NEW EDITION

New General Mathematics 2

A Junior Certificate Course

JB Channon A McLeish Smith
HC Head MF Macrae

With Answers

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HC Head
A McLeish Smith
MF Macrae

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Preface to 1992 edition

Books 1 and 2 of the New General Mathematics course have been revised to reflect the present content and philosophy of the mathematics syllabus of the Zimbabwe Junior Certificate. Book 2 completes the Junior Certificate course.

In Book 2, new material has been written to cover the following topics: Congruency (Chapter 12), Money transactions (Chapter 17), Ready reckoners (Chapter 23) and Calculator skills (Chapter 24). New sections on everyday consumer arithmetic have also been added. In addition, a full-scale Junior Certificate level practice examination is now included as part of the revision exercises and tests. To ensure that Book 2 keeps to the ZJC syllabus, topics such as trigonometry, Pythagoras' theorem, transformation geometry and matrices now appear in Book 3. Users should note that Chapter 24, Calculator skills, is included partly as a life skill for potential school leavers and partly as preparation for those who may go on to take the calculator option at School Certificate level. Although calculator skills are not necessary for the Junior Certificate, many teachers have indicated that they would like the topic included in Book 2 for the above reasons.

While revising Book 2, the opportunity was taken to make corrections and to update statistical information. The authors and publishers are grateful to the Central Statistical Office, Harare, for providing valuable data. Furthermore, we are grateful to the many readers who have made helpful suggestions and who have provided so much encouragement.

M F Macrae, 1990

Symbols

symbol	meaning
$=$	is equal to
$\neq$	is not equal to
$\simeq$	is approximately equal to
$\equiv$	is identical to
$\Leftrightarrow$	is equivalent to
$>$	is greater than
$<$	is less than
$\geqslant$	is greater than or equal to
$\leqslant$	is less than or equal to
$^\circ$	degrees (size of angle)
°C	degrees Celsius (temperature)
A, B, C, ...	points
AB	the line joining the point A and the point B *or* the distance between points A and B
$\triangle$ABC	triangle ABC
A$\hat{\text{B}}$C	the angle ABC
∟	lines meeting at right angles
π	pi (3,14 ...)
%	per cent
A = $\{p; q; r\}$	A is the set p; q; r
B = {1; 2; 3; . . .}	B is the infinite set 1; 2; 3 and so on
C = $\{x; x$ is an integer$\}$	set builder notation; C is the set of numbers x such that x is an integer
n(A)	number of elements in set A
$\in$	is an element of
$\notin$	is not an element of
A'	complement of A
{ } or $\varnothing$	the empty set
$\mathscr{E}$	the universal set
$A \subset B$	A is a subset of B
$A \supset B$	A contains B
$\not\subset$, $\not\supset$	negations of $\subset$ and $\supset$
$A \cup B$	union of A and B
$A \cap B$	intersection of A and B

Chapter 1

Number patterns

Factors, prime factors (revision)

$40 \div 8 = 5$ and $40 \div 5 = 8$
8 and 5 divide into 40 without remainder. 8 and 5 are **factors** of 40.
A **prime number** has only two factors, itself and 1.
2, 3, 5, 7, 11, 13, . . . are prime numbers. 1 is *not* a prime number.

Example 1

(a) Write down all the factors of 24. (b) State which of these factors are prime numbers. (c) Express 24 as a product of its prime factors.

(a) Factors of 24: 1; 2; 3; 4; 6; 8; 12; 24
(b) Prime factors of 24: 2 and 3
(c) $24 = 2 \times 2 \times 2 \times 3$

Exercise 1a (Revision)

For each number, (a) write down all its factors, (b) state which factors are prime numbers, (c) express the number as a product of its prime factors.

1 18	**2** 28	**3** 33	**4** 45
5 16	**6** 22	**7** 30	**8** 48
9 12	**10** 36	**11** 39	**12** 56
13 42	**14** 50	**15** 63	**16** 72

Example 2

Express 90 as a product of its prime factors in index form.

method: Divide 90 by the prime numbers 2; 3; 5; 7; . . . in turn until it will not divide further.

working:

2	90
3	45
3	15
5	5
	1

$90 = 2 \times 3 \times 3 \times 5$
$90 = 2 \times 3^2 \times 5$
Notice that $3 \times 3 = 3^2$ in index form.

Exercise 1b (Revision)

Express each number as a product of its prime factors in index form.

1 27	**2** 44	**3** 52	**4** 75
5 98	**6** 104	**7** 116	**8** 117
9 200	**10** 279	**11** 364	**12** 444

Highest common factor (revision)

14 is the **highest common factor** (HCF) of 28 and 42. It is the greatest number which will divide exactly into both 28 and 42.

Example 3

Find the HCF of 504 and 588.

method: Express each number as a product of its prime factors.

working:

2	504
2	252
2	126
3	63
3	21
7	7
	1

2	588
2	294
3	147
7	49
7	7
	1

$504 = 2^3 \times 3^2 \times 7$
$588 = 2^2 \times 3 \times 7^2$
Find the common prime factors.
$504 = (2^2 \times 3 \times 7) \times 2 \times 3$
$588 = (2^2 \times 3 \times 7) \times 7$
The HCF is the product of the common prime factors.
$\text{HCF} = 2^2 \times 3 \times 7$
$= 4 \times 3 \times 7$
$= 84$

Exercise 1c (Revision)

Find the HCF of the following.

1 28 and 42	**2** 30 and 45
3 24 and 40	**4** 18 and 30
5 54 and 105	**6** 24 and 78
7 60 and 108	**8** 216 and 168
9 36, 54 and 60	**10** 72, 108 and 54
11 324, 432 and 540	**12** 252, 567 and 378

Lowest common multiple (revision)

Multiples of 6: 6; 12; 18; 24; 30; 36; **42**; 48; ...

Multiplies of 14: 14; 28; **42**; 56; 70; ...

Notice that 42 is the lowest number which is a multiple of both 6 and 14. 42 is the **lowest common multiple** (LCM) of 6 and 14.

Example 4

Find the LCM of 22, 30 *and* 40.

method: Express each number as a product of its prime factors.

$22 = 2 \times 11$

$30 = 2 \times 3 \times 5$

$40 = 2^3 \times 5$

The prime factors in 22, 30 and 40 are 2, 3, 5 and 11.

The highest power of each prime factor must be in the LCM.

These are 2^3, 3, 5 and 11.

Thus, $\text{LCM} = 2^3 \times 3 \times 5 \times 11$

$= 8 \times 3 \times 5 \times 11$

$= 1\,320$

Exercise 1d (Revision)

Find the LCM of the following:

1 9 and 12	**2** 8 and 10
3 10 and 15	**4** 20 and 24
5 15 and 33	**6** 42 and 56
7 2, 3 and 7	**8** 4, 5 and 6
9 8, 10 and 12	**10** 12, 15 and 18
11 36, 45 and 60	**12** 20, 28 and 35

Number patterns

The multiples of 3 can be given in a row, or sequence:

3; 6; 9; 12; 15; 18; 21; ...

They can also be shown by shading on a 1–100 number square as in Fig. 1.1.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Fig. 1.1

These are both examples of **number patterns**.

Extending number patterns

Example 5

Find the next four terms in the sequence 1; 2; 4; 7; 11; 16; ...

method: Find the differences between one number and the next.

sequences: 1 ; 2 ; 4 ; 7 ; 11 ; 16

differences: 1 2 3 4 5

Notice the pattern in the differences. The differences increase by 1 each time. The next term in the sequence is found by adding 6 to 16. This gives 22. The next term is found by adding 7 to 22, and so on. The next four terms are: ...; 22; 29; 37; 46.

Exercise 1e

1 Complete the gaps in the following sequences.
 (a) Multiples of 4: 4; 8; 12; 16; ...; 100
 (b) Multiples of 6: 6; 12; 18; 24; ...; 96
 (c) Multiples of 8: 8; 16; 24; 32; ...; 96
 (d) Multiples of 9: 9; 18; 27; 36; ...; 99

2 Make four 1–100 number squares.
On the first number square, shade all the multiples of 4 which you found in question 1. Repeat on the other number squares for the multiples of 6, 8 and 9.

3 Find the next four terms of the following sequences.
 (a) 2; 5; 8; 11; 14; ...
 (b) 1; 6; 11; 16; 21; ...
 (c) 1; 12; 23; 34; 45; ...
 (d) 10; 9; 8; 7; 6; ...
 (e) 0; 1; 3; 6; 10; ...
 (f) 1; 2; 4; 8; 16; ...
 (g) 1; 3; 7; 13; 21; ...
 (h) 1; 2; 5; 10; 17; ...
 (i) 1; 4; 9; 16; 25; ...
 (j) 1; 1; 2; 3; 5; 8; 13; 21; ...

4 A trader stacks some tins in triangles as shown in Fig. 1.2 below.

Fig. 1.2

 (a) Copy and complete Table 1.1.

Table 1.1

Number of tins in bottom row	1	2	3	4
Number of tins altogether	1	3		

 (b) Extend the table for 5; 6; 7; 8 tins in the bottom row.

5 (a) Copy and complete the sequence of square numbers shown in Table 1.2.

Table 1.2

index form	1^2	2^2	3^2	4^2	...	10^2
number	1	4	9	16	...	100

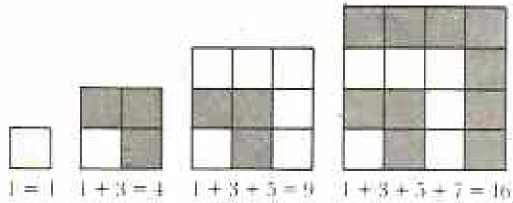


Fig. 1.3

 (b) Copy the pattern in Fig. 1.3 on to squared paper. Extend the pattern by drawing 5×5, 6×6 and 7×7 squares. Is it true that 7^2 = sum of the first seven odd numbers?

Table 1.3

number	pattern	total
1	1	1
2	1 + 2 + 1	4
3	1 + 2 + 3 + 2 + 1	9
4	1 + 2 + 3 + 4 + 3 + 2 + 1	16
5		
6		
7		

 (c) Copy the pattern in Table 1.3 and complete it for the numbers 5, 6 and 7. Write down the sequence formed by the total column. What do you notice?

Graphs of number patterns

A **graph** is a picture. The pictograms, bar charts and pie charts you drew when working with Book 1 were all examples of graphs.

Graphs are usually drawn on graph paper. There are two common kinds of graph paper as shown in Figs 1.4 and 1.5 overleaf.

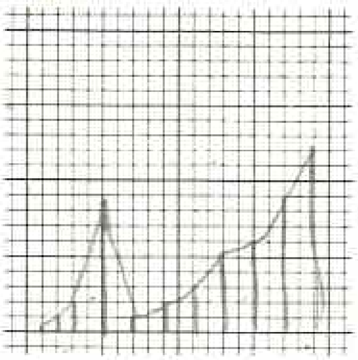


Fig. 1.4 2 mm graph paper – the small squares are 2 mm by 2 mm

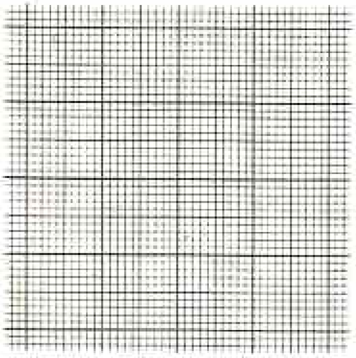


Fig. 1.5 1 mm graph paper – the small squares are 1 mm by 1 mm

The lines on the graph paper are either thick, medium or thin. These make big, medium and small squares. On *your* graph paper, find out the following:

1 the length of side of the big, medium and small squares;

2 the number of small squares inside a big square;

3 the width, in big squares, of your graph paper;

4 the length, in big squares, of your graph paper.

In this book most graphs will be drawn on 2 mm by 2 mm graph paper.

The following example shows how to draw a simple graph of a number pattern.

Example 6

Draw a graph to show the sequence 1; 2; 4; 7; 11; 16.

The graph is given in Fig. 1.6.

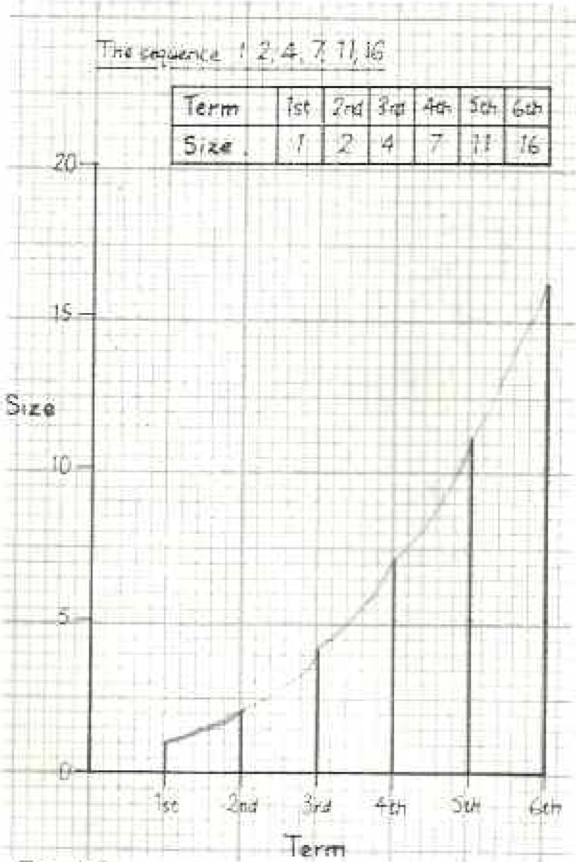


Fig. 1.6

Notice the following in Fig. 1.6.

(a) The lines represent the terms of the sequence.

(b) The length of each line represents the size of each term. For example, the 5th term, 11, is a line 11 units long.

(c) Two lines are labelled with numbers. These are called **axes**.

(d) The axis along the bottom of the graph shows the numbers of the terms. This is called the horizontal axis.

(e) The axis at the left-hand side of the graph gives a scale, or measure, of the sizes of the terms. This is called the vertical axis. In this case the scale uses 2 cm to represent 5 units.

(f) A table showing the data of the graph is given.

(g) The title, or name, of the graph is given at the top.

Every graph should show the following:

1 a title;
2 a table giving the data of the graph;
3 labelled axes with suitable scales;*
4 lines or points giving a picture of the data.

* Look at the highest numbers in the data when choosing scales. Further advice on choosing scales is given in Chapter 7.

Exercise 1f

In questions 1–6, use the same scales as those in Fig. 1.6.

1 Table 1.4 gives the factors of 18 in numerical order.

Table 1.4

numerical order	1st	2nd	3rd	4th	5th	6th
factors of 18	1	2	3	6	9	18

Draw a graph of the factors of 18.

2 Draw a graph of the first five multiples of 4: 4; 8; 12; 16; 20.

3 The numbers 1; 3; 6; 10; 15; 21; ... are known as the **triangle numbers** (see Exercise 1e, question 4). Draw a graph of the first six triangle numbers.

4 The numbers 1; 4; 9; 16; 25; ... are known as the **square numbers** (see Exercise 1e, question 5). Draw a graph of the first five square numbers.

5 Draw a graph of the first ten odd numbers.

6 Draw a graph of the first ten even numbers.

7 The sequence 1; 1; 2; 3; 5; 8; ... is known as the **Fibonacci sequence**. Each term is the sum of the previous two terms.
(a) Write down the first ten terms of the Fibonacci sequence.
(b) Draw a graph of the first ten terms of the Fibonacci sequence. Use a scale of 2 cm to 10 units on the vertical axis.

8 Draw a graph of the factors of 30.

9 Draw a graph of the decreasing sequence 16; 8; 4; 2; 1; $\frac{1}{2}$. Use a scale of 1 cm to 1 unit on the vertical axis.

10 Draw a graph of the sequence 32; 21; 12; 5; 0; 5; 12; 21; 32.

Chapter 2

Sets (2)

Sets (revision)

Example 1

Given $\mathscr{E} = \{f; o; r; m; u; l; a; e\}$, $A = \{f; r; a; m; e\}$ and $B = \{r; u; l; e\}$, (a) draw a Venn diagram showing A, B and $\mathscr{E}$; (b) list the elements of $A \cap B$; (c) find $n(A \cup B)$.

(a) Fig. 2.1 is the required Venn diagram:

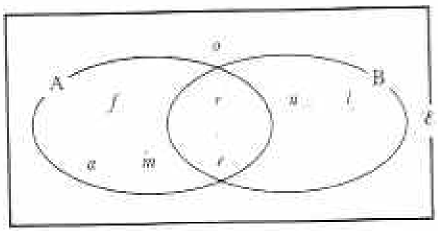


Fig. 2.1

(b) $A \cap B$ is the intersection of sets A and B, i.e. the set whose elements are members of both A and B.

$A \cap B = \{r; e\}$

(c) $A \cup B$ is the union of sets A and B, i.e. the set whose elements are members of A or B or both A and B.

$A \cup B = \{f; r; a; m; e; u; l\}$

Notice that although r and e are elements of both A and B, there is no need to write them down twice.

$n(A \cup B)$ is the number of elements in the set $A \cup B$.
$n(A \cup B) = 7$

Table 2.1 contains some of the symbols and language of sets that appeared earlier in the course, in Chapter 2 of Book 1.

Table 2.1

symbols	meaning
$P = \{a; b; c; d\}$	P is the set a; b; c; d
$Q = \{1; 2; 3; \ldots\}$	... means 'and so on'
$\varnothing$ or $\{\ \}$	the empty set
$\mathscr{E}$ or $\mathscr{U}$	the universal set
$\in$	is a member of
$\notin$	is *not* a member of
$\subset$	is a subset of
$\not\subset$	is *not* a subset of
$\supset$	includes
$P \cup Q$	union of P and Q
$P \cap Q$	intersection of P and Q
$n(S)$	number of elements in S

Exercise 2a (Revision)

1 Make each of the following true by writing either $\in$ or $\notin$ in place of the *.
- (a) $9 * \{2; 4; 6; 8; 10\}$
- (b) $15 * \{3; 6; 9; \ldots; 24\}$
- (c) $44 * \{5; 10; 15; 20; \ldots\}$
- (d) $R * \{a; b; c; d; \ldots; z\}$

2 Find n(X) when X =
- (a) $\{h; o; u; s; e\}$
- (b) {toes on your feet}
- (c) $\{0; 1; 2\}$
- (d) {months in a year}
- (e) $\{5; 5; 6; 6\}$
- (f) $\{11; 12; 13; \ldots; 22\}$

3 Give three examples of an empty set.

4 Give three examples of an infinite set.

5 Write down all the subsets of the following.
- (a) $\{3; 4; 5\}$
- (b) $\{x; y\}$
- (c) $\{0; 2\}$
- (d) $\{f; o; u; r\}$

6 Write down the following using symbols.
- (a) 2 and 6 form a subset of the factors of 18
- (b) {trees} is not a subset of {metal objects}
- (c) {vehicles} contains {buses}
- (d) children are members of the human race

7 If $\mathscr{E}$ = {months of the year}, F = {first eight months of the year}, Y = {months ending in y}, draw a Venn diagram to show the relationship between F, Y and $\mathscr{E}$.

8 Which of the following pairs of sets are disjoint? If the sets are not disjoint write down two members of the intersection.
 (a) {prime factors of 24}, {prime factors of 55}
 (b) {multiples of 5}, {multiples of 7}
 (c) {Zaire; Zomba; Zambia}, {countries of Africa}
 (d) {letters of *bull*}, {letters of *cow*}

9 If $\mathscr{E}$ = {2; 4; 6; 8; ...; 20}
 M = {multiples of 3}
 L = {numbers less than 14}
 write down the members of the following sets.
 (a) $M \cup L$ (b) $M \cap L$ (c) $M \cup \mathscr{E}$
 (d) $\mathscr{E} \cap M$ (e) $\mathscr{E} \cap L$ (f) $L \cup \mathscr{E}$

10 (a) Draw a Venn diagram to represent the data of question 9.
 (b) Hence find (i) n($M \cup L$), (ii) n($M \cap L$).

Sets of numbers

Here are some sets of numbers that were discussed in Book 1:

The numbers that people use for counting are called **natural numbers**. The set of natural numbers is usually called N.

N = {1; 2; 3; 4; 5; ...}

If zero, 0, is included with N, then the set becomes the set of **whole numbers**, usually called W.

W = {0; 1; 2; 3; 4; 5; ...}

If the negative whole numbers are included with W, then the set becomes the set of **integers**, usually called Z.

Z = {..., − 3; − 2; − 1; 0; + 1; + 2; + 3; ...}

Finally, any number which can be expressed as a fraction with a numerator which is a member of Z and a denominator which is a member of N is called a **rational number**. The set of rational numbers is usually called Q.

$Q = \{...; -2\frac{1}{4}; -1{,}2; -\frac{4}{5}; 0; +\frac{3}{4}; +0{,}933; 1\frac{11}{16}; 2; ...\}$

Notice that all the numbers in Q can be expressed in the form $\frac{a}{b}$ where $a \in Z$ and $b \in N$. For example, $-1{,}2 = -\frac{12}{10}$, $0{,}933 = \frac{933}{1000}$ and $2 = \frac{2}{1}$.

Exercise 2b

Given $A = \{+4; \frac{2}{3}; -8\frac{1}{2}; +1{,}5; 0; -19; -0{,}3; 3\frac{1}{7}; 20; \frac{18}{5}; -6\frac{2}{3}; -9{,}666; +3\frac{3}{10}\}$, refer to A when answering the questions in this exercise.

1 List those elements of A which are members of (a) N (b) W (c) Z (d) Q

2 List those elements of A which are integers but *not* natural numbers.

3 List those members of A which are integers but *not* whole numbers.

4 List those members of A which are rational but *not* whole numbers.

5 List those members of A which are both rational and whole numbers.

Venn diagrams, problem solving

Venn diagrams can sometimes be used to store numerical information. In such cases it is also possible to use the Venn diagrams to solve problems arising from the data.

Example 2

In a village everyone speaks either Ndebele or Shona or both. If 65% speak Ndebele and 89% speak Shona, what percentage speak both languages?

Let N = {Ndebele speakers}, S = {Shona speakers}. In Fig. 2.2 overleaf, the numbers in the regions represent the percentages of people in those regions. It is required to find $n(N \cap S)$. Let $n(N \cap S) = x$.

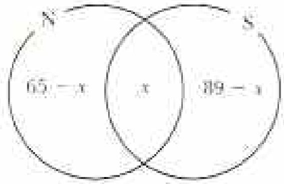


Fig. 2.2

In Fig. 2.2 check that $n(N) = 65$ and $n(S) = 89$.
Since $n(N \cup S) = 100$,

$$\begin{aligned} (65 - x) + x + (89 - x) &= 100 \\ \Leftrightarrow \quad 154 - x &= 100 \\ \Leftrightarrow \quad x &= 54 \end{aligned}$$

54% of the people can speak both languages. Note that this result means that 11% speak Ndebele only and 35% speak Shona only.

Example 3

50 students were asked what they did last night. 16 said they read a book, 41 said they watched television. If 7 said they did neither, how many did both?

Let $\mathscr{E}$ = {all students}, B = {book readers} and T = {television watchers}. It is required to find $n(B \cap T)$. Let $n(B \cap T) = x$.

Fig. 2.3 is a Venn diagram containing the given information. The numbers in the regions of the Venn diagram represent the numbers of elements in the regions.

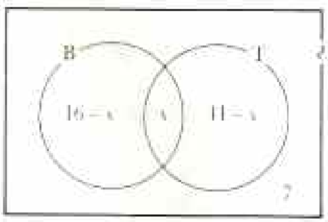


Fig. 2.3

In Fig. 2.3 check that $n(B) = 16$ and $n(T) = 41$.

The totals for the regions must add up to the number of people in the universal set:

$$\begin{aligned} x + (16 - x) + (41 - x) + 7 &= 50 \\ \Leftrightarrow \quad 64 - x &= 50 \\ \Leftrightarrow \quad x &= 14 \end{aligned}$$

14 students read a book *and* watched television.
Note: In this case, the 16 who read a book includes the 14 who also watched television.

In general:

1 Identify the sets, including the universal set.
2 Draw a Venn diagram.
3 Enter the data on the Venn diagram, starting with x, the unknown quantity.
4 Form an equation using the fact that the total number of elements in the regions equals the number of elements in the universal set.

In Exercise 2c the number of subsets will be restricted to two.

Exercise 2c

Draw a suitable Venn diagram in each question.

1 In Fig. 2.4 the numbers of elements in each region of the Venn diagram are as given.

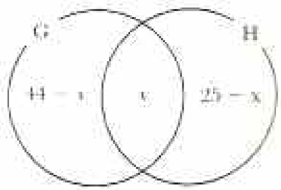


Fig. 2.4

If $n(G \cup H) = 52$, find x.

2 In Fig. 2.5 the numbers of elements in each region of the Venn diagram are as given.

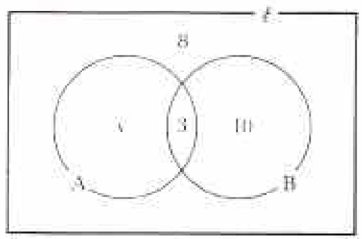


Fig. 2.5

If $n(\mathscr{E}) = 30$ find x.

3 In the Venn diagram in Fig. 2.6 the numbers of elements are as shown.

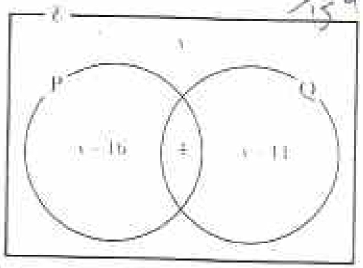


Fig. 2.6

Given that $n(P \cup Q) = x$, find x and hence find $n(\mathscr{E})$.

4 A company employs 100 people, 65 of whom are men. 60 people, including all the women, are paid weekly. How many of the men are paid weekly?

5 In a village all the people speak Tonga or English or both. If 97% speak Tonga and 64% speak English, what percentage speak both languages?

6 A job is applied for by 20 people. Everyone either has a school certificate or a diploma or both. If 14 have school certificates and 11 have diplomas, how many have a school certificate only (i.e. a school certificate but not a diploma)?

7 In a class of 35 students, everyone does history or economics or both. If 19 students do history and 27 students do economics, how many do both?

8 In a class of 36 students, everyone does biology or physics or both. If 9 do both subjects and 12 do physics but not biology, how many do biology but not physics?

9 In a school of 750 students, 320 are girls. 559 students do some kind of sport. If 101 girls do no sport, how many boys also do no sport?

10 Out of 25 teachers, 16 are married and 15 are women. If 6 of the men are married, how many of the women are not married?

Chapter 3

The cartesian plane

Points on a line

The number line is a **graph,** or picture, of all the positive and negative numbers (Fig. 3.1).

Fig. 3.1

If we draw points on the number line, we can say exactly where they are on the line.

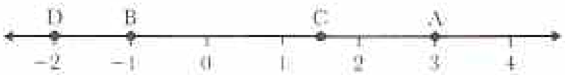


Fig. 3.2

In Fig. 3.2, A is 3 units to the right of zero and B is 1 unit to the left of zero. We can shorten this to A(3) and B(− 1). In the same way, C is the point C $(1\frac{1}{2})$ and D is the point D(− 2).

A(3) and B(− 1) give the **positions** of A and B. Notice that we are using brackets in a different way from the way we use them in algebra and arithmetic.

Exercise 3a

1 In Fig. 3.3 P(2) gives the position of P and Q(− 3) gives the position of Q. Give the positions of R, S, T, U and V in the same way.

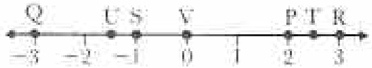


Fig. 3.3

2 In Fig. 3.4, A(0,7) describes the position of A. Describe the positions of B, C, D, E, F and G in the same way.

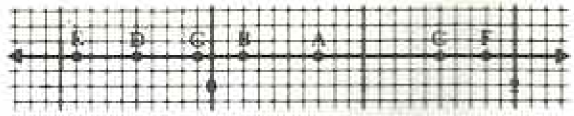


Fig. 3.4

3 Draw a number line from − 10 to 10. On the line, mark the points A(6), B(3), C(− 4), D(− 8), E(9), F(− 9), G(0), H$(7\frac{1}{2})$ and I$(-6\frac{1}{2})$.

4 Use graph paper to draw a number line like that of Fig. 3.4. On the line, mark the points P(0,8), Q(1,3), R(0,4), S(− 0,4), T(− 0,7), U(1,9) and V(1,0).

Points on a plane

Exercise 3b (Discussion)

1 Try to describe the positions of points P, Q and R in Fig. 3.5.

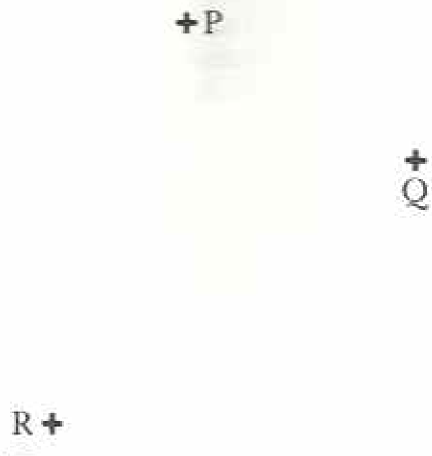


Fig. 3.5

Hint: one way is to measure the distances of P, Q and R from the edges of the page.

2 Fig. 3.6 shows the same points on a cm square grid. Starting at the cross, describe how to get to P, Q and R. Does this make it easier to describe the positions of the points?

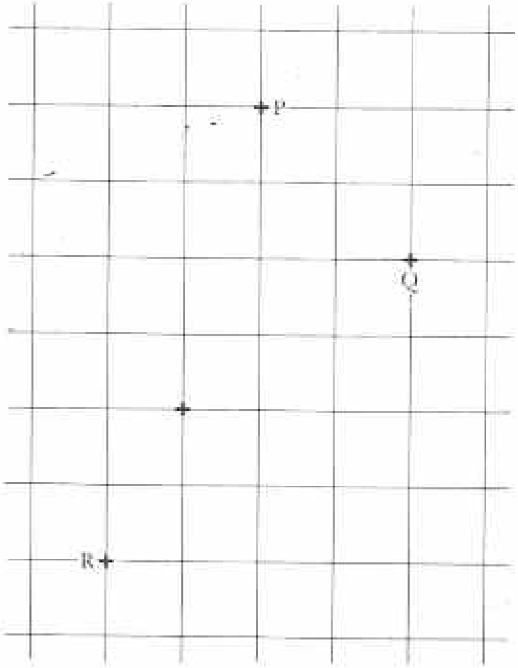


Fig. 3.6

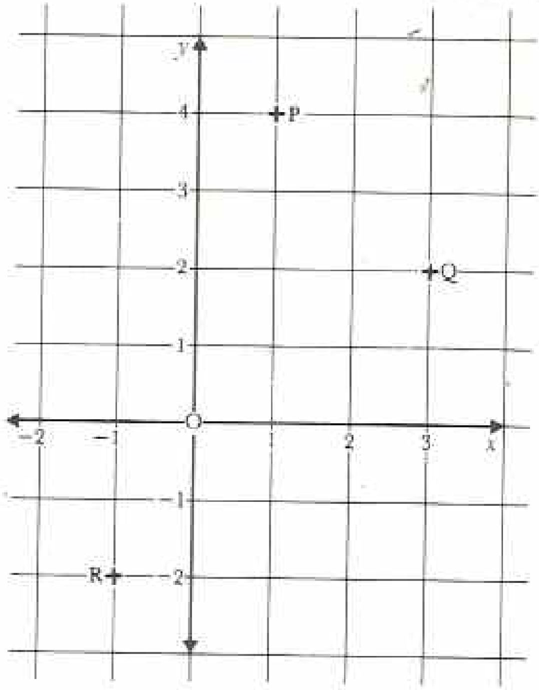


Fig. 3.7

Cartesian plane

The positions of points on a line are found by using a number line. The positions of points on a plane surface are found by using *two* number lines, usually at right angles. See Fig. 3.7.

In Fig. 3.7, starting from the zero point, P is in position 1 unit to the *right* and 4 units *up*; Q is in position 3 units to the *right* and 2 units *up*; R is in position 1 unit to the *left* and 2 units *down*.

We can shorten this to P(1; 4), Q(3; 2) and R(− 1; − 2). The position of each point is represented by a pair of numbers.

Fig. 3.7 is a graph, or picture, of the three points P, Q and R. In a graph like this, the number lines are called **axes**. They cross at the zero-point of each axis. This point is called the **origin**. The axis going across from left to right

Fig. 3.8

is called the **x-axis**. It has a positive scale to the right of O and a negative scale to the left of O. The axis going up the page is called the **y-axis**. It has a positive scale upwards from O and a negative scale downwards from O.

A plane surface with axes drawn on it, such as Fig. 3.7 and Fig. 3.8 on page 11, is called a **cartesian plane**. It is named after the French philosopher and mathematician, Descartes. His work made it possible to represent geometry in a numerical way.

Coordinates

Fig. 3.9 shows a cartesian plane with points A, B, C and D drawn on it.

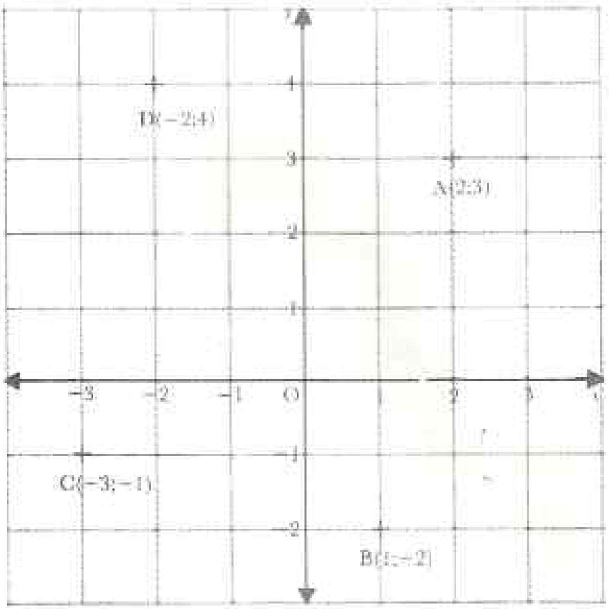


Fig. 3.9

The position of A is found by moving 2 units to the *right* of the origin and then 3 units *up* the page parallel to the y-axis. We can shorten this to A(2; 3). C is found by moving 3 units to the *left* of the origin and then 1 unit *down* the page. Its position is C(− 3; − 1). In the same way, B and D are the points B(1; − 2) and D(− 2; 4).

The position of each point is given by an **ordered pair of numbers**. These are called the **coordinates** of the point. The first number is called the **x-coordinate**. The x-coordinate gives the distance of the point along the x-axis. The second number is called the **y-coordinate**. The y-coordinate gives the distance of the point along the y-axis. The coordinates are separated by a semicolon.

Fig. 3.10

The *order* of the pair of numbers is very important. For example, the point (1; 4) is not the same as the point (4; 1). This is shown in Fig. 3.11.

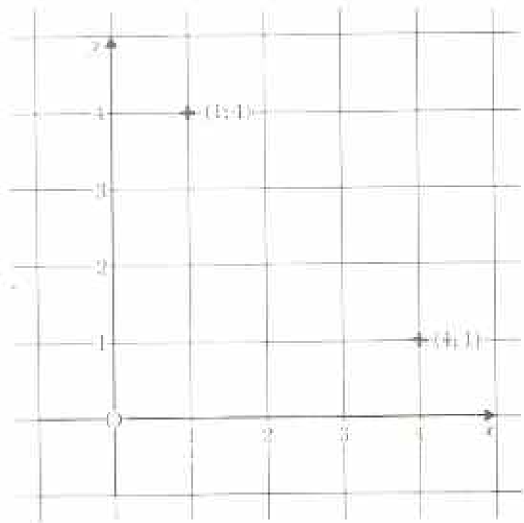


Fig. 3.11

Example 1

Write down the coordinates of the vertices of triangle ABC *and parallelogram* PQRS *in Fig. 3.12.*

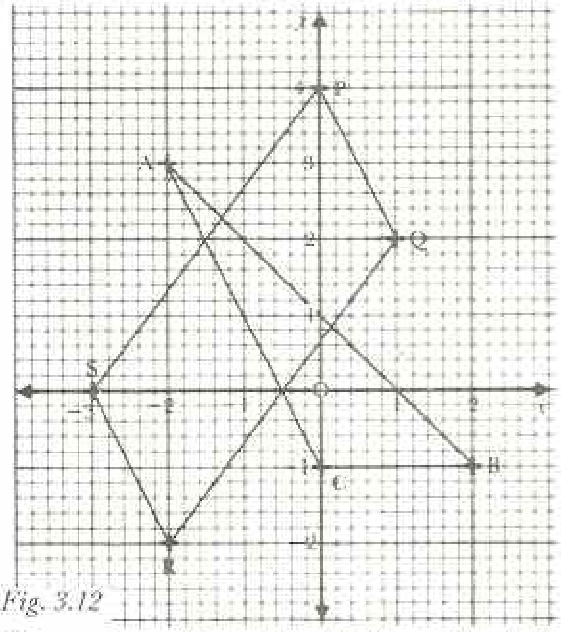


Fig. 3.12

The vertices of triangle ABC are A(− 2; 3), B(2; − 1) and C(0; − 1). The vertices of parallelogram PQRS are P(0; 4), Q(1; 2), R(−2; −2) and S(− 3; 0).

Notice that C and P are on the y-axis. Their x-coordinate is 0 (zero). S is on the x-axis. Its y-coordinate is 0.

Exercise 3c

1 What are the coordinates of the points A, B, C, D, E, F, G, H, I and J in Fig. 3.13?

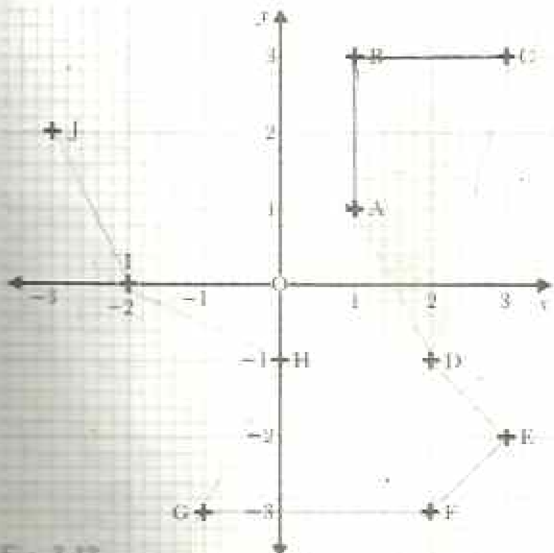


Fig. 3.13

2 In Fig. 3.14 name the points which have the following coordinates.

(a) (9; 5)	(g) (0; − 7)
(b) (5; − 8)	(h) (− 7; 0)
(c) (− 15; − 10)	(i) (14; − 11)
(d) (− 5; 8)	(j) (− 13; 15)
(e) (12; 0)	(k) (− 4; − 12)
(f) (0; 12)	(l) (14; 14)

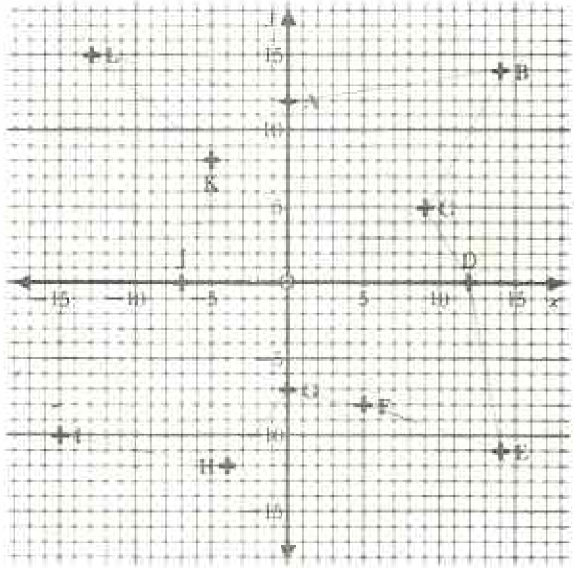


Fig. 3.14

3 What are the coordinates of the vertices T, U, V, W, X, Y and Z of the shape in Fig. 3.15?

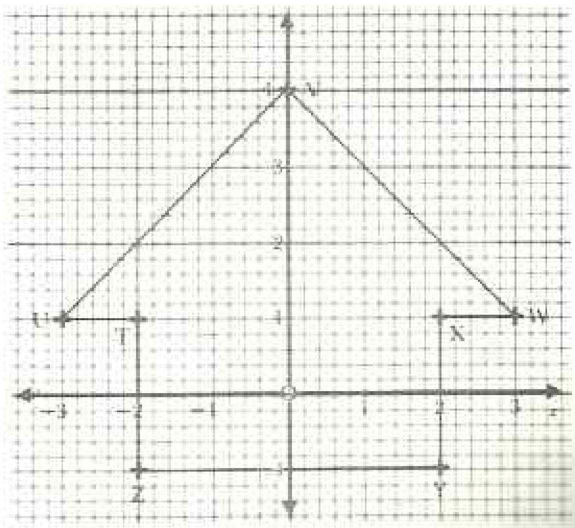


Fig. 3.15

4 What are the coordinates of the vertices of the 'elephant' in Fig. 3.16? Start where shown and work clockwise round the figure.

