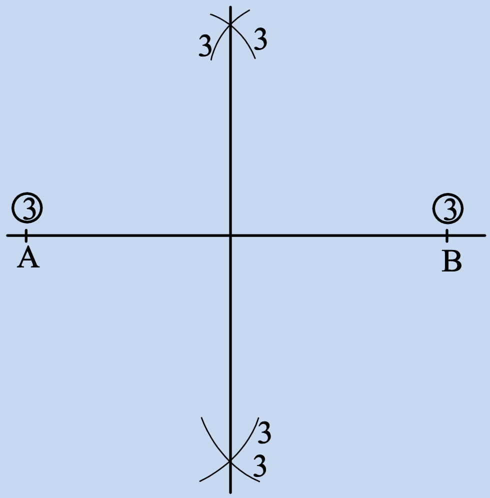
Syllabus Objectives
Leaner should be able to:
a) Draw a perpendicular bisector to a given straight line
b) Draw angles of 30^0, 45^0, 60^0;120^0, 150^0, 135^0
c) Draw a triangle given three side only and given on angel
d) Draw a quadrilateral
e) Draw the requires loci

Guide lines for accurate constructions

a) Use a sharp pencil and a smooth ruler
b) Tighten your compass so that it is not loose or too light
c) Draw a bigger line and measure your required length of line segment from that line
d) All constructions line should be visible and should not be erased.
e) Lines should pass right through the point

1) BISECTING A LINE SEGMENT (PERPENDICULAR BISECTOR)
BISECTING A LINE MEANS DIVIDING IT INTO TO EQUAL PARTS

Steps
1.Draw a big line and measure your line segment AB.
2.Measure about ¾ radius of your line
3.Placing compass point on A draw curves above and below the line. Do the same at point B.
4.Joint the two point of intersection of curves below and above the line.



Take note

-The cycles numbers show where compass needle point would be resting on a given step. Same number on different point means the same activity is done in the same step but at different points.
-Number out of cycles represent curves draw for a particular step. eg step 3 result in curve 3.

2) **Bisecting an angle**

It means dividing an angle into two equal parts.

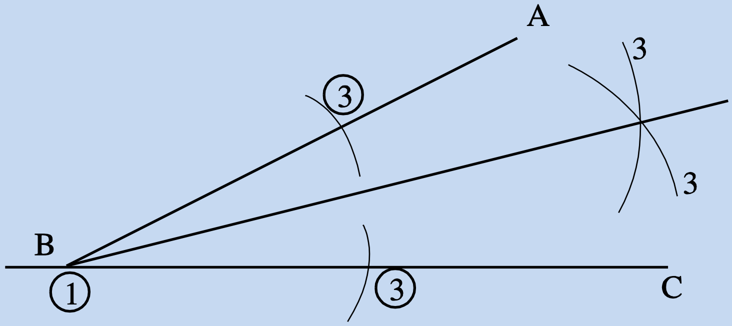
Steps

1) open any radius and place your compass point at point B

2) Draw curves which intersect line AB and BC.

3) At the points of intersection Place the compass point and Draw two intersecting curves.

4) Joint the point of intersection to B.



Practice exercise 1,1

1) Bisect a line segment of 10cm
2) Draw an angle of 50^0 and bisect it

To construct an angle of 90^0 to a given point

1) Mark your point A on your line
2) Place the compass point at that point
3) and use the same radius, draw curves which Intersect on either side of your point Place your compass at the two points of intersection and draw curves to intersect each other above the line.
4) Join the point of intersection to your point A.

Bisect an angle of 90^0 to construct an angle of 45^0

From an angle of 45^0 an obtuse angle of 135^0 (180^0-45^0)

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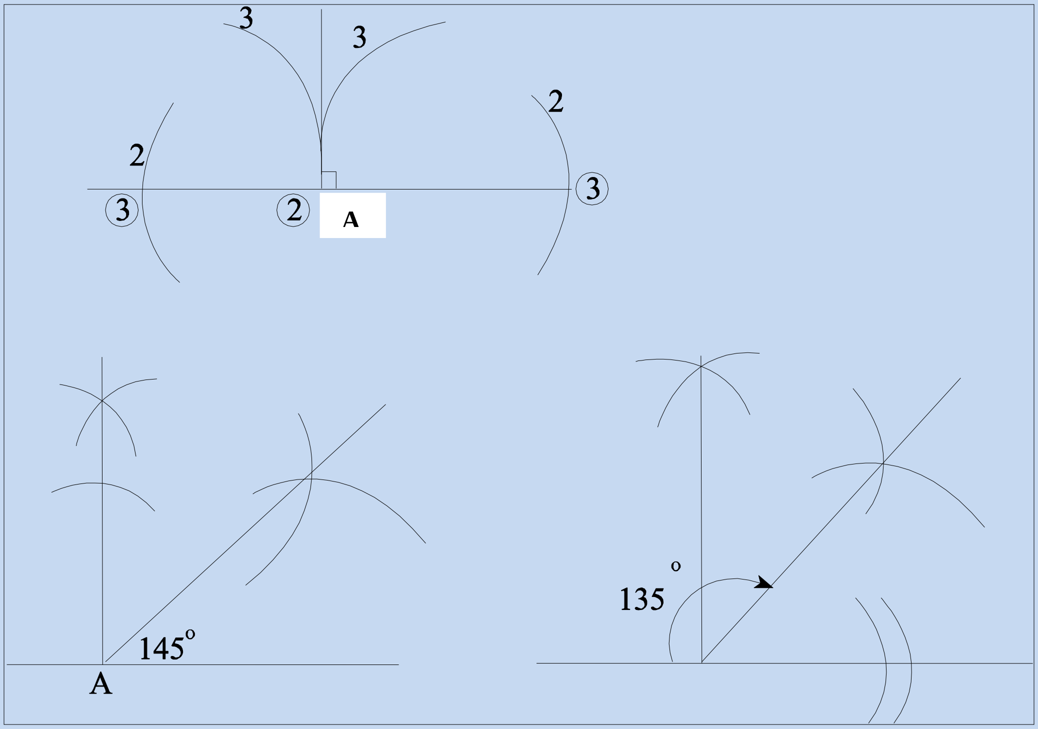
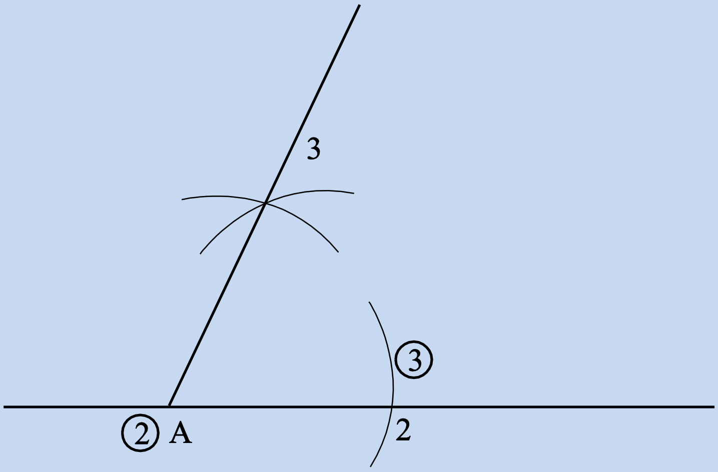


To construct an angle of 60^0 to a given point

1) Mark your point A on your line.
2) Place your compass point on A
   Then draw a curve from vertically
   above the point until it intersects
   The line.
3) Place your compass at the point
   Of intersection and draw a curve
   that intersects the first curve.
4) Joint point A to the point of
   Intersection of the curves



Bisect an angle of 60^0 to Construct an angle of 30^0

From an angle of 30^0 and 60^0 obtuse angles of 150^0 (180^0-30^0) and 120^0 (180^0-60^0) respectively are formed.

REVISIT DRAW

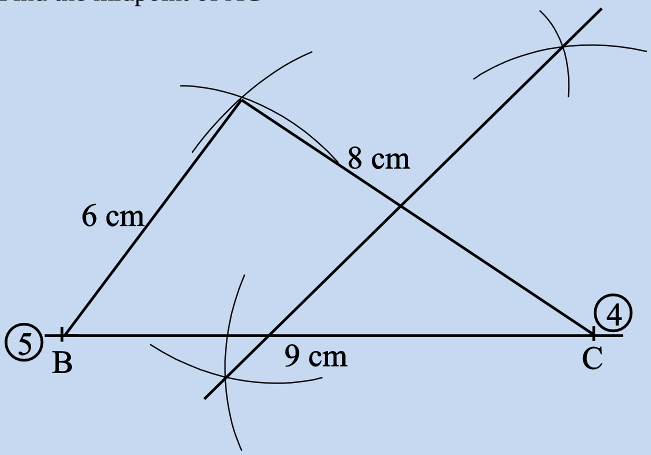
Practice exercise 1,2

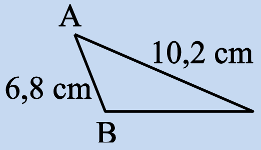
1) Construct the following angles

a)	90^0	b) 60^0		c) 45^0	d)	30^0	
e)	120^0	f)	135^0	g)150^0	h)	180^0	

Example 1

Construct triangle ABC with sides AB = 6cm, BC= 9, AC= 8cm
Find the midpoint of AC



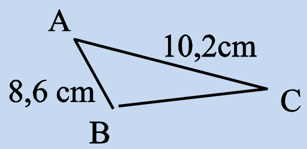


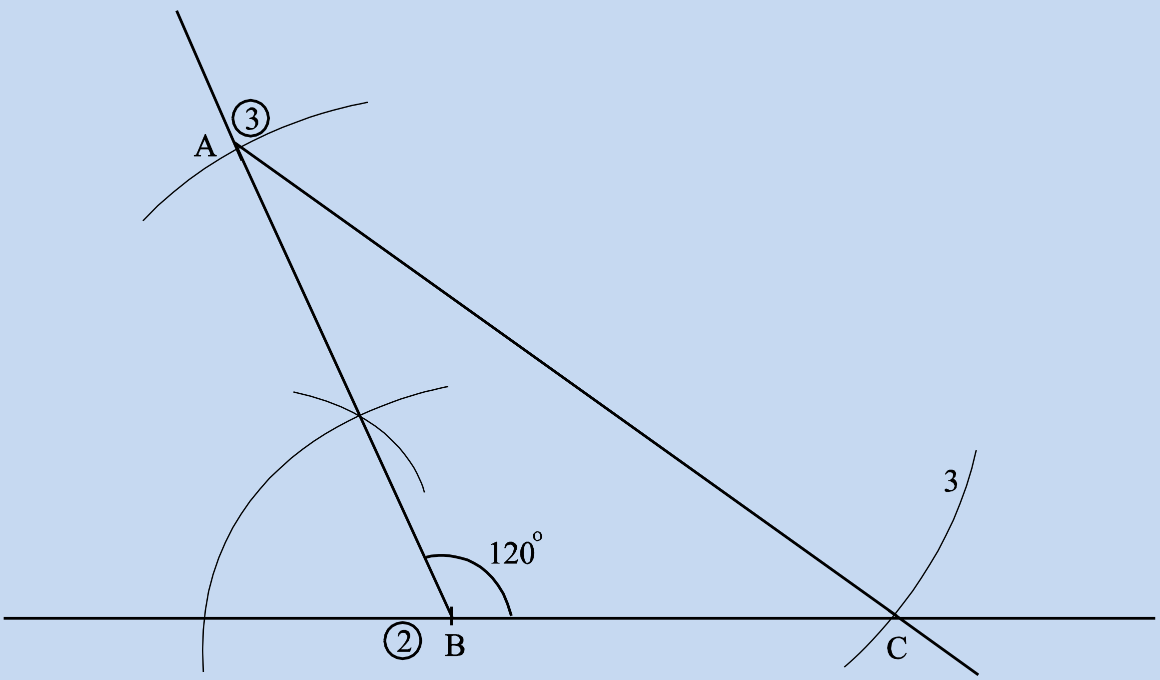
step

1. First draw a rough sketch of your triangle
2. Let you longest side be your base.
3. At point, B, measure a radius of 6cm
   And draw a curve.
4. At point, C, measure a radius of 8cm and
   draw a curve.
5. Join the points of intersection of two
   curves to the points B and
6. Midpoint of AC is: a perpendicular
   Bisector of line AC.

Example 2

Construct triangle ABC with AB= 6,8cm, AC= 10,2cm and B= 120





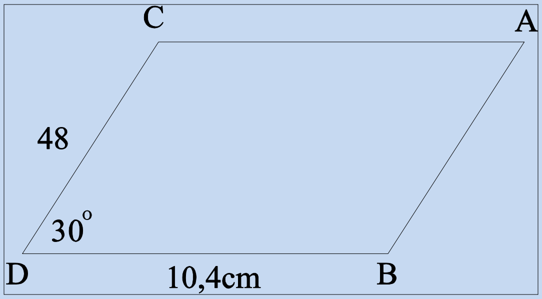
Steps

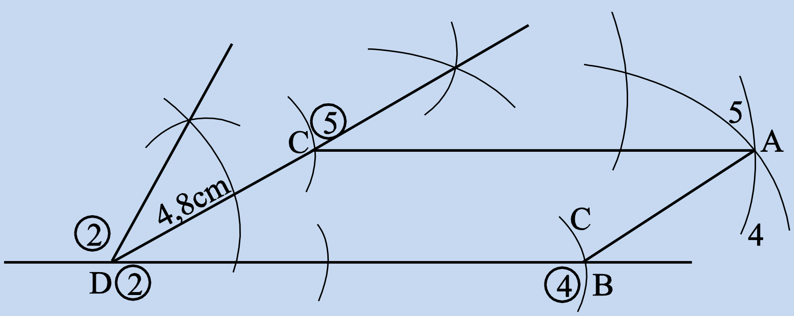
1) construct on angle of 60^0 at point B
2) Measure a radius of 6,8cm on your compass
   and with the campus point at B draw a curve which
   Intersect the line forming an angle.

3) Measure a radius of 10,2cm and place the compass at point at point of intersection (ie, A) and draw a curve that Intersect the line in which B lies. That point forms your point C.

Example 3

Construct a parallelogram ABCD with BD = 10,4cm, DC = 4,8cm and angle BDC = 30^0 Measure AC





1. Mark point D on your line and construct an angle of 30^0
2. Place compass at D and draw an arc with a radius of 4,8cm. Point C, is formed at the point of intersection of line and curve.
3. With a radius of 10,4cm and compass at D draw a curve which intersect the line on which D lie. Point of intersection forms point B.
4. Since opposite sides of a parallelogram are equal. From point B draw a curve of radius 4,8cm.
5. From point, C, draw a curve of radius 10,4cm. where the two curves meet is point A.

Exercise 1,1

1. Construct △ABC in which AB= 8cm, BC = 9 8cm and AC= 7,2cm.
b) Draw the perpendicular bisectors of all three sides. These should meet at one point O.
c) With centre O and radius OA, draw a circle
d) Measure the radius of the circle
2. Draw the△PQR with sides 90mm, 105mn and 85mm.
b) Draw the perpendicular bisectors of all three sides. These should meet at one point O.
c) With centre O and radius OA, draw a circle.
d) Measure radius of the circle.

3. Construct △ABC with BC= 8cm, AB= 6,4cm and ABC=60^0
b) Use ruler and compass to find the midpoint of AB.

4. Construct△ABC in which BC=6,5cm ABC= 90^0 and ACB= 30^0

b) Construct M the midpoint of AC
c) Measure AM.

5. Draw a quadrilateral ABCD in which AB= 4cm, BC = 6cm, (1)= 9cm and AD= 7cm.

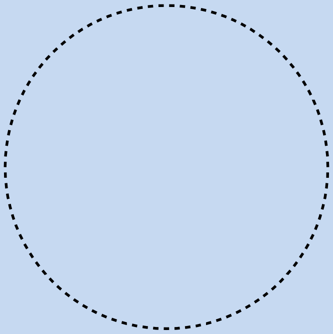
6. Construct a quadrilateral ABC (1) such that BC = 6cm, ABC $=90^0$, AB= 9cm, AD= 8cm and DC= 10cm.
7. Construct quadrilateral ABCD such that AB= 5cm, BD= DC= 8cm, ABD = 30^0 and BCD= 45^0
b) Measure the diagonal AC

<u>Locus</u>

A straight line can be said to be a set of point closely joined together to form that continuous set point called a straight line.

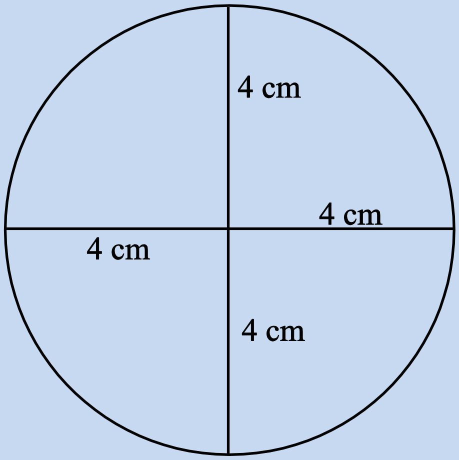
\- -

The same is true for the circumference of a circles. It is also a set of point closely grouped together.



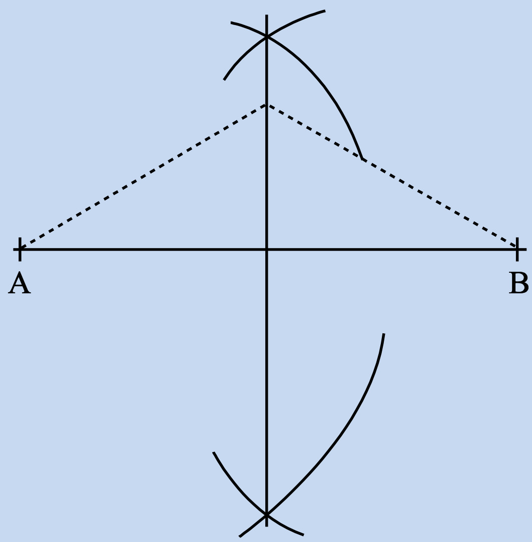
Locus refer to a set of points that will always be equidistant (equal distance) from a fixed point, or a fixed line (set of points).

1. **<u>Locus of points from a fixed point</u>**
Consider a circle of 4cm of centre O



Every point on along the circumference will always be 4cm from the centre O. All points along the circumference are equidistance from O. it means that the circle describes the locus of points equidistance from O.
***Therefore, the locus of points equidistance from a fixed point is a circle with that point as the centre.**

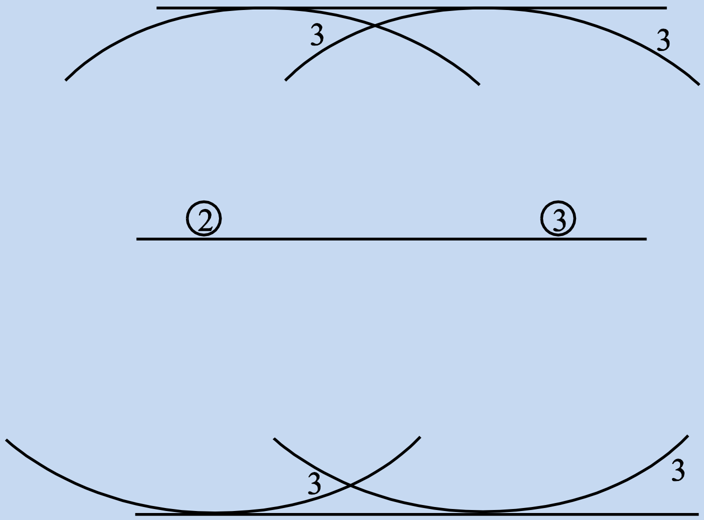
2) **<u>Locus of points equidistant from two fixed points</u>**
Consider a perpendicular bisector of line AB which is 4cm
Insert:
Every point along the bisector
Will always be equidistance from
Points, A and B. For example,
AX = BX= 4cm
AZ= BZ= 2,3cm.



The locus of points equidistant from two fixed points is a perpendicular bisector of that line.

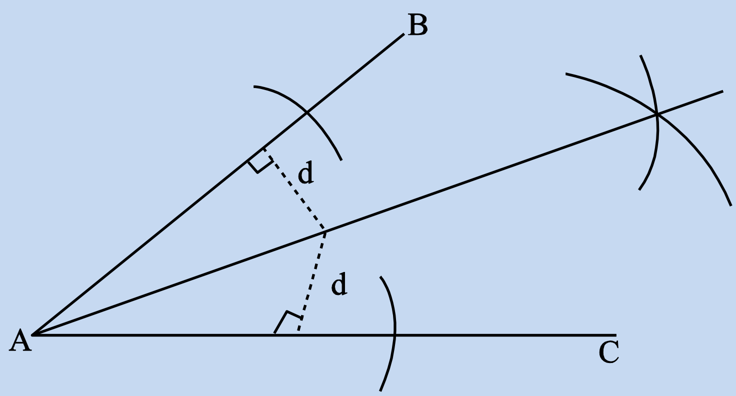
3) **<u>Locus of points equidistant from a straight line are lines parallel to that straight line</u>**
For example, showing the locus of point 2cm from line AB

Insert:
1) Draw line AB
2) Measure a radius of 5cm and place compass on any point along line AB and draw a curves above and below AB.
3)Place the same compass on
Another point along the line and
Draw a curve below and above AB
4)Above AB draw a line that just
Touches the turning points of your curve (draw a tangent)
5) Do the same with curves below the line.