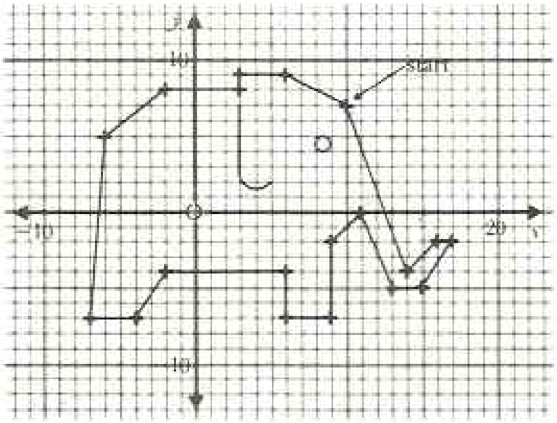


Fig. 3.16

5 Fig. 3.17 shows part of a map drawn on a cartesian plane.

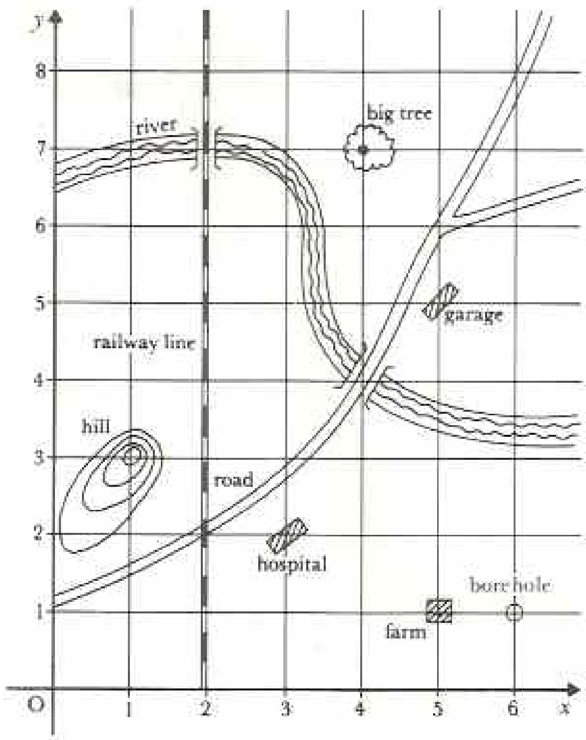


Fig. 3.17

Find the coordinates of
(a) the big tree (b) the garage
(c) the farm (d) the borehole
(e) the hospital (f) the top of the hill
(g) the point where the railway line crosses the road
(h) the point where the railway line crosses the river
(i) the point where the road crosses the river
(j) the point where the road branches to the right

Find the coordinates of any 4 points on the railway line. What do you notice?

6 Fig. 3.18 is the graph of lines *l* and *m*.
(a) Write down the coordinates of the points marked + on line *l*. What do you notice?
(b) Write down the coordinates of the points marked + on line *m*. What do you notice?

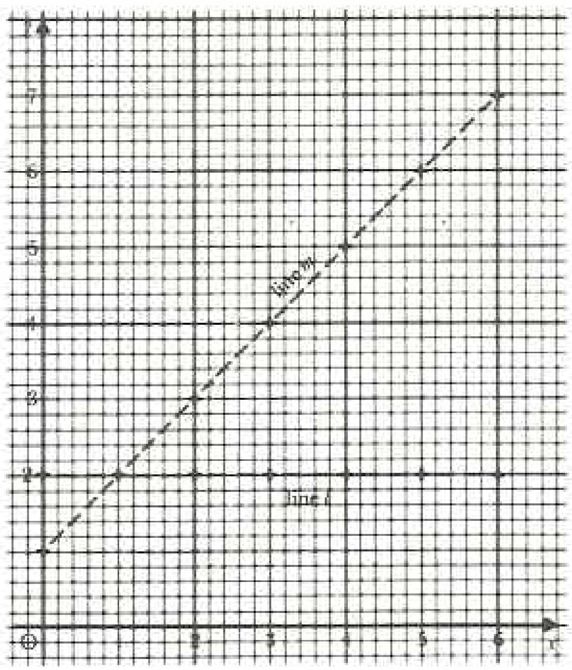


Fig. 3.18

Plotting points

To **plot** a point means to draw its position on a cartesian plane.
The easiest way to plot a point is as follows.

1 Start at the origin.
2 Move along the x-axis by an amount and in a direction given by the x-coordinate of the point.
3 Move up or down parallel to the y-axis by an amount and in a direction given by the y-coordinate.

Example 2

Plot the points $(-1; 2)$ *and* $(2{,}6; -1{,}8)$ *on a cartesian plane.*

The dotted arrows in Fig. 3.19 show the method of plotting.

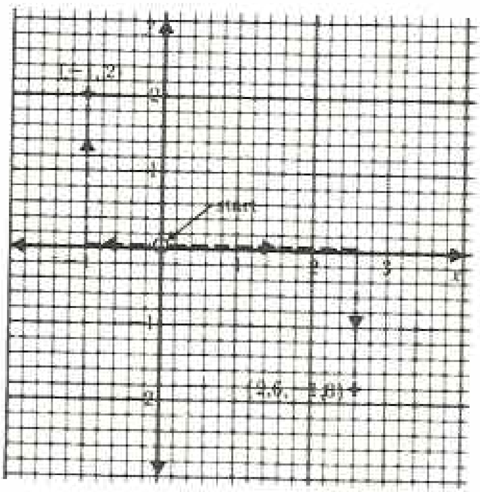


Fig. 3.19

For $(-1; 2)$:
Start at the origin. The x-coordinate is -1. Move 1 unit to the left along the x-axis. The y-coordinate is 2. Move 2 units up parallel to the y-axis. Plot the point.
For $(2{,}6; -1{,}8)$:
Start at the origin. The x-coordinate is 2,6. Move 2,6 units to the right on the x-axis. The y-coordinate is $-1{,}8$. Move 1,8 units down parallel to the y-axis. Plot the point.

Notes:
1 The dotted arrows in Fig. 3.19 are not normally put on the graph. They are given here to show the method only.
2 Use a small vertical cross (+) to plot points.

Example 3

The vertices of quadrilateral PQRS *have coordinates* P$(-3; 18)$, Q$(15; 14)$, R$(11; -4)$ *and* S$(-7; 0)$. A *and* B *are the points* A$(-3; -7)$ *and* B$(3; 0)$.
(*a*) *Using a scale of* 2 cm *to represent* 10 *units on both axes, plot points* P, Q, R, S, A *and B.*
(*b*) *Join the vertices of quadrilateral* PQRS. *What kind of quadrilateral is it?*
(*c*) *Find the coordinates of the point where the diagonals of* PQRS *cross.*
(*d*) *What do you notice about the points* A, B *and* Q?

(a) The scale is given. The highest x-coordinate is 15 and the lowest is -7. The x-axis must include these numbers. A scale from -10 to 20 on the x-axis will be suitable. The highest y-coordinate is 18 and the lowest is -7. A scale for -10 to 20 on the y-axis will be suitable. the points are plotted in Fig. 3.20.

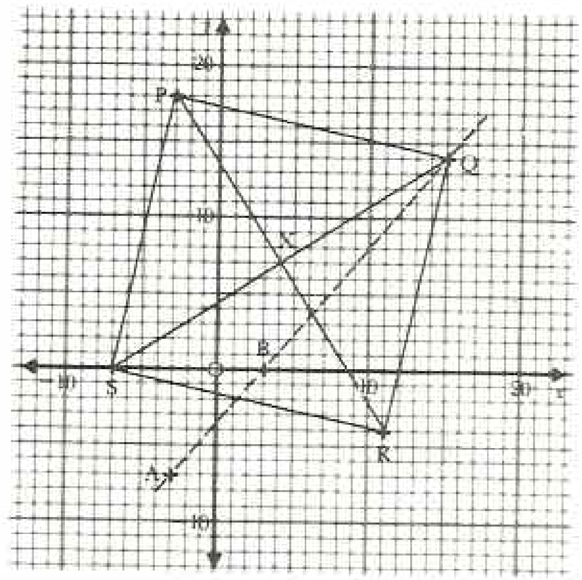


Fig. 3.20

(b) PQRS is a square.
(c) The diagonals of PQRS cross at X $(4; 7)$.
(d) B, A and Q lie on a straight line (dotted in Fig. 3.20).

When drawing cartesian graphs, *always*:
1 draw the axes;
2 label the origin O;
3 label the axes x and y;
4 write the scales along each axis.

Exercise 3d

Work on graph paper in this exercise.

1 Draw the origin, O, near the middle of a clean sheet of graph paper. Use a scale of 1 cm represents 1 unit on both axes. Plot the following points:
A(8; 10), B(− 8; − 10), C(3; − 5), D(− 6; 9), E(− 4; − 7), F(1; 8), G(2; 0), H(0; − 6), I(− 2,4; 5,2), J(− 4; 3,8), K(0; 6,6), L(0,8; − 7,8).

2 Draw the origin, O, near the middle of a sheet of graph paper. Use a scale of 2 cm to represent 1 unit on both axes. Plot the following points then join each point to the next in alphabetical order.
A(0; 1), B(1; 2), C(1; 1), D(2; 1), E(1; 0), F(2; − 1), G(1; − 1), H(1; − 2), I(0; − 1), J(− 1; − 2), K(− 1; − 1), L(− 2; −1), M(− 1; 0), N(− 2; 1), P(− 1; 1), Q(− 1; 2).
Finally, join Q to A.

3 Draw the origin, O, near the middle of a sheet of graph paper. Use a scale of 2 cm to represent 5 units on both axes. Plot the following points. Join each point to the next in the order they are given.
START (− 10; − 5), (− 5; 10), (0; 15), (5; 17), (3; 14), (3; 12), (15; 6), (14; 3), (11; 3), (13; 2), (5; 3), (6; − 6) FINISH
What does your graph show a picture of?

4 Take O near the middle of your graph paper and let 2 cm represent 1 unit on both axes.
(a) Plot the points P(4; 3), Q(4; 1), R(− 1; 1), S(− 1; 3), W(1; 2), X(1; − 1), Y(− 2; − 1) and Z(− 2; 2).
(b) Find the areas, in unit^2, of rectangle PQRS, square WXYZ, triangle SXY, triangle PYZ.

5 As in question 4, but plot the points A(0; 4), B(− 3; − 1), C(− 2; − 4), D(1; 1), E(3; 2), F(4; 0) and G(2; − 1).
(a) Draw quadrilateral ABCD. What kind of quadrilateral is it? Let its diagonals cross at X. Find the coordinates of X.
(b) What do you notice about points B, X, D and E?
(c) Draw quadrilateral DEFG. What kind of quadrilateral is it? Let its diagonals cross at Y. Find the coordinates of Y.

6 (a) Complete the ordered pairs in the following pattern: (0; 0), (1; 1), (2; 4), (3; 9), (4; 16), (5; 25), (6;), (7;), (8;), (9;), (10;).
(b) Draw the origin, O, at the bottom left-hand corner of a sheet of graph paper. Draw an x-axis with a scale of 1 cm to 1 unit. Draw a y-axis with a scale of 1 cm to 5 units.
(c) Plot the points in part (a).
(d) Join the points you have plotted by drawing as smooth a curve as you can.
(e) Use your graph to find $(8{,}4)^2$, $(6{,}5)^2$, $\sqrt{20}$, $\sqrt{90}$.

Chapter 4

Indices, powers, squares and square roots

Large numbers

There is no such thing as 'the biggest number in the world'. It is always possible to count higher. Science and economics use very large numbers. Table 4.1 gives the names and values of some large numbers.

Table 4.1

name	value
thousand	1 000
million	1 000 thousand = 1 000 000 $= 1\,000^2$
billion	1 000 million = 1 000 000 000 $= 1\,000^3$

How big is a million?

The following examples may give you some idea of the size of a million.

1 A 1 cm by 1 cm square of 1 mm graph paper contains one hundred small 1 mm × 1 mm squares.

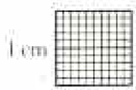


Fig. 4.1 100 small squares

A 1 m by 1 m square of the same graph paper contains 1 million of these small squares.

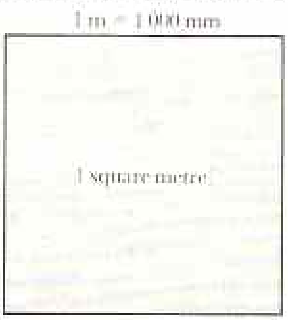


Fig. 4.2 1 000 mm × 1 000 mm = 1 000 000 mm²

2 A cubic metre measures 100 cm by 100 cm by 100 cm.

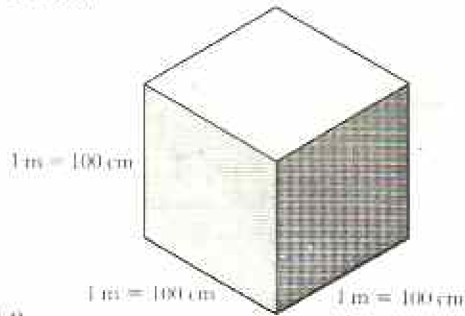


Fig. 4.3

Volume of cubic metre
$= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$
$= 1\,000\,000 \text{ cm}^3$
Thus 1 million cubic centimetres, cm^3, will exactly fill a 1 cubic metre box.

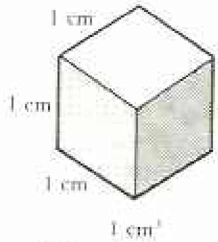


Fig. 4.4 One million of these make 1 cubic metre

Exercise 4a

1 What is the correct name for (a) a thousand thousand, (b) a thousand million?

2 A football field measures 80 m by 50 m.
   (a) Change the dimensions to cm and calculate the area of the field in cm^2.
   (b) A playing card packet measures 8 cm by 5 cm. Calculate how many playing cards would be needed to cover the football field.

3 A library has about 4 000 books. Each book has about 250 pages. Approximately how many pages are there in the library?

4 How long would it take to count to 1 million if it takes an average of 1 second to say each number? Give your answer to the nearest $\frac{1}{2}$ day.

5 Find out which of the following is nearest to the number of seconds in a year.

(a) 500 000 (b) 1 000 000
(c) 3 000 000 (d) 30 000 000
(e) 2 000 000 000

Writing large numbers

Grouping digits

Read the number 31556926 out aloud. Was it easy to do? It may have been quite difficult. You had to decide, 'Is the number bigger than a million or not?', 'Does it begin 3 million, 31 million or 315 million?'.

It is necessary to write large numbers in a helpful way. It is usual to group the digits of large numbers in threes from the decimal comma. A small gap is left between each group.

31556926 should be written 31 556 926. Now it is easy to see that the number begins with 31 million.

Exercise 4b

Write the following numbers, grouping digits in threes from the decimal comma.

1	1 million	**2**	59244	**3**	721568397
4	2312400	**5**	8 million	**6**	3 billion
7	9215	**8**	14682053	**9**	108412
10	12345	**11**	100000000	**12**	987654

Digits and words

Editors of newspapers know that large numbers sometimes confuse readers. They often use a mixture of digits and words when writing large numbers.

Example 1

What do the numbers in the following headlines stand for?

(*a*) **FOOD IMPORTS RISE TO $1 BILLION**

(*b*) **OIL PRODUCTION NOW 2,3 MILLION BARRELS DAILY**

(*c*) **FLOODS IN INDIA – 0,6 MILLION HOMELESS**

(*d*) **ROAD TO COST 22\frac{1}{4}$ MILLION**

(a) $1 billion is short for $1 000 000 000

(b) 2,3 million = 2,3 × 1 000 000
= 2 300 000

(c) 0,6 million = 0,6 × 1 000 000
= 600 000

(d) 22\frac{1}{4}$ million = $22,25 million
= $22 250 000

Example 2

Express the following in a mixture of digits and words.

(*a*) $3 000 000 (*b*) 6 800 000 000
(*c*) 240 000 000 (*d*) $500 000

(a) $3 000 000 = $3 × 1 000 000
= $3 million

(b) 6 800 000 000 = 6,8 × 1 000 000 000
= 6,8 billion

(c) 240 000 000 = 240 × 1 000 000
= 240 million

or

240 000 000 = 0,24 × 1 000 000 000
= 0,24 billion

(d) $500 000 = $0,5 × 1 000 000
= $0,5 million or $$\frac{1}{2}$ million

Exercise 4c

1 Express the following numbers in digits only.

(a) $2 million (b) 150 million km
(c) 3 billion (d) 5$\frac{1}{2}$ million
(e) $2,1 billion (f) 4,2 million litres
(g) 0,4 billion (h) 1\frac{1}{4}$ million
(i) 0,7 million tonnes (j) $$\frac{3}{4}$ million
(k) 0,45 million (l) $0,58 billion

2 Imagine you are a newspaper editor. Write the following numbers using a mixture of digits and words.

(a) 8 000 000 tonnes (b) $6 000 000
(c) 2 000 000 000 (d) $3 700 000 000
(e) $7 400 000 (f) $1 750 000
(g) 200 000 litres (h) 500 000 000
(i) 300 000 tonnes (j) 250 000
(k) 980 000 barrels (l) 490 000 000

Small numbers

Decimal fractions

Decimal fractions also have names.

8 tenths	= 0,8
8 hundredths	= 0,08
8 thousandths	= 0,008
8 ten thousandths	= 0,000 8
8 hundred thousandths	= 0,000 08

Notice that digits are grouped in threes from the decimal comma as before.

Example 3

Write the following as decimal fractions.

(*a*) 28 *thousandths* (*b*) $\frac{865}{100\,000}$

(*c*) 350 *millionths* (*d*) $\frac{400}{10\,000}$

(a) 28 thousandths = 1 thousandth × 28
= 0,001 × 28
= 0,028

(b) $\frac{865}{100\,000}$ = 0,008 65

There are 5 zeros in the denominator. The decimal fraction is obtained by moving the digits in the numerator 5 places to the right.

(c) 350 millionths = 1 millionth × 350
= 0,000 001 × 350
= 0,000 350 = 0,000 35

In a decimal fraction it is not necessary to write any zeros after the last non-zero digit.

(d) $\frac{400}{10\,000}$ = 0,040 0
= 0,04

Exercise 4d

Write the following as decimal fractions.

1 6 hundredths **2** 4 thousandths
3 9 tenths **4** 8 millionths
5 4 ten thousandths
6 6 hundred thousandths
7 $\frac{3}{1\,000}$ **8** $\frac{9}{100\,000}$
9 $\frac{7}{10\,000}$ **10** 16 hundredths
11 34 thousandths **12** 26 ten thousandths
13 $\frac{28}{100\,000}$ **14** $\frac{84}{1\,000}$
15 $\frac{750}{100\,000}$ **16** 27 tenths
17 65 hundredths **18** 402 thousandths
19 20 hundredths **20** 240 thousandths
21 700 thousandths **22** $\frac{620}{100\,000}$
23 $\frac{330}{100\,000}$ **24** $\frac{4\,020}{100\,000}$
25 90 hundredths **26** 900 thousandths
27 300 ten thousandths
28 $\frac{720}{1\,000}$
29 $\frac{720}{10\,000}$ **30** $\frac{720}{100\,000}$

Laws of indices

10^3 is short for 10 × 10 × 10. Similarly, x^5 is short for $x \times x \times x \times x \times x$. x can be any number.

Example 4

Multiply (*a*) x^5 *by* x^3, (*b*) a^3 *by* a^2, (*c*) y *by* y^4.

(a) $x^5 \times x^3 = (x \times x \times x \times x \times x) \times (x \times x \times x)$
$= x \times x \times x \times x \times x \times x \times x \times x$
$= x^8$

(b) $a^3 \times a^2 = (a \times a \times a) \times (a \times a)$
$= a \times a \times a \times a \times a$
$= a^5$

(c) $y \times y^4 = y \times (y \times y \times y \times y)$
$= y \times y \times y \times y \times y$
$= y^5$

Notice that the index in the result is the sum of the given indices:

$x^5 \times x^3 = x^{5+3} \quad = x^8$

$a^3 \times a^2 = a^{3+2} \quad = a^5$

$y \times y^4 = y^1 \times y^4 \quad y^{1+4} = y^5$

In general: $\boldsymbol{x^a \times x^b = x^{a+b}}$

Example 5

Simplify the following. (*a*) $10^4 \times 10^2$ (*b*) $4c^3 \times 7c^2$

In fully expanded form:

(a) $10^4 \times 10^2$
$= (10 \times 10 \times 10 \times 10) \times (10 \times 10)$
$= 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$= 10^6$

(b) $4c^3 \times 7c^2 = 4 \times c \times c \times c \times 7 \times c \times c$
$= 4 \times 7 \times c \times c \times c \times c \times c$
$= 28c^5$

Or, more quickly, by adding indices:

(a) $10^4 \times 10^2 = 10^{4+2} = 10^6$

(b) $4c^3 \times 7c^2 = 4 \times 7 \times c^{3+2} = 28c^5$

Exercise 4e (Oral)

1 Simplify the following by fully expanding the terms.

(a) $x^2 \times x^3$ (b) $10^2 \times 10^5$

(c) $a^4 \times a^3$ (d) $10^5 \times 10$

(e) $n^5 \times n$ (f) $10^4 \times 10^5$

(g) $3a^2 \times 8a^4$ (h) $5x^3 \times 4x^7$

(i) $3c^2 \times 2c^5$

2 Simplify the following by adding the indices.

(a) $m^3 \times m^5$ (b) $a^6 \times a^4$

(c) $c^5 \times c^9$ (d) $10^2 \times 10^7$

(e) $b^8 \times b^7$ (f) $x^7 \times x$

(g) $2e^4 \times 5e^{10}$ (h) $3 \times 10^6 \times 5 \times 10^3$

(i) $5y^5 \times 3y^3$

Example 6

Divide (a) x^5 by x^3, (b) a^7 by a^4, (c) p^6 by p^5.

(a) $x^5 \div x^3 = \dfrac{x^5}{x^3} = \dfrac{x \times x \times x \times x \times x}{x \times x \times x}$

$= \dfrac{x \times x \times (x \times x \times x)}{(x \times x \times x)}$

$= x^2$

(b) $a^7 \div a^4 = \dfrac{a^7}{a^4} = \dfrac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a \times a}$

$= \dfrac{a \times a \times a \times (a \times a \times a \times a)}{(a \times a \times a \times a)}$

$= a^3$

(c) $p^6 \div p^5 = \dfrac{p^6}{p^5} = \dfrac{p \times p \times p \times p \times p \times p}{p \times p \times p \times p \times p}$

$= \dfrac{p \times (p \times p \times p \times p \times p)}{(p \times p \times p \times p \times p)}$

$= p$

Notice that the index in the result is the index of the divisor subtracted from the index of the dividend:

$x^5 \div x^3 = x^{5-3} = x^2$

$a^7 \div a^4 = a^{7-4} = a^3$

$p^6 \div p^5 = p^{6-5} = p^1 = p$

In general: $\mathbf{x^a \div x^b = x^{a-b}}$

Example 7

Divide (a) 10^6 by 10^2, (b) $12a^7$ by $3a^2$.

In fully expanded form:

(a) $10^6 \div 10^2 = \dfrac{10^6}{10^2}$

$= \dfrac{10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10}$

$= 10 \times 10 \times 10 \times 10 = 10^4$

(b) $12a^7 \div 3a^2$

$= \dfrac{12 \times a \times a \times a \times a \times a \times a \times a}{3 \times a \times a}$

$= \dfrac{4 \times (3 \times a \times a) \times a \times a \times a \times a \times a}{(3 \times a \times a)}$

$= 4 \times a \times a \times a \times a \times a = 4a^5$

Or, more quickly, by subtracting indices:

(a) $10^6 \div 10^2 = 10^{6-2} = 10^4$

(b) $12a^7 \div 3a^2 = \frac{12}{3} \times a^{7-2} = 4a^5$

Exercise 4f (Oral)

1 Simplify the following by fully expanding the terms.

(a) $a^7 \div a^3$ (b) $10^5 \div 10^2$

(c) $c^4 \div c$ (d) $\dfrac{10^6}{10^3}$

(e) $\dfrac{d^6}{d^5}$ (f) $\dfrac{10^5}{10^3}$

(g) $12x^7 \div 4x^3$ (h) $10a^8 \div 5a^6$

(i) $4x^6 \div 4x$

2 Simplify the following by subtracting indices.

(a) $x^6 \div x^4$ (b) $b^8 \div b^5$

(c) $c^9 \div c^3$ (d) $\frac{a^{11}}{a^9}$

(e) $10^9 \div 10^7$ (f) $\frac{x^7}{x}$

(g) $18x^5 \div 9x^4$ (h) $\frac{24x^8}{6x^5}$

(i) $\frac{8 \times 10^9}{4 \times 10^6}$

Example 8

Simplify $x^3 \div x^3$ (a) by fully expanding each term, (b) by subtracting indices.

(a) $x^3 \div x^3 = \frac{x \times x \times x}{x \times x \times x} = 1$

(b) $x^3 \div x^3 = x^{3-3} = x^0$

From the results of parts (a) and (b) in Example 10, $x^0 = 1$.

In general: **any number raised to the power 0 has the value 1.**

Example 9

Simplify $x^2 \div x^5$ (a) by fully expanding each term, (b) by subtracting indices

(a) $x^2 \div x^5 = \frac{x \times x}{x \times x \times x \times x \times x}$

$$= \frac{1}{x \times x \times x} = \frac{1}{x^3}$$

(b) $x^2 \div x^5 = x^{2-5} = x^{-3}$

From the results of parts (a) and (b) in Example 11,

$$x^{-3} = \frac{1}{x^3}$$

In general, $\boldsymbol{x^{-a} = \frac{1}{x^a}}$

Example 10

Simplify the following. (a) 10^{-3} (b) $x^0 \times x^4 \times x^{-2}$ (c) $a^{-3} \div a^{-5}$ (d) $(\frac{1}{4})^{-2}$

(a) $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$

(b) $x^0 \times x^4 \times x^{-2} = 1 \times x^4 \times \frac{1}{x^2} = \frac{x^4}{x^2}$

$$= x^{4-2} = x^2$$

or

$$x^0 \times x^4 \times x^{-2} = x^{0+4+(-2)} = x^2$$

(c) $a^{-3} \div a^{-5} = \frac{1}{a^3} \div \frac{1}{a^5} = \frac{1}{a^3} \times \frac{a^5}{1}$

$$= a^{5-3} = a^2$$

or

$$a^{-3} \div a^{-5} = a^{(-3)-(-5)}$$

$$= a^{-3+5} = a^2$$

(d) $(\frac{1}{4})^{-2} = \frac{1}{(\frac{1}{4})^2} = \frac{1}{\frac{1}{16}} = 16$

Exercise 4g

Simplify the following.

1 10^{-2} **2** 10^{-4}

3 10^{-6} **4** $x^5 \times x^{-2}$

5 $a^{-2} \times a^{-3}$ **6** $m^0 \times n^0$

7 $a^2 \div a^7$ **8** $x^2 \div x^{-7}$

9 $p^{-2} \div p^{-7}$ **10** $b^3 \div b^0$

11 $r^7 \div r^7$ **12** $c^{-1} \times c^{-1}$

13 $(\frac{1}{2})^{-3}$ **14** $(\frac{1}{3})^{-2}$

15 $(\frac{1}{9})^{-1}$ **16** $2a^{-1} \times 3a^2$

17 $(2a)^{-1} \times 3a^2$ **18** $2a^{-1} \times (3a)^2$

Squares and square roots

$7^2 = 7 \times 7 = 49$.

In words, 'the square of 7 is 49'. We can turn this statement round and say, 'the square root of 49 is 7'.

In symbols, $\sqrt{49} = 7$

The symbol $\sqrt{\ }$ means 'the square root of'.

To find the square root of a number, first find its factors.

Example 11

Find $\sqrt{11\,025}$.

method: Try the prime numbers in turn.

working:

3	11 025
3	3 675
5	1 225
5	245
7	49
7	7
	1

$$11\,025 = 3^2 \times 5^2 \times 7^2$$
$$= (3 \times 5 \times 7) \times (3 \times 5 \times 7)$$
$$= 105 \times 105$$

Thus $\sqrt{11\,025} = 105$

It is not always necessary to write a number in its prime factors.

Example 12

Find $\sqrt{6\,400}$.

$$6\,400 = 64 \times 100$$
$$= 8^2 \times 10^2$$

Thus $\sqrt{6\,400} = 8 \times 10 = 80$

The rules for divisibility can be useful when finding square roots.

Example 13

Find $\sqrt{5\,184}$.

Use the rules for divisibility. Since 84 is divisible by 4, 5 184 is divisible by 4.

Since $5 + 1 + 8 + 4 = 18$, 5 184 is divisible by 9.

working:

4	5 184
4	1 296
4	324
9	81
9	9
	1

$$5\,184 = 4^2 \times 4 \times 9^2$$
$$= 4^2 \times 2^2 \times 9^2$$

Thus $\sqrt{5\,184} = 4 \times 2 \times 9 = 72$

Exercise 4h

Find by factors the square roots of the following.

1 225	**2** 196	**3** 324
4 441	**5** 484	**6** 400
7 900	**8** 1 600	**9** 2 500
10 4 900	**11** 576	**12** 784
13 729	**14** 625	**15** 1 225
16 1 936	**17** 1 764	**18** 2 025
19 2 304	**20** 2 916	**21** 3 025
22 3 600	**24** 3 969	**24** 8 100
25 3 136	**26** 4 356	**27** 5 625
28 6 561	**29** 7 056	**30** 7 744

Perfect squares

$$9 = 3 \times 3 \qquad \sqrt{9} = 3$$
$$25 = 5 \times 5 \qquad \sqrt{25} = 5$$
$$225 = 15 \times 15 \qquad \sqrt{225} = 15$$
$$9\,216 = 96 \times 96 \qquad \sqrt{9216} = 96$$

We say that 9; 25; 225; 9 216 are **perfect squares** because their square roots are whole numbers. A perfect square is a whole number whose square root is also a whole number.

It is always possible to express a perfect square in factors with even indices. For example,

$$9216 = 96^2 = 3^2 \times 32^2$$
$$= 3^2 \times 4^2 \times 8^2$$
$$= 3^2 \times 2^{10}$$

Example 14

Find the smallest number by which 540 *must be multiplied so that the product is a perfect square.*

2	540
2	270
3	135
3	45
3	15
5	5
	1

$$540 = 2^2 \times 3^3 \times 5$$

The index of 2 is even.
The indices of 3 and 5 are odd.
One more 3 and one more 5 will make all the indices even. The product will then be a perfect square. The number required $= 3 \times 5 = 15$.

Exercise 4i

Find the smallest numbers by which the following must be multiplied so that their products are perfect squares.

1 24	**2** 54	**3** 45
4 99	**5** 84	**6** 162
7 405	**8** 240	**9** 432
10 147	**11** 252	**12** 504

To find the square root of a fraction, find the square roots of its numerator and denominator.

Example 15

Find the values of the following.

(a) $\sqrt{\frac{4}{25}}$ (b) $\sqrt{\frac{27}{48}}$ (c) $\sqrt{5\frac{4}{9}}$

(a) $\sqrt{\dfrac{4}{25}} = \dfrac{\sqrt{4}}{\sqrt{25}} = \dfrac{2}{5}$

(b) Reduce the given fraction to its lowest terms.

$$\sqrt{\frac{27}{48}} = \sqrt{\frac{9 \times 3}{16 \times 3}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

(c) Express the mixed number as an improper fraction.

$$\sqrt{5\tfrac{4}{9}} = \sqrt{\frac{49}{9}} = \frac{\sqrt{49}}{\sqrt{9}} = \frac{7}{3} = 2\tfrac{1}{3}$$

Exercise 4j

Find the square roots of the following.

1 $\frac{9}{25}$	**2** $\frac{16}{49}$	**3** $\frac{1}{4}$
4 $\frac{36}{81}$	**5** $\frac{9}{16}$	**6** $\frac{48}{75}$
7 $\frac{8}{18}$	**8** $\frac{72}{200}$	**9** $\frac{72}{98}$
10 $\frac{75}{108}$	**11** $2\frac{1}{4}$	**12** $1\frac{7}{9}$
13 $1\frac{9}{16}$	**14** $3\frac{6}{25}$	**15** $20\frac{1}{4}$

Chapter 5

Equations (2)

Example 1 (Revision)
Solve $2x - 9 = 15$.

$2x - 9 = 15$
Add 9 to both sides.
$2x - 9 + 9 = 15 + 9$
$\Leftrightarrow \quad 2x = 24$
Divide both sides by 2.
$$\frac{2x}{2} = \frac{24}{2}$$
$\Leftrightarrow \quad x = 12$
Check: When $x = 12$,
LHS $= 2 \times 12 - 9 = 24 - 9 = 15 =$ RHS

Exercise 5a (Revision)
Solve the following equations.

1 $3x - 8 = 10$	**2** $2x + 5 = 17$
3 $4x - 1 = 1$	**4** $16 = 7x + 2$
5 $27 = 10x - 3$	**6** $20 = 9x + 11$
7 $4 + 5a = 19$	**8** $37 = 7 + 6z$
9 $51 = 3 + 8n$	**10** $10y - 7 = 27$
11 $9 + 2r = 16$	**12** $2 = 3d - 8$

Word problems

We can use equations to solve many word problems.

Example 2
I think of a number. I multiply it by 5. *I add* 15. *The result is* 100. *What is the number I thought of?*

Let the number be n.
I multiply n by 5: $5n$.
I add 15: $5n + 15$
The result is 100: $5n + 15 = 100$
Subtract 15 from both sides.
$5n + 15 - 15 = 100 - 15$
$\Leftrightarrow \quad 5n = 85$
Divide both sides by 5.
$$\frac{5n}{5} = \frac{85}{5}$$
$\Leftrightarrow \quad n = 17$
The number is 17.
Check: $17 \times 5 = 85$; $85 + 15 = 100$

Example 3
A rectangle is 8 m *long and its perimeter is* 30 m. *Find the breadth of the rectangle.*

Let the breadth of the rectangle be b metres.
Perimeter $= 8 + b + 8 + b$ metres
$= 16 + 2b$ metres
Thus $16 + 2b = 30$
Subtract 16 from both sides.
$2b = 30 - 16 = 14$
Divide both sides by 2.
$b = \frac{14}{2} = 7$
The breadth of the rectangle is 7 metres.
Check: 8 m + 7 m + 8 m + 7 m = 30 m

Notice the method in Examples 2 and 3.
1. Choose a letter for the unknown.
2. Write down the information of the question in algebraic form.
3. Make an equation.
4. Solve the equation.
5. Give the answer in written form.
6. Check the result against the information of the question.

Exercise 5b will help you to change written information into algebraic form.

Exercise 5b
1 How many altogether if,
(a) a number x is doubled?
(b) a number n is multiplied by 6?
(c) a number m is multiplied by 6 and then 4 is added?

(d) a number y is doubled and then 5 is taken away?

(e) a number is 3 less than a?

(f) a number d is added to another number 4 times as big?

(g) Rudo has h cents and Peter has twice as much?

(h) a number t is trebled and 7 is subtracted?

(i) one girl has k cents and another girl has 9 cents less?

(j) team X scores g goals and team Y scores 23 goals more than X?

2 What is the perimeter of

(a) a square of side x cm?

(b) a triangle with sides $2a$ metres, a metres and 4 metres?

(c) a regular hexagon of side c cm?

(d) a rectangle of breadth b metres and length 3 times as long?

(e) a rectangle of length 10 metres and breadth h metres?

(f) an isosceles triangle with two sides of length $2t$ cm and one side of length t cm?

Exercise 5c

The questions in this exercise correspond in order to the questions of Exercise 5b.

1 John thinks of a number. He doubles it. His result is 58. What number did John think of?

2 6 students each have the same number of sweets. The total number of sweets is 78. How many sweets did each student have?

3 A number is multiplied by 6 and then 4 is added. The result is 34. Find the first number.

4 A man has two boxes of matches. He uses 5 matches and has 75 matches left. How many matches were in each box?

5 I am thinking of a number. I take away 3. The result is 14. What number did I think of?

6 When a number is added to another number 4 times as big, the result is 30. Find the first number.

7 Rudo and Peter share 21 cents so that Peter gets twice as much as Rudo. How much does Rudo get?

8 Find the number such that when it is trebled and 7 is subtracted, the result is 8.

9 One girl has 9 cents less than another girl. They have 29 cents between them. How much does each girl have?

10 During a football season, one team scored 23 goals more than another. Between them they scored 135 goals. How many goals did each team score?

11 A square has a perimeter of 32 m. Find the length of one side of the square.

12 A triangle is such that the first side is twice the length of the second side. The third side is 4 m long. If the perimeter of the triangle is 13 m, find the lengths of the first and second sides.

13 A regular hexagon has a perimeter of 90 cm. Find the length of one side of the hexagon.

14 A rectangle is three times as long as it is broad. If the perimeter of the rectangle is 40 m, find its length and breadth.

15 A rectangle is 10 m long and its perimeter is 26 m. Find the breadth of the rectangle.

16 An isosceles triangle has 2 long sides and 1 short side. The short side is half the length of a long side. If the perimeter of the triangle is 15 cm, find the length of the short side.

Solving equations – further examples

It is possible to use operations with directed numbers when solving equations.

Example 4

Solve $25 - 9x = 2$.

$$25 - 9x = 2$$

Subtract 25 from both sides.

$$25 - 25 - 9x = 2 - 25$$

$$\Leftrightarrow \quad -9x = -23$$

Divide both sides by -9.

$$\frac{-9x}{-9} = \frac{-23}{-9}$$

$$\Leftrightarrow \quad x = \frac{23}{9} = 2\tfrac{5}{9}$$

Check: When $x = \frac{23}{9}$,
LHS $= 25 - 9 \times \frac{23}{9} = 25 - 23 = 2 =$ RHS

If an equation has unknown terms on both sides of the equals sign, collect the unknown terms on one side and the number terms on the other side.

Example 5
Solve $5x - 4 = 2x + 11$.

$$5x - 4 = 2x + 11$$
Subtract $2x$ from both sides.
$5x - 2x - 4 = 2x - 2x + 11$
$\Leftrightarrow \quad 3x - 4 = 11$
Add 4 to both sides.
$3x - 4 + 4 = 11 + 4$
$\Leftrightarrow \quad 3x = 15$
Divide both sides by 3.
$$x = 5$$
Check: When $x = 5$,
LHS $= 5 \times 5 - 4 = 25 - 4 = 21$
RHS $= 2 \times 5 + 11 = 10 + 11 = 21 =$ LHS

Exercise 5d
Solve the following equations and check the solutions.

1 $13 - 6a = 1$
2 $12 + 5a = -3$
3 $4b + 24 = 0$
4 $0 = 25 - 15x$
5 $12 = 9 - 3a$
6 $9 - 8y = 3$
7 $5 - 4n = 8$
8 $7 = 9 - 3m$
9 $7a = 3a + 20$
10 $20 - 2t = 3t$
11 $5n = 12 - n$
12 $7c - 6 = c$
13 $10q = 3q - 7$
14 $3x = 18 - 3x$
15 $3m + 8 = m$
16 $9x + 1 = 7x$
17 $4h - 2 = h + 7$
18 $5a + 6 = 2a + 20$
19 $18 - 5f = 2f + 4$
20 $11 - 3e = 2e - 19$
21 $6x + 1 = 26 - 2x$
22 $4x + 7 = 5x + 6$
23 $x + 7 = 19 + 2x$
24 $11 + 9n = 6n + 13$

Equations with brackets

Always remove brackets before collecting terms.

Example 6
Solve $3(3x - 1) = 4(x + 3)$.

$$3(3x - 1) = 4(x + 3)$$
Remove brackets.
$$9x - 3 = 4x + 12$$
Subtract $4x$ from and add 3 to both sides.
$9x - 4x - 3 + 3 = 4x - 4x + 12 + 3$
$\Leftrightarrow \quad 5x = 15$
Divide both sides by 5. $x = 3$
Check: When $x = 3$,
LHS $= 3(3 \times 3 - 1) = 3(9 - 1)$
$= 3 \times 8 = 24$
RHS $= 4(3 + 3) = 4 \times 6 = 24 =$ LHS

Example 7
Solve $5(x + 11) + 2(2x - 5) = 0$.

$5(x + 11) + 2(2x - 5) = 0$
Remove brackets.
$5x + 55 + 4x - 10 = 0$
Collect like terms.
$$9x + 45 = 0$$
Subtract 45 from both sides.
$$9x = -45$$
Divide both sides by 9.
$$x = -5$$
Check: When $x = -5$,
LHS $= 5(-5 + 11) + 2(2 \times (-5) - 5)$
$= 5 \times 6 + 2(-10 - 5)$
$= 30 + 2 \times (-15) = 30 - 30 = 0$
$=$ RHS

Exercise 5e
Solve the following equations and check the solutions.

1 $2(x + 5) = 18$
2 $15 = 3(x - 3)$
3 $55 = 5(2a - 1)$
4 $2(3y + 1) = 14$
5 $4(x + 7) + 12 = 0$
6 $0 = 7(x - 3)$
7 $6(2s - 7) = 5s$
8 $4b = 3(3b + 15)$
9 $3(f + 2) = 2 - f$
10 $3x + 1 = 2(3x + 5)$
11 $5(a + 2) = 4(a - 1)$
12 $5(b + 1) = 3(b + 3)$
13 $7(2e + 3) = 3(4e + 9)$
14 $8(2d - 3) = 3(4d - 7)$
15 $5(x + 1) = 7(2 - x)$
16 $2(4 - x) = 3(2 - x)$

17 $2(y - 2) + 3(y - 7) = 0$
18 $5(y + 8) + 2(y + 1) = 0$
19 $5(x - 4) - 4(x + 1) = 0$
20 $3(2x + 3) - 7(x + 2) = 0$
21 $5(5z - 2) - 9(3z - 2) = 2$
22 $3(6 + 7y) + 2(1 - 5y) = 42$
23 $5(v + 2) + 3(v + 5) = 1$
24 $4(3 - 5n) - 7(5 - 4n) + 3 = 0$

Word problems involving brackets

Example 8

I subtract 3 from a certain number, multiply the result by 5 and then add 9. If the final result is 54, find the original number.

Let the original number be x.
I subtract 3: this gives $x - 3$
I multiply by 5: this gives $5(x - 3)$
I add 9: this gives $5(x - 3) + 9$
The result is 54.
Thus $5(x - 3) + 9 = 54$
Clear brackets.
$$5x - 15 + 9 = 54$$
Collect terms.
$$5x = 54 + 15 - 9 = 60$$
$$\Leftrightarrow x = 60 \div 5 = 12$$
The original number is 12.

Example 9

Jabulani and Pius sell ballpoint pens at the same price. Jabulani increases his price by 2 cents and Pius reduces his price by 4 cents. Jabulani sells 6 pens and Pius sells 9 pens. If they both take the same amount of money, what was the original price of a pen?

Let the original price of a pen be x cents.
Jabulani's new price = $(x + 2)$ cents
Pius' new price = $(x - 4)$ cents
Jabulani's income = $6(x + 2)$ cents
Pius' income = $9(x - 4)$ cents
They both take in the same money, thus,
$$6(x + 2) = 9(x - 4)$$
Clear brackets.
$$6x + 12 = 9x - 36$$
Collect terms.
$$12 + 36 = 9x - 6x$$
$$48 = 3x$$
$$16 = x$$
The original price of a pen was 16 cents.

Exercise 5f

1 I add 12 to a certain number and then double the result. The answer is 42. Find the original number.

2 I subtract 8 from a certain number. I then multiply the result by 3. The final answer is 21. Find the original number.

3 I think of a number. I multiply it by 5. I then subtract 19. Finally, I double the result. The final number is 22. What number did I think of?

4 Find two consecutive whole numbers such that 5 times the smaller number added to 3 times the greater number makes 59. (*Hint:* let the numbers be x and $x + 1$).

5 Find two consecutive odd numbers such that 6 times the smaller added to 4 times the greater comes to 138. (*Hint:* let the numbers be x and $x + 2$.)

6 A rectangular room is 2 m longer than it is wide. Its perimeter is 70 m. If the width of the room is w m, express its length in terms of w. Hence find the width of the room.

7 A man has a body mass of m kg. He is 30 kg heavier than each of his twin children. Express the body mass of each child in terms of m. If the mass of the father and 2 children comes to 156 kg, find the mass of the father.

8 Black pencils cost 15c each and coloured pencils cost 21c each. If 24 pencils cost \$4,02, how many of them were black? (*Hint:* let there be x black pencils. Thus there are $24 - x$ coloured pencils. Work in cents.)

9 A worker gets 60c an hour for ordinary time and 90c an hour for overtime. If she gets \$32,40 for a 50-hour week, how many hours were overtime?

10 The cost of petrol rises by 2c a litre. Last week a man bought 20 litres at the old price. This week he bought 10 litres at the new price. Altogether, the petrol cost \$9,20. What was the old price for 1 litre?

11 A trader bought some oranges at 5c each. She finds that 6 of them are rotten. She sells the rest at 8c each and makes a profit of \$4,50. How many oranges did she buy? (*Hint:* let the number of oranges be x. Work in cents.)

12 In 1990 an egg cost 2c less than in 1991. In 1992 an egg cost 4c more than in 1991. The cost of 11 eggs in 1990 was the same as the cost of 8 eggs in 1992. Find the cost of an egg in 1991. (*Hint:* let the 1991 cost of an egg be n cents. Express the 1990 and 1992 costs in terms of n.)

Equations with fractions

Always clear fractions before collecting terms. To clear fractions, multiply both sides of the equation by the LCM of the denominators of the fractions.

Example 10

Solve the equation $\frac{4m}{5} - \frac{2m}{3} = 4$.

$$\frac{4m}{5} - \frac{2m}{3} = 4$$

The LCM of 5 and 3 is 15.
Multiply both sides of the equation by 15, i.e. multiply *every* term by 15.

$$15 \times \left(\frac{4m}{5}\right) - 15 \times \left(\frac{2m}{3}\right) = 15 \times 4$$

$$\Leftrightarrow \quad 3 \times 4m - 5 \times 2m = 15 \times 4$$
$$\Leftrightarrow \quad 12m - 10m = 60$$
$$\Leftrightarrow \quad 2m = 60$$

Divide both sides by 2.

$$m = 30$$

Check: When $m = 30$

$$\text{LHS} = \frac{4 \times 30}{5} - \frac{2 \times 30}{3} = \frac{120}{5} - \frac{60}{3}$$
$$= 24 - 20 = 4 = \text{RHS}$$

Example 11

Solve the equation $\frac{3x - 2}{6} - \frac{2x + 7}{9} = 0$.

The LCM of 6 and 9 is 18.
Multiply both sides of the equation by 18.

$$\frac{18(3x - 2)}{6} - \frac{18(2x + 7)}{9} = 18 \times 0$$

$$3(3x - 2) - 2(2x + 7) = 0$$

Clear brackets.

$$9x - 6 - 4x - 14 = 0$$

Collect terms.

$$5x - 20 = 0$$

Add 20 to both sides.

$$5x = 20$$

Divide both sides by 5.

$$x = 4$$

Check: When $x = 4$,

$$\text{LHS} = \frac{3 \times 4 - 2}{6} - \frac{2 \times 4 + 7}{9}$$
$$= \frac{12 - 2}{6} - \frac{8 + 7}{9}$$
$$= \frac{10}{6} - \frac{15}{9} = \frac{5}{3} - \frac{5}{3} = 0 = \text{RHS}$$

Exercise 5g

Solve the following equations.

1 $\frac{x}{3} = 5$ **2** $\frac{x}{5} = \frac{1}{2}$

3 $4 = \frac{a}{9}$ **4** $\frac{7a}{2} - 21 = 0$

5 $\frac{4}{3} = \frac{2z}{15}$ **6** $1\frac{1}{2} - \frac{3x}{4} = 0$

7 $\frac{x - 2}{3} = 4$ **8** $\frac{5 + a}{4} = 6$

9 $\frac{2 - a}{5} = 1$ **10** $5 = \frac{2y - 3}{7}$

11 $\frac{3n + 1}{8} = 2$ **12** $4 = \frac{9 + 5a}{6}$

13 $\frac{x + 18}{2} = 5x$ **14** $x = \frac{x - 24}{9}$

15 $\frac{5x - 8}{2} = 2x$ **16** $\frac{22 - 3x}{4} = 2x$

17 $\frac{4 - z}{7} = z$ **18** $\frac{2(8x + 7)}{3} = 5x$

19 $\frac{x}{2} - \frac{x}{3} = 2$ **20** $\frac{x}{2} + \frac{3x}{4} = 5$

21 $\frac{3m}{5} - \frac{m}{3} = \frac{8}{5}$ **22** $\frac{3x}{7} = \frac{2x}{3} - \frac{1}{3}$

23 $\frac{x-5}{2} = \frac{x-4}{3}$

24 $\frac{4t+3}{5} = \frac{t+3}{2}$

25 $\frac{5e-1}{4} - \frac{7e+4}{8} = 0$

26 $\frac{2d+7}{6} + \frac{d-5}{3} = 0$

27 $\frac{6m-3}{7} = \frac{2m+1}{7}$

28 $\frac{3(2a+1)}{4} = \frac{5(a+5)}{6}$

29 $\frac{2a-1}{3} - \frac{a+5}{4} = \frac{1}{2}$

30 $\frac{4x-3}{2} = \frac{9x-6}{8} + 2\frac{3}{4}$

Word problems involving fractions (1)

Example 12

I add 55 to a certain number and then divide the sum by 3. The result is 4 times the first number. Find the number.

Let the number be n.
I add 55 to n: this gives $n + 55$.

I divide the sum by 3: this gives $\frac{n+55}{3}$.

The result is $4n$.

Thus $\frac{n+55}{3} = 4n$

Multiply both sides by 3.

$\frac{3(n+55)}{3} = 3 \times 4n$

$n + 55 = 12n$

Collect terms.

$55 = 12n - n$

$55 = 11n$

$\Leftrightarrow \quad n = 5$

The number is 5.

Example 13

The body mass of a man is x kg. The body masses of his two children are $\frac{5}{6}$ and $\frac{4}{5}$ that of their father. (a) Express the children's masses in terms of x. (b) If the difference between the masses of the children is 2,3 kg find the mass of the father.

(a) One child is $\frac{5}{6}$ of x kg $= \frac{5x}{6}$ kg

The other child is $\frac{4}{5}$ of x kg $= \frac{4x}{5}$ kg

(b) $\frac{5x}{6} - \frac{4x}{5} = 2{,}3$

The LCM of 5 and 6 is 30.
Multiply both sides by 30.

$30 \times \frac{5x}{6} - 30 \times \frac{4x}{5} = 30 \times 2{,}3$

$\Leftrightarrow 5 \times 5x - 6 \times 4x = 69$

$\Leftrightarrow \quad 25x - 24x = 69$

$\Leftrightarrow \quad x = 69$

The mass of the father is 69 kg.

Exercise 5h

1 I think of a number. I double it. I divide the result by 5. My answer is 6. What number did I think of?

2 I subtract 17 from a certain number and then divide the result by 5. My final answer is 3. What was the original number?

3 I add 9 to a certain number and then divide the sum by 16. Find the number if my final answer is 1.

4 I add 45 to a certain number and then divide the sum by 2. The final result is 5 times the original number. Find the original number.

5 $\frac{1}{5}$ of an even number added to $\frac{1}{6}$ of the next even number makes a total of 15. Find the two numbers.
(*Hint:* let the numbers be x and $x + 2$.)

6 A mother is 24 years older than her daughter. If the daughter's age is x years,
(a) express the mother's age in terms of x;
(b) find x when the daughter's age is $\frac{1}{3}$ of her mother's age.

7 The price of a packet of salt goes up by 9 cents. The old price is $\frac{4}{5}$ of the new price. Find the old and new prices.
(*Hint:* let the old price be n cents. Thus the new price is $n + 9$ cents.)

8 A woman's weekly pay is $\$x$. She spends $\frac{1}{2}$ of her pay on food and $\frac{1}{3}$ on rent.

(a) Express the amount she spends on food in terms of x.

(b) Express the amount she spends on rent in terms of x.

(c) Find her weekly pay if she spends a total of \$80 on food and rent.

9 The distance between two villages is d km.

(a) Express $\frac{1}{4}$ of that distance in terms of d.

(b) Express $\frac{3}{4}$ of the distance in terms of d.

(c) If the difference between these distances is 1,5 km, find the value of d.

10 Kudzai is y years old.

(a) How old was he 3 years ago?

(b) How old will he be in 4 years time?

(c) Find his age, if $\frac{1}{2}$ of what he was 3 years ago is equal to $\frac{1}{3}$ of what he will be in 4 years time.

Fractions with unknowns in the denominator

Example 14

Solve $2\frac{3}{4} + \frac{33}{2x} = 0$.

$$2\tfrac{3}{4} + \frac{33}{2x} = 0$$

Express $2\frac{3}{4}$ as an improper fraction.

$$\frac{11}{4} + \frac{33}{2x} = 0$$

The denominators are 4 and $2x$. Their LCM is $4x$. Multiply each term in the equation by $4x$.

$$4x\left(\frac{11}{4}\right) + 4x\left(\frac{33}{2x}\right) = 4x \times 0$$

$$\Leftrightarrow \quad 11x + 66 = 0$$

$$\Leftrightarrow \quad 11x = -66$$

$$\Leftrightarrow \quad x = -6$$

Check: When $x = -6$,

$$\text{LHS} = 2\tfrac{3}{4} + \frac{33}{-12} = 2\tfrac{3}{4} - \frac{11}{4} = 0 = \text{RHS}$$

Example 15

Solve $\frac{1}{3a} + \frac{1}{2} = \frac{1}{2a}$

$$\frac{1}{3a} + \frac{1}{2} = \frac{1}{2a}$$

The denominators are $3a$, 2 and $2a$. Their LCM is $6a$. Multiply each term in the equation by $6a$.

$$6a \times \left(\frac{1}{3a}\right) + 6a \times \frac{1}{2} = 6a \times \left(\frac{1}{2a}\right)$$

$$\Leftrightarrow \quad 2 + 3a = 3$$

$$\Leftrightarrow \quad 3a = 1$$

$$\Leftrightarrow \quad a = \tfrac{1}{3}$$

Check: When $a = \frac{1}{3}$,

$$\text{LHS} = \frac{1}{3 \times \frac{1}{3}} + \frac{1}{2} = 1 + \tfrac{1}{2} = 1\tfrac{1}{2}$$

$$\text{RHS} = \frac{1}{2 \times \frac{1}{3}} = \frac{3}{2} = 1\tfrac{1}{2} = \text{LHS}$$

Examples 14 and 15 show that when unknowns, such as x or a, appear in the denominator, they are treated in the same way as numbers. Clear fractions by multiplying each term of the equation by the LCM of the denominators of the fractions. The equation can then be solved in the usual way.

Exercise 5i

Solve the following equations.

1 $\frac{1}{x} = \frac{1}{5}$

2 $\frac{1}{9} = \frac{1}{r}$

3 $\frac{1}{m} - \frac{1}{4} = 0$

4 $\frac{1}{y} = \frac{2}{7}$

5 $2\frac{1}{2} = \frac{1}{s}$

6 $2\frac{3}{4} + \frac{1}{n} = 0$

7 $\frac{2}{t} = \frac{6}{11}$

8 $\frac{9}{10} = \frac{3}{z}$

9 $\frac{4}{9} - \frac{3}{p} = 0$

10 $\frac{10}{3} = \frac{5}{a}$

11 $\frac{13}{x} = 5\frac{1}{5}$

12 $\frac{8}{q} + 3\frac{3}{7} = 0$

13 $\frac{1}{5b} = \frac{1}{30}$

14 $\frac{1}{40} = \frac{1}{8y}$

15 $\frac{1}{3r} - \frac{1}{24} = 0$

16 $\frac{7}{3c} = \frac{21}{2}$

17 $\frac{3}{10} = \frac{9}{2d}$

18 $\frac{16}{9} + \frac{4}{3s} = 0$

19 $3\frac{3}{8} = \frac{12}{25z}$

20 $\frac{33}{2r} = 3\frac{1}{7}$

21 $3\frac{3}{4} - \frac{5}{2t} = 0$

22 $\frac{1}{x} = \frac{1}{4} + \frac{1}{12}$

23 $\frac{1}{y} + \frac{1}{5} = \frac{1}{3}$

24 $\frac{1}{9} = \frac{1}{d} - \frac{1}{18}$

25 $\frac{1}{f} + \frac{1}{2} = \frac{5}{6}$

26 $2 = \frac{7}{2x} - \frac{1}{3}$

27 $\frac{3}{h} = \frac{1}{5} + \frac{8}{35}$

28 $\frac{5}{2x} - \frac{1}{x} = \frac{1}{6}$

29 $\frac{2}{x} + \frac{3}{2x} = \frac{7}{8}$

30 $\frac{9}{4x} - \frac{5}{x} + \frac{11}{3} = 0$

Word problems involving fractions (2)

Example 16

The students in a class have a total mass of 1 717 kg. *If the average mass per student is* $50\frac{1}{2}$ kg, *find the number of students in the class.*

The number of students is the unknown. Let there be n students in the class. Then,

average mass per student $= \frac{1\,717}{n}$ kg

(from first sentence in question).

Thus, $$50\tfrac{1}{2} = \frac{1\,717}{n}$$

(from second sentence in question).

$$\frac{101}{2} = \frac{1\,717}{n}$$

Multiply throughout by $2n$.

$$101n = 2 \times 1\,717$$

$$\Leftrightarrow \quad n = \frac{2 \times 1\,717}{101} = 2 \times 17$$

$$\Leftrightarrow \quad n = 34$$

There are 34 students in the class.

The problem in Example 16 could have been solved by simple arithmetic. The algebraic method was not really necessary. However, Example 17 gives a problem which is best solved by using algebra.

Example 17

A cow costs 7 times as much as a goat. For $840 I can buy 18 more goats than cows. How much does a goat cost?

The cost of a goat is the unknown. Let a goat cost $\$h$. Thus a cow costs $\$7h$ (from first sentence in question). For $840 I can buy

$\frac{840}{h}$ goats or $\frac{840}{7h}$ cows.

Thus, $\frac{840}{h} - \frac{840}{7h} = 18$ (from second sentence)

Multiply throughout by $7h$.

$$7h \times \frac{840}{h} - 7h \times \frac{840}{7h} = 7h \times 18$$

$$\Leftrightarrow 7 \times 840 - 1 \times 840 = 7 \times 18 \times h$$

$$\Leftrightarrow \quad 6 \times 840 = 7 \times 18 \times h$$

$$\Leftrightarrow \quad \frac{6 \times 840}{7 \times 18} = h$$

$$h = \frac{120}{3}$$

$$= 40$$

Thus a goat costs $40 (and a cow costs $280).

When solving problems of this kind:

1 find out what the unknown is;
2 choose a letter to stand for the unknown quantity;
3 change the statements in the question into algebraic expressions and make an equation;
4 solve the equation, leaving any numerical simplification until the last step of the working.

In Exercise 5j overleaf, some questions give a letter for the unknown; in other questions you must choose a letter for yourself.

Exercise 5j

1 A fisherman catches n fish. Their total mass is 14 kg.

(a) Write down the average mass of a fish in terms of n.

(b) If the average mass of a fish was $\frac{7}{8}$ kg, find the number of fish caught.

2 A trader buys x watches (all alike) for \$67,20.

(a) Write down the cost of one watch in terms of x.

(b) If the watches cost \$9,60 each, find the number of watches bought.

3 A girl walks 3 km at a speed of v km/h.

(a) Write down the time taken, in hours, in terms of v.

(b) If the journey takes 35 min, find the value of v.

4 A trader sells a number of books and takes in \$369 altogether. If the average selling price of a book is \$4,50, find the number of books sold.

5 A bag of mangoes has a total mass of 56 kg. If the average mass of a mango is $1\frac{3}{4}$ kg, find the number of mangoes in the bag. (Ignore the mass of the bag.)

6 A car travels 120 km at a certain average speed. If the journey takes $2\frac{1}{2}$ h, find the average speed.

7 A pencil costs x cents and a notebook costs $4x$ cents. I spend \$1,80 on pencils and \$1,80 on notebooks.

(a) Write down the number of pencils I get in terms of x.

(b) Write down the number of notebooks I get in terms of x.

(c) If I get 15 more pencils than notebooks, how much does a pencil cost?

8 A table costs 5 times as much as a chair. For \$600 a trader can buy 20 more chairs than tables. Find the cost of a chair.

9 A student walks 8 km at v km/h. She then cycles 15 km at $2v$ km/h. In terms of v, write down the time taken in hours (a) when walking, (b) when cycling. (c) If the total time for the journey is 2 h 35 min, find the student's walking speed.

10 A car travels for 15 km in a city at a certain speed. Outside the city it travels 72 km at twice its former speed. If the total travelling time is 1 h 8 min, find the average speed in the city.

11 Mary has n oranges of total mass 14,5 kg. Anu has $2n$ oranges with a total mass of 21 kg.

(a) What is the average mass of one of Mary's oranges in terms of n?

(b) What is the average mass of one of Anu's oranges in terms of n?

(c) If the average mass of Anu's oranges is 0,1 kg less than the average mass of Mary's oranges, find the number of oranges that Mary has.

12 A man caught 15 kg of fish on Monday and 23 kg of fish on Tuesday. On Tuesday there were twice as many fish as there were on Monday, but their average mass was $\frac{1}{2}$ kg less. How many fish did the man catch on Monday?

Example 18

Solve the equation $\frac{5}{x-3} = 2$.

$$\frac{5}{x-3} = 2$$

There is *one* denominator, $x - 3$. Notice that the *whole* of $x - 3$ is the denominator; it cannot be split into parts. Multiply both sides of the equation by $(x - 3)$.

$$(x-3) \times \frac{5}{x-3} = 2(x-3)$$

On the LHS, the $(x - 3)$'s divide, leaving 5; clear brackets on the RHS.

$$5 = 2x - 6$$

Add 6 to both sides

$$11 = 2x - 6$$

Divide both sides by 2

$$5\tfrac{1}{2} = x$$

Check: When $x = 5\frac{1}{2}$,

$$\text{LHS} = \frac{5}{5\frac{1}{2} - 3} = \frac{5}{2\frac{1}{2}} = 2 = \text{RHS}$$

Example 19

Solve $\frac{5}{7a-1} - \frac{4}{9} = 0$.

$$\frac{5}{7a-1} - \frac{4}{9} = 0$$

The denominators are $(7a - 1)$ and 9. Their LCM is $9(7a - 1)$. Multiply each term by $9(7a - 1)$.

$$9(7a-1) \times \frac{5}{7a-1} - 9(7a-1) \times \frac{4}{9} = 9(7a-1) \times 0$$

$$\Leftrightarrow \quad 9 \times 5 - 4(7a - 1) = 0$$

Clear brackets and collect terms.

$$45 - 28a + 4 = 0$$
$$\Leftrightarrow \quad 49 - 28a = 0$$
$$\Leftrightarrow \quad 49 = 28a$$
$$\Leftrightarrow \quad a = \frac{49}{28} = \frac{7}{4}$$
$$\Leftrightarrow \quad a = 1\tfrac{3}{4}$$

The solution is $a = 1\frac{3}{4}$. The check is left as an exercise.

In a fraction like $\frac{7}{2r+3}$, the division line acts like a bracket on the terms in the denominator: $\frac{7}{(2r+3)}$. Examples 18 and 19 show that the bracket must be kept in the working until it can be cleared properly.

Exercise 5k

Solve the following equations.

1 $\frac{12}{x-1} = 3$	**2** $\frac{4}{1+x} = 1$
3 $2 = \frac{7}{y+2}$	**4** $\frac{4}{t-2} = 3$
5 $\frac{1}{z+4} = 1$	**6** $5 = \frac{15}{1-r}$
7 $\frac{6}{x+7} + 3 = 0$	**8** $2 - \frac{1}{k-2} = 0$
9 $2 = \frac{2}{4-3a}$	**10** $\frac{13}{2x+1} = 5$
11 $\frac{1}{x+3} = \frac{1}{5}$	**12** $\frac{1}{7} = \frac{1}{a-3}$
13 $\frac{1}{2} - \frac{1}{y-5} = 0$	**14** $\frac{1}{b+5} + \frac{1}{4} = 0$
15 $\frac{3}{2-e} = \frac{3}{8}$	**16** $\frac{4}{9} = \frac{2}{c-8}$
17 $\frac{5}{2n-5} = \frac{3}{2}$	**18** $2\frac{1}{2} = \frac{10}{3d+7}$
19 $\frac{2}{5} + \frac{3}{a-8} = 0$	**20** $\frac{5}{3x+2} - \frac{1}{4} = 0$

Chapter 6

Scale drawing (1)

Scale

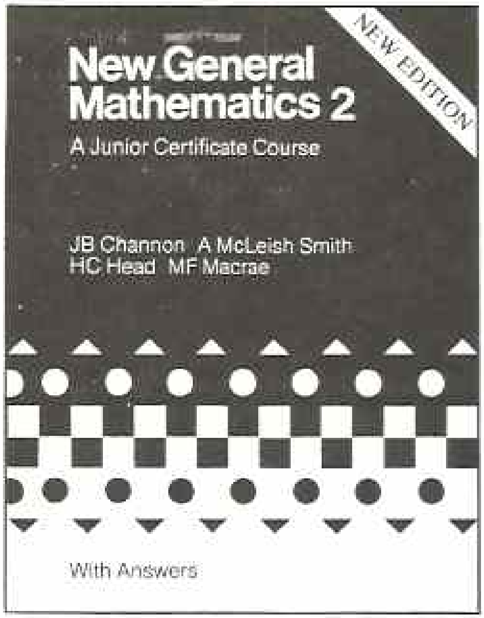


Fig. 6.1

Fig. 6.1 is a **scale drawing** of the front cover of this book. The only difference between Fig. 6.1 and the front cover is size. The scale drawing is smaller.

Check the following measurements:

width of Fig. 6.1 ≃ 6,3 cm

width of front cover ≃ 18,9 cm

where ≃ means 'is approximately equal to'.

We can write the ratio of the two widths as follows: 6,3 cm: 18,9 cm = 1:3.

We say that the **scale** of Fig. 6.1 is **1 to 3** or **1 cm to 3 cm** or **1 cm represents 3 cm.**

The scale of a drawing is found by comparing a length on the drawing with the corresponding length on the object which has been drawn.

$$\text{Scale} = \frac{\text{any length on the scale drawing}}{\text{corresponding length on the actual object}}$$

For example, the scale of Fig. 6.1 can be found by comparing the heights instead of the widths.

$$\text{Scale} = \frac{\text{height of Fig. 6.1}}{\text{height of front cover}}$$

$$\simeq \frac{8{,}2\text{ cm}}{24{,}6\text{ cm}} = \frac{1}{3}$$

Example 1

Fig. 6.2 (a) is a scale drawing of Fig. 6.2(b). Use measurement to find the scale of the drawing.

Width of Fig. 6.2(a) = 20 mm

Width of Fig. 6.2(b) = 40 mm

$$\text{Scale} = \frac{20\text{ mm}}{40\text{ mm}} = \frac{1}{2}$$

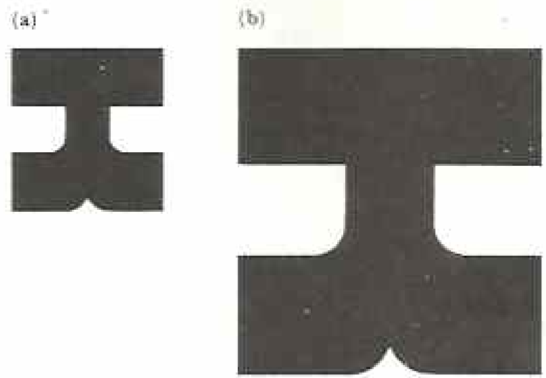


Fig. 6.2

The scale is 1 to 2, or 1 cm to 2 cm.

The result in Example 1 can be checked by comparing a different pair of corresponding lengths, e.g. the heights of the figures. Notice that the measurements must be in the same units.

Example 2

A plan of a school is drawn to a scale of 1 cm *represents* 5 m. (*a*) *If the football field is* 80 m *by* 53 m, *find its length and breadth on the drawing. (b) If the scale drawing of the hall is a 7 cm by 3,2 cm rectangle, find its actual length and breadth.*

In scale drawings, a **plan** is a drawing of the view from above the object.

(a) 5 m is represented by 1 cm
1 m is represented by $\frac{1}{5}$ cm
80 m is represented by $80 \times \frac{1}{5}$ cm = 16 cm
53 m is represented by $53 \times \frac{1}{5}$ cm = 10,6 cm
On the drawing, the football field will be 16 cm long and 10,6 cm wide.

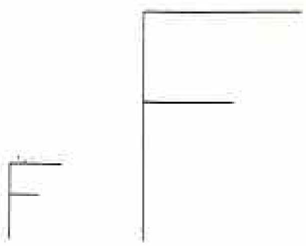
(b) 1 cm represents 5 m
7 cm represents 7×5 m = 35 m
3,2 cm represents $3{,}2 \times 5$ m = 16 m
The hall is 35 m long and 16 m wide.

Notice, in Example 2, that the scale is given in mixed units. 1 cm represents 5 m is the same as 1 cm represents 500 cm or 1 to 500.

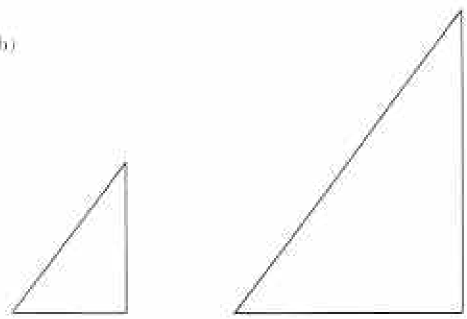
Exercise 6a

1 In each part of Fig. 6.3, the smaller diagram is a scale drawing of the larger diagram. Measure two corresponding lengths and give the scales of the drawings.

(a)



(b)



(c)

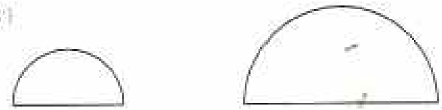


Fig. 6.3

2 Copy and complete Table 6.1. The first part has been done.

Table 6.1

true length	scale	length on drawing
(a) 90 m	1 cm to 10 m	9 cm
(b) 20 m	1 cm to 5 m	
(c) 8 m	1 cm to 2 m	
(d) 73 m	1 cm to 10 m	
(e) 65 m	1 cm to 5 m	
(f) 3 km	1 cm to 200 m	
(g) 450 m	1 cm to 100 m	
(h) 375 km	1 cm to 50 km	
(i) 1,53 km	10 cm to 1 km	
(j) 2,86 km	5 cm to 1 km	

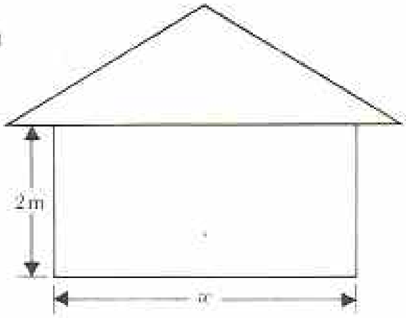
3 Copy and complete Table 6.2. The first part has been done.

Table 6.2

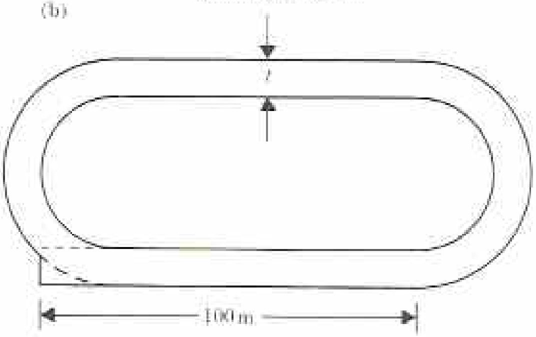
length on drawing	scale	true length
(a) 6 cm	1 cm to 10 m	60 m
(b) 11 cm	1 cm to 5 m	
(c) 5 cm	1 cm to 2 m	
(d) 7,5 cm	1 cm to 10 m	
(e) 8,2 cm	1 cm to 100 m	
(f) 9,3 cm	1 cm to 2 m	
(g) 8,6 cm	1 cm to 50 km	
(h) 14,8 cm	2 cm to 1 km	
(i) 11,3 cm	5 cm to 1 m	

4 Each part of Fig. 6.4 on page 36 is a scale drawing.
(a) For each part, use the given dimensions and a ruler to complete the statement, 'Scale: 1 cm represents ______'.
(b) Hence find the dimensions w, t, h, d.

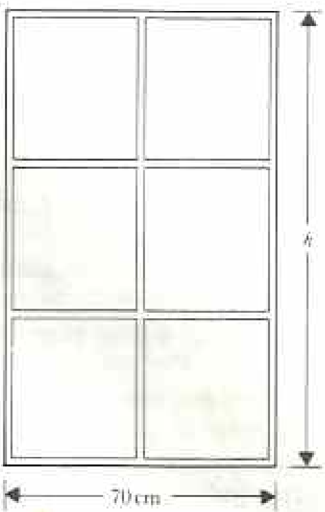
(a) *view of building*



(b) *plan of running track*



(c) *window frame*



(d) *Plan of roundabout*

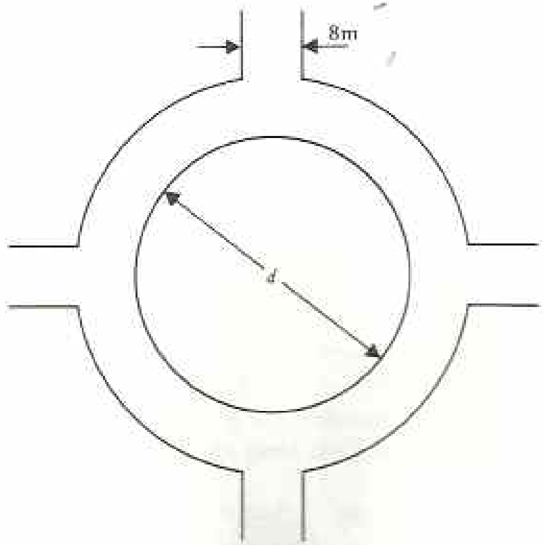


Fig. 6.4

Scale drawing

Example 3

A rectangular field measures 45 m *by* 30 m. *Draw a plan of the field. Use measurement to find the distance between opposite corners of the field.*

First, make a rough sketch of the plan. Enter the details on the rough sketch as in Fig. 6.5.

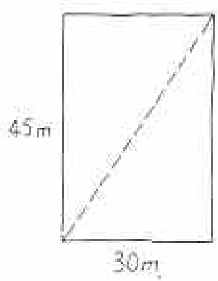


Fig. 6.5

Second, choose a suitable scale. As with graphs, the scale must suit the size of the page.

A scale of 1 cm to 1 m will give a 45 cm × 30 cm rectangle. This will be too big for the page. A scale of 1 cm to 5 m will give a 9 cm by 6 cm rectangle. This will be suitable.

Third, make an accurate drawing of the plan. This is shown in Fig. 6.6.

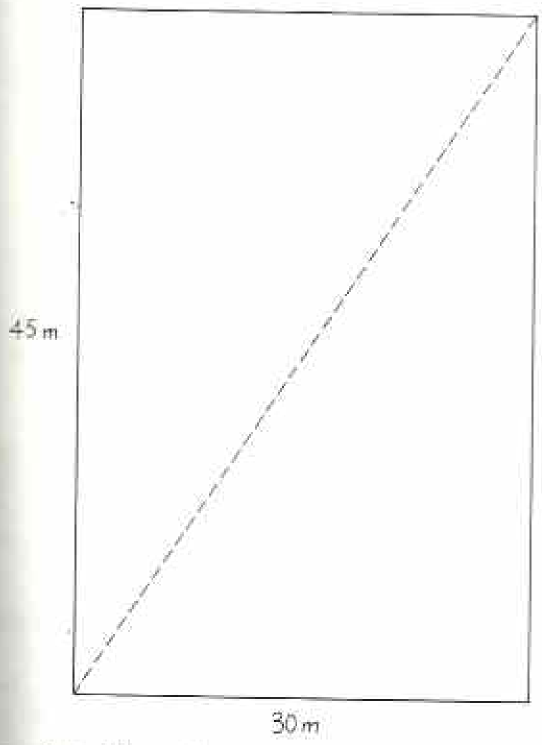


Fig. 6.6 Plan of field Scale: 1 cm to 5 m

The distance between opposite corners of the field is represented by the dotted line.

Length of dotted line ≃ 10,8 cm

Actual distance ≃ 10,8 × 5 m

= 54 m (to nearest metre)

Notice the following points.

1 Scale drawings should be made on plain paper.
2 Mathematical instruments are needed. For example, a pencil, a ruler and a set-square were used to draw Fig. 6.6.
3 The drawing has a title and the scale is given.
4 The dimensions of the actual object are written on the drawing.

Example 4

Fig. 6.7 shows a sketch of two paths **AX** *and* BX. *Points* A *and* B *are* 178 m *and* 124 m *from* X *respectively. The distance between* A *and* B *is* 108 m. *Make a scale drawing of the paths and find the angle between the paths at* X.

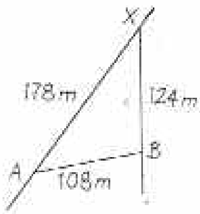


Fig. 6.7

It is necessary to construct a scale drawing of triangle AXB. Using a scale of 1 cm to 20 m, the sides of the triangle in the scale drawing will be as follows.

AX = $\frac{178}{20}$ cm = 8,9 cm

BX = $\frac{124}{20}$ cm = 6,2 cm

AB = $\frac{108}{20}$ cm = 5,4 cm

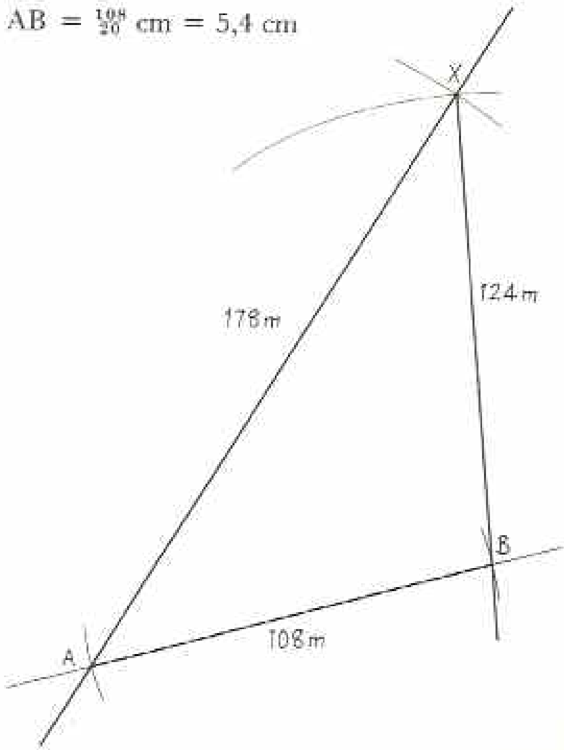


Fig. 6.8 Plan of paths AX *and* BX
Scale: 1 cm to 20 m

Using the method of constructing a triangle given its three sides, Fig. 6.8 on the previous page is the required scale drawing.

Using a protractor, $A\hat{X}B = 37°$ (to the nearest degree). The angle between the paths is 37°.

Exercise 6b

Make sketches where none are given. Choose suitable scales where none are given. All questions should be answered by taking measurements from an accurate scale drawing.

1 Find the distance between the opposite corners of a rectangular room which is 12 m by 9 m. Use a scale of 1 cm to 1 m.

2 A rectangular field measures 55 m by 40 m. Draw a plan of the field. Use measurement to find the distance between opposite corners of the field.

3 Measure the length and breadth of the top of your desk. Draw a plan of the top of your desk. Find the length of a diagonal from your drawing. Check your work by measuring the actual diagonal on your desk.

4 Measure the length and breadth of your classroom. Draw a plan of your classroom. Find a way of showing that your drawing is accurate.

5 A square field is 300 m × 300 m. Draw a plan of the field. Find the distance of the centre of the field from one of its corners.

6 Fig. 6.9 shows the plan of a room ABCD. PQ and XY are windows. HK is a door.

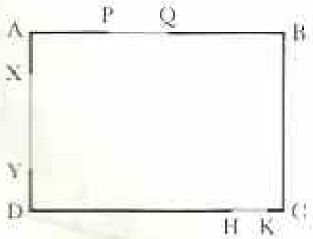


Fig. 6.9

AB = 10 m, XY = 4 m, KC = 0,75 m,
BC = 7 m, AP = 3 m, HK = 1,5 m.
AX = 1,5 m, PQ = 2,5 m,

Draw a plan on a scale of 1 cm to 1 m. Find the distances AC, XK, PH, and QY.

7 A football field measures 104 m by 76 m. Use a scale of 1 cm to 10 m to draw a plan of the field. Find the distance from the centre spot to a corner flag.

8 Fig. 6.10 is a sketch of a cross-section of a round hut. Use the dimensions on the figure to make an accurate scale drawing.

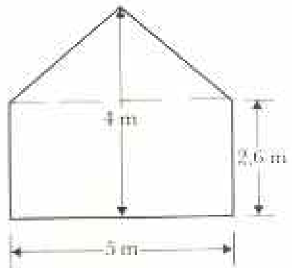


Fig. 6.10

Find the angle at the vertex of the roof.

9 Fig. 6.11 is a sketch of the end view of a house.

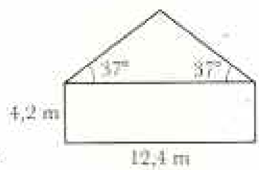


Fig. 6.11

Make a scale drawing and find the greatest height of the house.

10 A triangular plot ABC is such that AB = 120 m, BC = 80 m and CA = 60 m. P is the middle point of AB. Find the length of PC. Use a scale of 1 cm to 10 m.

11 In Fig. 6.12 A and D are on opposite sides of a river.

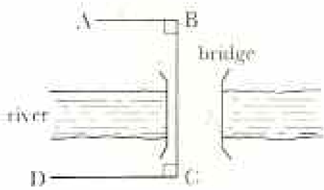


Fig. 6.12

AB = 52 m, BC = 119 m and CD = 86 m. Make a scale drawing and hence find the distance AD.

12 Two straight paths meet at an angle of 55° at a point X. Two students start together at X. One student runs down one path at

4 metres per second. The other student runs down the other path at 5 metres per second. If they start at the same time, how far apart are they after 11 seconds?

Reading scale drawings

Many professions use scale drawings. The most common scale drawings are **maps** and **technical drawings.** Surveyors and cartographers make maps. Maps are used by geographers, navigators, planners, the police and soldiers.

Fig. 6.13

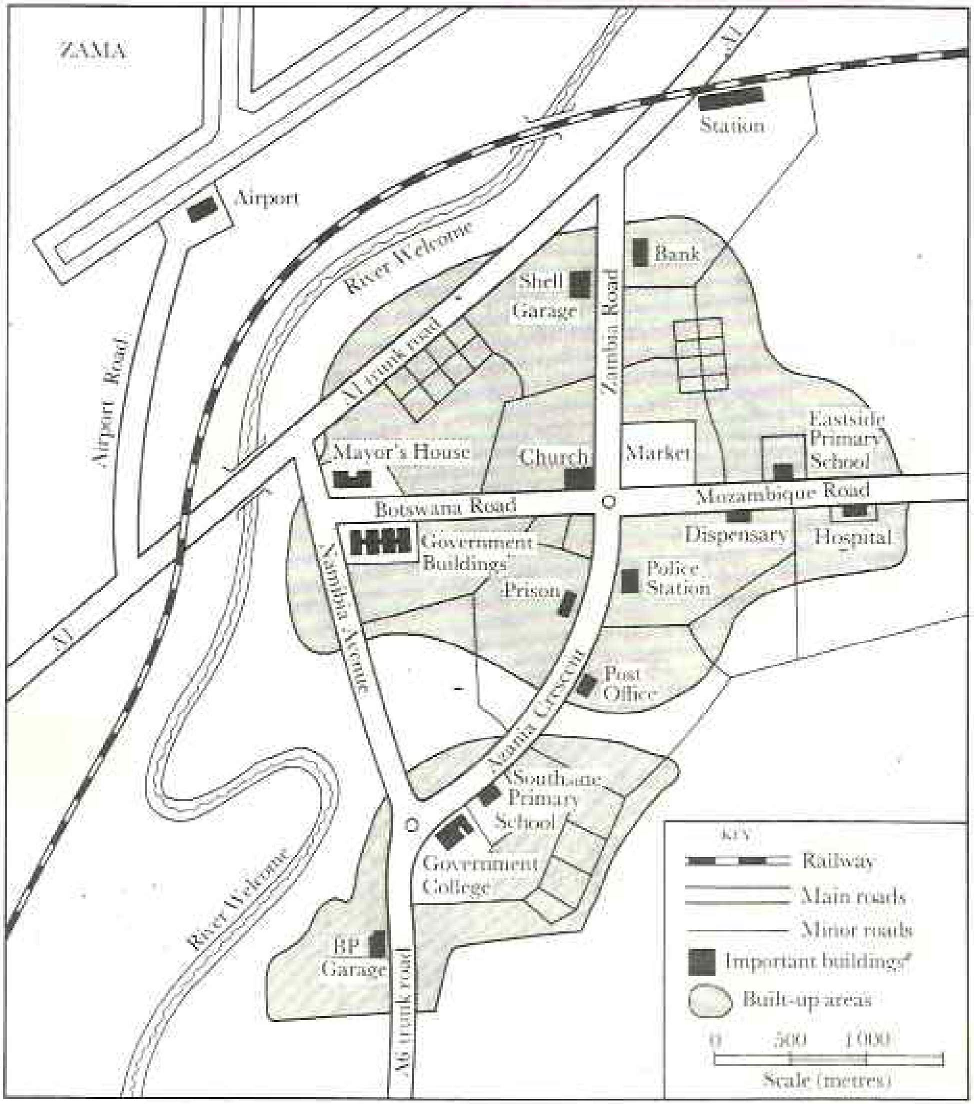


Fig. 6.14

Engineers and draughtsmen make technical drawings. Technical drawings are used by builders, mechanics, electricians and skilled tradesmen. A scale drawing is an accurate way of storing and giving information. It is important to be able to read scale drawings.