4) Locus of points equidistant from two straight lines is bisector of the angle formed by the two lines:

All point along the bisector will always be an equal
Perpendicular distance from AB and AC.



<u>Practice Exercise</u>

1) Draw the locus of points 2cm from a fixed point O.
2) Draw the locus of points of 2 points x and y which are 7cm apart.
3) Draw the locus of points that are 2,5cm from line PQ.
4) Draw the locus of points equidistance from two lines from an angle of 40^0.

<u>Example 4</u>

Draw accurately the $\triangle$ABC with base AB= 12cm, (CAB= 30^0 and AC= 10cm)

i) Measure and write down the length of BC.
ii) On the same diagram (a) draw the locus of points, within the triangle ABC, which are 3cm from AC.
b) Construct the locus of points equidistant from AB and BC.
ii) A point P lies inside the $\triangle$ABC. The position of P is such that it is more than 3cm from AC but its distance from BC is less than its distance from AB. Indicate clearly, by shading, the region in which P must lie.

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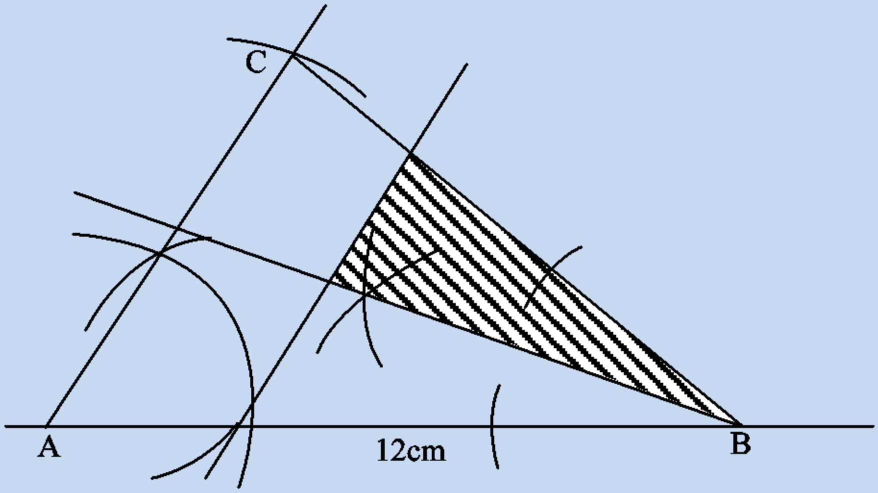

1) Draw $\triangle$ ABC

Construct locus of points 3cm from line AC. (only points inside the triangle).

b) Construct a bisector of angle ABC

iii) Point P is outside the quadrilateral formed by AC and the line parallel to it.

EXAMINATION QUESTIONS

1) Use ruler and compass only

i) Construct $\triangle$PQR in which PQ= 7,5cm, QR= 6cm and PQR = 90^0

ii) Measure and write down the length of PR.

iii) Draw the locus of points which are 5cm from the point R

iv) Draw the locus of points which are 2cm from the line QR and on the same side of QR as P.

v) Mark the two points, X inside the triangle and Y outside the triangle, which are 5cm from R and 2cm from QR.

vi) Calculate the area of $\triangle$QRY Comb Nov 199^0

2) On a single diagram construct

i) A line, 9cm long

ii) A circle centre O and radius 3,5cm

iii) The locus of points which are equidistant from O and P.

iv) The circle whose diameter is OP to cut the circle centre O at R and Q.

v) The two tangents to the circle centre O from the point P.

b) OP represents a certain locus. Describe this locus fully.

c) A point T lies inside the quadrilateral PQOR and is such that it is nearer PQ than PQ than PR and nearer O than P. Given also that OT ≥ 3,5cm, show by shading clearly the region in which T lies

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3. Construct the parallelogram ABCD in which AB= 9cm, AD= 5cm angle BAD =60^0. Measure, and write down, the length of AC. On the same diagram. Construct.

i) The locus of a point X which moves so that it is equidistant from A and C.

ii) The locus of a point Y which moves so that angle BYD = 90^0.

The position of a point P, which lies inside the parallelogram, is such that AP ≥ PC and angle BPD ≤90^0, Indicate clearly, by shading, the region in which the point P must lie.

4) Construct $\triangle$XYZ in which YZ = 10cm, XYZ =600 and XY= 5,2cm.

ii) the locus of points which are equidistant from XY and XZ

iii) the point P which is equidistant from XY and XZ and is such that YP is parallel to XZ. Measure and write down the length of XP.

5) Use ruler and compasses only for all constructions and show clearly all construction arcs and lines on a single diagram.

P,Q,R and T are four villages. P is due north of T. The bearing of Q from P is 240^0, P,Q and R are in a straight line in that order. PR = 10km, QR =2km and angle PRT = 45^0

a) Using a scale of 1cm to 2km construct a diagram to show the relative positions of P,Q,R and T.

b) Construct the locus of points

i) equidistant from P and R

ii) 5km from T.

c) A school is to be built such that it is equidistant from P and R and 10km from T, mark and label clearly S, and S_2 the possible positions of the school.

d) Use the diagram to find

i) the actual distance of Q from T,

ii) the bearing of R from T.

CHAPTER 32

Transformations: Isometric

A transformation is a change in position and, or size of a shape.

Isometric transformations result in change in position only. Images that are congruent to the original object are formed. Isometric transformations are translation, reflection and rotation.

Syllabus objectives

Learner should be able to:

a) Carryout a translation using a translation vector $\begin{pmatrix} x \\ y \end{pmatrix}$

b) Carryout a reflection in a given line of reflection

c) Find Matrix operators

d) Carryout rotation through a given angle about a given centre.

e) Describe a transformation fully

A translation is a transformation that moves every point of a given figure or points in the same direction by the same distance. This is done by a translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$

If a point P (x:y) on a Cartesian plane is translated by vector $\begin{pmatrix} a \\ b \end{pmatrix}$ its image P^1 (x+a ; y +b)

If P is the point (2,3) and it is translated by vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, its final position Q is shown below

REVISIT GRAPH

Further points on translation

1) No point is invariant, i.e., no point Stays in its original position
2) Direct isometry- the image is identical to the original, although in a different place.
3) A translation is the only transformation Described by a column vector. All the others are described by 2 x2 matrices.

Generally if T represent the translation $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ then

T (p)= q

Example 1

Given points, A (3,2); B (-1;4) and (1;-2), find the images A^1,B^1,C^1 under T= $\frac{2}{2}$

T (A)= A^1 T(B)=B^1 T(C)=C^1

$A^1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\quad B^1 = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ $\quad C^1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ $\quad = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ $\quad = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

:-A^1 (5,4) $\quad$ B^1 (1,6) $\quad$ C^1 (3,0)

Example 2:

Draw triangle ABC with A(5,4); B (1;6) and C(3,0)

b) Given that the image of C is point (2,2). Find the column vector of this translation

ii) Hence, find, the coordinates of the $\triangle A^1 B^1 C^1$ and show it on the graph.

Insert:

REVISIT GRAPHY

b) T (C) = C^1

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$3 + x = 2$

:- $x = -1$

y= 2

:- Translation vector= $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

ii) T (A) =A^1

$A^1 = \begin{pmatrix} 5 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

A^1= (4;6)

T (B) = B^1

$B^1 = \begin{pmatrix} 1 \\ 6 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 8 \end{pmatrix}$

:- B^1 (0,8)

Reflection

Suppose a triangle ABC is drawn on a sheet of paper and a mirror is placed vertically such that the horizontal distance from every point on the triangle is perpendicular to it. As shown below.

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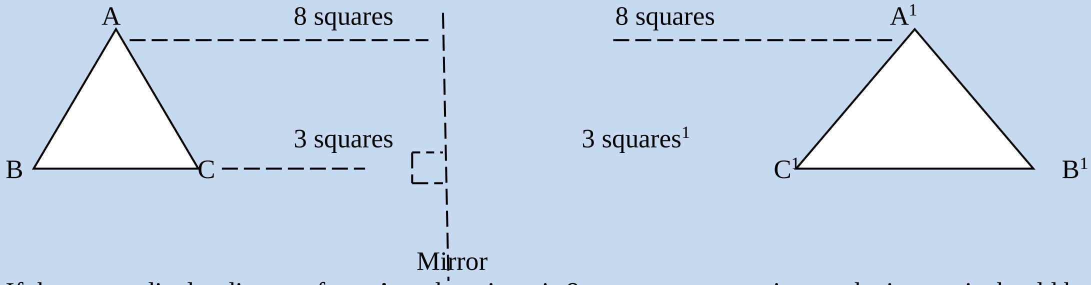
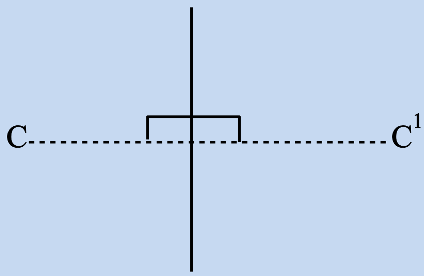


If the perpendicular distance from A to the mirror is 8 squares, on counting on the image, it should be noted that the image of A also forms a length of 8 squares. The same is true for C and C^1, and B and B^1.

In transformation $\triangle$ABC is reflected in the mirror line to form image A^1 B^1 C^1. The mirror line is called invariant line and it is stated specifically in the cartesion plane e.g. x axis, y axis etc.

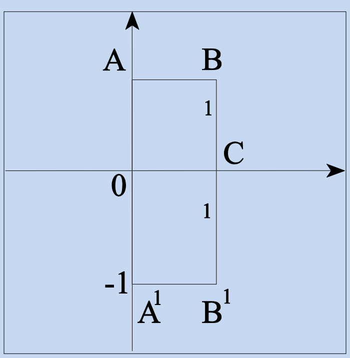
- The mirror line forms a perpendicular bisector between a point and its image.



mirror line

Finding Matrices for reflection

Consider a unit square being reflected in the x axis



C is invariant as it lies on the x axis.

if ,M represents the reflection in the x-axis, then considering what happens to OA and OC,

M(OA) = OA^1 and M (OC)= OC^1

If M is the matrix $\begin{pmatrix} w & x \\ y & z \end{pmatrix}$, then

$$\begin{pmatrix} w & x \\ y & z \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (1)$$

and $$\begin{pmatrix} w & x \\ y & z \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (2)$$

(1) $\begin{pmatrix} -x \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

:- x= 0 z=-1

(2) $\begin{pmatrix} w \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

:- w= 1 y= 0

Hence M = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$M^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

M^2 = **I** (identity)

Find the matrix when the unit square is reflected in the y axis, y= x and y = x.
If the Cartesian plane is given a reflection M then every point on the plane, except those on the line of reflection is transformed. The line of reflection is on invariant line.

<u>Summary of Matrices of reflection</u>

Mirror Line	Matrix
Reflection in y- axis	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Reflection in y axis	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
Reflection in y = x	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Reflection in y = -x	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

The mirror line is invariant under reflection.

<u>Example 3</u>

The △ PQR with vertices P (1,2), Q (3;2) and R (4;-1) is reflected in the x axis

i) Write down the matrix M of reflection.

ii) Find the co-ordinates of the vertices of the image $P^1 Q^1 R^1$ of the $\triangle$PQR under M.
iii) Find the value of the determinant of M.
i) Under reflection in the x –axis

$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\text{ii)} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{pmatrix} 1 & 3 & 4 \\ 2 & 2 & -1 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} P^1 & Q^1 & R^1 \end{array} \\ \begin{pmatrix} 1 & 3 & 4 \\ -2 & -2 & 1 \end{pmatrix} \end{array}$$

:- P^1 (1;-2), Q^1 (3;-2), R^1 (4, 1)

iii) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ det of M= -1

Exercise 1,1

1) Find the image of a triangle whose verticals are A (2,0), B (0,-2), C (3;-3) under the translation T= $= \frac{2}{3}$

2) Find the image of the quadrilateral A (0;0), B (6;-1), D (4,2) C (9,9), under the translation which maps D on to D^1 (-1,4).

i) Find the vector for translation

ii) Hence find, the images A^1 and B^1

3) The image of $\triangle$ PQR are P^1 (6;1), B^1 (1;2), C (7;3) under a translation vector $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ find the coordinate of $\triangle$ PQR.

4) Draw the figure and its image, and state the translation.

a) square PQRS where P (2,3), Q (5;3), R(5;1), S (2,1) translated so that point S (-2;-2).

b) Quadrilateral OABC, where O is (8; 12), A (10;10), B (10,13) (9;14) translated so that 0=B.

5) $\triangle$ PQR with vertices P (2;3), Q 4;3), R (5;0) is reflected in the y axis.

i) write down the matrix M of reflection

ii) Find the, coordinates of the vertices of the image $P^1 Q^1 R^1$ of the $\triangle$ PQR under M.

6. Find the image of the quadrilateral A (0;0), B (6;-1) D (-4,2), C (9,9) when reflected in the x axis.

7. Draw $\triangle$ ABC the reflection of $\triangle$ABC where A (-6;2), B (-4,6) and C (-2,4).

i) in the x axis

ii) in the line y =x

8. Draw $\triangle$ABC where A is (1,3), B (1,1) and C, (2,1). Triangle ABC is mapped onto triangle $A^1 B^1 C^1$ by a transformation given by the matrix M= $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Draw triangle $A^1 B^1 C^1$ in your diagram and describe the transformation

9. The image in question $\triangle$ $A^1 B^1 C^1$ is mapped onto triangle $A^{11} B^{11} C^{11}$ by the matrix k $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Draw triangle $A^{11} B^{11} C^{11}$ in your diagram and describe the transformation given by the matrix K.

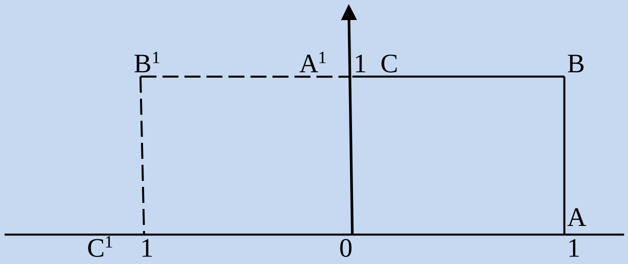
10. Apply $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ to the vertices of $\triangle$ PQR where P(1,0), Q(3,2), R(4,0). On a graph paper show

$\triangle$PQR and its image $\triangle P^1Q^1R^1$, Describe the transformation fully.

Rotation

It is a transformation that is achieved by a turning.

Matrices of 90^0 about the origin (anticlockwise)



After the 90^0 turn the position of the origin does not change. Point A $\begin{pmatrix}1\\0\end{pmatrix}$ moves to the image position A^1 $\begin{pmatrix}0\\1\end{pmatrix}$.

Point

C $\begin{pmatrix}1\\0\end{pmatrix}$ moves to the image position C^1 $\begin{pmatrix}-1\\0\end{pmatrix}$ thus:

$\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}-1\\0\end{pmatrix}$

:- Therefore the matrix for 90^0 rotation about the origin is $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$

Summary of some rotation Matrices

Transformation	Matrix
Rotation of 90^0 centre Origin anticlockwise (tve)	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$
Rotation of 90^0 centre Origin clockwise (-ve)	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
Rotation through 180^0 About origin	$\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$

It should be noted that rotation is an isometric transformation since the shape of an object or figure do not change after a rotation.

Example 4

△ABC has vertices A (1,1); B (1;6) C (5,6)

a) Draw and label △ABC .

b) find it images △ $A^1 B^1 C^1$ under a 90^0 rotation anticlockwise about the origin

Matrix for 90^0 anticlockwise centre origin (0,0), $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 1 & 1 & 5 \\ 1 & 6 & 6 \end{pmatrix} \end{array} \begin{array}{c} \begin{array}{ccc} A^1 & B^1 & C^1 \end{array} \\ \begin{pmatrix} -1 & -6 & -6 \\ 1 & 1 & 5 \end{pmatrix} \end{array}$$

△ A^1 B^1 C^1 has coordinates A^1 (-1,1), B (-6;1), C(-6,5)

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Rotation when the centre of rotation is not the origin.

Given the object and its image

Step 1: join the corresponding sides

Step 2: Bisect only two lines joining the corresponding Sides. The coordinates of the point of intersection of the lines forms the centre of rotation

Step 3: Measure the angle of rotation

Example 5

Mark the points A (2,0), B (4,1), A^1 (2,1) and B^1 (0,2) on graph paper. Find the Centre and the angle of the rotation which maps AB onto A^1 B^1

Insert:

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Step1. Join the corresponding sides i.e. A to A^1 B to B^1
Step 2. Bisect them and the point of intersection of the perpendicular bisectors is the centre
Step 3. Measure the angle of rotation
Centre
C (1,8; 0,5)
Angle BCB1 approx 127^0

Example 6

△ABC has vertices A (2,2), B (-2;4), C (0;8)
Find the coordinates of the vertices of the image of △ABC after rotation of 270^0 clockwise about the point (3;2) .
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Step

1) Draw △ABC.
2) Draw a straight line from the vertex A to the centre (ie (3;2).
3) Measure an angle of 270^0 clockwise from that line and Draw a straight line forming that angle
4) Measure the distance from x vertex A to the centre. Use that same measurement to locate the position of A^1 from the centre

5) Repeat stage 2,3 and 4 to locate the images B^1 and C^1

Exercise 1,2

1) Find the coordinates of the images of each of the following under 90^0 clockwise about the origin.
i) A (2;2) ii) P(-3;-2) iii) Q (6,0)

2) $\triangle$ ABC has the vertices A(1,1), B(1,6), C(5,6) Graph $\triangle$ABC and its image $\triangle A^1 B^1 C^1$ under a 90^0 anticlockwise rotation about the origin.

3) Rectangle ABCD has the vertices A(1,4), B(9,4), C(9,8) and D (1,8). Graph rectangle ABCD and its image under 180^0 about the origin.

4) State the matrices for rotation of the following

i) Rotation of 90^0 centre origin anticlockwise

ii) 270^0 rotation about the origin

5) $\triangle$ PQR has vertices P(11,0), Q (9,2), R (12,2). It is mapped into its image $\triangle P^1 Q^1 R^1$ with vertices P^1 (7,7), Q^1 (11,5), R1 (8,5). Using a scale of 2cm to 2 units on both axis draw triangle PQR and its image $\triangle P^1 Q^1 R^1$.

Bi) Hence, find the coordinates of its centre

ii) the angle of rotation

6) Answer this question on a sheet of graph paper. The triangle ABC vertices A(2;0), B (4;4) and C (0;1)
The triangle PQR has vertices P (8;-2), Q(4;0) and R(7;-4)
The triangle LMN has vertices L(-2;-7), M(-6;-9) and W (-3;-5).

Draw these triangles on graph paper using a scale of 1cm to unit on each axis, and label the vertices.
$\triangle$ABC can be mapped onto $\triangle$PQR by an anticlockwise rotation about the origin followed by a translation.

i) State the angle of rotation

ii) Find the matrix which represent this rotation

iii) Find the column vector of the translation.

iv) Given that $\triangle$ABC can be mapped onto $\triangle$PQR by a single rotation, find the coordinates of the centre of this rotation.

v) Given that $\triangle$ABC can be mapped onto $\triangle$LMN by a translation of $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ followed by a reflection in the mirror line m, draw the line on your graph and label it clearly.

7) $\triangle$ ABC whose vertices are A(2;1), B (5;2) and C (3;3) is transformed by a transformation U represented by the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

i) Find the, coordinates of the vertices of $\triangle$ $A^1 B^1 C^1$

ii) Describe, completely, the transformation U

8) The triangle ABC with vertices A(1,1), B(4;1) C(2;3) is mapped onto triangle T, by an anticlockwise rotation through 90^0 about the point (-2,0). Draw$\triangle$ ABC and its image T_1.

CHAPTER 33

Non-Isometric Transformation

These are transformations that do not retain their size when transformed, but retain some characteristics of the original shape. Among these transformations are:

c) Enlargement
d) Stretch
e) Shear

Syllabus Objectives
Leaner should be able to

a) Carryout an enlargement transformation
b) Carryout a stretch transformation
c) Carryout a shear transformation
d) Solve compound transformation (those involving more than one transformation)

Enlargement

An enlargement with centre O, scale factor k is a transformation which enlarges a given figure k times the original value.

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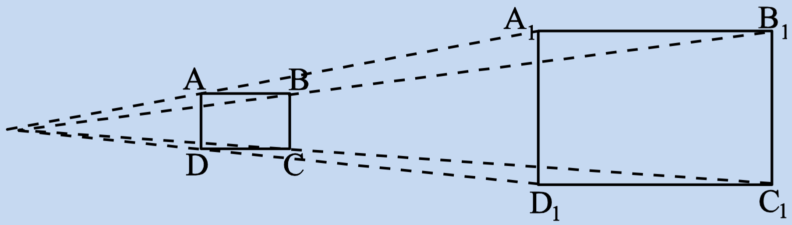

The diagram above show square $A_1B_1C_1D_1$ which is an enlargement of square ABCD, with O as the centre of enlargement and a scale factor of 3.