Maps

A map is a scale drawing of a piece of land. Distances on the map represent the horizontal distances between points on the land. Fig. 6.13 (page 39) is a small-scale map of Zimbabwe. Fig. 6.14 (page 39) is a large-scale map of an imaginary town, Zama. Look at the way the scale is given in Fig. 6.14. Measure the scale (repeated in Fig. 6.15). You will find that 1 cm represents 500 m.

Fig. 6.15

Exercise 6c

1 Use the map in Fig. 6.13. Find the following distances to the nearest 10 km.
- (a) Harare to Bulawayo.
- (b) Mutare to Hwange.
- (c) Kadoma to Masvingo.
- (d) Gweru to Harare.
- (e) Bulawayo to Kwekwe.
- (f) Mutare to Kadoma.
- (g) Masvingo to Hwange.
- (h) Kwekwe to Gweru.

2 Use the map in Fig. 6.14. Which roads would you travel on if you took the best route between the following? (Main roads only.)
- (a) The Mayor's house and the hospital.
- (b) The station and the airport.
- (c) The Shell garage and the BP garage.
- (d) The bank and the Post Office.
- (e) The two primary schools.
- (f) Government College and the dispensary.
- (g) The church and the airport.
- (h) The Police Station and the airport.
- (i) The hospital and the airport.
- (j) Government Buildings and the Post Office.

3 Use the map in Fig. 6.14. Find the following distances as accurately as possible.
- (a) From the Mayor's house to the church.
- (b) From the prison to the bank.
- (c) From Government Buildings to Eastside Primary School.
- (d) From the hospital to the dispensary.
- (e) From the Shell garage to the market.
- (f) The length of Botswana Road.
- (g) The length of Namibia Avenue.
- (h) The width of the river.
- (i) The width of the airport's runway.
- (j) The length of Zambia Road.

4 Use the map in Fig. 6.14. Find the following distances, (i) in a straight line, (ii) by going on the main roads, taking the best route.
- (a) From the Post Office to the Mayor's house.
- (b) From the station to the airport.
- (c) From the dispensary to the Post Office.
- (d) From Southside Primary School to Government Buildings.
- (e) From Government Buildings to the station.

Technical drawings

Fig. 6.16 is the plan of the wiring for an electric plug. It is drawn full size.

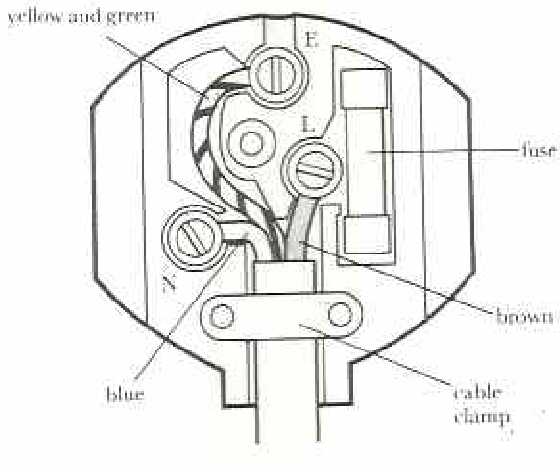


Fig. 6.16 Electric plug: wiring diagram, full size

Fig. 6.17 is the ground plan of a house. Some of the important dimensions are given on the drawing. All such dimensions are in mm.

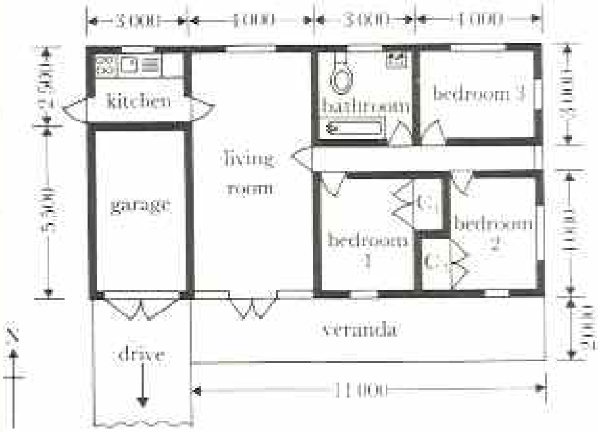


Fig. 6.17 House: ground plan

Exercise 6d

1 Use the wiring diagram in Fig. 6.16 to answer the following.

(a) Each wire is connected to a terminal. How many terminals are there?

(b) What is the colour of the wire which is connected to the terminal marked L?

(c) What is the colour of the wire which is connected to the terminal marked E?

(d) What is the colour of the wire which is connected to the terminal marked N?

(e) Which terminal is next to the fuse?

(f) How many screws are on the cable clamp?

(g) Find the greatest width of the plug.

2 Use the ground plan in Fig. 6.17 to answer the following.

(a) How many rooms has the house? Do not count the garage as a room.)

(b) Which is the biggest room in the house?

(c) Which is the smallest room in the house?

(d) If a person walked from the kitchen to the bathroom, how many doors would she pass through?

(e) Which room has most windows?

(f) How many windows does the house have altogether?

(g) Which room(s) is (are) north of bedroom 2?

(h) Which room(s) is (are) west of the bathroom?

(i) What do you think C1 and C2 stand for?

(j) What is the length and breadth of the living room in metres?

(k) What is the length and breadth of the garage in metres?

(l) What is the total area that the house covers? Give your answer in m^2 and include the garage and veranda.

Chapter 7

Straight-line graphs (1)

Continuous graphs

Consider the following example.

If 1 m of cloth costs \$5, then 2 m cost \$10, 3 m cost \$15, and so on. We can show lengths and costs in a **table of values** (Table 7.1). The values in the table form a set of ordered pairs: (1; 5), (2; 10), (3; 15), (4; 20), (5; 25).

Table 7.1

length (m)	1	2	3	4	5
cost (\$)	5	10	15	20	25

We can plot the ordered pairs on a cartesian plane. Fig. 7.1 shows the graph of the 5 points. Length (m) is on the horizontal axis and cost (\$) is on the vertical axis.

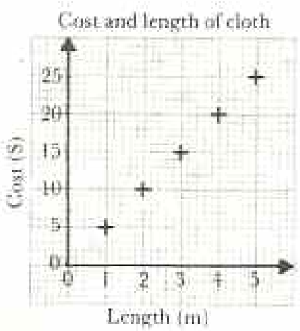


Fig. 7.1

It can be seen that the points in the graph lie in a straight line.

Other lengths of cloth, such 10 m, 1,4 m, 3,75 m would have corresponding costs. Thus it is possible to plot more points. Instead of this, we can draw a **continuous line** through the points which have been plotted. Starting at (0;0) (no cloth costs nothing!) we can continue the line as far as we want.

Fig. 7.2 shows the graph extended as far as the cost of 10 m of cloth.

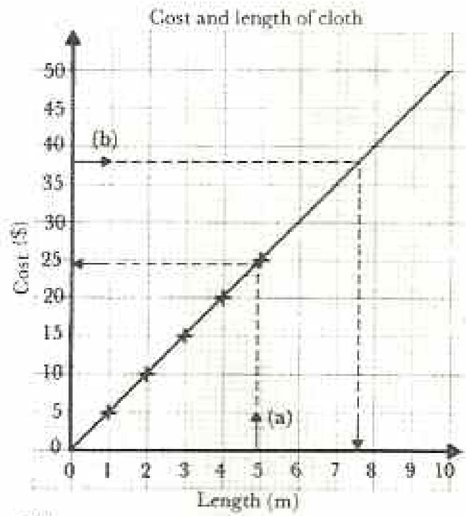


Fig. 7.2

The graph can be used to answer questions.

1 *How much would* 4,9 m *of cloth cost?*
Follow the dotted line (a) in Fig. 7.2. Start at 4,9 m on the length axis. Read the cost which corresponds to 4,9 m: \$24,50.

2 *How much cloth can be bought for* \$38?
Follow the dotted line (b) in Fig. 7.2. Start at \$38 on the cost axis. Read the length which corresponds to \$38: 7,6 m.

Example 1

A student walks at a speed of 120 m *per minute.*

(*a*) *Make a table of values showing how far the student has walked after* 0, 1, 2, 3, 4, 5, *minutes.*

(*b*) *Using a scale of* 1 cm *to* 1 min *on the horizontal axis and* 1 cm *to* 100 m *on the vertical axis, draw a graph of this information.*

(*c*) *Use the graph to find* (*i*) *how far the student has walked after* 2,6 min, (*ii*) *how long it takes to walk* 500 m.

(a) Table 7.2 is the table of values.

Table 7.2

time (min)	0	1	2	3	4	5
distance (m)	0	120	240	360	480	600

(b) See Fig. 7.3.

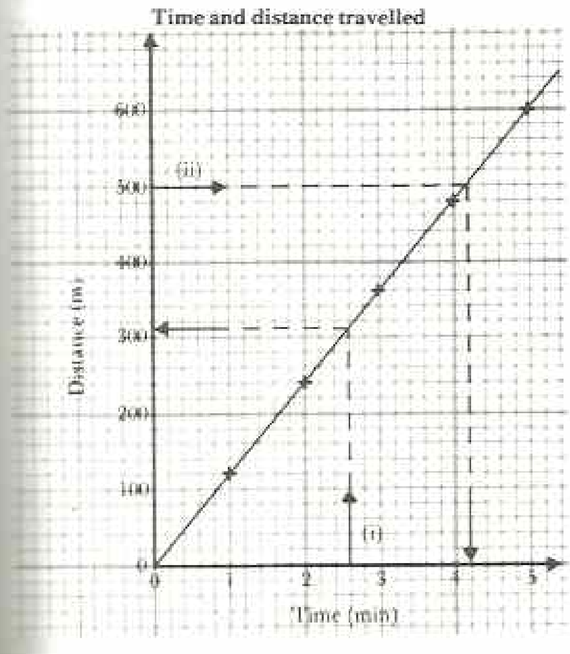


Fig. 7.3

(c) (*i*) See dotted line (i) on Fig. 7.3. 2,6 min corresponds to 310 m (approximately). The student has walked about 310 m after 2,6 min.
(*ii*) See dotted line (ii) on Fig. 7.3. 500 m corresponds to 4,2 min (approximately). The student takes about 4,2 min to walk 500 m.

Notice that graphs usually only give approximate results.
When drawing graphs, *always*
1 draw and name the two axes;
2 give a title to the graph.

Exercise 7a

1 Use Fig. 8.2 to find the following.
(a) The cost of 6 m, 3,5 m, 9 m, 8,2 m, 2,6 m, 7,1 m of cloth.
(b) How much cloth can be bought for $35, $40, $21, $9, $12,50, $46,50.

2 A car increases its speed steadily over 6 seconds as shown in Table 7.3.

Table 7.3

time (s)	0	1	2	3	4	5	6
speed (km/h)	0	15	30	45	60	75	90

(a) Use a scale of 2 cm represents 1 second on the horizontal axis and 2 cm represents 10 km/h on the vertical axis. Draw a graph of the information in Table 7.3.
(b) Use your graph to find, (i) the speed of the car after 2,5 s, (ii) the time taken to reach a speed of 80 km/h.

3 A girl walks along a road at a speed of 100 m per minute.
(a) Copy and complete Table 7.4.

Table 7.4

time (min)	0	1	2	3	4	5	6
distance (m)	0	100	200				

(b) Using a scale of 2 cm to 1 min on the horizontal axis and 2 cm to 100 m on the vertical axis, draw a graph of the information.
(c) Use your graph to find, (i) how far the girl has walked after 5,7 min, (ii) how long it takes her to walk 335 m.

4 Cloth cost $6 for 1 metre.
(a) Copy and complete Table 7.5.

Table 7.5

length (m)	1	2	3	4	5	6
cost ($)	6	12	18			

(b) Using a scale of 2 cm to 1 metre on the horizontal axis and 2 cm to \$5 on the vertical axis, draw a graph of the information.

(c) Use your graph to find, (i) the cost of 3,8 m of cloth, (ii) how much cloth can be bought for \$14.

5 A car travels 7 km on 1 litre of petrol.

(a) Copy and complete Table 7.6.

Table 7.6

petrol (litres)	0	10	20	30	40	50
distance (km)	0	70	140	210		

(b) Using a scale of 2 cm to 10 litres on the horizontal axis and 2 cm to 100 km on the vertical axis, draw a graph of the information.

(c) Use your graph to find, (i) the distance that the car will travel on 22 litres, (ii) how much petrol the car uses in travelling 230 km.

6 1 litre of petrol costs 40 cents.

(a) Copy and complete Table 7.7.

Table 7.7

petrol (litres)	0	10	20	30	40	50	60
cost (\$)	0	4	8	12			

(b) Using a scale of 2 cm to 10 litres on the horizontal axis and 2 cm to \$5 on the vertical axis, draw a graph of this information.

(c) Use the graph to find, (i) the cost of 22 litres of petrol, (ii) how much petrol can be bought for \$15.

7 Sugar costs 80 cents per kg.

(a) Copy and complete Table 7.8.

Table 7.8

sugar (kg)	1	2	3	4	5	6
cost (\$)	0,80	1,60	2,40			

(b) Using a scale of 2 cm to 1 kg on the horizontal axis and 2 cm to \$1 on the vertical axis, draw a graph of this information.

(c) Use your graph to find, (i) the cost of $2\frac{1}{2}$ kg of sugar, (ii) how much sugar can be bought for \$3.

8 The drill of an oil well drills downwards at a rate of 7,5 m/h.

(a) Copy and complete Table 7.9.

Table 7.9

time (h)	0	1	2	3	4	5
distance (m)	0	−7,5	−15	−22,5		

(b) Draw the origin near the top left corner of your graph paper. Using a scale of 2 cm to represent 1 hour on the horizontal axis and 1 cm represents 5 m on the vertical axis, draw a graph of the information.

(c) Use the graph to find, (i) how long it takes the drill to drill down through 25 m, (ii) the distance of the drill from ground level after 90 min.

9 A car travelling at 90 km/h covers 3 km in 2 min, 6 km in 4 m, 9 km in 6 min, and so on.

(a) Make a table of values showing how far the car travels in 2 min, 4 min, 6 min, 8 min, 10 min.

(b) Using a scale of 1 cm to 1 min on the horizontal axis and 1 cm to 1 km on the vertical axis, draw a graph of the information in your table.

(c) Use the graph to find, (i) how far the car travels in $3\frac{1}{2}$ min, (ii) how long it takes the car to travel 10 km.

10 A man cycles at a speed of 18 km/h.

(a) Make a table of values showing how far he travels in $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3 hours.

(b) Using a scale of 2 cm to represent 1 hour on the horizontal axis and 2 cm to represent 10 km on the vertical axis, draw a graph of the information.

(c) Use the graph to find, (i) how far the man cycles in 1,6 hours, (ii) how long it takes him to cycle 40 km.

Discontinuous graphs

Example 2
Glasses cost 90 cents each. (a) Make a table of values showing the cost of 1, 2, 3, 4, 5 glasses. (b) Draw a graph to show this information.

(a) See Table 7.10.

Table 7.10

number of glasses	1	2	3	4	5
cost ($)	0,90	1,80	2,70	3,60	4,50

(b) See Fig. 7.4

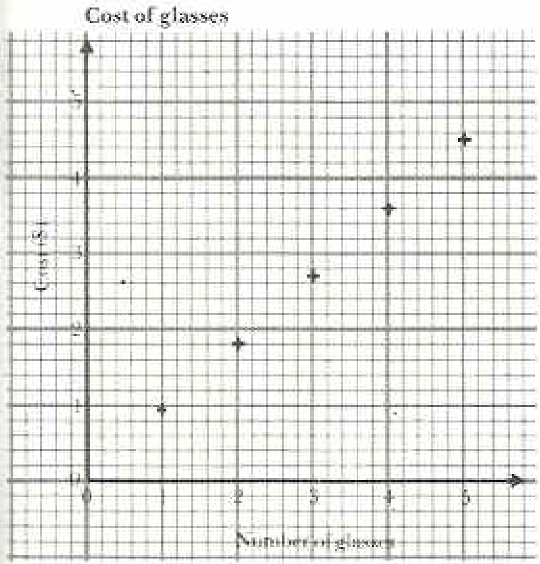


Fig. 7.4

The points in the graph in Fig. 7.4 lie in a straight line. However, we do *not* connect them. This is because it is impossible to buy a fraction of a glass such as $1\frac{1}{2}$ or 3,2. There is no part of the graph which corresponds to fractions. A graph like this is called a **discontinous graph**.

Exercise 7b
The graphs in this exercise are discontinuous. Plot the points only.

1 Cinema tickets cost $2 each.
(a) Copy and complete Table 7.11.

Table 7.11

number of tickets	0	1	2	3	4	5
cost ($)	0	2	4			

(b) Using a scale of 2 cm to 1 ticket on the horizontal axis and 1 cm to $1 on the vertical axis, draw a graph of this information.

2 A bottle of 60 pills costs $4,80.
(a) Copy and complete Table 7.12.

Table 7.12

number of pills	10	20	30	40	50	60
cost (cents)	80					480

(b) Using a scale of 2 cm to represent 10 pills on the horizontal axis and 2 cm to represent 100 cents on the vertical axis, draw a graph to show the information.
(c) Without doing any more calculation, plot the points representing the cost of 5, 15, 25, 35, 45, 55 pills.
(d) Hence *estimate* the cost of 17 pills.

3 The sum of the angles of an n-sided polygon is $(n - 2) \times 180°$.
(a) Use this formula to complete Table 7.13.

Table 7.13

number of sides of polygon	3	4	5	6	7
sum of interior angles (degrees)	180	360	540		

(b) Using a scale of 2 cm to 1 side on the horizontal and 2 cm to 100° on the vertical axis, draw a graph to show the information in the table.

4 Car tyres cost $96 each.
   (a) Make a table of values showing the costs of 1, 2, 3, 4, 5 tyres.
   (b) Using a scale of 2 cm to represent 1 tyre on the horizonatal axis and 2 cm to represent $100 on the vertical axis, draw a graph to show this information.

Choosing scales

Most of the graphs shown in this chapter are drawn to a small scale. This is to fit the sizes of the columns in the book. However, it is better to choose a big scale when drawing graphs. The scales given in Exercises 7a and 7b are all of a suitable size.

When choosing scales, first look at your table of values. For examples, look at Table 7.14.

Table 7.14

time (min)	0	10	20	30	40	50
temperature (°C)	− 8	− 1	6	13	20	27

This shows that the time scale on the horizontal axis must go from 0 min to 50 min, as shown in Fig. 7.5.

Fig. 7.5

The temperature scale on the vertical axis must go from − 8 °C to 27 °C. It is better to round these values to give a range from − 10 °C to 30 °C, as shown in Fig. 7.6.

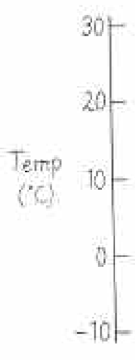


Fig. 7.6

Both axes meet at the origin. A rough sketch of the axes, such as in Fig. 7.7, can be made.

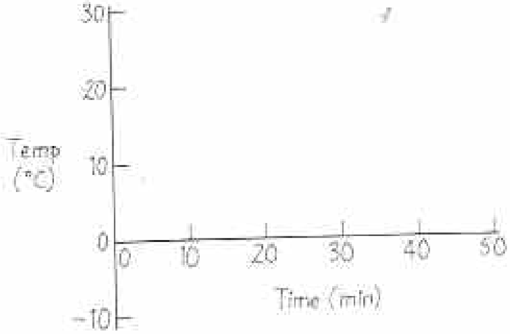


Fig. 7.7

Most graph paper is about 24 cm long by 18 cm wide. In this case, a scale of 2 cm to 10 units on both axes will be suitable.

Always look at the data and make a rough sketch as in Fig. 7.7. This will help you to place your graph on your graph paper.

On 2 mm graph paper, it is usual to let 2 cm represent 1, 2, 5, 10, 20, 50, 100, ... units (Fig. 7.8).

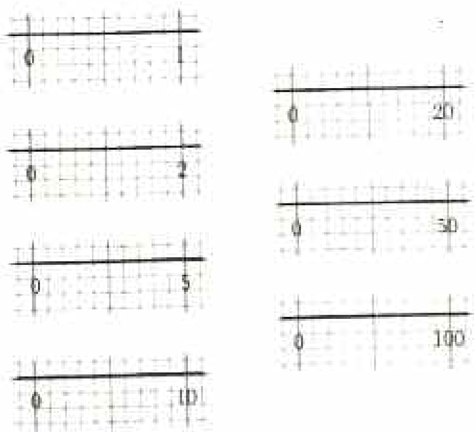


Fig. 7.8

The scale that you use will depend on the data. Do not use scales in multiples of 3 or 4.

Example 3

The labour charges for repairing a radio consist of: a standing charge of $5 *on all bills and an hourly rate of* $2 *per hour.*

(*a*) *Make a table showing the total labour charges for jobs which take* $\frac{1}{2}$ h, 1 h, 2 h, 3 h, 4 h.
(*b*) *Choose a suitable scale and draw a graph of the information.*
(*c*) *Find the total labour charges for a job which takes* (*i*) $2\frac{1}{2}$ hours, (*ii*) 24 min.

(a) The total labour charges are shown in Table 7.15.

Table 7.15

time (hours)	$\frac{1}{2}$	1	2	3
standing charge ($) hourly rate ($)	5 1	5 2	5 4	5 6
labour charge ($)	6	7	9	11

(b) Choosing scales:
Time goes from $\frac{1}{2}$ h to 3 h. Use a range 0 to 3 h (Fig. 7.9).

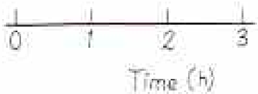


Fig. 7.9

Labour charges go from \$6 to \$11. Use a range 0 to \$15 (Fig. 7.10). Always include the origin if possible.

Fig. 7.10

Fig. 7.11 is a sketch of the axes.

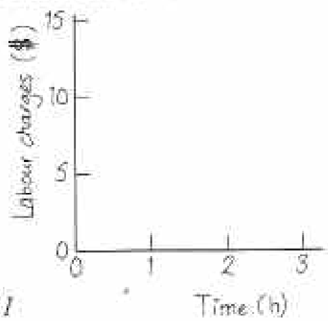


Fig. 7.11

Scales of 2 cm to 1 h on the horizontal axis and 2 cm to \$5 on the vertical axis will be suitable. The graph is given in Fig. 7.12.

(c) (*i*) Reading up from $2\frac{1}{2}$ h on the time axis corresponds to a charge of \$10 on the labour charges axis.

(*ii*) 24 min $= \frac{24}{60}$ hour $= 0{,}4$ h

0,4 h corresponds approximately to \$5,75. The charge for a 24-minute job is about \$5,75.

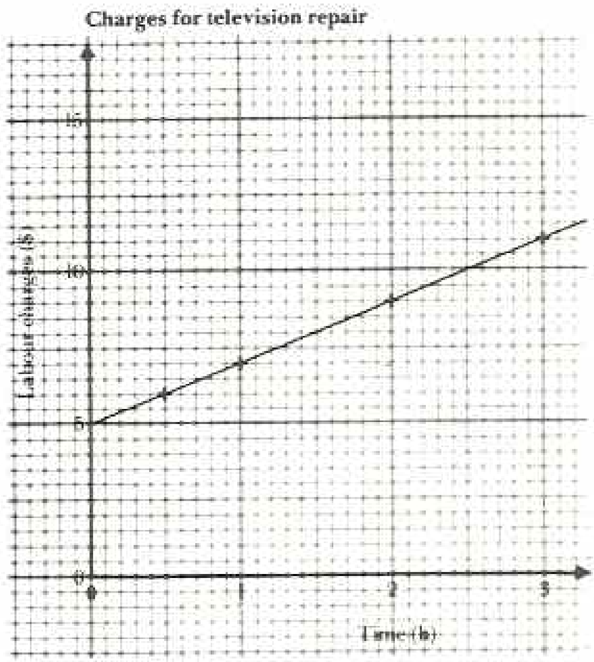


Fig. 7.12

Exercise 7c

1 A piece of meat is taken out of a freezer. Its temperature rises steadily as shown in Table 7.16 below.

Table 7.16

time (min)	0	10	20	30	40	50
temperature (°C)	−6	0	6	12	18	24

(a) Choose a suitable scale and draw a graph of this information.

(b) Use the graph to estimate, (i) the temperature of the meat after 13 min, (ii) the time taken for the meat to reach a temperature of 20 °C.

2 The unstretched length of a rubber band is 120 mm. When masses were hung on the rubber band, its total length changed as given in Table 7.17.

Table 7.17

mass (g)	0	200	400	600	800	1 000
length (mm)	120	170	220	270	320	370

(a) Choose a suitable scale and draw a graph of the data.

(b) Use your graph to find, (i) the length of the rubber band when a mass of 250 g is hung on it, (ii) the mass which stretches the rubber band to a length of 300 mm.

3 A baby was 3,4 kg when he was born. For his first 6 weeks, his mass increased by about 0,3 kg per week.

(a) Copy and complete Table 7.18.

Table 7.18

week number	0	1	2	3	4	5	6
mass (kg)	3,4	3,7	4,0	4,3			

(b) Choose a suitable scale and draw a graph of this information.

(c) Approximately how many days old was the baby when his mass was 5 kg?

4 An oil company sells petrol to garages at the rate of \$280 per kilolitre. There is also a delivery charge of \$60 on all orders.

(a) Copy and complete Table 7.19.

Table 7.19

amount of petrol (kℓ)	1	2	3	4	5
delivery charge (\$) basic charge (\$)	60 280	60 560	60 840	60	60
total cost (\$)	340	620	900		

(b) Choose a suitable scale for each of the axes and draw a graph of this information.

(c) Use the graph you have drawn to find, (i) the cost of 4 500 litres of petrol, (ii) how much petrol is delivered for \$1 000.

5 The basic cost of window glass is \$5 per m^2. There is also a handling and cutting charge of \$5 on all orders.

(a) Copy and complete Table 7.20.

Table 7.20

area of glass (m^2)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
handling/cutting (\$) basic charge (\$)	5 8	5 16	5 24	5	5
total cost (\$)	13	21	29		

(b) Choose a suitable scale for each of the axes and draw a graph of this information.

(c) A customer buys 9 panes of glass. Each pane measures 25 cm by 60 cm. (i) Calculate the total area, in m^2, of glass bought. (ii) Use the graph you have drawn to find out how much the glass costs.

Information from graphs

Conversion graphs

A **conversion graph** changes one set of units into another. Fig. 7.13 is a conversion graph for changing dollars into pounds and pounds into dollars.

The conversion graph was drawn using the exchange rate. The exchange rate for Fig. 7.13 is \$1 = £0,72. This is equivalent to \$100 = £72. The graph can be extended as far as we like. Fig. 7.13 can be used for values as high as \$150 and £108.

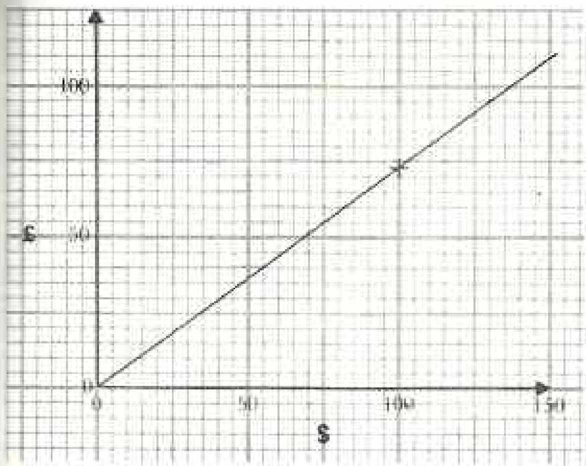


Fig. 7.13

Note: this graph is a kind of **ready reckoner.** Also see Chapter 23 and Fig. 8.1 on page 54.

Example 4

Use the conversion graph in Fig. 7.13 to find the following:

(*a*) *The British equivalent of* (*i*) $50, (*ii*) $140.
(*b*) *The Zimbabwean equivalent of* (*i*) £25. (*ii*) £90.

From the graph

(a) (*i*) $50 is approximately equivalent to £36.
   (*ii*) $140 is approximately equivalent to £100.
(b) (*i*) £25 is approximately equivalent to $35.
   (*ii*) £90 is approximately equivalent to $125.

A bigger scale would give a graph which could be read more accurately.

Distance-time graphs

Example 5

Sam and Nda leave home at the same time to walk to school 3 km away. Their journeys are shown in Fig. 7.14.

Travel graph for Sam and Nda

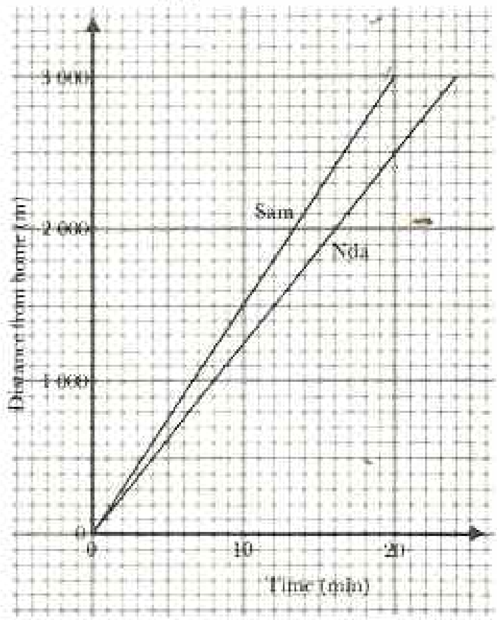


Fig. 7.14

(*a*) *How long did Sam take to walk to school?* (*b*) *How long did Nda take to walk to school?* (*c*) *Find Sam's and Nda's speeds in* m/min.

From the graph:

(a) Sam took 20 min.
(b) Nda took 24 min.
(c) In each case the journey was 3 000 m.
Sam took 20 min to walk 3 000 m.

$$\text{In 1 min Sam walked } \frac{3\,000}{20} \text{ m} = 150 \text{ m}$$

Sam's speed = 150 m/min.
Nda took 24 min to walk 3 000 m.

$$\text{In 1 min Nda walked } \frac{3\,000}{24} \text{ m} = 125 \text{ m.}$$

Nda's speed = 125 m/min.

A speed is a rate of change. It is the rate of change of distance with time. There is another way to find the speeds in Example 5. Fig. 7.15 overleaf is the graph of Nda's journey.

A right-angled triangle PQR has been drawn on part of Nda's line in Fig. 7.15. In going from P to Q the time changes from P to R and the distance changes from R to Q.

Travel graph for Nda's journey

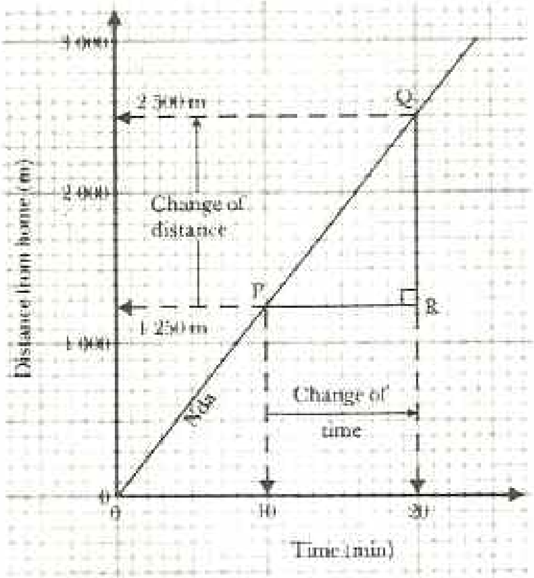


Fig. 7.15

$$\text{Nda's speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{\text{change of distance RQ}}{\text{change of time PR}}$$

$$= \frac{(2\,500 - 1\,250)\text{ m}}{(20 - 10)\text{ min}}$$

$$= 125\text{ m/min}$$

Any right-angled triangle can be drawn. The bigger the triangle, the more accurate the result.

Exercise 7d

1 Use Fig. 7.13 to find the British equivalent of the following.
(a) \$70 (b) \$105 (c) \$132,50 (d) \$17,50

2 Use Fig. 7.13 to find the Zimbabwean equivalent of the following.
(a) £40 (b) £65 (c) £82,50 (d) £37,50

3 Refer to Fig. 7.14 to answer the following.

(a) If Sam arrived at school just when the first lesson started, how many minutes late was Nda?

(b) At the moment Sam arrived at school, how far did Nda have to walk?

(c) When Sam was halfway to school how far was Nda (i) from home, (ii) from school?

(d) How far had Sam walked after 14 min?

(e) How long did it take Nda to walk 1,8 km?

A test is marked out of 30. Fig. 7.16 is a conversion graph for changing the marks into percentages.

Conversion graph: marks – percentages

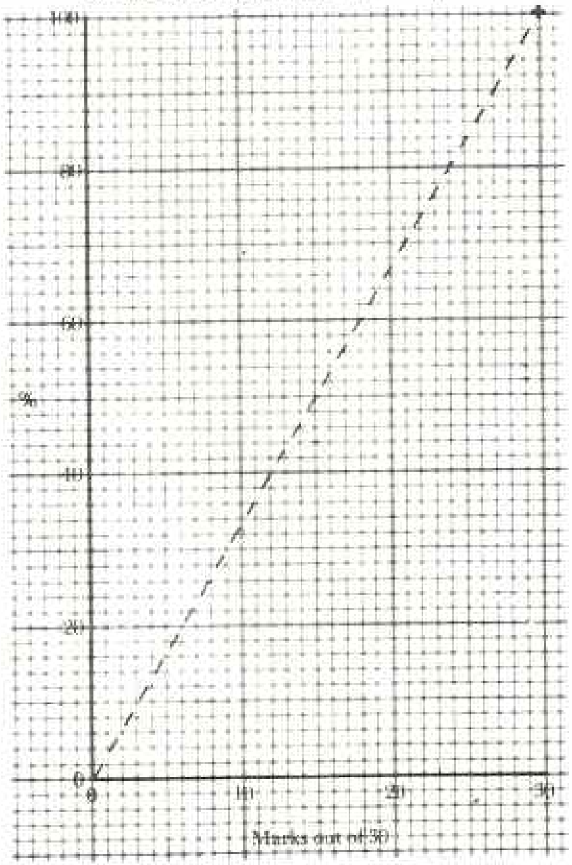


Fig. 7.16

4 Use Fig. 7.16 to change the following marks out of 30 to percentages. Give your answers to the nearest whole per cent.

(a) 15	(b) 12	(c) 27	(d) 18
(e) 10	(f) 20	(g) 5	(h) 25
(i) 4	(j) 13	(k) 16	(l) 29
(m) $7\frac{1}{2}$	(n) $22\frac{1}{2}$	(o) $11\frac{1}{2}$	(p) $24\frac{1}{2}$

5 Use Fig. 7.16 to change the following percentages to marks out of 30. Give your answers to the nearest whole mark.

(a) 80% (b) 20% (c) 30% (d) 70%
(e) 53% (f) 47% (g) 63% (h) 13%

Car A and car B leave Kwekwe at the same time. They travel 200 km to Harare. Their journeys are shown in Fig. 7.17. Use Fig. 7.17 to answer questions 6 and 7.

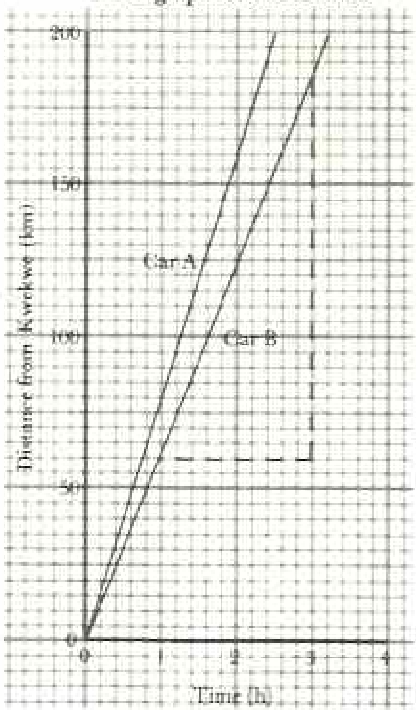


Fig. 7.17

6 (a) How long did (i) car A (ii) car B take to get to Harare?
(b) Find the speeds of car A and car B.
(c) Use the triangle (dotted) to check your result for car B.

7 (a) When car A reached Harare, how far behind was car B?
(b) After 1 hour, how far was each car from Harare?
(c) After 2 h how far apart were they?
(d) How long did it take each car to travel the first 50 km?

Fig. 7.18 is a graph of the journeys of two students, Mary and Gono. They leave their university at different times and travel 6 km to hospital. Mary walks (line ABCD). Gono cycles (line FD).

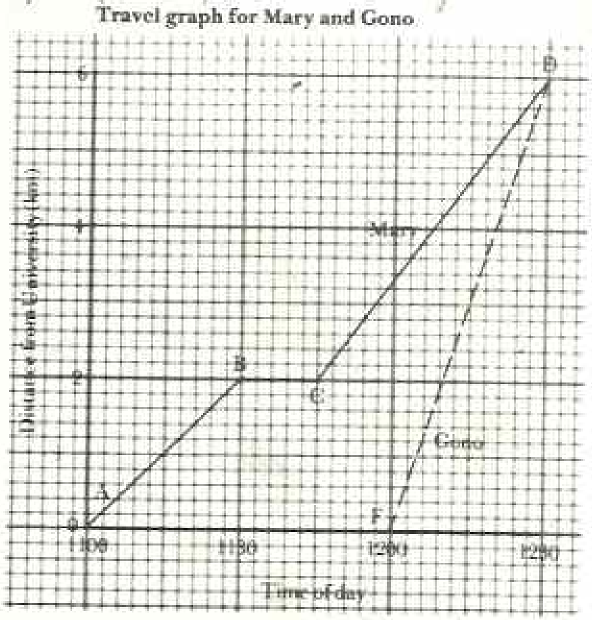


Fig. 7.18

8 (a) When did Mary leave the university?
(b) When did Mary arrive at the hospital?
(c) How long did Mary stop for?
(d) During part AB of Mary's journey, how far did she walk?
(e) How long did part AB take?
(f) During part CD of Mary's journey, how far did she walk?
(g) How long did part CD take?
(h) How far was Mary from the hospital at 1115?

9 Use Fig. 7.18 to answer the following.
(a) When did Gono leave the university?
(b) When did Gono arrive at the hospital?
(c) When Gono leaves the university, how far is Mary from the hospital?
(d) How far apart were they at 1215?

10 Use your answers to questions 8 and 9 to find the following speeds.
(a) Mary's speed between A and B.
(b) Mary's speed between C and D.
(c) Mary's average speed for the whole journey.
(d) Gono's average speed for his whole journey.

Chapter 8

Direct and inverse proportion

Direct proportion

If a student walks with a steady speed, then the more time taken, the greater the distance travelled. Table 8.1 gives corresponding times and distances for a student walking with a steady speed of 5 km/h.

Table 8.1

time (h)	1	2	3	4	5
distance (km)	5	10	15	20	25

In Table 8.1, the ratio of any two times is equal to the ratio of the corresponding distances. For example, in 5 hours the student travels 25 km and in 2 hours he or she travels 10 km:

$$\frac{5\text{ h}}{2\text{ h}} = \frac{5}{2} \text{ and } \frac{25\text{ km}}{10\text{ km}} = \frac{5}{2}$$

Thus the distance travelled is in **direct proportion** to the time taken, or, distance **varies directly** with time.

Inverse proportion

If a person is to travel a certain distance, then the greater her speed, the *less* time it will take. Table 8.2 gives the corresponding speeds and times for a journey of 100 km.

Table 8.2

speed (km/h)	10	20	25	50
time (h)	10	5	4	2

In Table, 8.2, when the speed doubles from 25 km/h to 50 km/h the time taken is *halved* from 4 hours to 2 hours:

$$\frac{50\text{ km/h}}{25\text{ km/h}} = \frac{2}{1} \text{ and } \frac{2\text{ h}}{4\text{ h}} = \frac{1}{2}$$

The ratio of any two speeds is equal to the *reciprocal* of the ratio of the corresponding times. This is an example of **inverse proportion.** The time taken is **inversely proportional** to the speed, or, time **varies inversely** with speed.

Example 1

2 boxes of matches cost 32 c *and* 5 *boxes of matches cost* 80 c. (*a*) *Does the cost of the matches vary directly or inversely with the number of boxes?* (*b*) *Find the cost of* 8 *boxes of matches.*

(a) Find the ratios of the corresponding numbers of boxes and costs.

$$\frac{5\text{ boxes}}{2\text{ boxes}} = \frac{5}{2} \text{ and } \frac{80\text{ c}}{32\text{ c}} = \frac{5}{2}$$

Thus the cost is in direct proportion to the number of boxes.

(b) Let the cost of 8 boxes be x cents.

$$\text{Then } \frac{x}{32} = \frac{8}{2}$$

$$x = \frac{32 \times 8}{2}$$

$$= 128$$

8 boxes cost 128 c = $1,28

Example 1 could easily be answered by finding the cost of 1 box. However, the method given above shows that it is not necessary to find the cost of 1 box.

Example 2

A woman travels 40 *km between two villages. (a) Make a table showing her speed if the journey takes 1 h, 2 h, 4 h. (b) Is her speed directly or inversely proportional to the time taken? (c) If she travels at* 18 km/h *find how long the journey takes.*

(a) Table 8.3 is the required table:

Table 8.3

time(h)	1	2	4
speed (km/h)	40	20	10

(b) From the table, if the time is doubled, the speed is halved. Thus speed is inversely proportional to time.

(c) *either:*

Let the time be t hours. Then comparing a time of t hours and a speed of 18 km/h with a time of 4 hours and a speed of 10 km/h (the ratio of corresponding times is equal to the reciprocal of the ratio of corresponding speeds),

$$\frac{t}{4} = \frac{1}{\frac{18}{10}} \Leftrightarrow \frac{t}{4} = \frac{10}{18}$$

$$t = \frac{4 \times 10}{18} = \frac{20}{9} = 2\tfrac{2}{9}$$

The journey takes $2\frac{2}{9}$ hours.

or:

Using time $= \dfrac{\text{distance}}{\text{speed}}$,

time taken $= \dfrac{40}{18} = 2\frac{2}{9}$ hours

Example 3

A length of wire can be cut into 5 *pieces each* 24 cm *long. How many pieces each* 15 cm *long can be cut from the wire?*

First decide whether the example is one of direct proportion or inverse proportion.

From the data of the question, the greater the number of pieces, the smaller their length. The number of pieces is inversely proportional to the length of each piece. Let there be n pieces.

Then,

$$\frac{n}{5} = \frac{1}{\frac{15}{24}} = \frac{24}{15}$$

$$n = \frac{5 \times 24}{15} = \frac{24}{3} = 8$$

There will be 8 pieces.

Exercise 8a

1 In each of the following, say whether the two quantities in italics are in direct proportion or inverse proportion to each other.
- (a) the *radius* and *diameter* of a circle;
- (b) the *number* and *cost* of pencils;
- (c) the *number* and *size of angle* of equal sectors in a circle;
- (d) the *time* and *distance* when travelling at a steady speed;
- (e) the *speed* and *time* when travelling a certain distance;
- (f) the *cost* per item and the *number* of items that can be bought for a fixed sum;
- (g) the *volume* and *cost* of petrol;
- (h) the two numbers x and y such that their product is always 100.

2 5 rubbers cost \$1,20 and 8 rubbers cost \$1,92.
- (a) Does the cost of the rubbers vary directly or inversely with the number bought?
- (b) Find the cost of 9 rubbers.

3 A bottle of water can fill 5 cups of capacity 200 ml or 4 cups of capacity 250 ml. (a) Does the number of cups vary directly or inversely with their capacity? (b) How many cups of capacity 100 ml could the bottle fill?

4 1 metre of cloth costs \$13,50. (a) Make a table showing the cost of 2 m, 4 m, 8 m of cloth. (b) Is the cost of cloth directly or inversely proportional to the length?

5 A woman has \$30 to spend. She buys items which cost the same amount each. (a) Make a table showing the number of items she can buy if they cost \$2, \$3, \$6 each. (b) Is the cost per item directly or inversely proportional to the number of items?

6 A cake has a mass of 2 kg. It is cut into pieces of equal mass. (a) Make a table showing the number of pieces if they are each of mass 200 g, 125 g, 50 g. (b) Does the number of pieces vary directly or inversely with the mass of each piece?

7 A roll of cloth is cut into pieces of equal length. Table 8.4 shows the length of each piece and the corresponding number of pieces that can be cut from the roll.

Table 8.4

length (m)	5	8	20
number	8	5	2

(a) How many pieces of length 4 m could be cut from the roll? (b) If 80 pieces are cut from the roll, what is the length of each piece?

8 A man travels a distance of 50 km. (a) Make a table showing his speed if the journey takes 2 h, 5 h, 10 h. (b) If the journey takes 3 hours, find his speed.

9 A girl cycles a distance of 30 km. (a) Make a table to show the time she takes if her speed is 10 km/h, 15 km/h, 20 km/h. (b) If she cycles at 18 km/h, find the time she takes.

10 A roll of cloth is 40 m long. It is cut into pieces of equal length. (a) Make a table to show the number of pieces if they are each of length 5 m, 8 m, 20 m. (b) If the cloth is cut into 16 pieces of equal length, find the length of each piece.

11 A car travels 42 km on 7 litres of petrol. How far will it travel on 12 litres?

12 A length of string can be cut into 9 pieces of length 20 cm. How many pieces each 6 cm long can be cut from the string?

13 A student has enough money to buy 14 pencils at 45 cents each. How many rubbers costing 30 cents each can the student buy for the same money?

14 A car factory produces 700 cars in 5 work-days. How many cars will it produce in 12 work-days?

15 A lorry can carry safely 90 sacks each of mass 75 kg. How many boxes each of mass 27 kg can it carry safely?

16 A bus, travelling at a steady speed, takes $2\frac{1}{2}$ hours for a certain journey. How long will a car take if it travels at 3 times the speed of the bus?

Graphical representation

Direct proportion

Using the data of Table 8.1 on page 52, Fig. 8.1 is a distance/time graph for a student walking with a steady speed of 5 km/h.

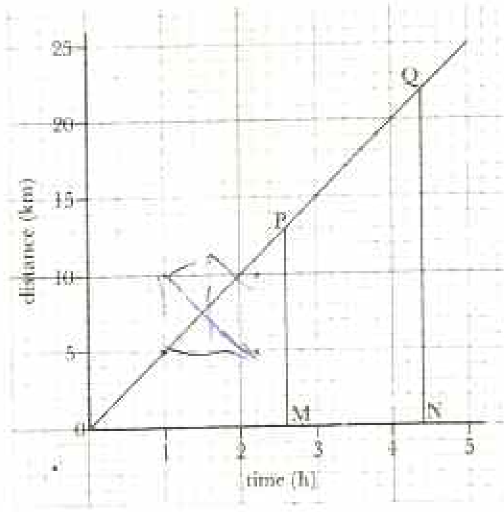


Fig. 8.1

Since distance is directly proportional to time, Fig. 8.1 is a straight line graph through the origin O. If P and Q are two points on the graph, then △ OPM is similar to △ OQN and

$$\frac{PM}{OM} = \frac{QN}{ON}$$

similarly, $\frac{PM}{QN} = \frac{OM}{ON}$

The first of these ratios gives the speed of the student, the second shows that distance is directly proportional to time. The graph can be used as a **ready reckoner** since corresponding times and distances can be read directly from it.

Inverse proportion

Using the data of Table 8.2 on page 52, Fig. 8.2 is a speed/time graph for a journey of 100 km.

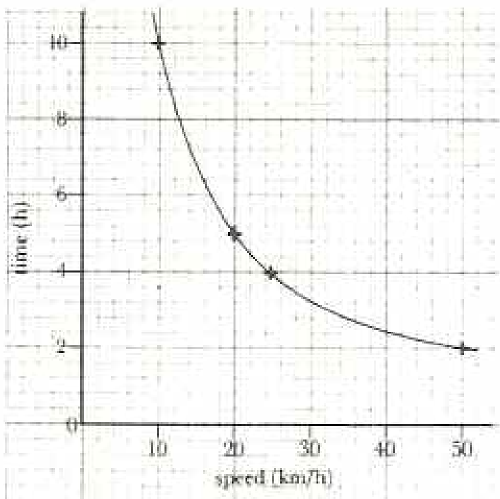


Fig. 8.2

Notice that as the speed increases, the time decreases. This gives a graph in the shape of a curve. This can be difficult to draw accurately, especially if only a few points are given.

However, since speed and time are inversely proportional, speed is *directly* proportional to $\frac{1}{\text{time}}$. Thus a straight line graph will be obtained by plotting $\frac{1}{\text{time}}$ against speed.

Table 8.5 gives the values of $\frac{1}{\text{time}}$ which correspond to the given speeds.

Table 8.5

speed (km/h)	10	20	25	50
time (h)	10	5	4	2
$\frac{1}{\text{time}}$	0,1	0,2	0,25	0,5

Fig. 8.3 is the corresponding graph of $\frac{1}{\text{time}}$ against speed.

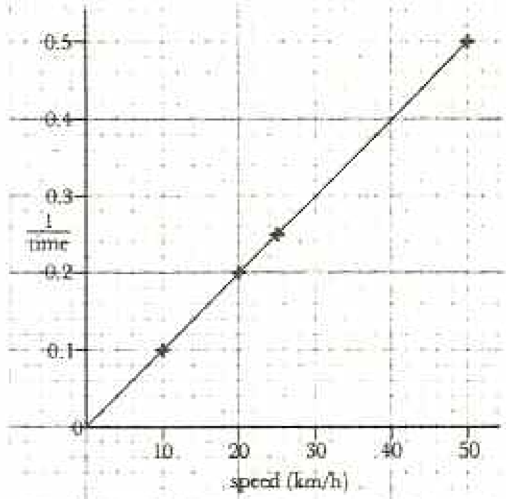


Fig. 8.3

• The advantage of a straight line graph is that it is much easier to draw accurately. Also, only 3 points need to be plotted (i.e. 2 necessary points and a 3rd point as a check).

Example 4

4 people can do a piece of work in 10 days. (a) Make a table of values showing the time it would take 4, 8, 20 people to do the work. (b) Find the corresponding values of $\frac{1}{\text{time}}$ and plot these values against the number of people to give a straight line graph. (c) Use the graph to find the number of days it would take 16 people to do the work.

It is assumed that all people work at the same rate.

(a) If 4 people take 10 days, then 8 people would take half as long, 5 days. 20 people would take one-fifth as long, 2 days. These values are shown in Table 8.6.

Table 8.6

number of people	4	8	20
time (days)	10	5	2
$\frac{\mathbf{1}}{\textbf{time}}$	0,1	0,2	0,5

(b) Corresponding values of $\frac{1}{\text{time}}$ are given in Table 8.6. Fig 8.4 is the required graph.

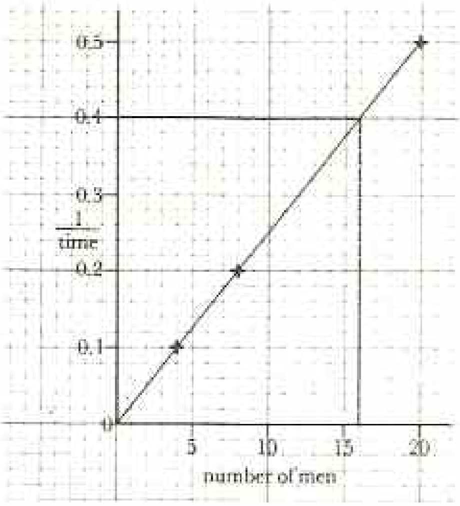


Fig. 8.4

(c) From the graph,
when the number of people = 16,

$$\frac{1}{\text{time}} = 0{,}4$$

Take the reciprocal of both sides,

$$\text{time} = \frac{1}{0{,}4} = \frac{10}{4} = 2\tfrac{1}{2} \text{ days}$$

Exercise 8b

1 Given that \$1 is equivalent to 350 yen, draw a ready reckoner graph to exchange amounts up to \$1 into yen. Use the graph to find the value of (a) 21 c, 46 c, in yen; (b) 120 yen, 280 yen in cents.

2 Three people start together and travel at speeds of 5 km/h, 15 km/h and 18 km/h respectively. (a) On one set of axes, draw graphs to show the distance they travel for any time up to 5 hours. (Take time in hours on the horizontal axis and distance in km on the vertical axis.) (b) Find the distance that each person has travelled after $3\frac{1}{2}$ hours.

3 Three cyclists travel a distance of 60 km at speeds of 12 km/h, 15 km/h and 20 km/h respectively, starting together. (a) On one set of axes, draw graphs to show their positions at any time. (b) Find the time that each cyclist takes to travel 24 km.

4 Given that 50 litres of petrol cost \$48,50; draw a graph and from it read off, (a) the cost of 11, 32, 45 litres; (b) the number of litres that can be bought for \$18,43; \$26,19; \$39,77.

5 Given that the mass of 100 cm^3 of a certain metal is 254 g, draw a graph connecting mass with volume up to 100 cm^3. Read off
(a) the mass of 37 cm^3, 64 cm^3 of the metal;
(b) the volume which has a mass of 100 g, 208 g.

6 Table 8.7 gives the average speeds and corresponding times for a journey.

Table 8.7

speed (km/h)	30	60	90
time (h)	4	2	$1\frac{1}{3}$

(a) Copy Table 8.7 and add an extra line for $\frac{1}{\text{time}}$. Complete your table by calculating the corresponding values of $\frac{1}{\text{time}}$.

(b) Taking speed on the horizontal axis and $\frac{1}{\text{time}}$ on the vertical axis, draw a graph connecting speed and reciprocal of time.

(c) Use your graph to find the time taken if the speed is 48 km/h.

7 A lorry travels a distance of 60 km.

(a) Copy and complete Table 8.8.

Table 8.8

speed (km/h)	6	12	60
time (h)			
$\frac{1}{\text{time}}$			

(b) Draw a graph connecting speed and reciprocal of time.

(c) Use your graph to find (i) the time taken at a speed of 15 km/h, (ii) the speed which corresponds to a time of $7\frac{1}{2}$ hours.

8 A bus travels a distance of 80 km. (a) Copy and complete Table 8.9.

Table 8.9

time (h)	8	4	2
speed (km/h)			
$\frac{1}{\text{speed}}$			

(b) Draw a graph connecting time with reciprocal of speed.

(c) Use your graph to find (i) the speed if the time taken was 5 hours, (ii) the time at a speed of 25 km/h.

9 A woman spends $6. She buys n items, each costing the same price.

(a) Copy and complete Table 8.10.

Table 8.10

price/item (c)	30	60	100
n	20		
$\frac{1}{n}$	0,2		

(b) Draw a graph connecting price/item with $\frac{1}{n}$.

(c) Use it to find (i) the number of items that can be bought if they cost 24 c each, (ii) the price/item when $n = 15$.

10 $20 is shared equally between n people.

(a) Make a table of values of n and $\frac{1}{n}$ showing each person getting $2, $4, $5.

(b) Draw a graph connecting $ per person with the reciprocal of n.

(c) Use the graph to find (i) n when each person gets $2,50, (ii) the amount each person gets when there are 9 people.

Revision exercises and tests

Chapters 1–8

Revision exercise 1 (Chapters 1, 4)

1 Find the next four terms in the following patterns.
 (a) 6; 13; 20; 27; 34; . . .
 (b) 5; 6; 8; 11; 15; 20; . . .
 (c) 3; 6; 12; 24; . . .
 (d) 0; 2; 6; 12; 20; . . .

2 Find the smallest number by which 350 must be multiplied to give a perfect square.

3 (a) Write down the factors of 24 in ascending order of size.
 (b) Draw a graph to show these factors.

Fig. R1 shows a number pattern where the numbers 1, 2, 3, . . ., 64 are arranged in a spiral. Use Fig. R1 to answer questions 4 and 5.

64	37	38	39	40	41	42	43
63	36	17	18	19	20	21	44
62	35	16	5	6	7	22	45
61	34	15	4	1	8	23	46
60	33	14	3	2	9	24	47
59	32	13	12	11	10	25	48
58	31	30	29	28	27	26	49
57	56	55	54	53	52	51	50

Fig. R1

4 The shaded boxes contain the sequence . . ., 57, 31, 13, 3, 1, 7, 21, 43, . . . Extend the sequence by two terms at each end.

5 In Fig. R1, two rows and one column are shown by arrows. In each case, investigate whether there are any patterns in the numbers. If possible extend the patterns by one term in both directions.

6 Express the following large numbers in digits, grouping them in threes from the decimal comma.
 (a) four and three-quarter million
 (b) eight hundred and sixty thousand
 (c) twenty-two and a half billion

7 Find the square roots of the following.
 (a) $3\frac{1}{16}$ (c) 39,69

8 Find the HCF and LCM of 84 and 210.

9 Which of the numbers 5, 6 and 8 will divide into 6 320 without leaving a remainder?

10 Divide 2,647 by 0,9 and give the answer correct to 2 d.p.

Revision test 1 (Chapters 1, 4)

1 The next term in the sequence 2; 3; 5; 8; 12; 17; . . . is
 A 18 B 19 C 22 D 23 E 24

2 The square root of $12\frac{1}{4}$ is
 A $1\frac{3}{4}$ B $3\frac{1}{2}$ C $3\frac{3}{4}$ D $6\frac{1}{4}$ E $6\frac{1}{8}$

3 Express 0,8 million in digits.
 A 80 000 000 B 8 000 000 C 800 000
 D 80 000 E 8 000

4 Calculate $82{,}5 \div 0{,}025$.
 A 0,003 3 B 0,33 C 3,3
 D 330 E 3 300

5 What is 0,003 867 to 3 significant figures?
 A 0,004 B 0,003 86 C 0,003 87
 D 386 E 387

6 Given $1 \times 2 = 2^2 - 2$
 $2 \times 3 = 3^2 - 3$
 $3 \times 4 = 4^2 - 4$
 $4 \times 5 = 5^2 - 5$
 (a) Write an expression for $n \times (n + 1)$.
 (b) Use this to calculate $1\,001^2 - 1\,001$.

7 Write 1 296 as a product of its prime factors. Hence find $\sqrt{1\,296}$.

8 Simplify the following. Express each answer in digits, grouped in threes.
 (a) 75% of 7 million

(b) The difference between a billion and a million.

9 Find the LCM and the HCF of 66 and 165.

10 A student writes 78 words in 8 lines.

(a) Find, to the nearest whole number, the average number of words per line.

(b) Estimate how many lines of writing it will take to write a 1 500-word essay.

Revision exercise 2 (Chapters 2, 5)

1 Given $\mathscr{E} = \{2; 3; 4; 5; 6; 7; 8\}$, $P = \{2; 4; 6; 8\}$, $Q = \{2; 3; 5; 8\}$ and $R = \{4\}$

(a) List the elements of (i) $P \cup Q$, (ii) $P \cap Q$.

(b) What kind of set is $P \cap Q \cap R$?

(c) Find (i) $n(P \cup R)$, (ii) $n(P \cap R)$, (iii) $n(Q \cup R)$.

2 Make four copies of the Venn diagram in Fig. R2.

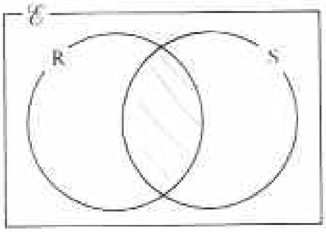


Fig. R2

On each copy shade the region which represents the set

(a) $R \cup S$, (b) $R \cap S$,

(c) $\mathscr{E} \cap R$, (d) $S \cup \mathscr{E}$.

3 If N = {natural numbers}

W = {whole numbers}

Z = {integers}

Q = {rational numbers}

(a) List the elements of W which are *not* members of N.

(b) List the elements of Z which are *not* members of W.

(c) Write down three members of Q and express each of them in the form $\frac{a}{b}$ where $a \in Z$ and $b \in N$.

4 There are 83 cattle on a farm. All of them have either been dehorned or vaccinated or both. 39 have been dehorned and 55 have been vaccinated. How many have been dehorned but not vaccinated?

5 Solve the following.

(a) $-3x = 12$ (b) $8n = -32$

(c) $-5d = -35$ (d) $\frac{1}{6}m = \frac{5}{36}$

(e) $\frac{3a}{2} = \frac{9}{14}$ (f) $-2\frac{1}{3}x = -14$

6 Solve the following.

(a) $12 - 5a = 2$

(b) $11 - 5x = 4x - 16$

(c) $2(3x + 1) = 4(x + 5)$

(d) $5(2a - 3) - 3(2a + 1) = 0$

7 One person earns \$17 more than another person. Between them they earn a total of \$105. How much does each person earn?

8 Find two consecutive even numbers such that three times the smaller added to eight times the greater comes to 170.
(*Hint*. let the numbers be x and $x + 2$.)

9 Solve the following.

(a) $\frac{3r}{5} - 15 = 0$ (b) $\frac{3x}{2} - 2 = \frac{2x}{3}$

(c) $\frac{m-3}{2} + \frac{m-8}{3} = 0$

(d) $\frac{3(2x-1)}{4} = \frac{4(x+2)}{3} - 3$

(e) $\frac{18}{2x-1} = 3$ (f) $\frac{7}{a-4} = \frac{5}{a-2}$

10 A student walks 6 km at a speed of v km/h. (a) Write the time taken in hours in terms of v. (b) Find v if the journey takes 1 h 20 min.

Revision test 2 (Chapters 2, 5)

1 Fig. R3 is a Venn diagram showing a universal set, $\mathscr{E}$, with subsets X and Y.

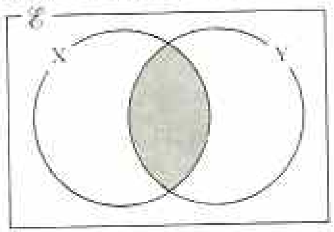


Fig. R3

Which one of the following is represented by the shaded region?

A $X \cup Y$ B $X \cap \mathscr{E}$ C $Y \cup \mathscr{E}$

D $X \cup \mathscr{E}$ E $X \cap Y$

2 $\mathscr{E} = \{3; 6; 9; 12; \ldots; 30\}$, $P = \{\text{numbers less than } 19\}$ and $Q = \{\text{factors of } 30\}$. List the members of $P \cap Q$.

A $\{1; 2; 3; \ldots; 28\}$
B $\{1; 2; 3; 5; 6; 10; 15\}$
C $\{3; 6; 9; 12; 15; 18\}$
D $\{3; 6; 15; 30\}$
E $\{3; 6; 15\}$

3 Express the following statement as an algebraic equation: 'The result of taking 2 from n then multiplying by 4 is the same as multiplying n by 3 and taking away 5.'

A $4(n - 2) = 5(n - 3)$
B $4(n - 2) = 3n - 5$
C $4(n - 2) = 5 - 3n$
D $4(2 - n) = -2n$
E $4(2 - n) = 3(n - 5)$

4 Solve the equation $\frac{x + 2}{3} + 2x = 10$. $x =$

A $9\frac{3}{4}$ B 5 C $4\frac{4}{5}$ D 4 E $1\frac{1}{4}$

5 If $\frac{6}{5} = \frac{3}{d}$ then $d =$

A $\frac{5}{18}$ B $\frac{2}{5}$ C $2\frac{1}{2}$ D $3\frac{3}{5}$ E 90

6 If $W = \{5; 6; 7; \ldots; 14; 15\}$, list the members of the following subsets of W.

(a) {numbers < 10}
(b) {perfect squares}
(c) {factors of 30}
(d) {even numbers} $\cap$ {multiples of 5}

7 If W = {all athletes}, F = {female athletes} and S = {sprinters}, show by shading on a Venn diagram the set of all sprinters who are men.

8 Solve the following.

(a) $-3x = 8$ (b) $\frac{1}{5}a = \frac{7}{15}$
(c) $\frac{n}{2} = -5$ (d) $\frac{7}{2}d = 42$
(e) $-\frac{7m}{10} = -2\frac{4}{5}$ (f) $\frac{5}{8}y = 1\frac{1}{4}$
(g) $9 - 4x = 11 - 7x$
(h) $6a - 3 = 25 + 5a$
(i) $17y - 2(6y + 1) = 8$
(j) $\frac{7z}{2} - 18 = \frac{z}{2}$

9 A man is 10 times as old as his son. In 6 years time he will be 4 times as old as his son. Find their present ages. (*Hint:* let the son's present age be x years.)

10 Solve the following.

(a) $\frac{2}{x - 10} + \frac{1}{3} = 0$ (b) $\frac{23 - 3x}{x + 1} = \frac{4}{3}$

Revision exercise 3 (Chapters 3, 7)

1 Draw a number line from -10 to 10. On the line mark the points A(8), B(2), C(-5), D(-2), E(0), F($3\frac{1}{2}$), G($-7\frac{1}{2}$), H($5\frac{1}{2}$).

2 Write down the coordinates of the points P, Q, R, S, T, U, V, W, X, Y, Z in Fig. R4.

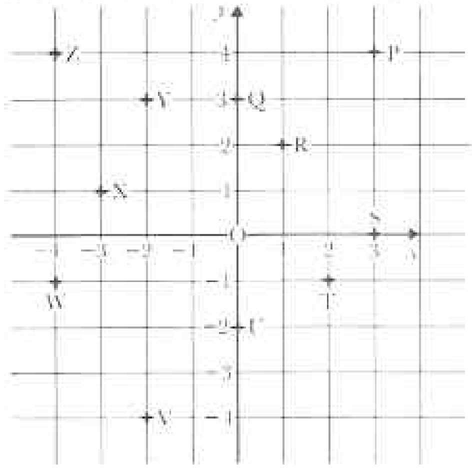


Fig. R4

3 In Fig. R5 name the points which have coordinates (3; -4), (-7; -3), (15; -6), (3; 4), (-4; 3), (6; 9), (-9; 8), (17; -2).

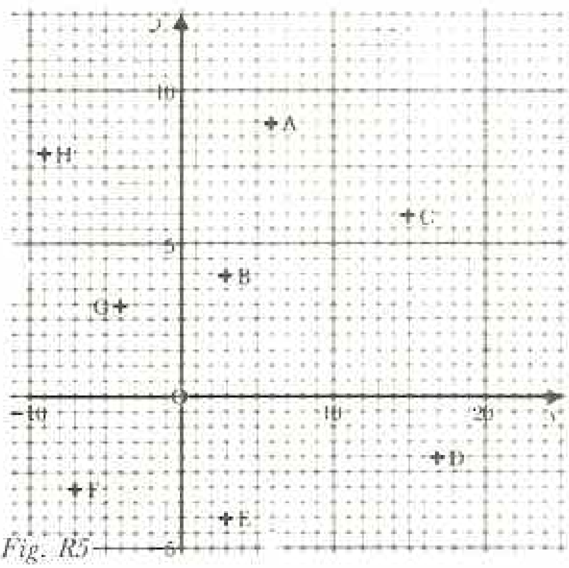


Fig. R5

4 (a) Choose a suitable scale and plot the points P(− 2; 1), Q(0; 2), R(2; 0) and S(0; − 1).
(b) What kind of quadrilateral is formed by the points P, Q, R and S?
(c) Find the coordinates of the point where the diagonals of quadrilateral PQRS cross each other.

5 Given the following sets of points:
(i) A(−2; 2), B(− 4; 2), C(− 4; − 2), D(− 2; 2)
(ii) E(− 1; 2), F(− 1; − 2), G(1; − 2), H(1; 2)
(iii) I(2; 0), J(4; 0), K(4; 2), L(2; 2), M(2; − 2)

draw an origin O near the middle of a sheet of graph paper. Use a scale of 1 cm represents 1 unit on both axes.
(a) Plot the points in (i). Join the points by straight lines in alphabetical order.
(b) Repeat for the points in (ii) and (iii).
(c) What word is formed?

6 A new candle is 15 cm long. When lit it burns steadily as shown in Table R1.

Table R1

burning time (h)	0	1	2	3	4	5
length of candle remaining (cm)	15	13,8	12,6	11,4	10,2	9

Choose a suitable scale and draw a graph of the information in Table R1. Use your graph to answer the following.
(a) Find the length of the candle remaining after it has burned for 2,6 h.
(b) How long does it take to burn 5 cm of candle?
(c) Hence estimate how long the candle will last altogether.

7 Fig. R6 is a sketch of a graph which shows the distances from A and the time taken by a car going from A to B and back again. Find
(a) the distance from A to B,
(b) how long the car stops at B,
(c) the average speed in km/h from (i) A to B, and (ii) B to A.

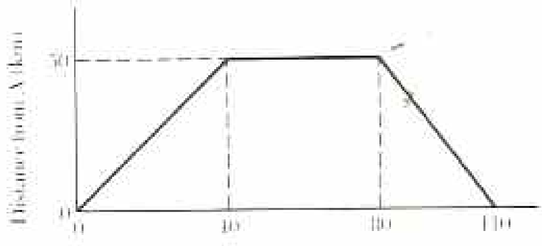


Fig. R6

8 The sum of the angles of a straight line is 180°. In Fig. R7, $x + y = 180$.

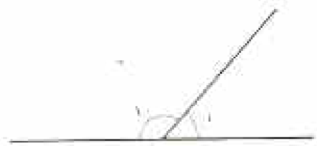


Fig. R7

(a) Copy and complete Table R2.

Table R2

x	0	45	90	135	180
y	180	135			

(b) Choose a suitable scale and draw a graph of the information in your table.
(c) Use your graph to find (i) y when $x = 40$, (ii) x when $y = 128$.

9 The cost of car insurance is $60 per $1 000 worth of insurance. To this is added a standing charge of $50.
(a) Copy and complete Table R3.

Table R3

value insured (× $1 000)	1	2	3	4	5
standing charge ($)	50	50	50	50	50
basic rate ($)	60	120	180		
total cost ($)	110	170	230		

(b) Choose a suitable scale and draw a graph of this information.

10 Use the graph you drew in question 9 to answer the following.

(a) What is the cost of insuring a car worth \$3 800?

(b) A sports car costs \$260 to insure. How much does it cost to insure a car half the value of the sports car?

Revision test 3 (Chapters 3, 7)

1 The position of point X is given by X(− 0,9). Which of the points A, B, C, D, E in Fig. R8 is in same position as X?

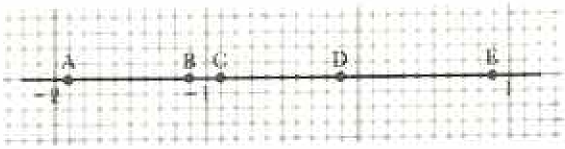


Fig. R8

2 Which of the points A, B, C, D, E in Fig. R9 has coordinates (− 2; 3)?

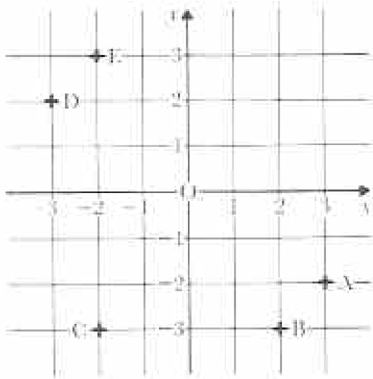


Fig. R9

3 A straight line PO joins the point P(5; − 3) to the origin, O. PO is extended to Q so that PO = OQ. What are the coordinates of Q?

A (− 5; − 3) B (− 3; − 5) C (− 5; 3)
D (− 3; 5) E (3; − 5)

4 The point (− 2; 7) is reflected in the y-axis. What are the coordinates of its image?

A (2; 7) B (7; − 2) C (2; − 7)
D (− 7; 2) E (− 2; − 7)

Fig. R10 is a discontinuous graph which gives the postal rates in cents (c) for letters up to 500 g in mass.

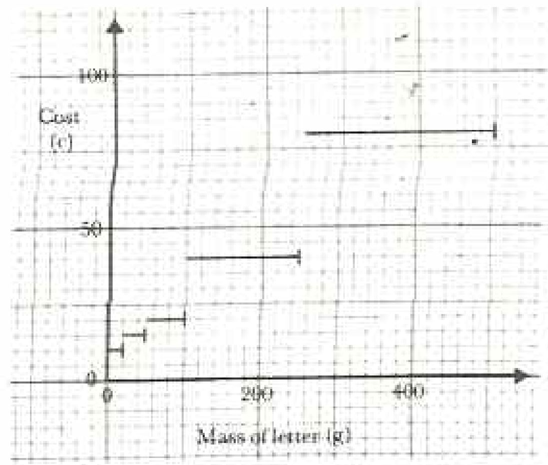


Fig. R10

Use Fig R10 to answer questions 5 and 6.

5 How much does it cost to send a letter of mass 80 g?

A 10 c B 15 c C 20 c D 40 c E 80 c

6 Each step in the graph is in the form of a line with a cross-mark at the right-hand end. Take this to mean that the value at the right-hand end is included but that the value at the left-hand end is *not* included. A letter of mass x g can be sent for 40 c. Express the range of values of x in the form $a < x \leqslant b$.

7 Choose a suitable scale and plot the points A(1; 3), B(6; 3), C(3; − 1), D(− 2; − 1).

(a) AC and BD cross at P. Find the coordinates of P.

(b) What is the size of $A\hat{P}D$?

(c) What kind of quadrilateral is ABCD?

8 A student walks with a speed of 6 km/h.

(a) Make a table of values to show how far the student walks in $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2 hours.

(b) Choose a suitable scale and draw a graph of the information in the table.

9 Use the graph drawn in question 8 to find:

(a) how far the student walks in 69 min,

(b) how long it takes the student to walk 10 km.

10 The exchange rate between Zimbabwean and British currencies is Z\$1,00 = UK£0,20.

(a) Find how many pounds (£) are equivalent to (i) Z\$10, (ii) Z\$100.

(b) Hence draw a conversion graph for changing up to Z$100 to pounds.

(c) Read off the equivalent amounts in the other currency of (i) Z$80, (ii) £14.

Revision exercise 4 (Chapters 6, 8)

1 A map is drawn to a scale of 1 cm to 20 km. On the map a river is 6,3 cm long. What is the true length of the river?

2 A map is drawn to a scale of 5 cm to 1 km. A straight road is 1,58 km long. How long will the road appear on the map?

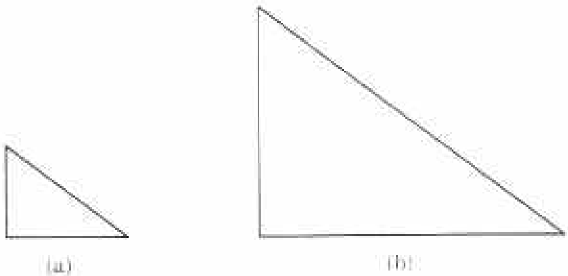


Fig. R11

3 Use measurement to find the scale of △ (a) to △ (b) in Fig. R11.

4 Use Fig. 6.13 on page 39 to estimate the distance of Masvingo from Mutare.

5 A triangular plot PQR is such that PQ = 100 m, QR = 70 m and RP = 50 m. X is the mid-point of side PQ.

Use a scale of 1 cm to 10 cm to make a scale drawing of the plot. Hence find the true length of XR.

6 A car travels 70 km on 8 litres of petrol. Estimate how far it will travel on a full tank containing 60 litres.

7 A boy walks at a steady speed and takes 90 min for a certain journey. How many minutes will a girl take if she cycles at $2\frac{1}{2}$ times the speed of the boy?

8 A tank will be completely filled by 84 buckets of water if each bucket contains 11 litres. However, if 12-litre buckets are used, then only 77 are required to fill the tank.

(a) Does the number of buckets needed to fill the tank vary directly or inversely with their capacity?

(b) How many 14-litre buckets would it take to fill the tank?

9 Use Fig. 8.2 on page 55 to estimate the time taken when the speed is 38 km/h.

10 Given that 16 000 m^2 of registered building land costs $900 000, draw a graph and from it read off (a) the cost of 4 000 m^2 of land and of 11 000 m^2 of land, (b) the area of land that can be bought for $360 000 and for $500 000.

Revision test 4 (Chapter 6, 8)

1 A road is 90 km long. On a map it appears as a line 6 cm long. The scale of the map is 1 to

A 15 B 150 C 1 500
D 15 000 E 1 500 000

2 A map is drawn to a scale of 2 cm to 100 km. If the distance between two towns is 374 km, what is this distance as measured on the map?

A 1,97 km B 3,74 km C 7,48 cm
D 19,7 cm E 74,8 cm

3 Refer to the map in Fig. 6.14 on page 39. Which one of the following is on Azania Crescent?

A Bank B Prison C Dispensary
D Church E Station

4 A shelf can hold 84 books each 3 cm wide. How many books of width 4 cm can it hold?

A 56 B 63 C 84 D 105 E 112

5 A long rope is cut into pieces of equal length. Table R4 shows various lengths of rope and the corresponding number of pieces that can be cut.

Table R4

Length (m)	15	40	45
No of pieces	24	9	8

If the rope was cut into 30 pieces, how long would each piece be?

A 6 m B 10 m C 12 m D 18 m E 48 m

6 A photograph measures 8 cm by 10 cm. It is enlarged so that the shorter side becomes 12 cm. What is the length of the longer side?

7 A plan is drawn on a scale of 1 cm to 5 m.
   (a) If a wall is 24 m long, find its length on the plan.
   (b) If the scale drawing of a round house is a circle of diameter 0,7 cm, find the actual diameter of the round house.

8 A rectangular sheet of paper measures 60 cm by 40 cm. Use a scale of 1 cm to 10 cm and make a scale drawing of the sheet of paper. Hence find the length of a diagonal of the sheet of paper.

9 A cooperative farmer has enough money to buy 8 bags of corn seed, costing \$39 per bag. How many rolls of chicken netting costing \$52 per roll can the farmer buy for the same money?

10 Table R5 gives some average speeds and corresponding times for a certain journey.

Table R5

speed (km/h)	15	30	50
time (h)	10	5	3

   (a) Copy Table R5 and add an extra line for values of $\frac{1}{\text{time}}$.
   (b) Draw a graph connecting speed and reciprocal of time.
   (c) Use your graph to find the time taken when the speed is 45 km/h.

General revision test A (Chapters 1–8)

1 What is the next term in the sequence 1; 3; 7; 15; 31; ...?
   A 33   B 47   C 51   D 59   E 63

2 Find the least number by which 112 must be multiplied to give a perfect square.
   A 2   B 3   C 5   D 7   E 11

3 Calculate 45 ÷ 900 000.
   A 500 000   B 500   C 5
   D 0,05   E 0,000 05

4 In Fig. R12, what are the coordinates of the point where the diagonals of kite PQRS cross?

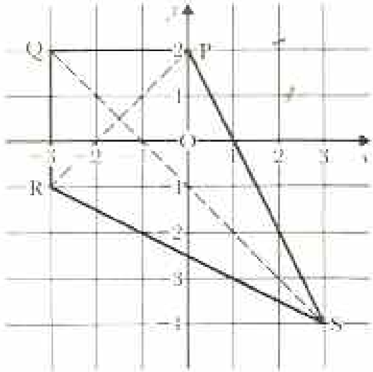


Fig. R12

   A $(-1\frac{1}{2}; -\frac{1}{2})$ B $(-1\frac{1}{2}; \frac{1}{2})$ C $(-\frac{1}{2}; -1\frac{1}{2})$
   D $(\frac{1}{2}; -1\frac{1}{2})$ E $(1\frac{1}{2}; -\frac{1}{2})$

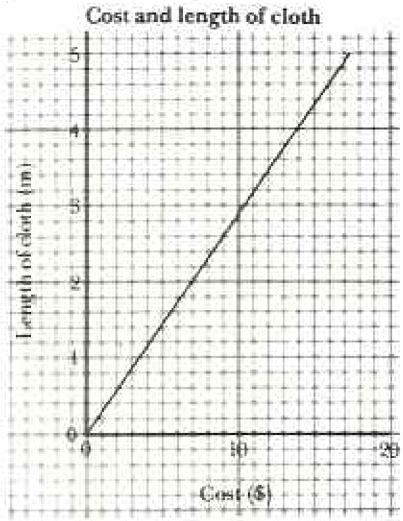


Fig. R13

Fig. R13 is a graph giving the cost in \$ of cloth in metres. Use it to answer questions 5 and 6.

5 What is the cost of 3 m of cloth?
   A \$0,90   B \$1,80   C \$10,50
   D \$11   E \$15

6 Approximately what length of cloth can be bought for \$4,50?
   A 1,3 m   B 2,5 m   C 4 m
   D 15,75 m   E 17,5 m

7 Solve the equation $5 - \frac{2x}{3} = 1$. $x =$
   A 1   B 4   C 6   D 7   E 9

8 A map is drawn to a scale of 1:40 000. What distance, in km, will a line on the map 2,8 cm long represent?
   A 11,2 km   B 7 km   C 1,12 km
   D 0,7 km   E 0,112 km

9 If $\mathscr{E} = \{a, b, c, d, e\}$, $B = \{b, e, d\}$ and $C = \{c, a, b\}$, then $B \cup C =$

A $\varnothing$ B $\{b\}$ C $\{a, c\}$ D $\{d, e\}$ E $\mathscr{E}$

10 A lorry takes 1 h 20 min for a certain journey. How many minutes will a car take if its average speed is $2\frac{1}{2}$ times that of the lorry?

A 8 min B 32 min C 50 min
D 80 min E 200 min

11 Given

$$1 = 1^2$$
$$1 + 3 = 2^2$$
$$1 + 3 + 5 = 3^2$$
$$1 + 3 + 5 + 7 = 4^2$$

(a) what will be the value of x if $1 + 3 + 5 + \ldots + x = 23^2$?,

(b) what will be the value of y if $1 + 3 + 5 + \ldots + 99 = y^2$?

12 \$90 is shared between n students.

(a) Make a table of values of n and $1/n$ corresponding to each student getting \$3, \$5, \$6.

(b) Draw a graph connecting the amount of money that each student gets with $1/n$.

(c) Use the graph to find (i) n when each of the students gets \$4,50, (ii) the amount each gets when there are 24 students.

13 Choose a suitable scale and plot the following points:

A (11; −15), B (−4; 0), C (−7; −3), D (−10; 0), E (−4; 6), F (5; 6), G (−2; 2), H (13; −13).

Join the points in alphabetical order and join H to A. What have you drawn a picture of?

14 Fig. R14 is part of a conversion graph. It is used to find the car insurance premiums to be paid according to the value of the cars being insured.

(a) What is the insurance premium for a car of value \$5 500?

(b) If a woman pays an insurance premium of \$106 on her car, what is the value of her car?

(c) What is the difference in premiums paid by a man who insures his car for \$3 000 and a man who insures his car for \$8 000?

Car Insurance graph

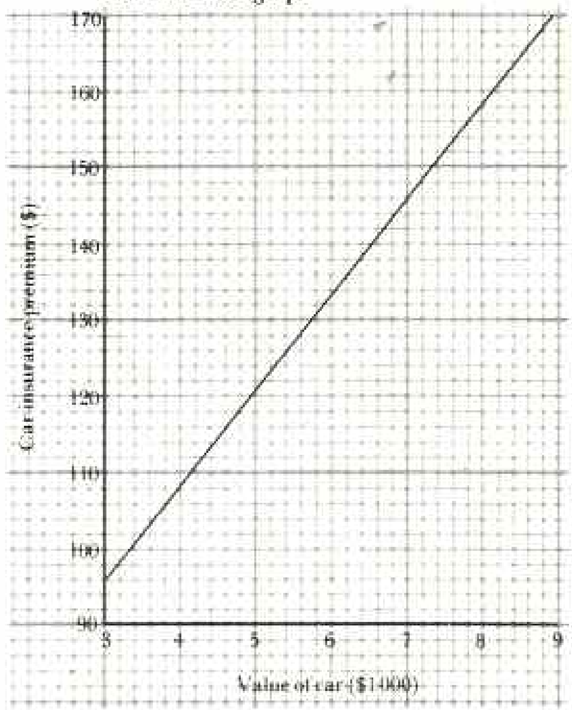


Fig. R14

15 (a) Solve $3(x - 6) = 4(1 - 2x)$.

(b) Solve $\frac{c - 5}{4} - \frac{6 - c}{8} = 1$.

(c) 6 times a number added to 7 times 4 less than the number comes to 11. Find the number.