To find the Centre enlargement produce any two lines of corresponding vertex of original shape and vertex of image, their point of intersection, forms the centre of enlargement e.g. AA_1 and DD_1

Generally, an enlargement centre (0;0) scale factor k is $\begin{pmatrix} k & 0 \\ O & k \end{pmatrix}$

Take note

- If K < 0 i.e negative, the image is on the opposite side of the centre of enlargement to the original figure and the image is inverted (upside down)
- If k>0, the image is on the same side of the centre of enlargement as the original figure
- The centre of enlargement is invariant under enlargement

Example 1

$\triangle$ABC with A(2;-2), B(2:2) C(0;2) is transformed by an enlargement M, with scale factor -2

i) Find the co-ordinates of $\triangle A^1B^1C^1$
ii) Plot the $\triangle$ABC and $\triangle A^1B^1C^1$
iii) Find the centre of enlargement

The matrix representing M is

$$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

i) E (ABC) = $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ -2 & 2 & 2 \end{pmatrix}$

$= \begin{pmatrix} -4 & -4 & 0 \\ 4 & -4 & -4 \end{pmatrix}$

A^1-4;4); B^1 (4;-4) C^1 (0;-4)
Insert:

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Stretch

This is a transformation where the shape and size of the original figure are all change

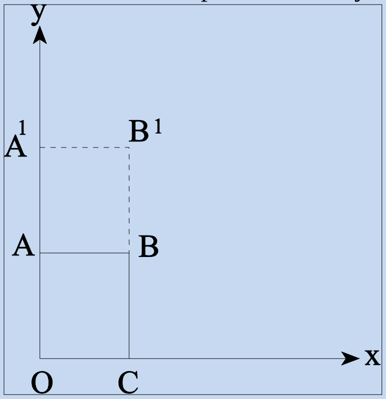
To specify a stretch. We need to state:

i) the invariant line. (i.e fixed line from which distances are measured
ii) the scale factor (i.e magnitude of the stretch.
iii) the direction of stretch

One way stretch

In one way stretch, there is one line perpendicular to the direction of stretch and that line remains invariant under a stretch.

Consider a stretch parallel to the y-axis



It can be seen that the a-axis remain invariant and perpendicular to y axis, the direction of the stretch.

$\frac{A^1O}{AO} = \frac{B^1C}{BC} = k$

For all other points the distance from the image point to the invariant line is proportional to the distance of the original point from the image line. k is called the scale factor.

Stretch with y invariant. Stretch with x axis invasions.
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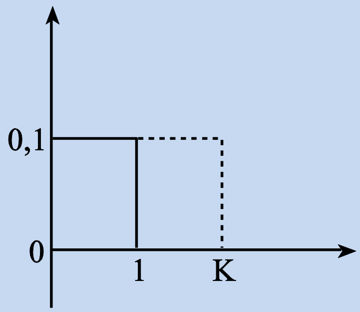
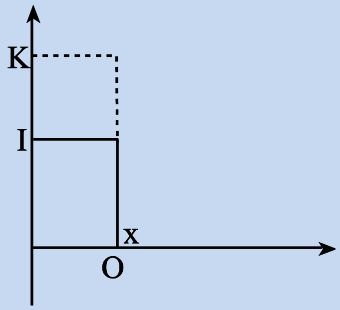




$$\begin{pmatrix}1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}k\\0\end{pmatrix} \qquad \begin{pmatrix}1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}1\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}0\\1\end{pmatrix} \qquad \begin{pmatrix}0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}0\\k\end{pmatrix}$$

Therefore stretch with y axis invariant

$$\begin{pmatrix}k & 0\\0 & 1\end{pmatrix}$$

stretch with x axis invariant

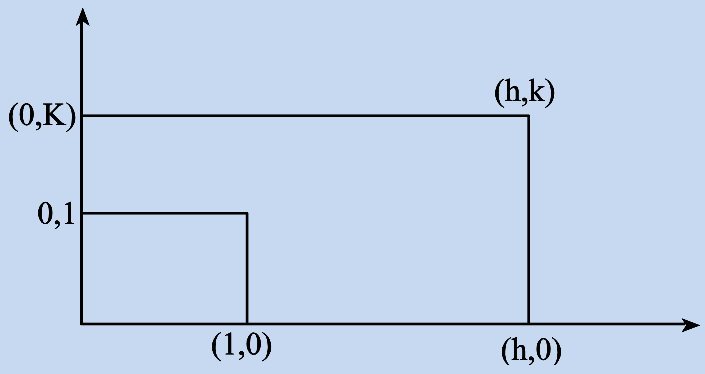
$$\begin{pmatrix}1 & 0\\0 & k\end{pmatrix}$$

To specify a one way stretch, specify
i) the invariant line
ii) the scale factor

A one way stretch is an enlargement in one direction only.

Two way stretch

This is a result of a combination of two one –way stretch
Diagram below show two way stretch parallel to the x axis and y axis.



$$\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}h\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}0\\k\end{pmatrix}$$

A two way stretch with scale factor h parallel to the a-axis and k parallel to the y axis with the origin remaining invariant is represented by the matrix.

$$\begin{pmatrix}h & o\\o & k\end{pmatrix}$$

If h=k, the transformation becomes an enlargement with scale factor k
If the values of both h and k is **I**, the stretch become a one way stretch.

Shear

A shear can be explained by considering a rectangle shape made by tying loosely four wire of their vertices.
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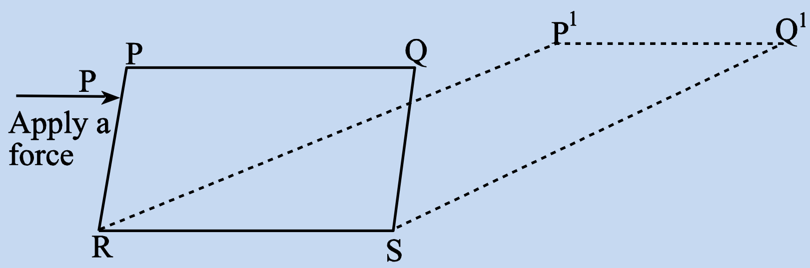


Applying a force on P moves it to its image P^1 and Q to Q^1 with RS remaining invariant
The area remains unchanged under a shear.

$$\frac{P^1P}{PR} = \frac{Q^1Q}{QS}$$

The constant of proportionality k is called the shearing constant

To specify a shear we need to state

i) The variant line or axis of shear (that is, the line on which no points move e.g RS

ii) The shearing constant

If x axis is the axis of shear, then the matrix for the shear is $\begin{pmatrix} 1 & k \\ 0 & I \end{pmatrix}$

If y axis is the axis for shear, then the matrix for the shear is represented by $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$

Example 2

A shear is represented by the matrix

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

a) Calculate the coordinates of the image of the point P (4,-4) under S

b) Calculate the coordinates of the point Q, which will be mapped into (9;6) by S

c) Write down the equation of the invariant line

a) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

$$\begin{pmatrix} x+3y \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

y = 6 $\quad x$ + 3y=9

x + 3(6) =9

x = -9

x = -9, y=6

points (-9,6)

Example 3

$\triangle$ABC has vertices A(2;1) B(3,5) and C(5,1). If S is a stretch such that the y axis is invariant and the point (1,0) is mapped onto (3,0) plot the image of $\triangle$ABC under S

a) Find the matrix for S and state the coordinates

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

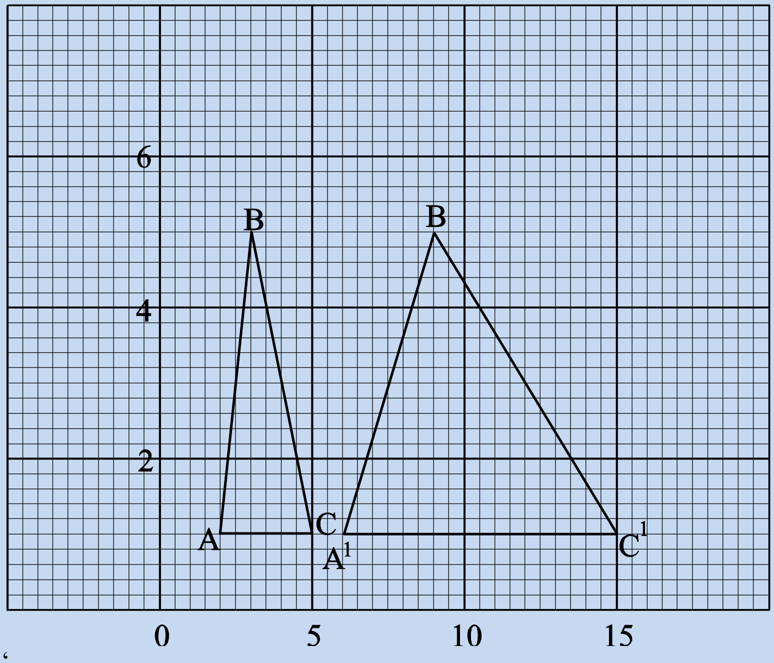
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

Matrix $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 5 \\ 1 & 5 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 9 & 15 \\ 1 & 5 & 1 \end{pmatrix}$$

A^1 (6;1), B^1 (9;5), C^1 (15;1)

a) Matrix for S $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ REVISIT



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Summary

Transformation	**Matrix**
Translation	$\begin{pmatrix} a \\ b \end{pmatrix}$
Reflection in the x axis	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Reflection in the y axis	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Reflection in the line y-x	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Reflection in the line y= -x	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
Rotation of 90^0 centre origin clockwise (+ve)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
Rotation of 900 of centre origin clockwise (-ve)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
Rotation through 180^0 about origin	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
Enlargement centre origin, scale factor k	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$
Shear x-axis invariant, scale factor k	$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$
Shear y-axis invariant, shear factor	$\begin{pmatrix} 1 & 0 \\ k & 0 \end{pmatrix}$
One way stretch parallel to x-axis scale factor k	$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$
One way stretch parallel to y axis scale factor k	$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$
Two way stretch scale factors h and k	$\begin{pmatrix} h & 0 \\ 0 & k \end{pmatrix}$

Exercise 1,1

1) Draw diagram to show the images of the unit square under the transformations given by the following matrices and describe the transformation.

a) $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

2) Draw diagrams to show the images of the unit square under the following transformation and hence write down the matrices representing the transforming

a) R, the clockwise rotation of 90^0 about the origin
b) S, the shear with the y axis invariant and shearing constant 2
c) the transformation R.S (i.e start with R followed by S)