16 A man leaves home and cycles to his office 12 km away. His journey is shown in Fig. R15.

Travel graph of cyclist

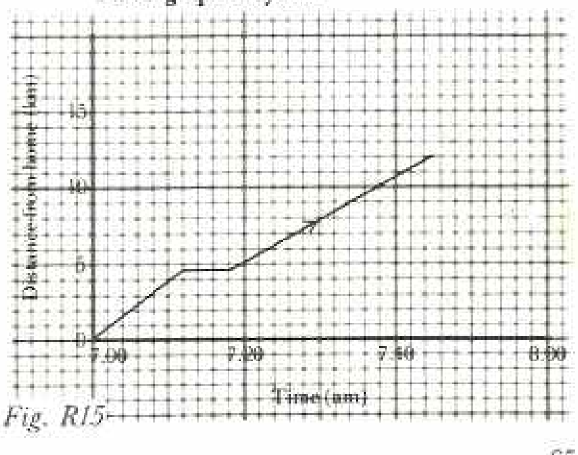


Fig. R15

(a) What time did he arrive at his office?
(b) How long did the journey take altogether?
(c) Find his average speed for the whole journey.
(d) During the journey he stopped to mend a broken pedal. How long did this take?
(e) How far was he from home when the pedal broke?

17 Solve the following.

(a) $\frac{15}{k} = \frac{3}{k} + 2$ (b) $\frac{15}{k} = \frac{3}{k+2}$

(c) $\frac{1}{x+5} - \frac{1}{4x-7} = 0$

18 Using paper and pencil only, calculate the square roots of the following.
(a) 44 100 (b) 0,000 484 (c) $182\frac{1}{4}$

19 Express each of the following as a number in digits, grouped in threes from the decimal comma.
(a) three millionths;
(b) one tenth of one percent;
(c) two million plus nineteen million;
(d) twenty percent of nine hundred thousand.

20 A class of 32 students contains 15 boys. 20 of the students walk to school each day. 8 of the girls do not walk to school. How many boys from the class walk to school each day?

Chapter 9

Inequalities (1)

Greater than, less than

Mathematicians probably use the equals sign, =, more than any other. For example, $5 + 3 = 8$.

However, many quantities are *not* equal:

$$5 + 5 \neq 8$$

where $\neq$ means *is not equal to*. We can also write:

$$5 + 5 > 8$$

where $>$ means *is greater than*. Similarly we can write the following:

$$3 + 3 \neq 8$$
$$3 + 3 < 8$$

where $<$ means *is less than*.

$\neq$, $>$ and $<$ are **inequality symbols.** They tell us that quantities are not equal. The $>$ and $<$ symbols are more helpful than $\neq$. They tell us more. For example, $x \neq 0$ tells us that x does not have the value 0; x can be any positive or negative number. However, $x < 0$ tells us that x is less than 0; x must be a negative number.

Exercise 9a

1 Re-write each of the following, using either $>$ or $<$ instead of the words.

(a) 6 is less than 11
(b) -1 is greater than -5
(c) 0 is greater than $-2{,}4$
(d) -3 is less than $+3$
(e) x is greater than 12
(f) y is less than -2
(g) 4 is greater than a
(h) a is less than 4
(i) 15 is less than b
(j) b is greater than 15

2 State whether each of the following is true or false.

(a) $13 > 5$ (b) $19 < 21$
(c) $-2 < -4$ (d) $-15 > 7$
(e) $3 + 9 < 10$ (f) $0 > -4 - 3$
(g) $14 - 6 > 8$ (h) $30 > -50 + 20$

3 Find which symbol, $>$ or $<$, goes in the box to make each statement true.

(a) $9 + 8 \square 10$ (b) $7 - 2 \square 7$
(c) $6 \square 12 - 5$ (d) $0 \square 3 - 6$
(e) $16 \square 2 \times 10$ (f) $29 \div 7 \square 3{,}6$
(g) $13 \square 3 \times 4{,}9$ (h) $(-5)^2 \square (2)^2$

The symbols $>$ and $<$ can be used to change English statements into algebraic statements.

Example 1

The distance between two villages is over 18 km. *Write this as an algebraic statement.*

If the distance between the villages is d km, then, $d > 18$.

A statement like $d > 18$ is called an **inequality.**

Example 2

I have x cents. *I spend* 20 c. *The amount I have left is less than* 5 c. *Write an inequality in* x.

I spend 20 c out of x cents.
Thus I have $x - 20$ cents left.
Thus $x - 20 < 5$.

Example 3

The area of a square is less than 25 cm². *What can be said about (a) the length of one of its sides, (b) its perimeter?*

(a) Let the length of a side of the square be a cm. Then,

$$a^2 < 25$$
$$\Leftrightarrow \quad \sqrt{a^2} < \sqrt{25}$$
$$a < 5$$

(b) Perimeter $= 4a$

$$a < 5$$
$$\Leftrightarrow \quad 4a < 4 \times 5$$
$$\Leftrightarrow \quad 4a < 20$$

The length of a side of the square is less than 5 cm. Its perimeter is less than 20 cm.

Exercise 9b

1 For each of the following, write an inequality in terms of the given unknown.
   (a) The height of the building, h m, is less than 5 m.
   (b) The mass of the boy, m kg, is less than 50 kg.
   (c) The cost of the meal, \$$x$, was over \$5.
   (d) The time taken, t min, was under 5 minutes.
   (e) The number of pages, n, was less than 24.
   (f) The mass of the letter, m g, was under 20 g.
   (g) The cost of the stamp, s cents, was less than \$1.
   (h) The time for the journey, t min, was over 2 hours.

2 For each of the following, choose a letter for the unknown and write an inequality.
   (a) The boy is less than 1,5 m tall.
   (b) The capacity of the bottle is less than 800 ml.
   (c) The book cost more than \$12.
   (d) The girl got over 60% in the mathematics exam.
   (e) The car used more than 28 litres of petrol.
   (f) Her mass was less than 55 kg.
   (g) The light was on for over 6 hours.

3 7 lorries each carry a load of over 4 tonnes. The total mass carried by the lorries is m tonnes. Write an inequality in m.

4 A boy saved over \$5. His father gave him \$2. The boy now had \$$y$ altogether. Write an inequality in y.

5 In 3 years time a girl will be over 18 years of age. If her age now is x years, write an inequality in x.

6 There are x goats on a field of area 3 ha. There are less than 20 goats on each hectare. Write an inequality in x.

7 A square has an area of more than 36 cm^2. What can be said about (a) the length of one of its sides, (b) its perimeter?

8 The perimeter of a square is less than 28 cm. What can be said about (a) the length of one of its sides, (b) its area?

Not greater than, not less than

In most towns there is a speed limit of 50 km/h. If a car, travelling at s km/h, is within the limit, then s is not greater than 50. If $s < 50$ or if $s = 50$, the speed limit will not be broken. This can be written as one inequality:

$$s \leqslant 50$$

where $\leqslant$ means *is less than or equal to*. Thus, not greater than means the same as less than or equal to.

In most countries, voters in elections must not be less than 18 years of age. If a person of age a years is allowed to vote, then a is not less than 18. The person can vote if $a > 18$ or if $a = 18$. This can be written as one inequality:

$$a \geqslant 18$$

where $\geqslant$ means *is greater than or equal to*. Thus, not less than means the same as greater than or equal to.

Example 4

Notebooks cost 60 c each. David has d cents. It is not enough to buy a notebook. Tsitsi has t cents. She is able to buy a notebook. What can be said about the values of d and t?

The question tells us that: d is less than 60.
$d < 60$
If Tsitsi gets no change, then $t = 60$
If Tsitsi gets some change, then $t > 60$
It is not known if Tsitsi gets change or not, so, $t \geqslant 60$.
Finally, Tsitsi clearly has more money than David.
Thus, $t > d$
Similarly, $d < t$

In conclusion, the following can be said about d and t:

$d < 60$	$t \geqslant 60$
$t > d$	$d < t$

Notice that $t > d$ and $d < t$ are two ways of saying the same thing. It is like saying $9 > 4$ and $4 < 9$.

Exercise 9c

1 For each of the following, write an inequality in terms of the given unknown.
   (a) The age of the girl, a years, was 12 years or less.
   (b) The number of goals, n, was 5 or more.
   (c) The temperature, t °C, was not greater than 38°C.
   (d) The selling price, \$$s$, was not less than \$24.
   (e) The number of students, n, was less than 36.
   (f) The speed of the car, v km/h, was never more than 120 km/h.

2 For each of the following, choose a letter for the unknown and then write an inequality.
   (a) The lorry can carry a load of not more than 7 tonnes.
   (b) My car cannot go faster than 140 km/h.
   (c) To join the police force, you must be not less than 160 cm tall.
   (d) The taxi cannot carry more than 5 passengers.
   (e) The borehole must be not less than 6 m deep.
   (f) Each cow needs at least 100 m^2 of grazing land.

3 The pass mark in a test was 27. One person got x marks and failed. Another got y marks and passed. What can be said about x and y?

4 Pencils cost 12c each. a cents is not enough to buy a pencil. A person with b cents is able to buy a pencil. Write down 3 different inequalities in terms of a or b or both.

5 The radius of a circle is not greater than 3 m. What can be said about (a) its circumference, (b) its area? Give any inequalities in terms of π.

Graphs of inequalities

Line graphs

The inequality $x < 2$ means that x can have any value less than 2. We can show these values on the number line in Fig. 9.1.

The heavy arrowed line in Fig. 9.1 shows the range of values that x can have. The empty circle at 2 shows that the value 2 is not included.

Fig. 9.1

x can have any value to the left of 2.

The inequality $x \geqslant -1$ means that x can have the value -1 or any value greater than -1. Its graph is given in Fig. 9.2.

Fig. 9.2

The shaded circle in Fig. 9.2 shows that the value -1 is included. x can have the value -1 and any value to the right of -1.

Example 5

Fig. 9.3 shows the graph of an inequality. What is the inequality?

Fig. 9.3

The shaded circle above 4 shows that the value $x = 4$ is included. The heavy line to the left of 4 shows that x can have values in the range $x < 4$. Thus Fig. 9.3 is the graph of $x \leqslant 4$.

Exercise 9d

1 Write down the inequalities in the following graphs.

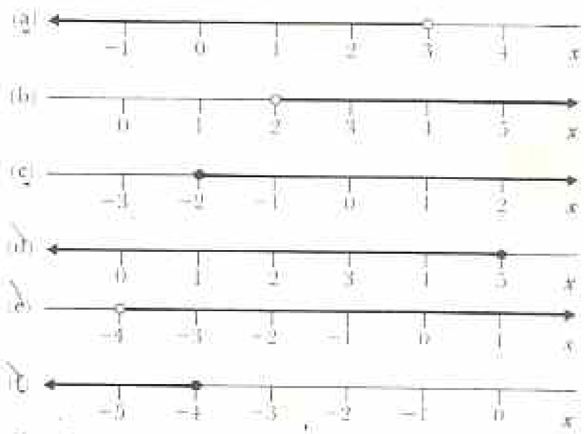


Fig. 9.4

2 Sketch graphs of the following inequalities.

(a) $x > 1$	(b) $x > -2$
(c) $x \geqslant -3$	(d) $x \leqslant 0$
(e) $x < 3$	(f) $x \geqslant -2$
(g) $x \geqslant 4$	(h) $x < -1$

Cartesian graphs

$(x; y)$ represents any point on the cartesian plane which has coordinates x and y. If $(x; y)$ is such that $x \geqslant 2$ then $(x; y)$ may lie anywhere in the unshaded region in Fig. 9.5.

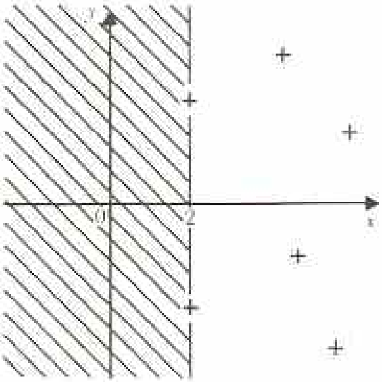


Fig. 9.5

Small crosses show some possible positions of $(x; y)$. The region on the left is shaded to show that it is *not* wanted.

In Fig. 9.5 the boundary of the shaded region is a line through the x-axis where $x = 2$. $x = 2$ at every point on this line. We say that the **equation of the line** is $x = 2$.

The points shown by crosses are members of a set of points, P, where $P = \{(x; y): x \geqslant 2\}$. I.e. P is the set of points $(x; y)$ such that $x \geqslant 2$. Similarly the line $x = 2$ is composed of the set of points, L, where $L = \{(x; y): x = 2\}$.

Example 6

On a cartesian plane, sketch the region which contains the set of points Q, where $Q = \{(x; y): y > -3\}$.

The unshaded part of Fig. 9.6 is the required region.

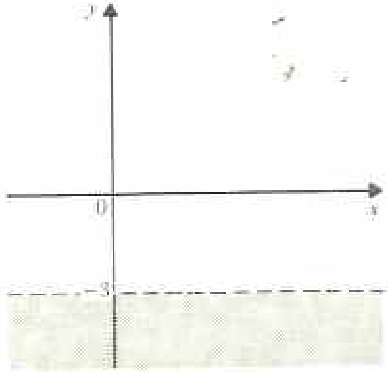


Fig. 9.6

Note: The boundary line has the equation $y = -3$. In Fig. 9.6 this line is broken to show that the points on the line are *not* included in the required region.

Example 7

Combine Fig. 9.5 with Fig. 9.6 to show the region which represents the set

$\{(x; y) : x \geqslant 2\} \cap \{(x; y) : y > -3\}$.

In Fig. 9.7 the unshaded part is the required region. The other parts are shaded to show that they are not wanted.

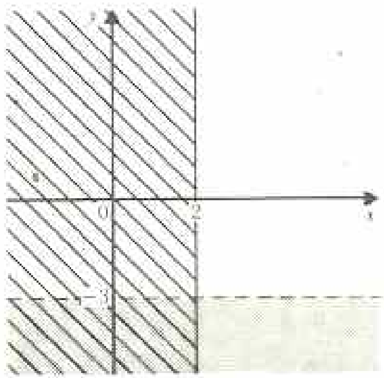


Fig. 9.7

Note: Fig. 9.7 shows the intersection of sets P and Q. The unshaded region contains those points which belong to both P and Q.

Exercise 9e

1 On a cartesian plane, sketch and label the lines represented by the following equations and sets of points.

(a) $x = 3$ (b) $\{(x; y) : y = 5\}$
(c) $y = -4$ (d) $\{(x; y) : x = -2\}$
(e) $x = 0$ (f) $\{(x; y) : y = 0\}$
(g) $y = -1$

2 In Fig. 9.8 the points belonging to the shaded region are not wanted. State the set of points represented by the unshaded region.

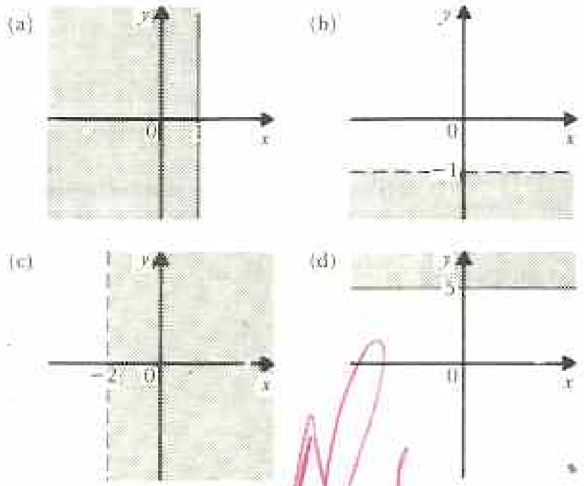


Fig. 9.8

3 On a cartesian plane sketch the regions which represent the following sets of points. Use the rule that shaded regions contain unwanted points and that solid boundary lines mean that the equality is included.

(a) $x \geqslant -1$ (b) $x < 2$
(c) $y > -2$ (d) $y \leqslant 3$
(e) $x \leqslant 0$ (f) $y \geqslant 0$
(g) $y < 6$

4 In each part of Fig. 9.9 the unshaded region represents the intersection of two sets of points. Use set notation to write out the sets.

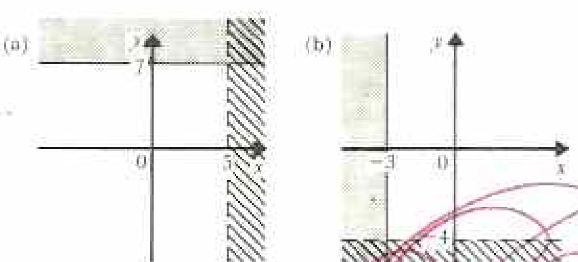


Fig. 9.9

5 On a cartesian plane sketch the region which represents the set of points in each of the following intersections.

(a) $\{(x; y) : x < 2\} \cap \{(x; y) : y \geqslant 5\}$
(b) $\{(x; y) : x > 1\} \cap \{(x; y) : y < -1\}$

Solution of inequalities

Solution sets

Consider a bus which can hold 46 people. At any one time there may be x people in the bus. If the bus is full, then $x = 46$. This is an equation.

If the bus is not full, then $x < 46$. This is an inequality.

The equation has only one solution: $x = 46$. The inequality has many solutions: if $x < 46$, then x can have the values 0; 1; 2; 3; ...; 44; 45. Notice that negative and fractional values of x are impossible in this example. The set of values of x which make $x < 46$ true is called the **solution set** of the inequality.

If $x < 46$, then $x \in \{0; 1; 2; 3; \ldots; 45\}$. The solution set is $\{0; 1; 2; \ldots; 45\}$.

Equations may also have solution sets. The solution set of $x = 46$ is $\{46\}$.

Inequalities are solved in much the same way as equations using the balance method. However, there is one important exception; this will be shown in Examples 10 and 11. Meanwhile, read the following examples carefully.

Example 8

Solve $6 \leqslant 2x - 1$ and show the solution on a line graph.

$$6 \leqslant 2x - 1$$

Add 1 to both sides.

$$7 \leqslant 2x$$

Divide both sides by 2.

$$\frac{7}{2} \leqslant \frac{2x}{2}$$

$$\Leftrightarrow 3\tfrac{1}{2} \leqslant x$$

If $3\frac{1}{2} \leqslant x$, then $x \geqslant 3\frac{1}{2}$.

$x \geqslant 3\frac{1}{2}$ is the solution of $6 \leqslant 2x - 1$. Fig. 9.10 overleaf is the graph of the solution set.

Fig. 9.10

Note: Arrange for the unknown variable to be positioned on the left-hand side of the inequality. This convention makes it easier to sketch the graph.

Example 9

Given that x is an integer, find the solution set of $3x - 3 > 7$.

$3x - 3 > 7$

Add 3 to both sides.

$3x > 10$

Divide both sides by 3.

$x > 3\frac{1}{3}$

Since x is an integer,

$x \in \{4; 5; 6; \ldots\}$

Therefore

$\{4; 5; 6; \ldots\}$ is the solution set of the equation $3x - 3 > 7$.

Exercise 9f

1 Solve the following inequalities. Sketch a line graph of each solution.

(a) $x - 2 < 3$ (b) $x + 3 \geqslant 6$
(c) $2 > x - 4$ (d) $7 < x + 2$
(e) $x + 9 \leqslant 3$ (f) $0 > x + 5$
(g) $2x < 6$ (h) $5x \geqslant 45$
(i) $12 \geqslant 3x$ (j) $-10 < 5x$
(k) $4x \geqslant -9$ (l) $8 \leqslant 3x$
(m) $3x + 1 < 13$ (n) $5x - 2 \geqslant 8$
(o) $-5 > 4x + 15$ (p) $3 \leqslant 17 + 2x$
(q) $4x - 2 > 19$ (r) $3 \leqslant 3x + 5$

2 Find the solution sets of the following inequalities, given that x is an integer in each case.

(a) $2x > 9$ (b) $3x < 7$
(c) $4x < -11$ (d) $4x > -14$
(e) $3 > 5x$ (f) $-8 < 3x$
(g) $2x + 1 < 12$ (h) $5x - 7 > 9$
(i) $7x > 5x - 9$ (j) $8x < 5x - 10$
(k) $6 > 4x + 1$ (l) $3x + 20 > 4$
(m) $2x + 4 \leqslant 2$ (n) $5x - 8 \geqslant 12$
(o) $1 \geqslant 6x - 11$ (p) $3x - 8 \leqslant 5x$
(q) $8x + 16 \leqslant 0$ (r) $x + 4 \geqslant 10x - 23$

Multiplication and division by negative numbers

Consider the following true statement: $5 > 3$. Multiply both the 5 and the 3 by -2 to obtain -10 and 6. Clearly $-10 < -6$, so multiplication by a negative number reverses the inequality. Similarly if 5 and 3 are both divided by -2, they become $-2\frac{1}{2}$ and $-1\frac{1}{2}$ respectively. But $-2\frac{1}{2} < -1\frac{1}{2}$; again the inequality is reversed.

In general, if both sides of an inequality are multiplied or divided by a negative number, the inequality sign must be reversed. For example, if $-2x > 10$ is true, then, on division throughout by -2, $x < -5$ will be true.

Example 10

Solve $5 - x > 3$.

Either:

$5 - x > 3$

Subtract 5 from both sides

$-x > -2$

Multiply by -1 and reverse the inequality.

$(-1) \times (-x) < (-1) \times (-2)$

$\Leftrightarrow \quad x < 2$

or:

$5 - x > 3$

Add x to both sides.

$5 > 3 + x$

Subtract 3 from both sides

$2 > x$

$\Leftrightarrow \quad x < 2$

The second method in Example 10 shows that the rule of reversing the inequality sign when multiplying by a negative number gives correct results.

Example 11

Solve $19 \geqslant 4 - 5x$.

$19 \geqslant 4 - 5x$.

Subtract 4 from both sides.

$15 \geqslant -5x$

Divide by -5 and reverse the inequality.

$$\frac{15}{-5} \leqslant \frac{-5x}{-5}$$

$-3 \leqslant x$

$\Leftrightarrow \quad x \geqslant -3$

Exercise 9g

Solve the following inequalities.

1 $-2x < 8$	**2** $-3a < -6$
2 $12 \geqslant -4m$	**4** $40 \leqslant -5d$
5 $3 - y \leqslant 7$	**6** $5 - z \geqslant 1$
7 $5 - 2a > 1$	**8** $2 - n \leqslant 3$
9 $2r \geqslant 5r + 6$	**10** $9 \leqslant 3 - 4t$

Word problems involving inequalities

Example 12

A triangle has sides of x cm, $(x + 4)$ cm *and* 11 cm, *where* x *is a whole number of* cm. *If the perimeter of the triangle is less than* 32 cm, *find the possible values of* x.

Perimeter of triangle $= x + (x + 4) + 11$

Thus, $x + (x + 4) + 11 < 32$

$\Leftrightarrow \quad 2x + 15 < 32$

$\Leftrightarrow \quad 2x < 17$

$\Leftrightarrow \quad x < 8\frac{1}{2}$

Also, in any triangle the sum of the lengths of any two sides must be greater than the length of the third side.

Thus, $x + (x + 4) > 11$

$\Leftrightarrow \quad 2x + 4 > 11$

$\Leftrightarrow \quad 2x > 7$

$\Leftrightarrow \quad x > 3\frac{1}{2}$

Hence $x < 8\frac{1}{2}$ and $x > 3\frac{1}{2}$. But x must be a whole number of cm. Thus the possible values of x are 4; 5; 6; 7; or 8.

Check:

When $x = 4$, perimeter $= 4+8+11$ cm $= 23$ cm

When $x = 8$, perimeter $= 8+12+11 = 31$ cm

The lowest and highest values of x have been checked. The perimeters in both cases are less than 32 cm. There is no need to check the other values.

Exercise 9h

In each question, first make an inequality, then solve the inequality.

1 If 9 is added to a number x, the result is greater than 17. Find the values of x.

2 If 7,3 is subtracted from y, the result is less than 3,4. Find the values of y.

3 Three times a certain number is not greater than 54. Find the range of values of the number.

4 5 times a whole number, x, is subtracted from 62. The result is less than 40. Find the three lowest values of x.

5 x is a whole number. If $\frac{3}{4}$ of x is subtracted from 1, the result is always greater than 0. Find the four highest values of x.

6 A man gets a monthly pay of \$x. His monthly rent is \$80. After paying his rent, he is left with less than \$200. Find the range of values of x.

7 A book contains 192 pages. A student reads x complete pages every day. If she has not finished the book after 10 days, find the highest possible value of x.

8 A rectangle is x cm long and 10 cm wide. Find the range of values of x if the area of the rectangle is not less than 120 cm^2.

9 A triangle has a base of length 6 cm and an area of less than 12 cm^2. What can be said about its height?

10 Last month a woman had a mass of 53 kg. She reduced this by x kg so that her mass is now below 50 kg. Assuming that $x < 6$, find the range of values of x.

11 A rectangle is 8 cm long and b cm broad. Find the range of values of b if the perimeter of the rectangle is not greater than 50 cm and not less than 18 cm.

12 The sides of a triangle are x cm, $x + 3$ cm and 10 cm. If x is a whole number of cm, find the lowest value of x.
Hint: the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

13 An isosceles triangle has sides of length x cm, x cm and 9 cm. Its perimeter is less than 24 cm and x is a whole number.
(a) Find the lowest value of x.
(b) Find the highest value of x.

14 On a journey of 120 km, a motorist averages less than 60 km/h. Will the journey take more or less than 2 hours?

15 A man cycles a distance of 63 km in less than 3 hours. What is his average speed?

Chapter 10

Similarity (1)

Similarity

Look at the photographs in Fig. 10.1.

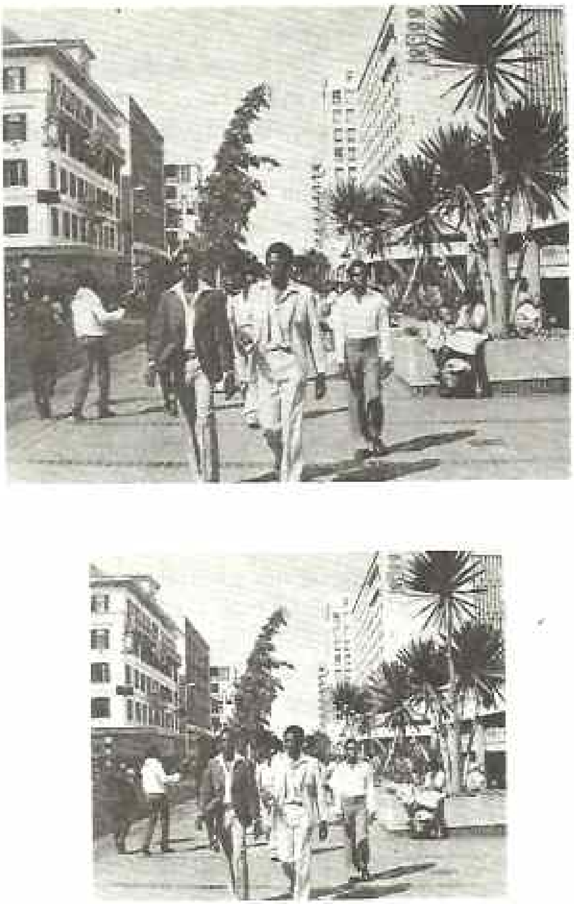
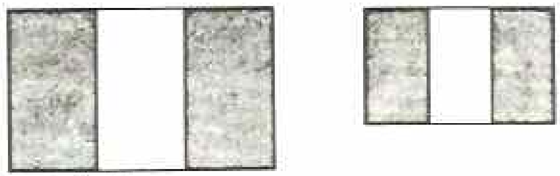


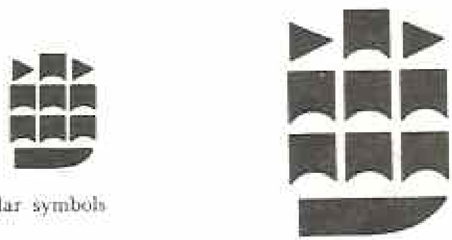
Fig. 10.1

Both photographs show the same picture but the pictures are different sizes. We say that the pictures are **similar** to each other. Fig. 10.2 shows further examples of similar shapes.

(a) similar cuboids



(b) similar flags



(c) similar symbols

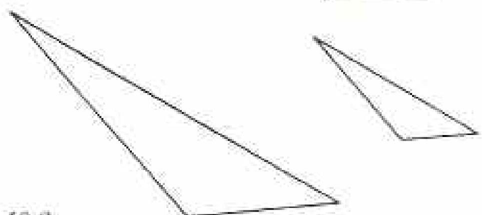


Fig. 10.2

Notice that the pairs of shapes on page 74 are the same but their sizes are different. Fig. 10.3 shows two rectangles ABCD and PQRS.

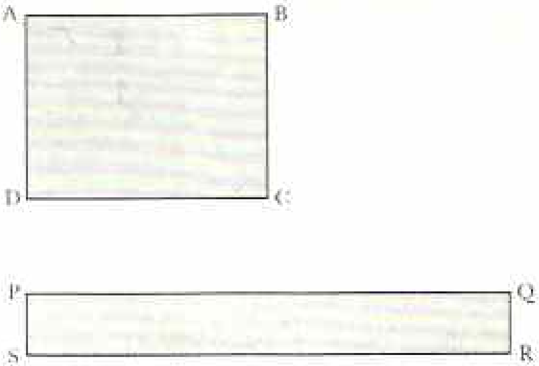


Fig. 10.3

ABCD has a shape like that of an envelope. PQRS has a shape like that of a ruler. Although the shapes are both rectangles they are *not* similar. PQRS is long and thin, but ABCD is not.

Example 1

(a) Measure the length and breadth of the flags in Fig. 10.2(b). (b) Find the ratio length: breadth *for each flag. (c) What do you notice?*

(a) Large flag:
length = 35 mm, breadth = 21 mm
Small flag:
length = 25 mm, breadth = 15 mm

(b) Large flag: $\frac{\text{length}}{\text{breadth}} = \frac{35}{21} = \frac{5}{3}$

Small flag: $\frac{\text{length}}{\text{breadth}} = \frac{25}{15} = \frac{5}{3}$

(c) The ratio of corresponding sides in each figure is the same, $\frac{5}{3}$.

Example 2

(a) Measure the lengths of all the sides of both shapes in Fig. 10.4. What do you notice? (b) Measure the angles of the shapes in Fig. 10.4. (c) Are the two shapes similar? Give reasons.

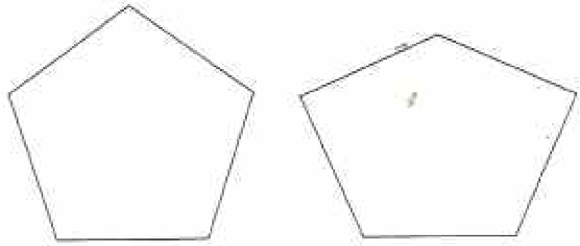


Fig. 10.4

(a) In both shapes, the sides are each 2 cm. Both shapes have sides of the same length.

(b) Each angle in the first shape = 108°
In the second figure there are 2 angles of 114°, 2 angles of 90° and 1 angle of 132°.

(c) The shapes are *not* similar. Although the sides in both figures are of equal length, their angles are different.

Example 3

(a) Use measurement to find the ratio longest side: shortest side *for the two triangles in Fig. 10.5. What do you notice? (b) Measure the angles of both triangles. What do you notice? (c) Are the two triangles similar? Give reasons.*

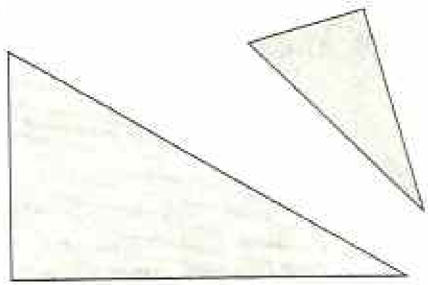


Fig. 10.5

(a) Large triangle:
$\frac{\text{longest side}}{\text{shortest side}} = \frac{6 \text{ cm}}{3 \text{ cm}} = \frac{2}{1}$

Small triangle:
$\frac{\text{longest side}}{\text{shortest side}} = \frac{32 \text{ mm}}{16 \text{ mm}} = \frac{2}{1}$

The ratio of corresponding sides in each triangle is the same.

(b) In each triangle the angles are 30°, 60°, 90°. The angles are the same in both triangles.

(c) The triangles are similar. Their angles are equal and their corresponding sides are in the same ratio.

Notice in Example 3 that the triangles are similar even although one of the triangles has been turned round.

From Examples 1, 2 and 3 we can conclude that two shapes are similar if corresponding angles are equal *and* if corresponding sides are in the same ratio.

Exercise 10a

1 (a) Measure the heights of the cuboids in Fig. 10.2(a) and find the ratio of the two heights. You will find Fig. 10.2 on page 74.

(b) Measure the widths of the cuboids in Fig 10.2(a). Find the ratios of the two widths.

2 Measure the angles of the triangles in Fig. 10.2(d). What do you notice?

3 Use a ruler to find the ratio *longest side: smallest side* for each of the triangles in Fig. 10.2(d). What do you notice?

4 (a) Write down the sizes of the angles of the two quadrilaterals ABCD and PQRS in Fig. 10.3 on page 75. What do you notice?

(b) Use a ruler to find the ratio $\frac{AB}{BC}$ and the ratio $\frac{PQ}{QR}$ for the rectangles in Fig. 10.3. What do you notice?

5 (a) Measure the length and breadth of (i) this text book, (ii) your desk top. Are the two shapes similar?

(b) If not similar, make a scale drawing of a shape which is similar to the shape of your desk.

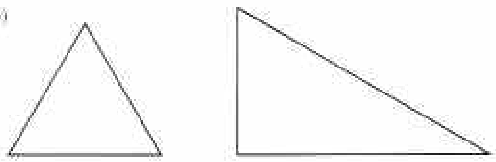
6 Compare the shape of a matchbox with that of a chalkbox. Are the shapes of the two boxes similar?

7 Each of the triangles in Fig. 10.5 is similar to the shape of a 30°/60° set square. Use a 30°/60° set square to check that the two triangles are similar. (You may have to slide the set square and turn it over.)

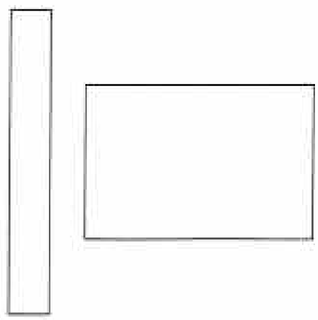
8 Look at each pair of diagrams in Fig. 10.6. Say whether they are similar or not. You may use measurement to help you if you are not sure.

(a)

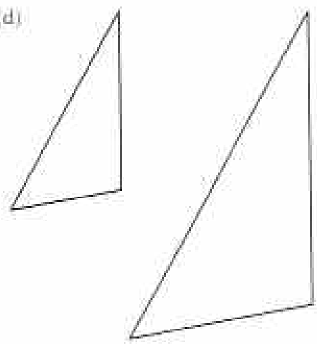
(b)



(c)



(d)



(e)

Fig. 10.6

Similar triangles

Fig. 10.7 shows two similar triangles ABC and DEF.

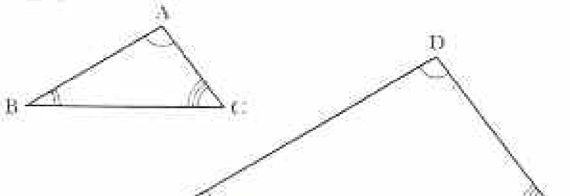


Fig. 10.7

Notice that Â, B̂, Ĉ in △ABC are equal respectively to D̂, Ê, F̂ in △DEF. The two triangles are **equiangular**. This means that the angles of one are equal to the angles of the other. Equiangular triangles are always similar. This is true even if one of the triangles is turned round as in Fig. 10.8.

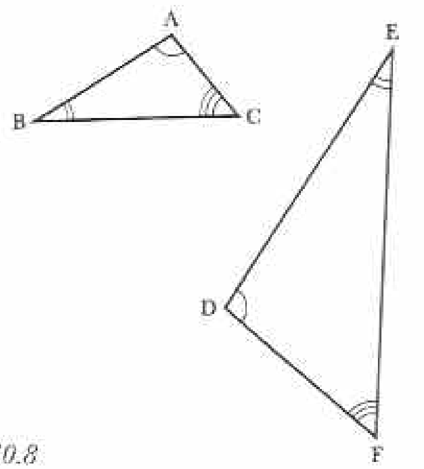


Fig. 10.8

In Fig. 10.9 △ABC is similar to △DEF.

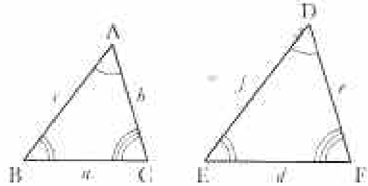


Fig. 10.9

Since the two triangles are alike in everything but size, it can be said that △DEF is a scale drawing of △ABC. Thus, if AB is $\frac{2}{3}$ of DE, then BC is $\frac{2}{3}$ of EF and CA is $\frac{2}{3}$ of FD. Hence AB and DE, BC and EF, CA and FD are all in the ratio 2:3,

i.e. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2}{3}$

It is quite common to use small letters for the lengths of sides of a triangle. For example side BC is opposite A; its length is *a* units. Thus,

$\frac{c}{f} = \frac{a}{d} = \frac{b}{e} = \frac{2}{3}$

Also, since $\frac{c}{f} = \frac{a}{d}$,

multiplying both sides by *fd*:

$\frac{c}{f} \times fd = \frac{a}{d} \times fd$

$\Leftrightarrow \quad cd = af$

Dividing both sides by *ad*:

$$\frac{cd}{ad} = \frac{af}{ad}$$

$$\Leftrightarrow \quad \frac{c}{a} = \frac{f}{d}$$

i.e. $\frac{AB}{BC} = \frac{DE}{EF}$

Similarly $\frac{BC}{CA} = \frac{EF}{FD}$ and $\frac{CA}{AB} = \frac{FD}{DE}$

These results can be more easily seen by taking an example with numbers. See Fig. 10.10 on page 78.

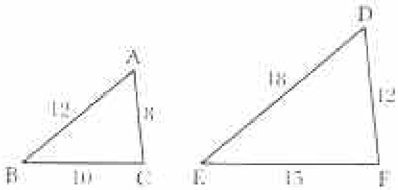


Fig. 10.10

If AB, BC, CA are 12, 10, 8 cm respectively, then, since AB is $\frac{2}{3}$ of DE, etc., DE, EF, FD are 18, 15, 12 cm respectively.

Hence $\frac{AB}{BC} = \frac{12}{10} = \frac{6}{5}$ and $\frac{DE}{EF} = \frac{18}{15} = \frac{6}{5}$

$\Leftrightarrow \quad \frac{AB}{BC} = \frac{DE}{EF}$

Similarly $\frac{BC}{CA} = \frac{EF}{FD}$ and $\frac{CA}{AB} = \frac{FD}{DE}$

Example 4

In Fig. 10.11, show that the two triangles are similar. Name the triangles, giving the letters in corresponding order. Hence calculate BC *and* RQ.

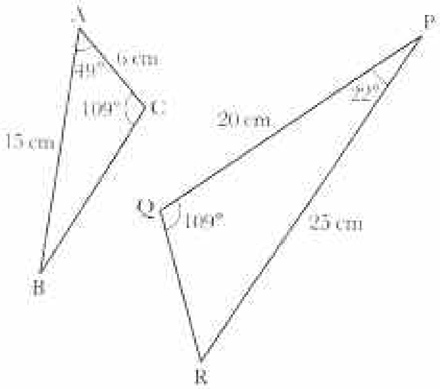


Fig. 10.11

$\hat{B} = 180° - 49° - 109° = 22°$
$\hat{R} = 180° - 22° - 109° = 49°$

Since the triangles are equiangular, they are similar. A corresponds to R, B to P and C to Q. Hence △ABC is similar to △RPQ.

Thus $\frac{AB}{RP} = \frac{AC}{RQ} = \frac{BC}{PQ}$

$\frac{15}{25} = \frac{6}{RQ} = \frac{BC}{20}$

$\Leftrightarrow \quad \frac{3}{5} = \frac{6}{RQ}$ and $\frac{3}{5} = \frac{BC}{20}$

$\Leftrightarrow \quad 3RQ = 30$ and $5BC = 60$

$\Leftrightarrow \quad RQ = 10$ cm and $BC = 12$ cm

Example 5

In Fig. 10.12, calculate MN *and* MY.

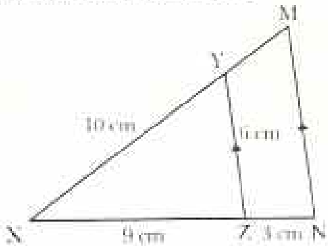


Fig. 10.12

Since YZ ∥ MN, $X\hat{Y}Z = X\hat{M}N$ and $X\hat{Z}Y = X\hat{N}M$ (corresponding angles). Thus △s XYZ and XMN are equiangular and similar.

Thus $\frac{XY}{XM} = \frac{XZ}{XN} = \frac{YZ}{MN}$

$\frac{10}{XM} = \frac{9}{12} = \frac{6}{MN}$

$\Leftrightarrow \quad \frac{10}{XM} = \frac{3}{4}$ and $\frac{6}{MN} = \frac{3}{4}$

$\Leftrightarrow \quad 3XM = 40$ and $3MN = 24$

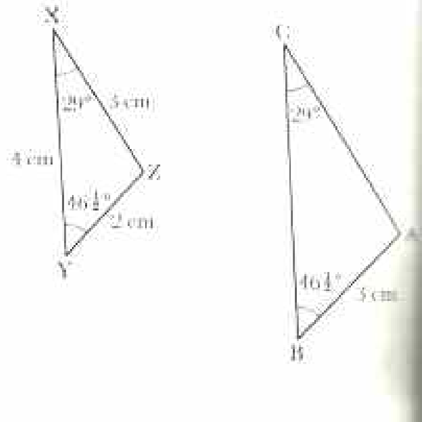
$\Leftrightarrow \quad XM = 13\frac{1}{3}$ cm and $MN = 8$ cm

$\Leftrightarrow \quad YM = 3\frac{1}{3}$ cm and $MN = 8$ cm

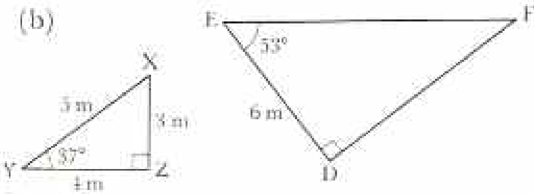
Exercise 10b

1 In each part of Fig. 10.13, (i) name the triangle that is similar to △XYZ, giving the letters in corresponding order, (ii) calculate those sides and angles which are not given.