3)a) Describe fully the transformation that maps ABDC into PQRS

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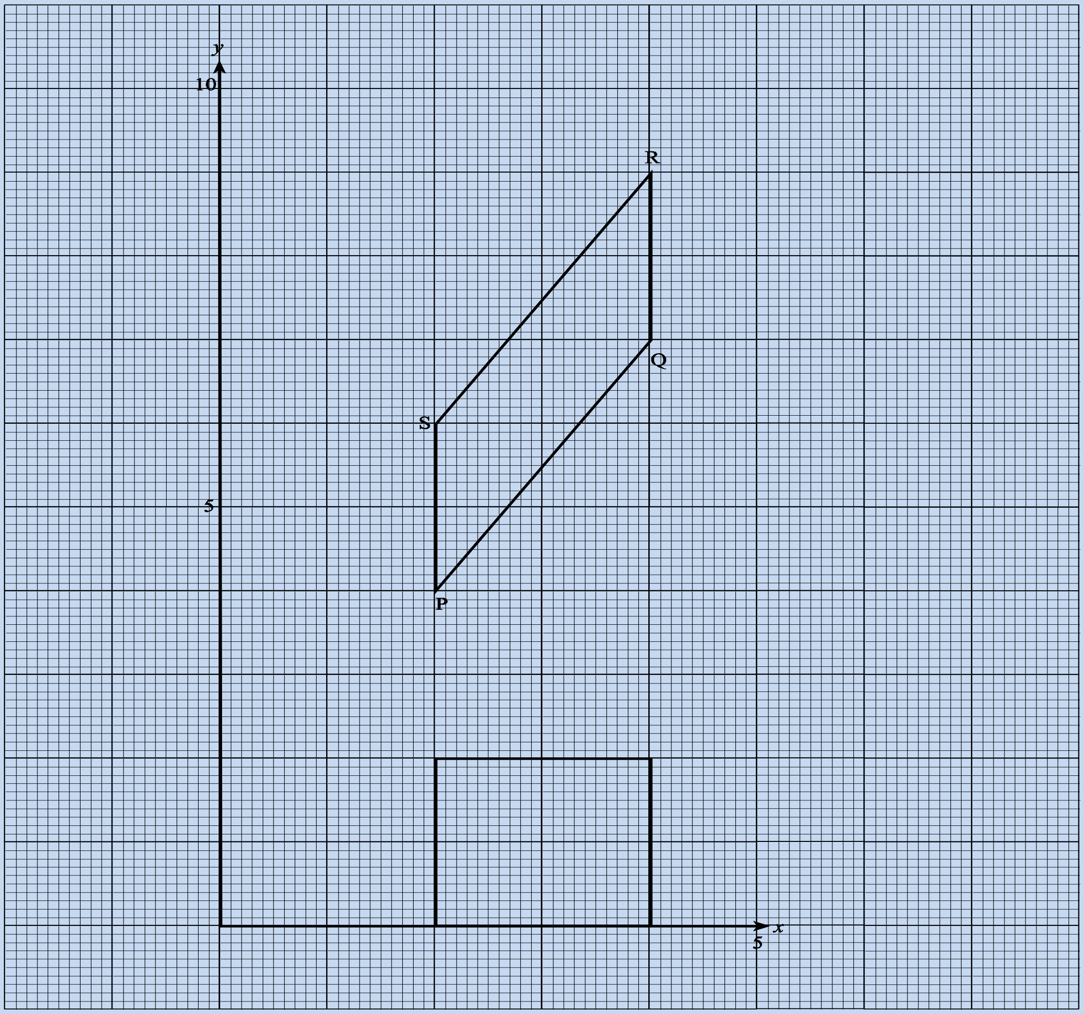

4) The transformation P,Q and K are represented by the matrices

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \quad \text{respectively}$$

a) Describe respectively the transformation P,Q and R

5) The diagram below shows a square OABC and its image $O^1A^1B^1C^1$ under a shear

a) What is the invariant line

b) calculate the shearing constant

c) Hence, write down the matrix representing the shear.

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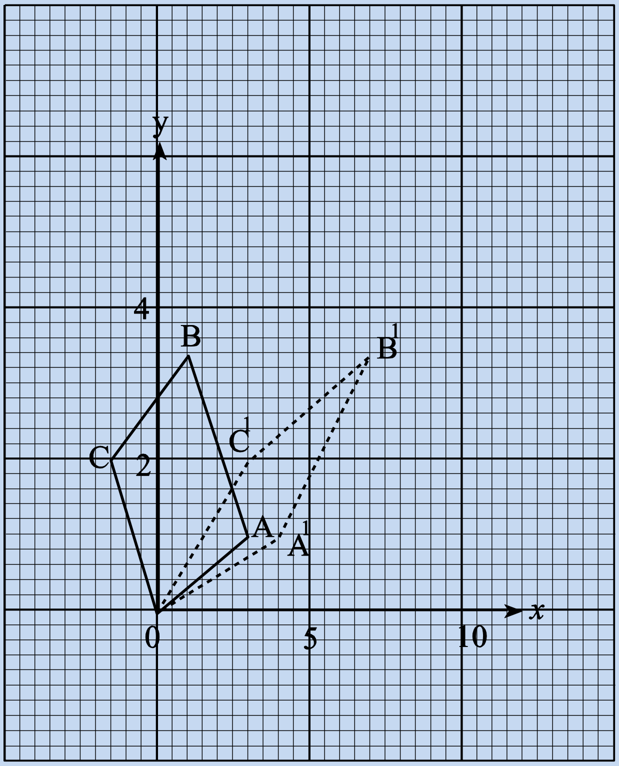

Examination Questions

1) A quadrilateral E with vertices (-8;-4), (-4;-4), (-6; -12) and (-10;-8) is the image of quadrilateral. A with vertices (4,2), (2;2); (2;2) (3;6) and (5;4).
Using a scale of 1cm to represent 1 unit on both axis or both axes, draw the x and y axes for $-10 \leq x \leq 6$ and $-12 \leq y \leq 8$.

a)i) Draw and label clearly the quadrilateral E
ii) Draw and label clearly the quadrilateral A
iii) Write down the matrix which represents the transformation which maps E onto A.

b) Quadrilateral T with vertices (0;6), (0;4); (-4;5) and (-2;7) is the image of quadrilateral A under a certain transformation.

i) Draw and label clearly the quadrilateral T.
ii) Describe completely the single transformation which maps A onto T.

c) A one- way stretch represented by $\begin{pmatrix} 1 & 0 \\ 0 & -1\frac{1}{2} \end{pmatrix}$

Maps quadrilateral A onto quadrilateral S.
Draw and label clearly the quadrilateral S

2) Using a scale of 1cm to represent 1 unit on each axis, draw x and y axes for $-8 \leq x \leq 10$ and $-4 \leq y \leq 18$.
Draw and label the triangle whose vertices are A (1,4), B (2;4) and (2;1).
b) The enlargement E has center the origin and maps ABC on to $\triangle A_1 B_1 C_1$

given that A, is the point (4;16)

i) Draw and label the $\triangle A_1 B_1 C_1$

ii) Write down the scale factor of E

c) The point B2 (-4;-2) is the image of B under a reflection in line L. Find the equation of L.

d) The transformation R, a clockwise rotation of 90º about the origin maps $\triangle$ABC onto $\triangle A_3 B_3 C_3$. Draw and lab el $\triangle A_3 B_3 C_3$ and find the matrix representing R

e) The transformation X is represented by the matrix $\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}$ and it maps $\triangle$ABC onto $\triangle A_4 B_4 C_4$

Find the coordinates of A_4, B_4 and C_4

Draw and label this triangle.

f) Describe fully the transformation X

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3) Using a scale of 2cm to represent 2 units on each axis draw the x and y-axis for $-4 \le x \le 16$ and $-12 \le y \le 8$

a) $\triangle$ABC has vertices at A (0; 3), B (2;1) and C (4;5) Draw and label ABC.$\triangle$

b) $\triangle$ABC is mapped onto $\triangle A_1 B_1 C_1$ by a reflection in the line y = x.

i) Draw and label $\triangle A_1 B_1 C_1$

ii) Write down the matrix that represents this reflection.

c) $\triangle A_2 B_2 C_2$ has vertices at A_2 (6;3), B_2 (4;1), and C_2 (14;5)

i) Draw and label triangle $A_2 B_2 C_2$

ii) Describe fully the single transformation which maps $\triangle$ABC onto $\triangle A_2 B_2 C_2$

d) $\triangle$ABC is mapped onto triangle $A_3 B_3 C_3$ by an enlargement of scale factor- 2 and center (5;0). Draw and label $\triangle A_3 B_3 C_3$

e) Find the ratio $\dfrac{\text{Area of } \triangle ABC}{\text{Area of } \triangle D_3 B_3 C_3}$

CHAPTER 34

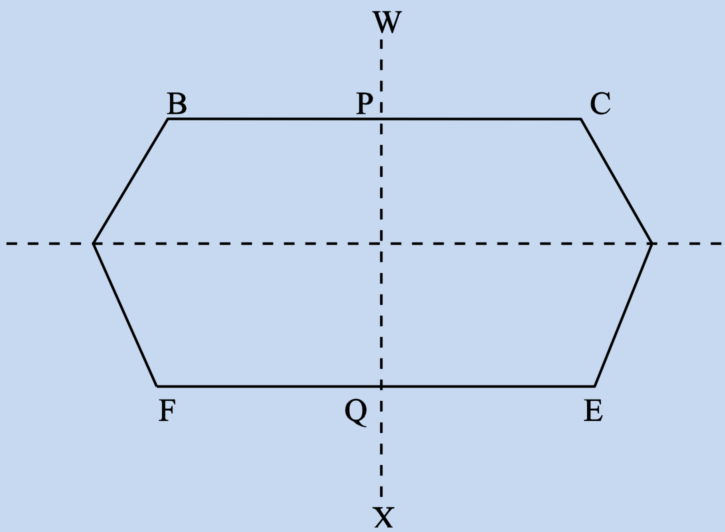
SYMMETRY

Shapes that can be cut to produce two filling parts are said to be in symmetry.

Syllabus Objectives

a) Identify shapes with point symmetry.
b) Identify shapes with rotational symmetry and state order
c) State the number of line symmetry on a given shape

Bilateral (or line symmetry)
A shape is said to have a line symmetry if it can be folded along a line to exactly fit on itself.
- The line which divides the shape into two identical parts is called a line of symmetry.



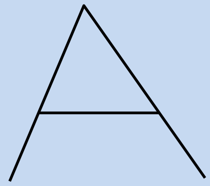
The regular hexagon ABCDEF is symmetrical. WX and AD are lines of symmetry. If the hexagon is folded about WX shape ABPQF fits exactly onto shape PCDEQ. If WX is regarded as a mirror line then E is the image of F, D is the image of A and C is the image of B.

In a symmetrical shape, for every point P on one side of the axis there is a corresponding point Q on the other side. PQ is perpendicular to the axis and P and Q are equal distance from it. e.g. B and C, A and D E and F.

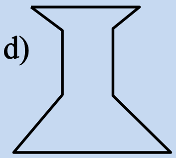
Exercise 1,1

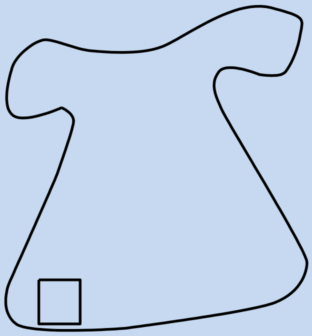
1) Which of the shapes have line symmetry?

a) 

b) 

c) 

d) 


e)

2) Draw a rough sketch of each of the symmetrical shape in question 1

b) Show the line(s) of symmetry on each of your sketches.

Short summary

The line of symmetry is sometimes called mirror-lie and bilateral or line symmetry is sometimes called mirror symmetry.

Completing figures- with given lines of symmetry

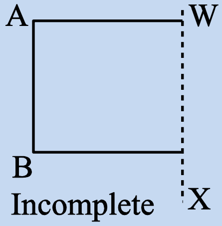


Diagram 1

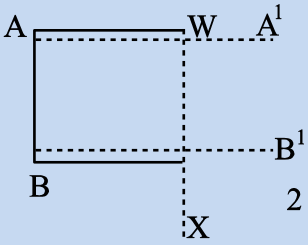
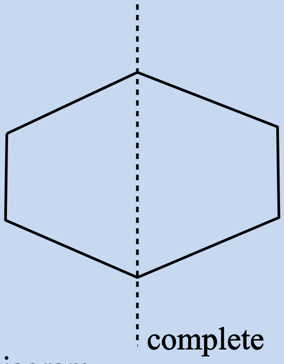


Diagram 2



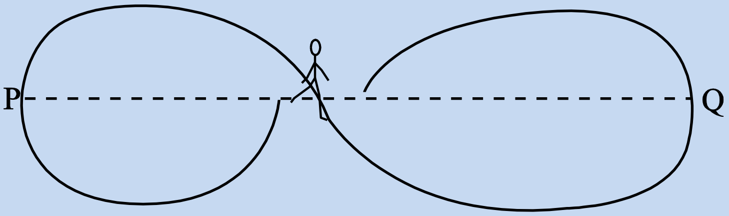
Diagram

Diagram 3

To complete the shape so that it is symmetrical about WX. We draw lines from A and B which are perpendicular to WX, and extend then the same distances on the, other side of WX to obtain points A^1 and B^1. These points are joined.

Point symmetry

A shape has point symmetry about a fixed point when every point of the object has a corresponding image such that the line joining the image through the fixed point to the object is equidistant

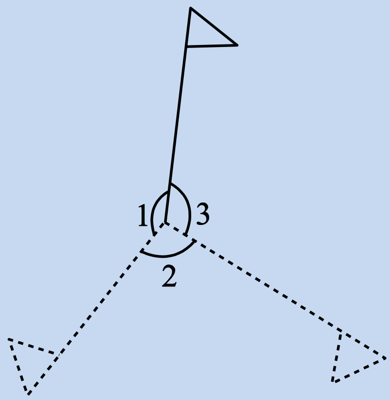


The curve has symmetry about the point O. For every point P on the curve there is a corresponding point Q, such that POQ is a straight line and QO= PQ.

Rotational symmetry

A shape has rotational symmetry if it can be rotated about some position to give the original status.

The number of such rotation is its order of rotation. (As every plane figure can be rotated through 360^0 onto itself, it has order of symmetry of at least 1.



The flag is part of a figure with rotational symmetry of order 3.

- Order 3 means 3 rotations about O are possible so each must be of 120^0. The dotted lines show the completion of the figure.

Note that the figure has no line symmetry.

<u>Exercise 1,2</u>

1a) Write down the letters in the name ROAD which have:
i) Line symmetry
ii) Point Symmetry
iii) Rotational symmetry of order 2.

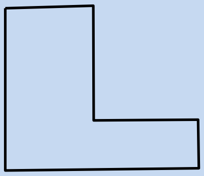
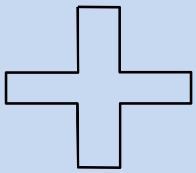
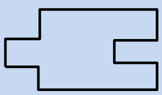
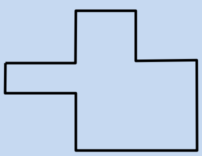
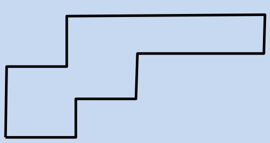
2) Fill in the blanks in the label below, which is about plane straight-sided figures.

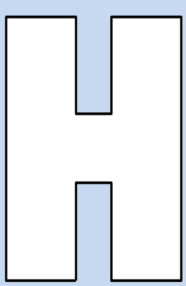
Name of Figure	**No. of Lines of Symmetry**	**Is symmetrical about a point**
Regular hexagon	i)………..	Yes
Kite	ii)……….	No
Triangle	iii)……….	No
Rhombus	iv)………..	(v)…………….
Square	v)…………	………………..

3) Copy and Complete

Name of Figure	**No. of Lines of Symmetry**	**Its Rotational Symmetry**
Equilateral △	i)………..	Yes
ii)…………….	4	No
Isosceles △	iv)……….	No
Rectangle	…………..	(v)…………….

4)
Figure 4 **No of line of symmetry** **symmetry about q point**

CHAPTER 35

Pythagoras's Theorem

Pythagoras's theorem is the of one the greatest Greek Mathematics known as Pythagoras. The theory states that the square on the longest side of a right angled triangle (hypotenuse) is equal to the sum of the squares on the other two sides i.e. $h^2 = a^2 + b^2$.

Syllabus objectives

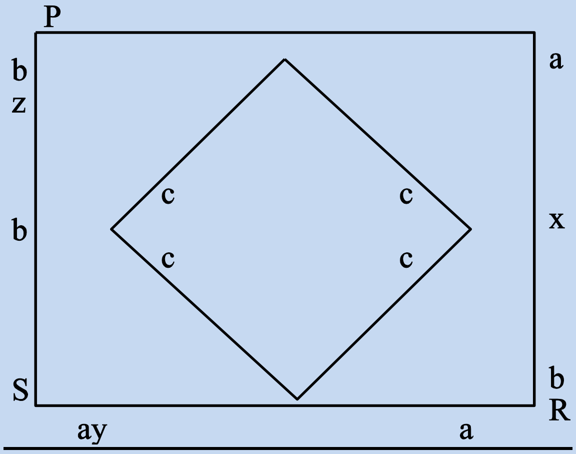
Leaner should be able to:

a) State the Pythagoras theorem

b) Use the Pythagoras theorem to calculate the unknown length

c) Apply Pythagoras theorem in given situations

Proof of Pythagoras theorem (Chinese Proof)



Give the square PQRS of side a+ b, W is a point on PQ such that PW= a units and WQ= 6 units similarly for X, Y and Z. Lines joining these points give a square of side c and 4 right-angled triangles (shaded) within the square. The area of square PQRS can be found in two ways.

1 Area of PQRS = length x breadth

$= (a+ b) (a +b)$

$= a^2 + 2\ ab + b^2$

2. Area pf {QRS = area of square WXYZ + area of 4 triangles

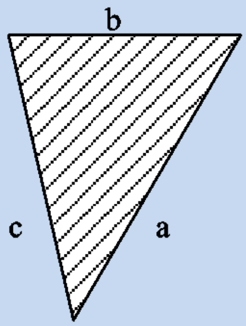
$= c^2 + 4 \times {}^1/_2{}^{ab}$

$= c^2 + 2ab$

Equating $c^2 + 2ab = a^2 + 2ab + b^2$

$c^2 = a^2 + b^2$

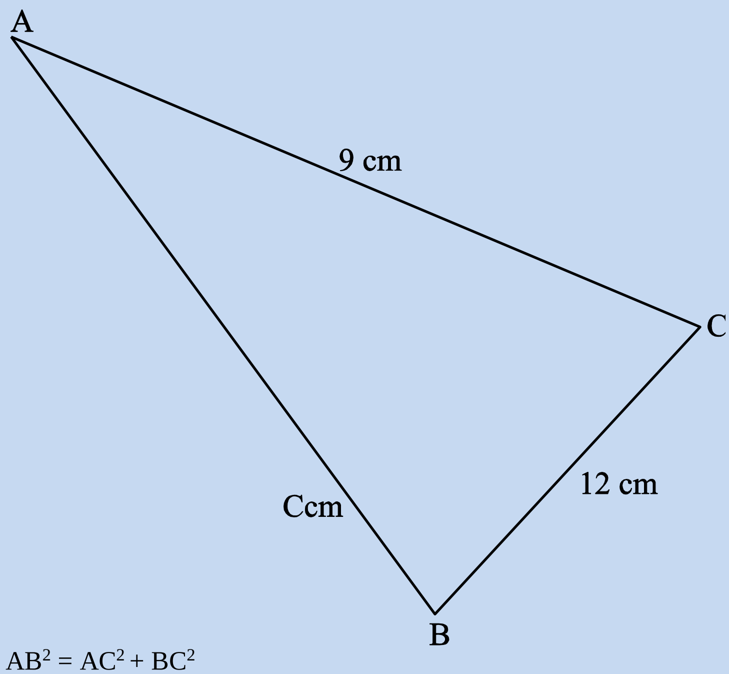
Taking triangle WQX in the diagram c is the hypotenuse of triangle WQX and a and b its other two sides.



For any right-angled triangle with hypotenuse c and other sides a and b, $c^2 = a^2 + b^2$

Example 1

Give the data below, calculate the value of c



$AB^2 = AC^2 + BC^2$

$c^2 = 9^2 + 12^2$

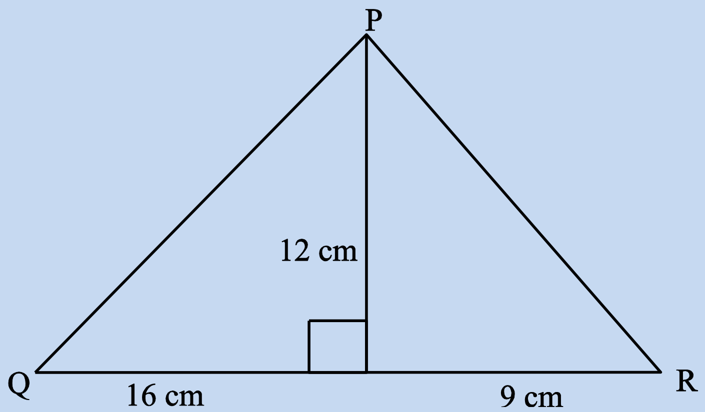
$= 81 + 144$

$c^2 = 225$

$\div c = \sqrt{225}$

$C = 15cm$

Examples 2



In the diagram above, calculate PQ and PR. Hence show that triangle PQR has a right angle.