(a)



(b)



(c)

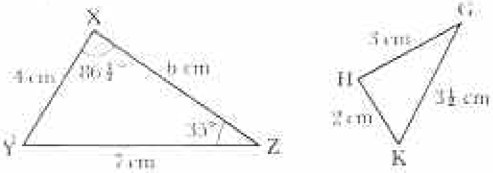


Fig. 10.13

2 Construct △s ABC, PQR as in Fig. 10.14 so that $\hat{B} = \hat{Q} = 35°$, $\hat{C} = \hat{R} = 80°$, BC = 8 cm and QR = 10 cm.

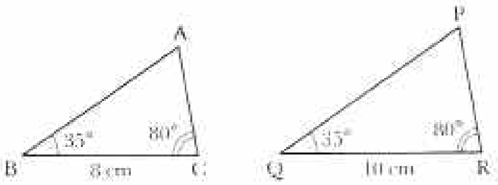


Fig. 10.14

Measure AB, AC, PQ, PR and complete the following:

$\frac{BC}{QR} = \frac{8}{10} = 0{,}8;\quad \frac{AB}{PQ} = \_\_\_\_ = \quad;$

$\frac{AC}{PR} = \_\_ =$

3 Construct triangles with angles as in Fig. 10.14 but with BC = 6 cm, QR = 9 cm. Measure AB, AC, PQ, PR and complete the following in decimal form.

(a) $\frac{AB}{BC} = \frac{\_}{6} = \quad;\qquad \frac{PQ}{QR} = \frac{\_}{9} =$

(b) $\frac{AC}{BC} = \frac{\_}{6} = \quad;\qquad \frac{PR}{QR} = \frac{\_}{9} =$

(c) $\frac{AB}{AC} = \frac{\_}{\_} = \quad;\qquad \frac{PQ}{PR} = \frac{\_}{\_} =$

4 Construct two triangles with sides of 6, 8, 10 cm and 9, 12, 15 cm respectively. Measure all the angles. What can you say about the triangles?

5 In Fig. 10.15, state why △s ABC and ADE are similar. If AB = 8 m, AC = 9 m, BC = 6 m, AD = 12 m, calculate AE and DE.

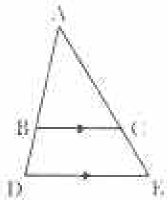


Fig. 10.15

6 In Fig. 10.15 if AB = 8 cm, BD = 2 cm, AC = 10 cm, DE = $6\frac{1}{4}$ cm, calculate AE and BC.

7 In Fig. 10.16, which triangle is similar to △YOQ and why? If OP = 4 m, OX = 7 m, PX = 6 m, YQ = $4\frac{1}{2}$ m, calculate OY and OQ.

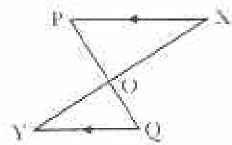


Fig. 10.16

8 In Fig. 10.16, if OP = 6 cm, PX = 9 cm, OY = 6 cm, YQ = $5\frac{2}{3}$ cm, calculate OX and OQ.

9 In Fig. 10.17, name the triangle which is similar to △OAB, and give reasons. If OA = 10 cm, OB = 8 cm, OK = 6 cm, AB = 7 cm, calculate OH and HK.

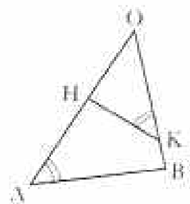


Fig. 10.17

10 In Fig. 10.17, if OH = 9 m, HA = 5 m, HK = 6 m, AB = 8m, calculate OK and KB.

Similar plane shapes and solids

For triangles to be similar it is sufficient that their corresponding angles are equal. However, Figs 10.3 and 10.4 on page 75 show that for all other plane shapes to be similar, their corresponding angles must be equal *and* the ratio of corresponding sides must be constant.

Example 6

Rectangle ABCD *is similar to rectangle* WXYZ. AB = 4 cm *and* WX = 5 cm. *If* BC = 9 cm, *calculate the length of* XY.

Assume that the letters of the rectangles are given in corresponding order.

The ratio $\frac{WX}{AB} = \frac{5}{4}$

The ratio $\frac{XY}{BC}$ must also be $\frac{5}{4}$.

Hence $\frac{XY}{9} = \frac{5}{4}$

$\Leftrightarrow \quad XY = \frac{5 \times 9}{4}$ cm

$= \frac{45}{4}$ cm $= 11\frac{1}{4}$ cm

For two solids to be similar, their corresponding angles must be equal *and* the ratio of the lengths of corresponding edges must be constant.

Example 7

A matchbox is in the shape of a cuboid 6 cm *long*, 3 cm *wide and* 2 cm *high. Matchboxes are packed in a similar box, a cuboid* 45 cm *wide. Calculate the length and breadth of the box.*

Comparing the widths of the matchbox and packing box:

$$\frac{\text{width of packing box}}{\text{width of matchbox}} = \frac{45 \text{ cm}}{3 \text{ cm}} = \frac{15}{1}$$

Each edge on the packing box is 15 times the corresponding edge on the matchbox.
Length of box = 15 × 6 cm = 90 cm
Height of box = 15 × 2 cm = 30 cm

Exercise 10c

1 Fig. 10.18 shows two sets of rectangles.

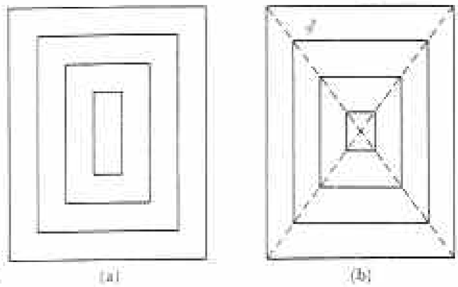


Fig. 10.18

Use a ruler to find which set contains rectangles which are similar to each other.

2 Rectangles ABCD and PQRS are similar. CD = 3 cm and RS = 5 cm. If AD = 12 cm, calculate the length of PS.

3 In rectangles ABCD and WXYZ, AB = 5 cm, BC = 15 cm, WX = 8 cm, XY = 18 cm. Is ABCD similar to WXYZ? Why?

4 A cuboid is 4 cm long, 7 cm wide and 10 cm high. A similar cuboid is 25 cm high. Calculate its length and width.

5 A small tin is of height 12 cm and diameter 10 cm. If the radius of a similar large tin is 7,5 cm, calculate its height.

6 Write down the dimensions of any two cubes. Are the two cubes similar?

7 A car is 4,30 m long and 1,72 m wide. If a toy model of the car is 2,6 cm long, calculate the width of the model.

8 A water tank is in the shape of a cuboid 2 m high, 3 m wide and 4 m long. A similar tank is 1,2 m high. Calculate the width and length of the smaller tank.

9 A rectangle is such that the lengths of two of its adjacent sides are in the ratio 1:3. A similar rectangle has one side of length 6 cm. Find two possible values for the length of its adjacent side.

10 State whether the following are *true* or *false*.
(a) all equiangular triangles are similar
(b) all isosceles triangles are similar
(c) all equilateral triangles are similar
(d) all squares are similar
(e) all rectangles are similar
(f) all cubes are similar
(g) all cuboids are similar

Chapter 11

Statistics (2) Averages, graphs

Averages

The **average** of a set of numbers is a very important statistic. The average is typical of the set of numbers and, therefore, provides information about them. For example:

(a) If a football team's **average score** is 5,2 goals, we know that the team is good at scoring goals.

(b) If two classes have **average ages** of 8,7 years and 16,9 years, we expect that the first is a primary school class and the second is a secondary school class.

(c) If the **average life** of a battery is 20 hours, we expect a new battery to last about 20 hours, maybe a little more or a little less.

Arithmetic mean

The arithmetic mean, or just **mean** for short, is the most common average. The averages in (a), (b) and (c) above are all examples of arithmetic means.

If there are n numbers in a set, then,

$$\text{arithmetic mean} = \frac{\text{sum of the numbers in the set}}{n}$$

Example 1

In 5 tests a student's marks were 13, 17, 18, 8 *and* 10. *What is her average mark?*

$$\text{Average (mean) mark} = \frac{13 + 17 + 18 + 8 + 10}{5}$$

$$= \frac{66}{5} = 13,2$$

Example 2

A hockey team has played 8 *games and has a mean score of* 3,5 *goals per game. How many goals has the team scored?*

$$\text{Mean score} = \frac{\text{total number of goals}}{\text{number of games}}$$

$$3,5 = \frac{\text{total number of goals}}{8}$$

Multiply both sides by 8

$$3,5 \times 8 = \text{total number of goals}$$

Total number of goals scored = 28

Exercise 11a

1 Calculate the mean of the following sets of numbers.

(a) 9; 11; 13 (b) 7; 8; 12

(c) 1; 9; 4; 6 (d) 15; 3; 5; 9

(e) 1; 8; 6; 8; 7 (f) 4; 6; 2; 1; 7

(g) 5; 12; 3; 9; 10; 3

(h) 8; 9; 11; 12; 15; 17

(i) 3; 1; 9; 8; 2; 3; 0; 7; 2; 5

(j) 8; 2; 3; 1; 7; 8; 8; 4; 1; 1

2 Calculate the mean of the following.

(a) 4 cm; 7 cm; 1 cm; 6 cm

(b) \$6; \$7; \$7; \$9; \$12

(c) 3,9 kg; 5,2 kg; 5,3 kg

(d) $1\frac{1}{4}$, $3\frac{3}{4}$, $4\frac{3}{4}$

(e) 0,9; 0,8; 0,6; 0,4; 0,9; 1,1; 0,2; 0,3; 0,5; 0,6

3 A market trader's profit after five days of trading was \$191,55. Calculate her mean profit per day.

4 On six working days a garage mended 6; 5; 2; 0; 3; 2 punctures. Calculate the mean number of punctures mended per day.

5 In four successive days a trader sold 24, 48, 12 and 60 oranges. Calculate her mean daily sale of oranges.

6 The temperatures at midday during a week in Dete were 23°, 25°, 24°, 26°, 25°, 26°, 26°C. Find, to the nearest degree, the average midday temperature for the week.

7 In the first six days of the month of June, the rainfall was 39 mm, 21 mm, 17 mm, 11 mm, 0 mm, 2 mm. It didn't rain on any of the other days of the month. Calculate (a) the mean daily rainfall for the first six days, (b) the mean daily rainfall for the whole month. (June has 30 days.)

8 After 15 matches a football team's goal average was 1,8. How many goals has the team scored?

9 The average age of a mother and her three children is 10 years. If the ages of the children are 1, 4 and 7 years, how old is the mother?

10 If the 'average man' has a mass of 77 kg, find, approximately, how many men together have a mass of 1 tonne.

11 In a test out of 40, the marks of 15 students were 31; 18; 6; 26; 36; 24; 23; 14; 29; 28; 32; 9; 11; 22; 21.
   (a) Calculate the mean mark for the test.
   (b) Express the mean mark as a percentage.

12 Ten *Atlas* batteries were tested to find their average life. The times, in hours, that the batteries lasted were as follows: 10,8; 10,6; 11,4; 8,9; 10,1; 10,6; 9,8; 12,6; 10,5; 11,9.
   (a) Find, to the nearest tenth of an hour, the average life of the ten batteries.
   (b) Which of the following advertisements is more accurate?
   (i) *Atlas batteries* are guaranteed to last 10 hours.
   (ii) *Atlas batteries* have an average life of over 10 hours.

The median

The **median** of a set of numbers is the *middle* number when the numbers are arranged in order of size.

Example 3

Find the median of 17; 34; 13; 22; 27; 44; 8; 31;

Arrange the numbers in order of increasing size.

8; 13; 13; 17; 22; 27; 31; 34; 44

There are 9 numbers. The 5th number is in the middle. The 5th number is 22. Median = 22. Note that the result would be the same if the numbers were arranged in order of decreasing size (i.e. rank order). Also notice that every number is written down even if some numbers appear more than once. In Example 3, there are two 13's; each is written down and counted.

If there is an even number of terms in the set, find the mean of the middle two terms. Take this to be the median.

Example 4

Find the median of 8,3; 11,3; 9,4; 13,8; 12,9; 10,5

Arrange the set of numbers in order of size:
8,3; 9,4; 10,5; 11,3; 12,9; 13,8
There are 6 numbers. The median is the mean of the 3rd and 4th numbers:

$$\text{median} = \frac{10{,}5 + 11{,}3}{2} = \frac{21{,}8}{2} = 10{,}9$$

The mode

In many examples of statistical data, some numbers appear more than once. The **mode** is the number that appears *most often*. In Example 3, the number 13 appears twice. 13 is the mode of this data.

Example 5

The following are the number of days absent during a term for a class of 21 *students:* 7; 5; 0; 5; 0; 3; 0; 15; 0; 2; 2; 0; 1; 3; 5; 32; 1; 0; 0; 1; 2. *Find the mode, median and mean days absent.*

Arrange the absences in order: 0; 0; 0; 0; 0; 0; 0; 1; 1; 1; 2; 2; 2; 3; 3; 5; 5; 5; 7; 15; 32
0 appears most often. Mode = 0 days.
The median is the 11th number.
Median = 2 days.

$$\text{mean} = \frac{\text{total number of days absent}}{\text{total number of students}}$$

$$= \frac{0 + 0 + \ldots + 15 + 32}{21}$$

$$= \frac{84}{21} = 4 \text{ days}$$

The **frequency** is the number of times that a piece of data appears. Hence the mode is the piece of data with the greatest frequency.

Example 6

The distribution of ages of a group of 30 Teacher Training College students is given in Table 11.1.

Table 11.1

age in years	20	21	22	23
frequency	4	11	9	6

Find the mode, median and mean ages of the students.

(a) The greatest frequency is 11. 11 students are aged 21. The mode is 21 years; we can say the **modal** age is 21 years.

(b) There are 30 students. The median age is the mean of the ages of the 15th and 16th students. In this case it is not necessary to make an ordered list of all the ages. Since there are 4 students aged 20 years and 11 students aged 21 years, the 15th student is aged 21 years (4 + 11 = 15). The 16th student is the first of the 22 year age group.

$$\text{Median age} = \frac{21 + 22}{2} = 21{,}5 \text{ years}$$

(c) $$\text{Mean age} = \frac{\text{total ages of all the students}}{\text{number of students}}$$

Using the frequency table,
4 students are aged 20:
sum of their ages = 20 × 4 = 80 years
11 students are aged 21:
sum of their ages = 21 × 11 = 231 years
9 students are aged 22:
sum of their ages = 22 × 9 = 198 years
6 students are aged 23:
sum of their ages = 23 × 6 = 138 years

$$\text{Mean age} = \frac{80 + 231 + 198 + 138}{30}$$

$$= \frac{647}{30} = 21{,}57 \text{ years}$$

Notice, in Example 6, that the mode, median and mean are quite close in value to each other. This usually happens when the frequency figures rise and fall fairly smoothly. It is possible for some of these averages to be equal to each other.

Exercise 11b

1 Find the mode, median and mean of the following sets of numbers.
(a) 7; 7; 9; 12; 15
(b) 4; 5; 5; 7; 8; 10
(c) 4; 8; 11; 11; 12; 12; 12
(d) 2; 3; 6; 6; 7; 7; 7; 8; 8; 9
(e) 15; 13; 13; 12; 11; 11; 10; 10; 10; 10; 9; 9; 8; 8; 7

2 Arrange the following numbers in order of size. Find the mode, median and mean of each set.
(a) 2; 4; 3; 4
(b) 7; 5; 2; 9; 5; 8
(c) 1; 0; 14; 0; 5; 10
(d) 7; 5; 11; 7; 12; 8; 6; 9; 7
(e) 6; 5; 3; 6; 3; 2; 4; 6; 4; 5; 6; 4

3 In a test, the grades go from A (best) to E (poorest). The bar chart in Fig. 11.1 shows the number of students getting each of these grades.

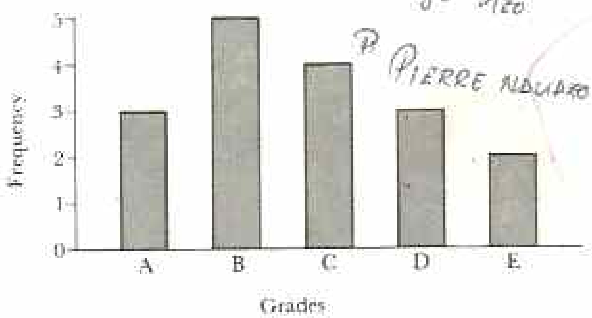


Fig. 11.1

Find (a) the number of students who took the test; (b) the mode for the test; (c) the median grade for the test.

4 16 people were asked which size of shoe they took. Their answers are shown in a frequency table (Table 11.2).

Table 11.2

shoe size	5	6	7	8	9	10
frequency	1	2	5	4	3	1

Find (a) the modal shoe size, (b) the median shoe size.

5 Ten students walk to school each day. The distances they walk, to the nearest kilometre, are given in a frequency table (Table 11.3).

Table 11.3

distance in km	1	2	3	4	5
frequency	4	2	2	1	1

Find the mode, median and mean distances walked.

6 Table 11.4 gives the ages and frequencies of girls in a choir.

Table 11.4

age in years	14	15	16	17
frequency	3	4	5	4

Find (a) the number of girls in the choir; (b) the modal and median ages of the choir; (c) the mean age of the choir.

Example 7

Nine boys take a test. Their marks are Nda 4, Ben 8, Dan 7, Rex 9, Joe 4, Ron 3, Bob 6, Tom 4, Sam 7. Place the marks in rank order. Hence find the mean, median and mode for the test.

Table 11.5 gives the names, marks and positions of the boys in rank order, i.e. from highest mark to lowest mark.

Table 11.5

name	mark	position
Rex	9	1
Ben	8	2
Dan	7	3 =
Sam	7	3 =
Bob	6	5
Nda	4	6 =
Joe	4	6 =
Tom	4	6 =
Ron	3	9

$$\text{mean} = \frac{9 + 8 + 7 + 7 + 6 + 4 + 4 + 4 + 3}{9}$$

$$= \frac{52}{9} = 5\tfrac{7}{9}$$

Bob is in the middle position. His mark is 6.
Median = 6.
4 marks is the most common score.
Mode = 4.

Example 8

A woman buys three bottles of cooking oil at $1,90 per bottle. Next week she buys 2 bottles of the same oil at $2,00 per bottle. Find the average cost per bottle for the two weeks.

3 bottles at $1,90 cost 3 × $1,90 = $5,70
2 bottles at $2 cost 2 × $2 = $4,00

Total cost of 5 bottles = $9,70

Average cost of 1 bottle $= \frac{\$9{,}70}{5}$

= $1,94

Example 9

A motorist travelled 96 km *at an average speed of* 60 km/h. *She returned at an average speed of* 48 km/h. *What was her average speed for the whole journey?*

96 km at 60 km/h takes $\frac{96}{60}$ hours = 1,6 h

96 km at 48 km/h takes $\frac{96}{48}$ hours = 2,0 h

Altogether she travelled 192 km in 3,6 h.

$$\text{Average speed} = \frac{192}{3,6}\text{ km/h} = \frac{1\,920}{36}\text{ km/h}$$

$$= \frac{160}{3}\text{ km/h} = 53\tfrac{1}{3}\text{ km/h}$$

Always remember that

$$\textbf{average speed} = \frac{\textbf{total distance travelled}}{\textbf{total time taken}}$$

Example 10

Find the median of the following percentages.
43%, 76%, 64%, 37%, 76%, 54%.

First, arrange the percentages in rank order.
76%, 76%, 64%, 54%, 43%, 37%
The median is the middle value. However, there is no single middle value in the list. The 3rd and 4th percentages are in the middle. Take the median as the mean of these values.

$$\text{Median} = \frac{64 + 54}{2}\% = \frac{118}{2}\% = 59\%$$

Exercise 11c

1 A car travels 80 km at an average speed of 63 km/h. Which average is this: mean, median or mode?

2 20 people apply for jobs as police officers. There are only 10 jobs available. They are given to the 10 people who are above the average height of those who applied. Which average is this: mean, median or mode?

3 A trader sells shoes. He wants to know the average size of shoe that people buy. Which average is most useful?

4 Arrange the following sets of numbers in order of size. Find the mean, median and mode of each set.
(a) 7; 10; 7; 9; 7
(b) 5; 3; 0; 7; 3; 6
(c) 1; 9; 5; 6; 1; 4
(d) 8; 3; 1; 7; 3; 4; 8; 3; 4; 9; 5
(e) 0,1; 0; 1,5; 0; 0,6; 1,1
(f) 159,5; 155,8; 153,7; 157,2; 155,8

5 The weekly wages of 5 local government trainees are \$62,49, \$69,27, \$56,13; \$87,57; \$80,49. Find (a) the mean, (b) the median wage.

6 6 people have a total mass of $\frac{1}{2}$ tonne. What is the average mass per person in kg?

7 A man caught four fish of mass 1,9 kg, 1,1 kg, 0,9 kg, 0,7 kg. Find (a) the mean, (b) the median mass of the fish.

8 The mean age of 4 people is 19 yr 11 mo. When a fifth person joins them, the average age of all 5 is 20 yr 7 mo. How old is the fifth person?

9 There are 8 men and 1 woman in a boat. The average mass of the 9 people is 79 kg. Without the woman, the average is 81,5 kg. What is the mass of the woman?

10 A trader mixes 3 kg of sugar at 86 c/kg with 2 kg of sugar at 76 c/kg. What is the cost/kg of the mixture?

11 The mean daily rainfall for a week was 5,5 mm. For the first 6 days the mean rainfall was 1 mm. How much rain fell on the 7th day?

12 A bridge $1\frac{1}{4}$ km long cost \$$7\frac{1}{2}$ million. What was the average cost per metre?

13 The ages of a family of six children are 16.4; 14.8; 13.6; 11.10 and the twins are 9.1. Find the mean, median and modal ages of the family. (Take 16.4 to mean 16 years 4 months).

14 A car travelled 30 km at an average speed of 40 km/h. It returned at an average speed of 60 km/h. Find its average speed for the whole journey.

15 A car travelled for $\frac{1}{2}$ hour at an average speed of 40 km/h. For the next $\frac{1}{2}$ hour its average speed was 60 km/h. What was its average speed for the whole time?

16 Of a journey of 216 km, 56 km were on untarred road. For this part of the journey, a motorist could average only 28 km/h. If her average speed for the whole journey was 48 km/h, what was her average speed for the part of the journey on tarred road?

17 A lorry travelled 80 km at 40 km/h, 64 km at 48 km/h and 40 km at 60 km/h. Find its average speed for the whole distance.

18 In a test the average marks for three classes were 74, 58 and 51. If the classes contained 25, 22, and 23 students respectively, what was the average mark for the three classes together?

19 Some students were asked how many brothers and sisters they had. The bar chart in Fig. 11.2 shows the number of students who had 0, 1, 2, 3, 4, 5, 6, 7 or 8 brothers and sisters.

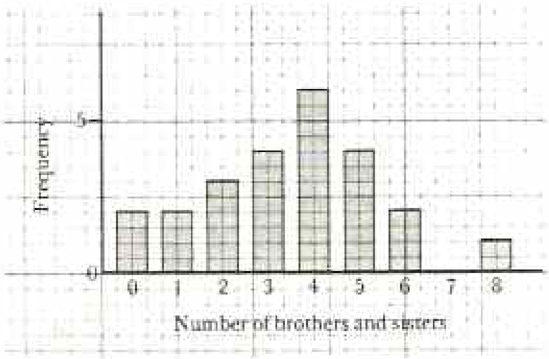


Fig. 11.2

Find (a) the number of students in the survey, (b) the modal number of brothers and sisters, (c) the median, and (d) the mean number of brothers and sisters.

20 Five students took a test in four subjects. Their results are given in Table 11.6.

Table 11.6

	English	History	Maths	Science
Rudo	52	69	54	57
Sola	68	60	67	73
Tembo	80	73	49	42
Urban	26	14	37	35
Vera	34	44	38	48

Find (a) the mean mark of each student, (b) the mean mark in each subject, (c) the median mark in each subject.

Deviation from the mean

The heights of three children are 147 cm, 148 cm and 155 cm. Their mean height is $\frac{1}{3}(147 + 148 + 155)$ cm $= \frac{1}{3}(450)$ cm $= 150$ cm.

147 cm is 3 cm below the mean height. We say that the **deviation** from the mean is -3 cm. Similarly, the deviation of 148 cm is -2 cm and the deviation of 155 cm is $+5$ cm. Notice that the deviations of numbers below the mean are negative and the deviations of numbers above the mean are positive.

Adding the deviations gives $(-3) + (-2) + (+5) = 0$. For any set of numbers, the sum of the deviations from the mean is zero.

Assumed mean

When finding the mean of a set of numbers, it is often easier to estimate the mean and work with deviations of the numbers from this estimation. The estimated mean is called the **assumed mean**. Read Examples 11 and 12 carefully.

Example 11

Find the mean of 121,7; 122,8; 124,2; 125,8; 127,5.

1st step: Look at the given numbers. They go from 121,7 to 127,5. The true mean will lie roughly half-way between those values. Take an assumed mean of 124.

2nd step: Make a column of the given numbers and their deviations from the assumed mean. Find the sum of the deviations from the assumed mean. See Table 11.7.

Table 11.7

assumed mean = 124

	deviation −	deviation +
121,7	2,3	
122,8	1,2	
124,2		0,2
125,8		1,8
127,5		3,5
Sum of deviations	3,5	5,5
		2,0

In Table 11.7, the sum of the negative deviations is 3,5; the sum of the positive deviations is 5,5. This gives a total deviation of $+2,0$ for the 5 numbers.

$$\text{Thus, mean} = 124 + \frac{(+2{,}0)}{5}$$
$$= 124 + 0{,}4$$
$$= 124{,}4$$

When a large set of numbers is given, it is not necessary to list each number separately. The working can be done by making a list of the negative and positive deviations as they arise. See Example 12.

Example 12

The ages of 17 *students in years and months are* 14.9; 15.2; 14.3; 13.9; 14.10; 14.11; 14.8; 15.3; 14.6; 15.6; 15.8; 16.1; 15.4; 14.4; 14.3; 14.9; 15.3. *Find the mean age of the students.*

The ages range from 13.9 to 16.1. Take 15.0 as the assumed mean. Make two columns of numbers. The one marked (−) shows all the deviations in months for ages less than 15.0; the other (+) gives the deviations in months for ages over 15.0.

assumed mean = 15.0

−	+
3	2
9	3
15	6
2	8
1	13
4	4
6	3
8	
9	
3	
60	39
21	

The result shows that the total deviation for the 17 students is 21 months less than 15,0. Thus,

$$\text{mean age} = 15\text{ yr }0\text{ mo} + \left(\frac{-21}{17}\right)\text{ mo}$$
$$\simeq 15\text{ yr }0\text{ mo} - 1{,}2\text{ mo}$$
$$\simeq 14\text{ yr }11\text{ mo}$$

The method in Examples 11 and 12 is useful when given a large set of numbers and when the numbers are of roughly the same size.

Exercise 11d

1 Use the method of Example 11 to find the mean of each of the following sets of numbers.

(a) 146; 147; 151; 153; 154
(b) 36,4; 36,9; 37,3; 38,0; 39,1; 39,7
(c) \$12,20; \$11,40; \$11,70; \$12,10
(d) 35 min, $1\frac{1}{4}$ h, 50 min, $1\frac{1}{3}$ h, $1\frac{1}{2}$ h (Work in minutes.)
(e) 15.8; 15.10; 15.11; 16.3; 16.5; 16.6; 16.7, these numbers being ages in years and months.

2 Use the method of Example 12 to find the mean of each of the following sets of numbers.

(a) 223; 231; 229; 228; 222; 223; 230; 227; 222; 225
(b) 121; 128; 126; 118; 119; 122; 116; 113; 115; 121; 128; 119; 130; 118
(c) 33,4; 33,6; 33,2; 33,1; 33,7; 33,8; 33,5; 33,5; 33,3; 33,7; 33,4; 33,8
(d) 14.2; 14.5; 14.4; 14.0; 13.10; 13.11; 13.9; 14.1; 14.2; 15.1; 13.11; 14.3; 14.4; 13.3; 13.6; 14.1, these numbers being ages in years and months.
(e) \$3,78; \$3,82; \$3,86; \$3,89; \$3,74; \$3,81; \$3,74; \$3,76; \$3,80; \$3,88; \$3,90; \$3,74; \$3,77; \$3,85; \$3,81

Statistical graphs

Bar charts, pie charts, pictograms (revision)

Example 12

Table 11.8 shows how class 2A travel to school.

Table 11.8

method	bicycle	walk	bus	car
frequency	9	15	7	5

Draw (a) a pictogram, (b) a bar chart, (c) a pie chart to show this information.

Table 11.8 is a **frequency table.** The frequency is the number of students using each method of transport.

Fig. 11.3 shows the required graphs.

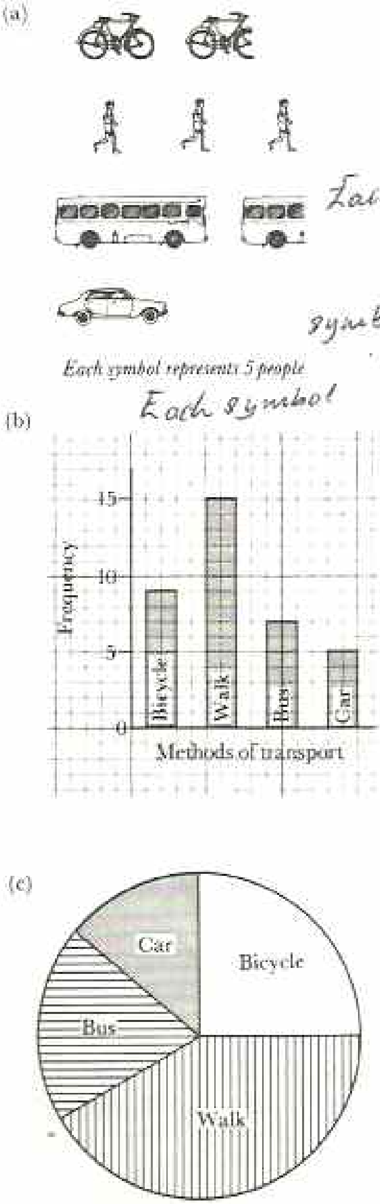


Fig. 11.3

Revision notes:

1 **Pictogram:** each method of transport is represented by a suitable picture. In this case each complete picture represents 5 students.

2 **Bar chart:** each bar represents a method of transport. The height of each bar is proportional to the number of students using that method of transport.

3 **Pie chart:** each sector represents a method of transport. The angle of each sector is proportional to the number of students who use the method of transport shown in that sector.

Line graphs

Fig. 11.4 shows a temperature chart for a hospital patient.

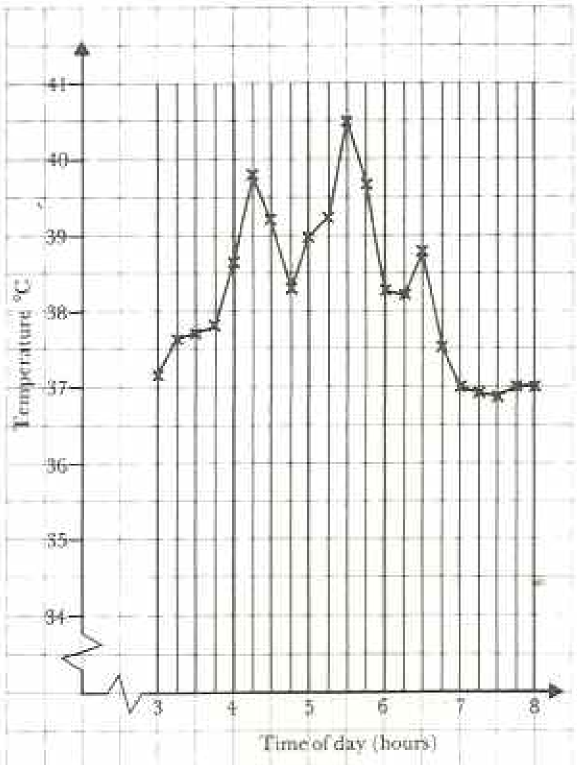


Fig. 11.4

In Fig. 11.4 the temperature is recorded every $\frac{1}{4}$ hour. To make the points easier to see, they are joined by straight lines. In **straight line graphs** of this kind, the lines are given for convenience; they do not really tell us what happens between the points.

Frequency polygons

A **frequency polygon** is a line graph in which points represent frequencies. The points are then joined by straight lines to form a polygon.

Example 2

A class of 30 students scored the following marks in a test.

5 4 7 5 6 8 9 5 6 5 9 7 5 6 5

5 7 5 6 7 8 6 5 8 4 7 5 5 4 6

Use tally marks to make a frequency table. Hence draw a frequency polygon.

Table 11.9 shows how the tally marks are used to find the frequencies of the marks.

Table 11.9

mark	tally	frequency
4	|||	3
5	卌 卌 |	11
6	卌 ||	7
7	||||	4
8	|||	3
9	||	2

Fig. 11.5 is the frequency polygon.

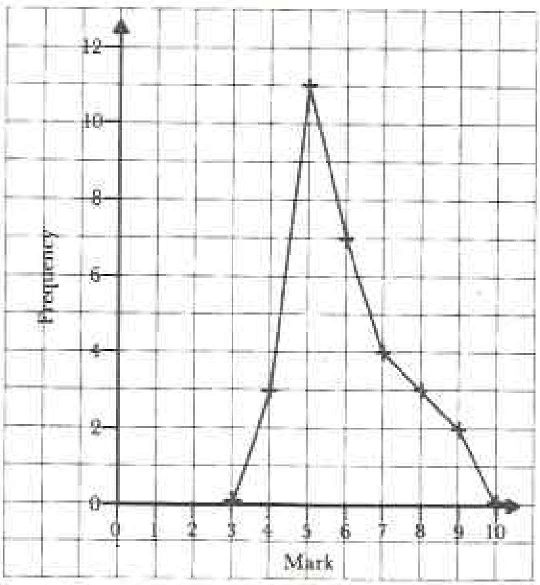


Fig. 11.5

Exercise 11e

1 Fig. 11.6 is a pictogram of a traffic survey.
   (a) Which kind of vehicle is the most common?
   (b) Which kind of vehicle is the least common?
   (c) Approximately how many bicycles were counted?
   (d) Approximately how many lorries were counted?

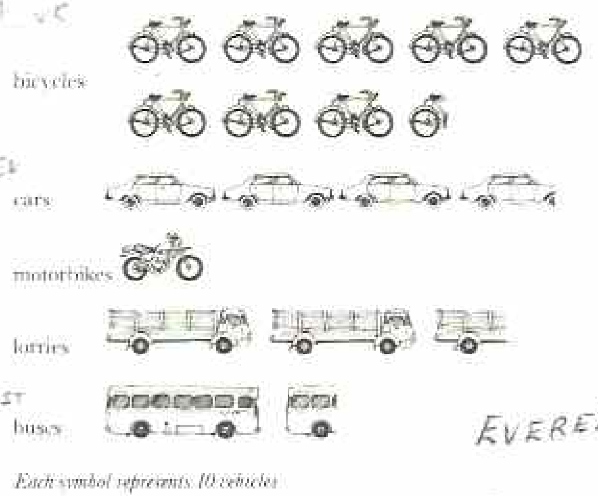


Fig. 11.6

2 Fig 11.7 is a bar chart showing the heights and names of the highest mountains of Asia, South America, North America, Africa and Europe respectively.

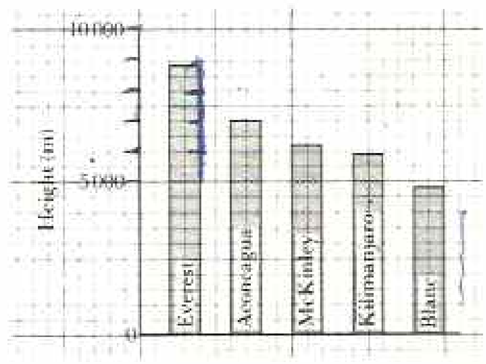


Fig. 11.7

(a) What is the height of Kilimanjaro?
(b) What is the height of Mount McKinley?
(c) Which mountain is 4 800 m high?
(d) Find the difference in height between Everest and Aconcagua?

3 Fig. 11.8 is a pie chart showing how a typist spent the first hour of a day after getting out of bed.

Fig. 11.8

(a) What fraction of the hour was spent eating?
(b) What percentage of the hour was spent washing?
(c) How many minutes were spent travelling to work?

4 In Fig. 11.4 on page 88:
(a) What is the temperature of the patient at 0645?
(b) What is the highest temperature shown?
(c) How many of the recorded temperatures are above 39°C?
(d) At what time is the patient's temperature 39°C exactly?

5 A farmer has 100 cows, 40 sheep and 65 goats. Draw a pictogram to show these animals. (Let one symbol represent 10 animals.)

6 The ages of 36 students are given below.

13 16 16 15 12 14 13 15 16
16 14 15 12 16 13 14 16 12
15 13 15 16 13 13 15 14 16
13 14 16 15 15 12 14 12 13

Make a frequency table and hence represent the age distribution of the students on (a) a bar chart, (b) a frequency polygon.

7 Table 11.10 gives the production of coffee in tonnes in 2-year intervals from 1982 to 1990.

Table 11.10

year	1982	1984	1986	1988	1990
tonnes	750	1 600	3 800	3 200	4 100

Show the increase in production on (a) a bar chart, (b) a line graph.

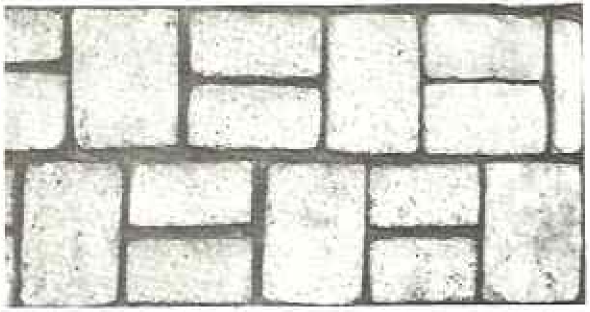
8 During a 24-hour period the sun shone for 10 hours, it was cloudy for 3 hours and it was dark for the remainder of the time. Draw a pie chart to show the periods of sunshine, cloudiness and darkness during the day.

Chapter 12

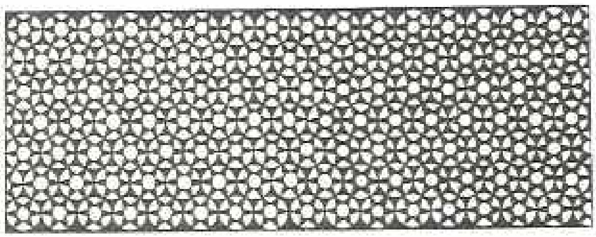
Congruency

Congruency

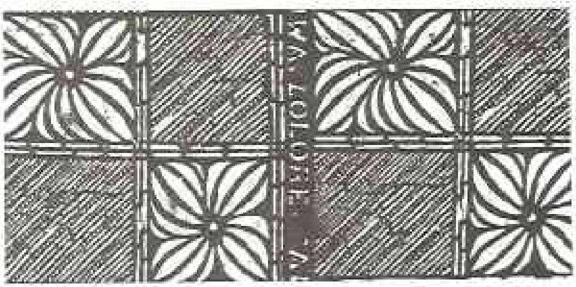
Look at the patterns in Fig. 12.1.



(*a*) *brick wall*



(*b*) *wall pattern*



(*c*) *cloth pattern*

(*d*) *print*

Fig. 12.1

The patterns in Fig. 12.1 all have something in common. They are made by taking a **basic shape** and repeating it to build up the pattern.

Look at the patterns in Fig. 12.2.

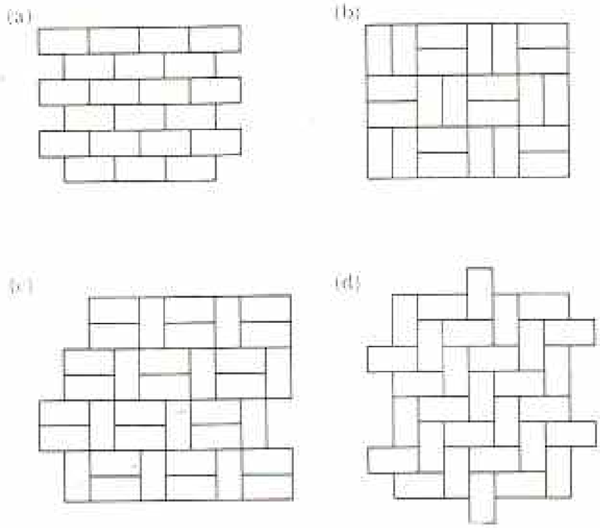


Fig. 12.2

The basic shape which makes each pattern is a 2 × 1 rectangle. The patterns appear different because the rectangles have been arranged in different ways.

Exercise 12a

You will need graph paper and a ruler for this exercise.

1 Copy patterns (*a*), (*b*) and (*c*) of Fig. 12.2 on to graph paper. Make each rectangle 2 cm by 1 cm. Extend each pattern by drawing more rectangles until the patterns are about 10 cm wide by 8 cm long.

2 (a) Name the basic shapes which make the patterns in Fig. 12.3.

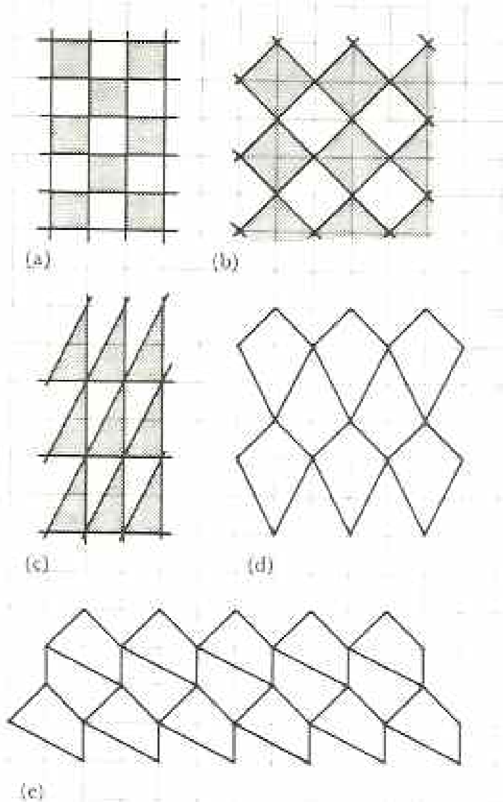


Fig. 12.3

(b) Copy each pattern on to graph paper. Draw more shapes until each pattern is about 8 cm wide and 6 cm long.