$PQ^2 = 12^2 + 16^2$
$= 144 + 256$
$PQ^2 = 400$
$PQ = \sqrt{400}$
$= 20cm$

$PR^2 = 12^2 + 9^2$
$= 144 + 81$

$PR^2 = 225$

$:-PR = \sqrt{225}$
$= 15cm$

If triangle PQR has a right angle then

$QR^2 = PR^2 + QP^2$
$25^2 = 15^2 + 20^2$
$25^2 = 225 + 400$
625 = 625 shown

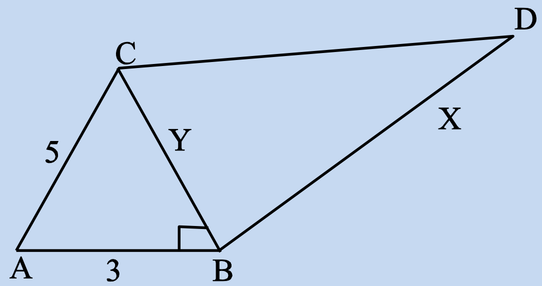
Exercise 1,1

1) PQR is a triangle in which Q = 90. In each of the following, draw and label a sketch the calculate the length of the third side of the triangle.

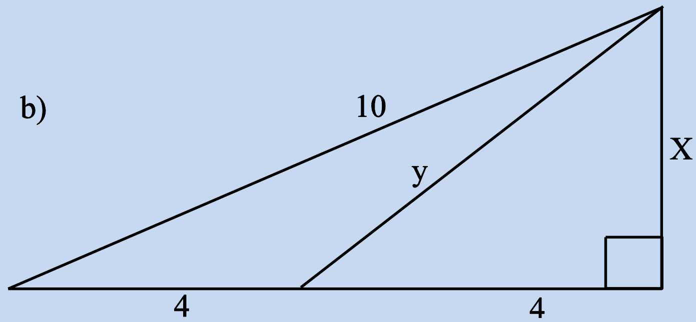
a)	AB= 3m	BC= 4m
b)	AB=30cm	BC= 40cm
c)	AC= 5cm	AB= 3cm
d)	AC= 13cm	BC =5m
e)	AC= 10m	AB= 8m

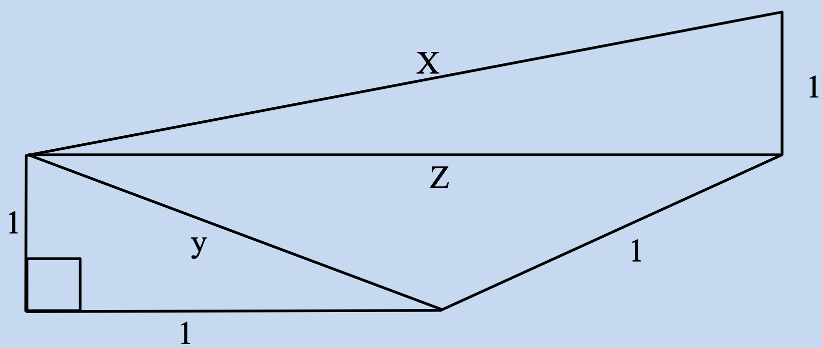
2) Find the value of the unknown in each of the diagrams below. It will be necessary to find a value for y^2 before finding x.

a)



b)





Using square root tables

Example 3

Use square roots table to find a) √8,512 b) √851,2 c) √8512

a) √8,512 = 2,917 For square roots less than
= 2,92 100. Find the square root
Directly In 8,512. Look

1) Across 8,5
2) Under 1 on the next
Column but on the same line across
8,5. Take that number i.e 2,917.
3) Under difference column under 2 add that number
i e 2,917 to = 2, 917.

b) √851,2 = √8,512 x 10^2 8 512 I greater than 100

	=	√8,512 x x √100	1)	First express in the given form
			2)	Distribute square root
	=	2,917 x 10	3)	Find √8,512 directly From tables
	=	29,17		

:- √851,2 = 29,17

c) √8 512 = $\sqrt{8\ 5{,}12 \times 10^2}$

$\sqrt{85{,}12} \times \sqrt{10^2}$

= 9,226 x 10

= 92,26

Exercise 1,2

Find the square roots of the following

a)	11	b)	151	c)	284
d)	1 870	e)	7,51	f)	352,4
g)	72 100	h)	63 945		

Using squares tables

Example 4

Use table to find the following square

a) 5, 812 b) 132 c) 150^2

a) 5,812 = 33,76
1) Look across 58 under (33.76)
2) Put comma using inspection Since 5,81 is nearer to 5 whose Square is 25.

b) 132 = 169

c) 1502 = 22 500 Use inspection to place comma

Exercise 1,3

Find the value of the following.

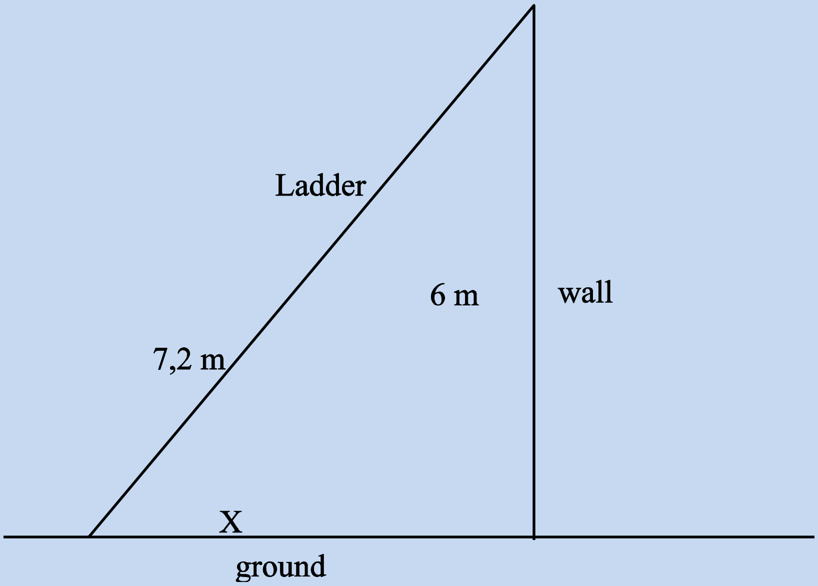
a)	$6,1^2$	b)	$8,28^2$	c)	$63,4^2$
d)	$634,8^2$	e)	890^2	f)	$8\ 350^2$

Word problems involving Pylhagorous Theorem

Example 5

A ladder 7,2cm in length leans on a vertical hall 6m above the ground. Find the horizontal distance on the grown between the ladder and the wall.

First make a rough sketch



x represents the required distance

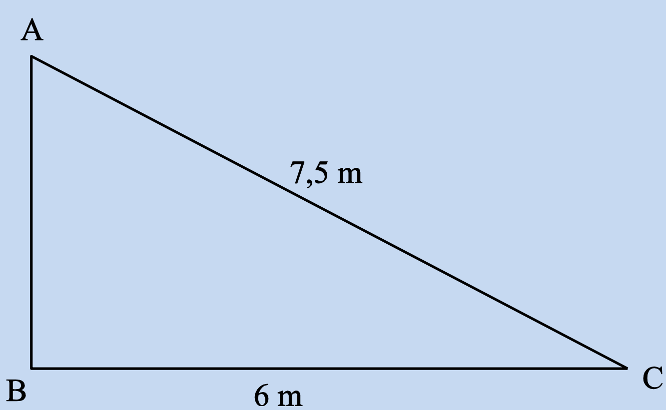
According to Pythagoras's theorem

$$7,2^2 = x^2 + 6^2$$
$$x^2 = 7,^2 - 6^2 \quad \text{making } x^2 \text{ the subject}$$
$$= 51,84 - 36$$
$$x^2 = 15,84$$

$$x = \sqrt{18,84}$$
$$= 3,979$$
$$= 3,98 \text{ to 3 s.f}$$

<u>Exercise 1,4</u>

1) The diagram shows a side view of a wedge shaped piece of wood.



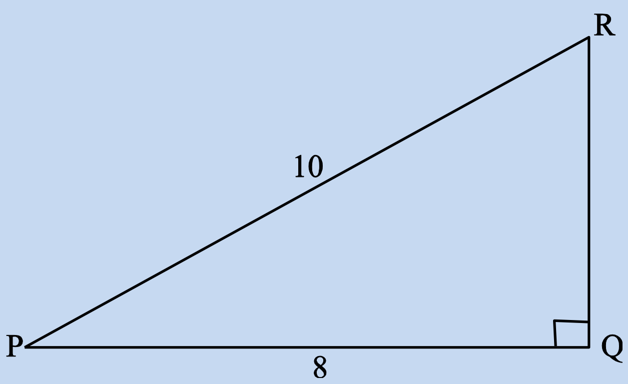
The wedge is placed on a level floor in the position shown. Calculate its height AB.

2) A wooden ladder of length8m lean against a wall 6,5m above the ground.

a) Find the horizontal distance on the ground between the ladder and the wall.
b) A man reduces its height above the ground by increasing its horizontal distance from the wall by 0,6m . Find the new height of the ladder.

3) A telegraph pole of height 8m stands vertically above the ground. It is supported by a wire which is attached three quarters its height above the ground. When the wire is tightly ties on the ground a horizontal distance of 3,1m from the pole,
4) A square has side of 12,2m. calculate the length of a diagonal
5) A ship soils 13m due south and then 7m due. How far is from its starting point.

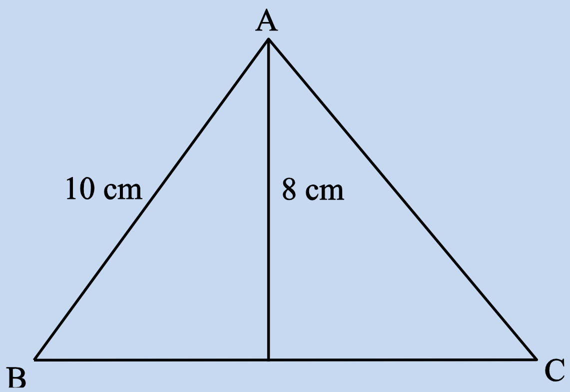
Examination Questions



In the diagram, PQR is a right angled triangle. PQ= 8cm, PQ= 10cm and PQR = 90=900. Calculate the area of triangle PQR, giving your answer in square meters. (ZIMSEC NOV 2005)

2) A rectangle is twice as long as it wide.
a) the length of the rectangle
b) the length of a diagonal of the rectangle

3) The diagram below shows isosceles triangle ABC with AP its line of symmetry, AB = 10cm and AP = 8cm.



A ladder, 8m long, leans against a vertical wall. If the top of the ladder is 7,5m above the ground, how far is the bottom of the ladder from the wall? Give your answer correct to 3 s.f.