When the position or dimensions (or both) of a shape changes, we say that it is **transformed.** The **image** of a shape is the figure which results after transformation (Fig. 12.4).

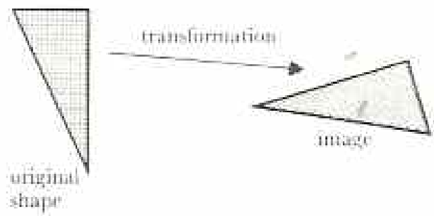


Fig. 12.4

The patterns in Exercise 12a were made by building up images of basic shapes on a plane surface. In every case, each image has the same dimensions as the original given shape. Transformations of this kind are called **congruencies.** Two shapes are congruent if their corresponding dimensions are identical. From this it can be seen that congruency is a special case of similarity. Plane shapes are *similar* if corresponding angles are equal and corresponding sides are in the same ratio. Plane shapes are *congruent* if corresponding angles and sides are equal. This is summarised in Table 12.1.

Table 12.1

	congruent shapes	similar shapes
corresponding angles	equal	equal
corresponding sides	equal	equal ratio

Congruent triangles

Two figures are **congruent** if they have exactly the same shape and size.

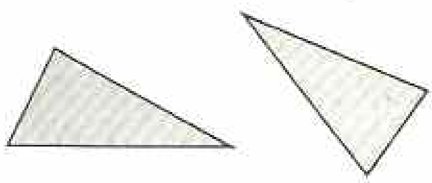


Fig. 12.5

In Fig. 12.5, the two triangles are congruent, although at first sight they may appear different. If one of the triangles were cut out and turned over it could be arranged to fit exactly over the other.

The shape and size of a triangle depend on the sizes of its angles and sides. Given sufficient information, it is possible to draw one, and *only one*, triangle.

Two sides and the included angle

If two triangles LMN and XYZ are drawn in which LM = XY, MN = YZ and $L\hat{M}N = X\hat{Y}Z$, the triangles will be congruent. As Fig. 12.6 shows, this amounts to drawing the same triangle twice with different lettering.

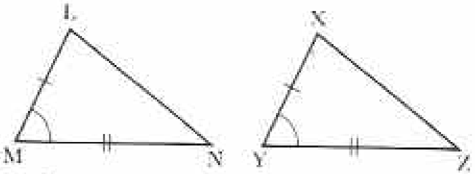


Fig. 12.6

$L\hat{M}N$ is called the **included angle** because it lies between, or is included between, the given sides LM and MN.

Since the triangles are congruent, the other corresponding angles and sides are also equal: $M\hat{L}N = Y\hat{X}Z$, $L\hat{N}M = X\hat{Z}Y$ and LN = XZ.

If the given angle is **not included** between the given sides, it is possible to draw two different triangles. In Fig. 12.7, the $\triangle$s A_1BC and A_2BC both have the same angle C, the same side BC and $A_1B = A_2B$. However, it is clear that the triangles are *not* congruent. This is called the **ambiguous case.**

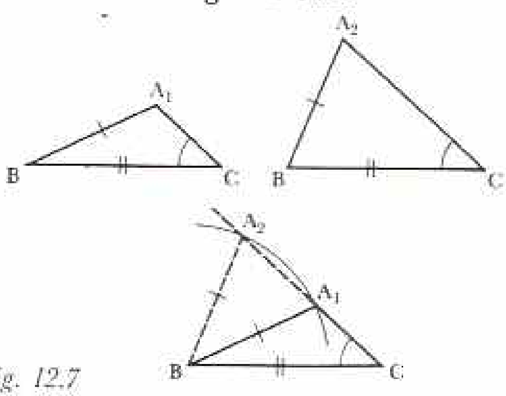


Fig. 12.7

Hence if a triangle has two sides and a non-included angle equal to two corresponding sides and the corresponding angle of another, the two triangles may not be congruent.

Two angles and corresponding side

If two triangles DEF and HJK are drawn in which $D\hat{E}F = H\hat{J}K$, $E\hat{F}D = J\hat{K}H$ and EF = JK, the triangles will be congruent (Fig. 12.8).

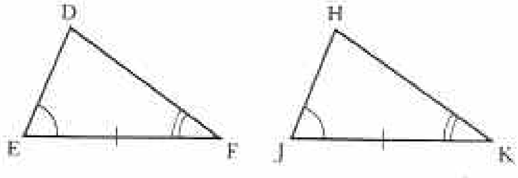


Fig. 12.8

It follows that $E\hat{D}F = J\hat{H}K$, DE = HJ and DF = HK.

In Fig. 12.8 the corresponding sides are between the pairs of given angles. However, it is not necessary that the sides should be between the angles. If two angles of one triangle are given equal to two angles of another triangle, then, from the sum of the angles of a triangle, their third angles will be equal. Hence it doesn't matter which two angles are given so long as the sides correspond. For example, in Fig. 12.9, $\triangle$ ABC is congruent to $\triangle$ PQR.

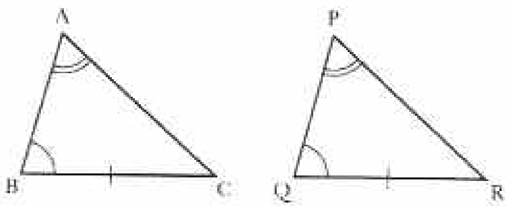


Fig. 12.9

Three sides

If two triangles RST and UVW are drawn in which RS = UV, RT = UW and ST = VW, the triangles are congruent (Fig. 12.10 on page 94).

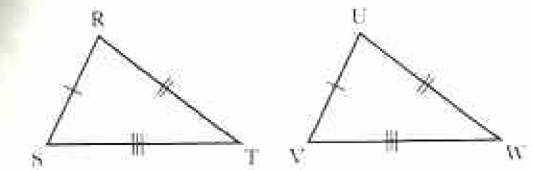


Fig. 12.10

Since the triangles are congruent, corresponding angles are also equal: $R\hat{S}T = U\hat{V}W$, $R\hat{T}S = U\hat{W}V$ and $S\hat{R}T = V\hat{U}W$.

Right angle, hypotenuse and side

If two triangles EFG and KLM are drawn in which $E\hat{F}G = K\hat{L}M = 90°$, EG = KM and FG = LM, the triangles are congruent (Fig. 12.11).

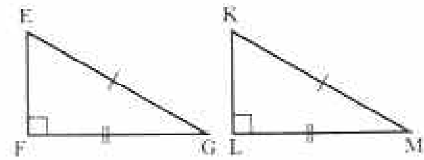


Fig. 12.11

It follows that EF = KL, $F\hat{E}G = L\hat{K}M$, $E\hat{G}F = K\hat{M}L$.

Compare this with the ambiguous case in Fig. 12.7 on page 93. This is the only case in which two triangles will be congruent if two sides and a non-included angle of one are correspondingly equal to two sides and a non-included angle of the other (i.e. when the non-included angle is a right angle).

Naming congruent triangles

When naming congruent triangles, give the letters in the correct order so that it is clear which letters of the triangles correspond to each other. For example, in Fig. 12.11, △FGE is congruent to △LMK, *not* △KLM or △LKM, etc. When congruent triangles are properly named, it is possible to find pairs of equal sides or equal angles without looking at the figure.

The symbol ≡ means 'is identically equal to', or 'is congruent to'. Thus △EFG ≡ △KLM is short for 'triangle EFG is congruent to triangle KLM'.

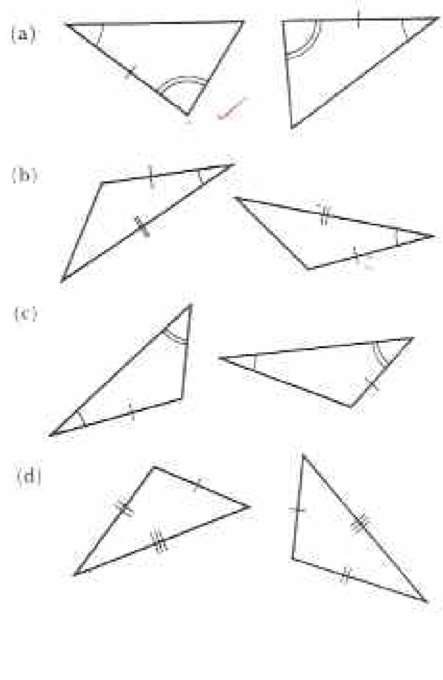
The following summary gives the four sets of conditions for congruency of two triangles.

Two triangles are congruent if

1 **two sides and the included angle** of one are respectively equal to two sides and the included angle of the other (abbreviation SAS).
2 **two angles and a side** of one are respectively equal to two angles and the **corresponding** side of the other (ASA or AAS).
3 the **three sides** of one are respectively equal to the three sides of the other (SSS).
4 they are **right-angled,** and have the **hypotenuse and another side** of one respectively equal to the hypotenuse and another side of the other (RHS).

Exercise 12b

1 In each part of Fig. 12.12, pairs of triangles have equal sides or equal angles shown with marks. In each case, state whether the triangles are congruent, not congruent or not necessarily congruent. If congruent, state the condition of congruency, RHS, SSS, SAS, ASA or AAS.



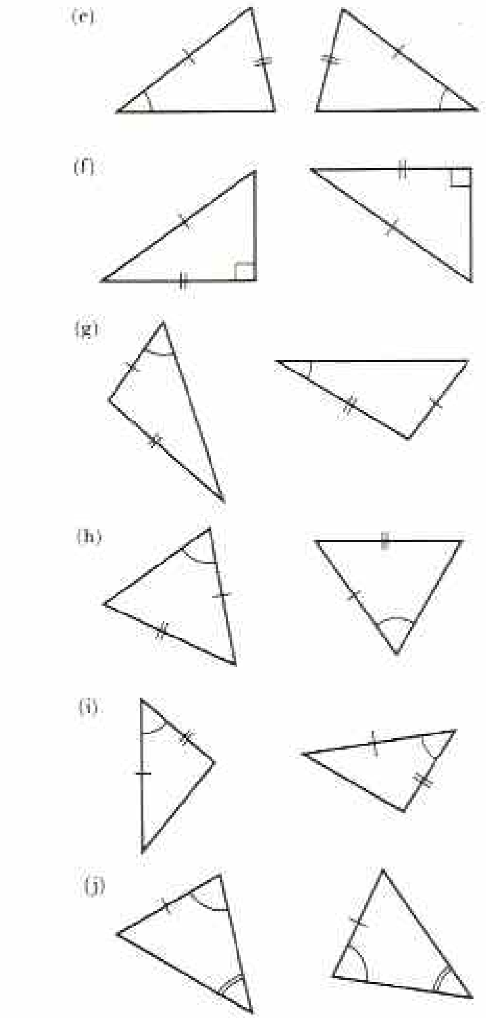
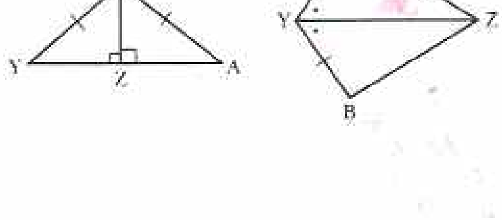


Fig. 12.12

2 In each diagram in Fig. 12.13, name the triangle which is congruent to $\triangle XYZ$, giving the letters in the correct order. In each case state the condition of congruency using the abbreviations RHS, SSS, SAS, ASA or AAS.



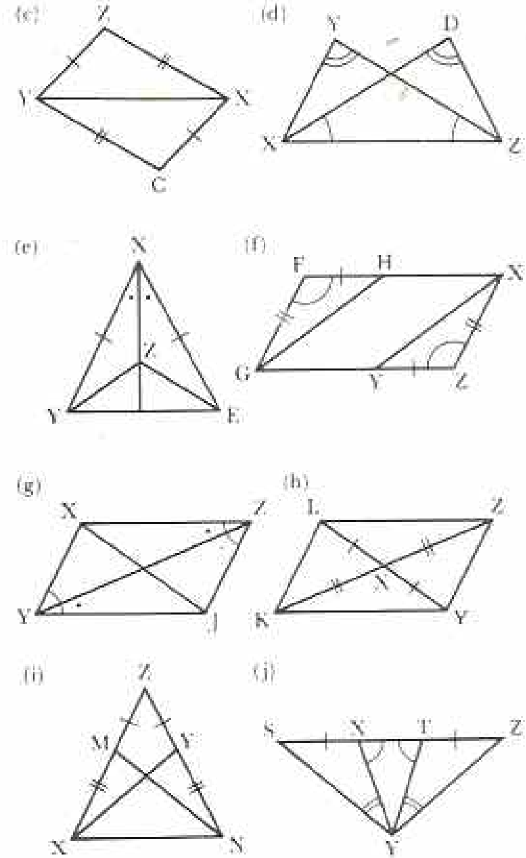


Fig. 12.13

3 In each of the following, the statements refer to $\triangle$s ABC and PQR. In each case sketch the triangles and mark in what is given. If the triangles are congruent, state three other pairs of equal elements and give the condition of congruency.

(a) $AB = PQ$, $BC = QR$, $\hat{B} = \hat{Q}$
(b) $AB = RQ$, $\hat{B} = \hat{Q}$, $\hat{C} = \hat{P}$
(c) $AC = PR$, $\hat{B} = \hat{R}$, $\hat{C} = \hat{Q}$
(d) $AB = QP$, $BC = PR$, $CA = RQ$
(e) $AC = PQ$, $BC = RQ$, $\hat{A} = \hat{P} = 90°$
(f) $AB = RP$, $AC = RQ$, $\hat{B} = \hat{P}$
(g) $AB = QR$, $AC = QP$, $\hat{B} = \hat{P}$
(h) $BC = QP$, $\hat{B} = \hat{Q}$, $\hat{C} = \hat{P}$
(i) $AC = QR$, $BC = PR$, $\hat{C} = \hat{R}$
(j) $BC = RQ$, $\hat{A} = \hat{P}$, $\hat{C} = \hat{Q}$

The use of congruent triangles is shown in the sections on isosceles and equilateral triangles and parallelograms which follow.

Isosceles and equilateral triangles

An **isosceles triangle** has two equal sides. The third side is called the **base**. The equal sides meet at the **vertex.** The angle at the vertex is called the **vertical angle.**

Theorem:
The base angles of an isosceles triangle are equal.

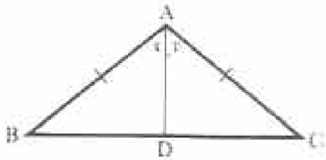


Fig. 12.14

Given: △ABC in which AB = AC.
To prove: $\hat{B} = \hat{C}$
Construction: Draw the bisector of $\hat{A}$ to meet BC at D.

Proof:
In △s ABD and ACD,

AB = AC	*(given)*
AD = AD	*(same side)*
$x = y$	*(construction)*
∴ △ABD ≡ △ACD	*(SAS)*
∴ $\hat{B} = \hat{C}$	*(corresp. angles in △s ABD, ACD)*

Notice the importance of the order of the letters in the first line of the proof. In the two triangles, A corresponds to A, B to C and D to D. AB and x come before the = signs because they are parts of △ABD which is mentioned first.

There are other properties of isosceles triangles which follow from the fact that △ABD ≡ △ACD in Fig. 12.14. For example:

(a) The bisector of the vertical angle bisects the base (BD = CD).
(b) The bisector of the vertical angle meets the base at right angles.

$$(A\hat{D}B = A\hat{D}C \text{ and } A\hat{D}B + A\hat{D}C = 180°,$$
$$\therefore A\hat{D}B = A\hat{D}C = 90°)$$

An **equilateral triangle** is a special isosceles triangle in which the three sides are of equal length. Each angle in an equilateral triangle is 60°.

Example 3

Isosceles triangles ABC *and* ABD *are drawn o[n] opposite sides of a common base* AB. *If* $A\hat{B}C$ = 7[0°] *and* $A\hat{D}B$ = 118° *calculate* $A\hat{C}B$ *and* $C\hat{B}D$.

Fig. 12.15 contains the data of the questio[n].

In △ ABC,

$A\hat{B}C = 70°$ *(give[n])*
$\therefore B\hat{A}C = 70°$ *(base angles of isos. △)*
$\therefore A\hat{C}B = 180° - 70° - 70°$ *(angle sum of △)*
$= 40°$

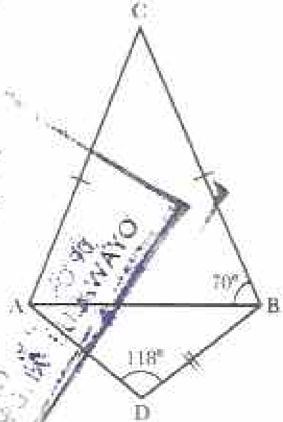


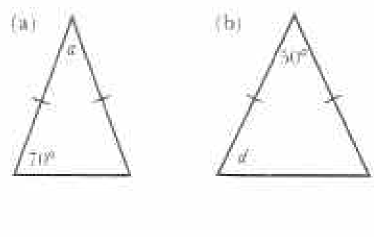
Fig. 12.15

In △ ABD,

$A\hat{D}B = 118°$ *(give[n])*
$\therefore A\hat{B}D + B\hat{A}D = 180° - 118°$ *(angle sum of △)*
$= 62°$
$\therefore 2 \times A\hat{B}D = 62°$ *(base angles of isos. △)*
$\therefore A\hat{B}D = 31°$
$C\hat{B}D = C\hat{B}A + A\hat{B}D$
$= 70° + 31°$
$= 101°$
$A\hat{C}B = 40°$ and $C\hat{B}D = 101°$

Exercise 12c

1 In Fig. 12.16, calculate the angles mar[ked] with letters.



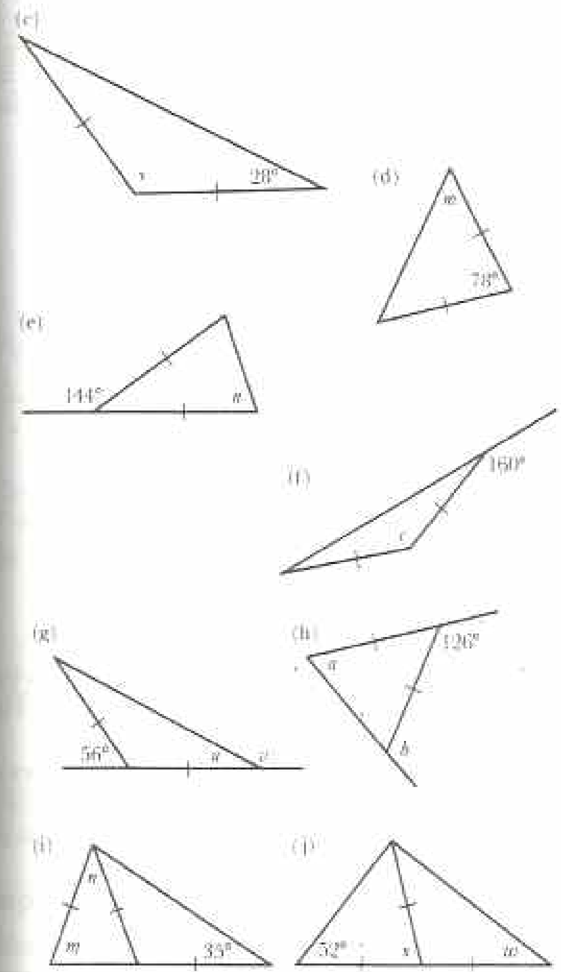


Fig. 12.16

2 In isosceles △ABC, AB = AC. If $\hat{B} = 55°$, calculate $\hat{A}$.

3 PQR is an isosceles △ in which PQ = PR and $\hat{P} = 58°$. Calculate $\hat{Q}$.

4 In isosceles △XYZ, $\hat{X} = 117°$. Calculate $\hat{Z}$.

5 The base JK of isosceles △HJK is produced to L. If $\hat{J} = 69°$, calculate $H\hat{K}L$.

6 ABC is an isosceles triangle with its base BC produced to D. If $\hat{A} = 75°$, calculate $A\hat{C}D$.

7 The base QR of isosceles △PQR is produced to S. If $P\hat{R}S = 102°$, calculate $\hat{Q}$.

8 The base VW of isosceles △UVW is produced to X. If $U\hat{W}X = 121°$, calculate $\hat{U}$.

9 In Fig. 12.17, ABC is an equilateral triangle. P is a point on AC such that $P\hat{B}C = 46°$. Calculate $A\hat{P}B$.

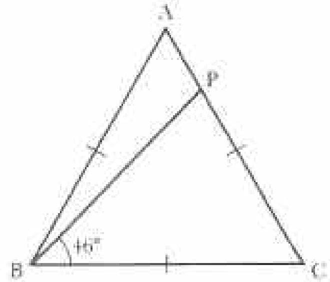


Fig. 12.17

10 PQR is an equilateral triangle and QP is produced to S so that PS = QP. Calculate $Q\hat{R}S$.

11 In △ABC, $\hat{B} = 67°$ and $\hat{C} = 46°$. Prove that CA = CB.

12 Given the data of Fig. 12.18, prove that △PQR is isosceles.

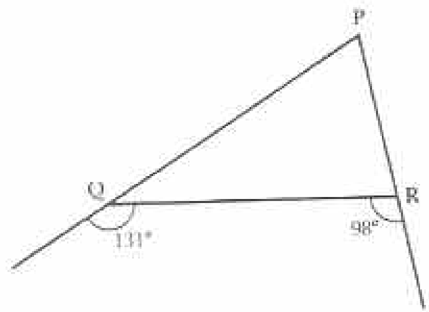


Fig. 12.18

13 An isosceles triangle is such that the vertical angle is 4 times the size of a base angle. What is the size of a base angle?

14 Calculate the angles of an isosceles triangle in which each base angle is 4 times the vertical angle.

15 In Fig. 12.19, △XYZ is equilateral. Use the data in the figure to calculate $N\hat{M}X$.

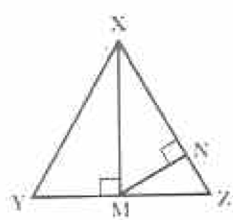


Fig. 12.19

16 In Fig. 12.20, XY = YZ, XZ = ZH and $X\hat{Y}Z = 52°$. Calculate $Z\hat{H}X$.

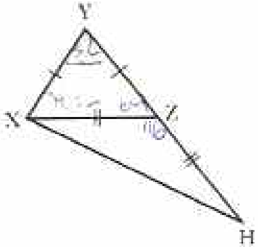


Fig. 12.20

17 In Fig. 12.21, PR = PQ, QS = QR and $R\hat{P}Q = 40°$. Calculate $P\hat{Q}S$.

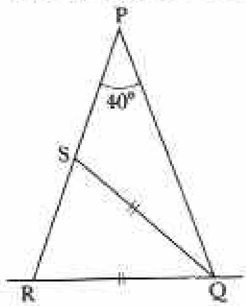


Fig. 12.21

18 In Fig. 12.22, AD = DB = BC and $A\hat{D}B = B\hat{D}C = 62°$. Calculate $A\hat{B}C$.

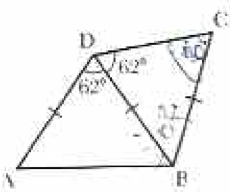


Fig. 12.22

19 In Fig. 12.23, ABC is an isosceles triangle with AB = AC. BC is produced to D until AC = CD. If $A\hat{B}C = 2x°$, $B\hat{A}C = x°$ and $A\hat{D}C = y°$, prove that $x = y$.

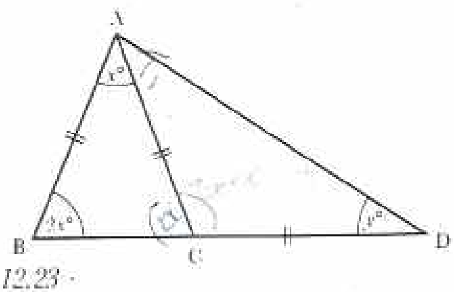


Fig. 12.23

20 In Fig. 12.24, DEF is a triangle with $E\hat{D}F = 2x°$. DE and DF are produced to G and H respectively so that EF = EG = FH. EH and FG intersect at K. Show that $E\hat{K}G = 90° - x°$.

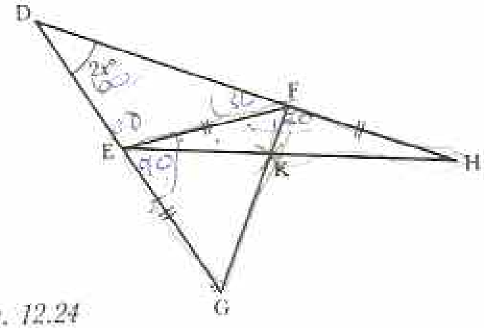


Fig. 12.24

Parallelograms

Definitions

A **parallelogram** is a quadrilateral which has both pairs of opposite sides parallel (Fig. 12.25(a)).

A **rhombus** is a parallelogram with sides of equal length (Fig. 12.25(b)).

A **rectangle** is a quadrilateral which has all its angles right angles (Fig. 12.25(c)).

A **square** is a rectangle with sides of equal length (Fig. 12.25(d)).

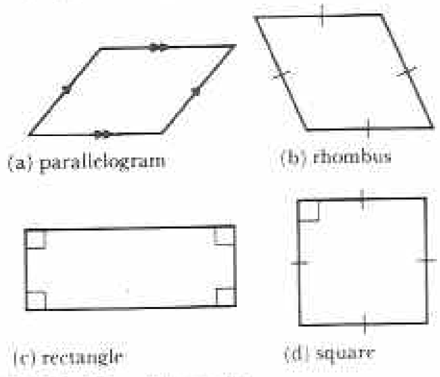


Fig. 12.25 Parallelograms

From the definitions it can be seen that th rhombus, rectangle and square are all speci examples of parallelograms. Any properti which can be proved for a parallelogram wi also be true of a rhombus, rectangle or squar

Theorem:

In a parallelogram, (i) the opposite sides are equal, (ii) the opposite angles are equal.

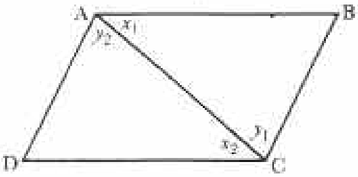


Fig. 12.26

Given: parallelogram ABCD

To prove: (i) AB = CD, BC = DA,

(ii) $\hat{B} = \hat{D}$, $\hat{A} = \hat{C}$.

Construction: Draw the diagonal AC.

Proof:

In △s ABC and CDA,

$x_1 = x_2$ (*alt. angles*, AB ∥ DC)

$y_1 = y_2$ (*alt. angles*, AD ∥ BC)

AC is common

∴ △ABC ≡ △CDA (*ASA*)

∴ (i) AB = CD, BC = DA (*corresp. sides*)

(ii) $\hat{B} = \hat{D}$ (*corresp. angles*)

$\hat{A} = x_1 + y_2$

$= x_2 + y_1$

$= \hat{C}$

Notice that since △ABC ≡ △CDA, a diagonal bisects a parallelogram into two triangles of equal area.

Theorem:

The diagonals of a parallelogram bisect one another.

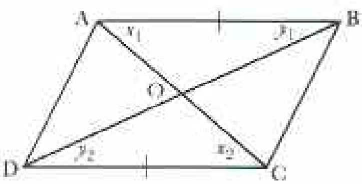


Fig. 12.27

Given: parallelogram ABCD with diagonals AC and BD intersecting at O.

To prove: AO = OC, BO = OD.

Proof:

In △s AOB and COD,

$x_1 = x_2$ (*alt. angles*, AB ∥ DC)

$y_1 = y_2$ (*alt. angles*, AB ∥ DC)

AB = CD (*opp. sides of* ∥^{gm})

∴ △AOB ≡ △COD (*ASA*)

∴ AO = CO (*corresp. sides*)

and BO = DO (*corresp. sides*)

Exercise 12d

Give a formal proof of the following theorems.

1 A quadrilateral having one pair of opposite sides equal and parallel is a parallelogram. (Construct a diagonal.)

2 If both pairs of opposite sides of a quadrilateral are equal, the quadrilateral is a parallelogram. (Construct a diagonal.)

3 The diagonals of a rhombus, (i) bisect each other at right angles, (ii) bisect the angles of the rhombus.

Summary of properties

In a **parallelogram:**

1 the opposite sides are parallel;

2 the opposite sides are equal;

3 the opposite angles are equal;

4 the diagonals bisect one another.

In a **rhombus:**

1 all four sides are equal;

2 the opposite sides are parallel;

3 the opposite angles are equal;

4 the diagonals bisect one another at right angles;

5 the diagonals bisect the angles.

In a **rectangle:**

1 all of the properties of a parallelogram are found;

2 all four angles are right angles.

In a **square:**

1 all of the properties of a rhombus are found;

2 all four angles are right angles.

Exercise 12e

1 In Fig. 12.28, how many parallelograms are there altogether?

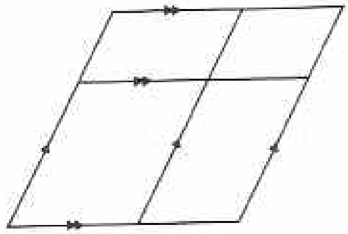


Fig. 12.28

2 In Fig. 12.29, AB = DC, AB || DC, CQ || PA and QAB and DCP are straight lines.

(a) Name all the parallegrams in the figure.

(b) Name all the pairs of congruent triangles in the figure.

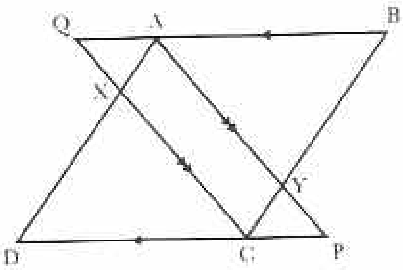


Fig 12.29

3 In Fig. 12.30, ABCD and CDEF are parallelograms and ABEF is a straight line. If BE = 2 cm and DC = 3 cm, find AF.

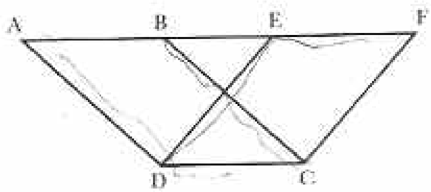


Fig. 12.30

4 Given the data of Fig. 12.30, if AF = 11 cm and BE = 3 cm, find DC.

5 In Fig. 12.31, ABCD is a rhombus and APCQ is a square. If $P\hat{A}B = 21°$, calculate the four angles of ABCD.

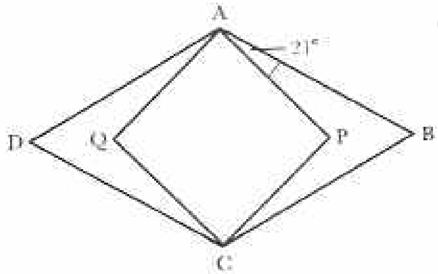


Fig. 12.31

6 In Fig. 12.32, ABCDE is such that ABCD is a parallelogram and ABDE is a square MD = 3 cm and AD = 8.5 cm. Calculate the perimeter of ABCDE.

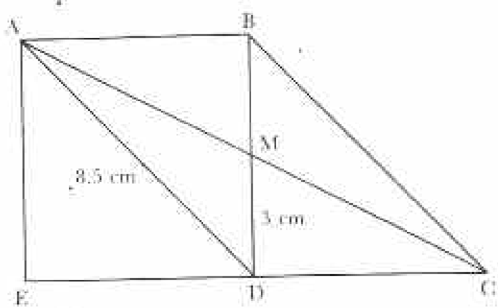


Fig. 12.32

7 ABCDEF is a regular hexagon with O at its centre. What kind of quadrilaterals are ABCO and ACDF?

8 ABCD is a square with centre O. P is a point on AB such that AP = AO. Calculate $P\hat{O}B$.

9 ABC is a triangle and M is the mid-point of BC. A line through C parallel to AB cuts AM produced at X. Prove MX = MA.

10 PQRS is a parallelogram. SP is produced to a point X so that PX = PS. XR cuts PQ at Y. Prove that Y is the mid-point of PQ.

Chapter 13

Factorisation (1) Common factors

Removing brackets (revision)

The expression $3(2x - y)$ means 3 times $(2x - y)$.

Example 1

Remove brackets from (*a*) $3(2x - y)$,
(*b*) $(3a + 8b)5a$, (*c*) $-2n(7y - 4z)$.

(a) $3(2x - y) = 3 \times 2x - 3 \times y = 6x - 3y$

(b) $(3a + 8b)5a = 3a \times 5a + 8b \times 5a$
$= 15a^2 + 40ab$

(c) $-2n(7y - 4z) = (-2n) \times 7y - (-2n) \times 4z$
$= -14ny - (-8nz)$
$= -14ny + 8nz$
$= 8nz - 14ny$

Exercise 13a (Oral revision)

Remove brackets from the following.

1 $2(x + y)$	**2** $5(7 - a)$
3 $(n + 9)3$	**4** $8(2a - b)$
5 $5(-x - 3y)$	**6** $(-3p + q)4$
7 $-2(m + n)$	**8** $-3(a - b)$
9 $(p + q)(-4)$	**10** $-7(3d - 2)$
11 $-9(-2k - 3r)$	**12** $(-7s + t)(-6)$
13 $x(x + 2)$	**14** $y(y - 1)$
15 $(a + b)a$	**16** $n(3n - 2)$
17 $p(2s + 3t)$	**18** $(5 - 3n)m$
19 $2a(5a - 8b)$	**20** $3x(x + 9)$
21 $5p(9r - 8s)$	**22** $-6a(2a - 7b)$
23 $(3a - 4b)3b$	**24** $2\pi r(r + h)$

Common factors

Example 2

Find the HCF of $6xy$ *and* $18x^2$.

$6xy = 6 \times x \times y$
$18x^2 = 3 \times 6 \times x \times x$
The HCF of $6xy$ and $18x^2$ is $6 \times x = 6x$

Exercise 13b (Oral revision)

Find the HCF of the following.

1 $5a$ and $5z$	**2** $6x$ and $15y$
3 $7mnp$ and mp	**4** $5xy$ and $15x$
5 $12a$ and $8a^2$	**6** $13ab$ and $26b$
7 ab^2 and a^2b	**8** $6d^2e$ and $3de^2$
9 $8pq$ and $24p^2$	**10** $10ax^2$ and $14a^2x$
11 $9xy$ and $24pq$	**12** $30ad$ and $28ax$

Factorisation by taking out common factors

To **factorise** an expression is to write it as a product of its factors.

Example 3

Factorise the following. (*a*) $9a - 3z$ (*b*) $5x^2 + 15x$
(*c*) $2mh - 8m^2h$

(a) The HCF of $9a$ and $3z$ is 3.

$$9a - 3z = 3\left(\frac{9a}{3} - \frac{3z}{3}\right)$$
$$= 3(3a - z)$$

(b) The HCF of $5x^2$ and $15x$ is $5x$.

$$5x^2 + 15x = 5x\left(\frac{5x^2}{5x} + \frac{15x}{5x}\right)$$
$$= 5x(x + 3)$$

(c) The HCF of $2mh$ and $8m^2h$ is $2mh$.

$$2mh - 8m^2h = 2mh\left(\frac{2mh}{2mh} - \frac{8m^2h}{2mh}\right)$$
$$= 2mh(1 - 4m)$$

The above examples show that factorisation is the opposite of removing brackets. *Note:* It is not necessary to write the first line of working as above; this has been included to show the method.

Exercise 13c (Oral revision)

Factorise the following. Questions 1–12 correspond to questions 1–12 in Exercise 13b.

1 $5a + 5z$	**2** $6x - 15y$
3 $7mnp - mp$	**4** $5xy + 15x$
5 $12a + 8a^2$	**6** $13ab - 26b$
7 $ab^2 - a^2b$	**8** $6d^2e - 3de^2$
9 $8pq + 24p^2$	**10** $10ax^2 + 14a^2x$
11 $9xy + 24pq$	**12** $30ad - 28ax$
13 $5am - 20bm$	**14** $5a^3 - 3a^2b$
15 $\pi r^2 + \pi rs$	**16** $7d^2 - d$
17 $33bd - 3de$	**18** $9pq + 12t$
19 $ab - 2b$	**20** $3dh + 15dk$
21 $x^2 + 9xy$	**22** $2a^2 + 10a$
23 $am + a$	**24** $24x^2y - 6xy$

Simplifying calculations by factorisation

Example 4

By factorising, simplify $79 \times 37 + 21 \times 37$.

37 is common factor of 79×37 and 21×37.

$$\begin{aligned} 79 \times 37 + 21 \times 37 &= 37(79 + 21) \\ &= 37 \times 100 \\ &= 3\,700 \end{aligned}$$

Example 5

Factorise the expression $\pi r^2 + 2\pi rh$. *Hence find the value of* $\pi r^2 + 2\pi rh$ *when* $\pi = \frac{22}{7}$, $r = 14$ *and* $h = 43$.

$\pi r^2 + 2\pi rh = \pi r(r + 2h)$

When $\pi = \frac{22}{7}$, $r = 14$ and $h = 43$,

$$\begin{aligned} \pi r^2 + 2\pi rh &= \pi r(r + 2h) \\ &= \tfrac{22}{7} \times 14(14 + 2 \times 43) \\ &= 22 \times 2(14 + 86) \\ &= 44 \times 100 \\ &= 4\,400 \end{aligned}$$

Exercise 13d

Simplify questions 1–15 by factorising.

1 $34 \times 48 + 34 \times 52$
2 $61 \times 87 - 61 \times 85$
3 $128 \times 27 - 28 \times 27$
4 $693 \times 7 + 693 \times 3$
5 $\frac{8}{13} \times 125 + \frac{5}{13} \times 125$
6 $\frac{22}{7} \times 10 + \frac{22}{7} \times 4$
7 $121 \times 67 + 79 \times 67$
8 $67 \times 23 - 67 \times 13$
9 $\frac{22}{7} \times 3\frac{1}{4} - \frac{22}{7} \times 2\frac{1}{4}$
10 $53 \times 49 - 53 \times 39$
11 $\frac{3}{4} \times 133 - \frac{3}{4} \times 93$
12 $35 \times 29 + 35 \times 11$
13 $27 \times 354 + 27 \times 646$
14 $\frac{22}{7} \times 1\frac{1}{4} + \frac{22}{7} \times 2\frac{3}{4}$
15 $762 \times 87 - 562 \times 87$
16 Factorise the expression $\pi R^2 - \pi r^2$. Hence find the value of the expression when $\pi = \frac{22}{7}$, $R = 9$ and $r = 5$.
17 Factorise the expression $2\pi r^2 + 2\pi rh$. Hence find the value of the expression when $\pi = \frac{22}{7}$, $r = 5$ and $h = 16$.
18 Factorise the expression $\pi r^2h + \frac{1}{3}\pi r^2H$. Hence find the value of the expression when $\pi = \frac{22}{7}$, $r = 3$, $h = 10$ and $H = 12$.

Factorisation of larger expressions

Example 6

Factorise $2x(5a + 2) - 3y(5a + 2)$.

In the given expression,

$2x(5a + 2) = 2x$ times $(5a + 2)$

and $3y(5a + 2) = 3y$ times $(5a + 2)$

Hence the products $2x(5a + 2)$ and $3y(5a + 2)$ have the factor $(5a + 2)$ in common. Thus,

$2x(5a + 2) - 3y(5a + 2) = (5a + 2)(2x - 3y)$

Note in Example 6 that the given expression is like $2xm - 3ym$ where $m = (5a + 2)$.

Example 7

Factorise $2d^3 + d^2(3d - 1)$.

$2d^3$ and $d^2(3d - 1)$ have the factor d^2 in common. Thus,

$$\begin{aligned} 2d^3 + d^2(3d - 1) &= d^2[2d + (3d - 1)] \\ &= d^2(2d + 3d - 1) \\ &= d^2(5d - 1) \end{aligned}$$

Example 8

Factorise $(a + m)(2a - 5m) - (a + m)^2$.

The two parts of the expression have the factor $(a + m)$ in common. Thus,

$(a + m)(2a - 5m) - (a + m)^2$

$= (a + m)[(2a - 5m) - (a + m)]$
$= (a + m)(2a - 5m - a - m)$
$= (a + m)(a - 6m)$

Example 9
Factorise $(x - 2y)(z + 3) - x + 2y$.

Notice that -1 is a factor of the last two terms. The given expression may be written as follows.

$$(x - 2y)(z + 3) - 1(x - 2y)$$

The two parts of the expression now have $(x - 2y)$ as a common factor.

Expression $= (x - 2y)[(z + 3) - 1]$
$= (x - 2y)(z + 3 - 1)$
$= (x - 2y)(z + 2)$

Exercise 13e
Factorise the following.

1 $3m + m(u - v)$ **2** $2a - a(3x + y)$
3 $x(3 - a) + bx$ **4** $(4m - 3n)p - 5p$
5 $a(m + 1) + b(m + 1)$
6 $a(n + 2) - b(n + 2)$
7 $ax - x(b - 4c)$
8 $5x(a - b) - 2y(a - b)$
9 $3h(5u - v) + 2k(5u - v)$
10 $m(u - v) + m^2$
11 $d(3h + k) - 4d^2$
12 $5a^2 + a(b - c)$
13 $4x^2 - x(3y + 2z)$
14 $3d^3 - d^2(e - 4f)$
15 $a(3u - v) + a(u + 2v)$
16 $(5x - y)a - (3x + 5y)a$
17 $3(3u + 2v) - a(3u + 2v)$
18 $(4a - b)3x + (4a - b)2y$
19 $h(2a - 7b) - 3k(2a - 7b)$
20 $m(3m - 2) + 2m^2$
21 $a^2(5a - 3b) - 3a^3$
22 $5x^2 - x(x + 4)$
23 $2d(3m - 4n) - 3e(3m - 4n)$
24 $(a + 2b)(x - y) - 3(x - y)$
25 $p(2m + n) + (q - r)(2m + n)$
26 $(h + k)(r + s) + (h + k)(r - 2s)$
27 $(3x - y)(u + v) + (x + 2y)(u + v)$
28 $(b - c)(3d + e) - (b - c)(d - 2e)$
29 $(a + 2b)^2 - 3(a + 2b)$
30 $(3m - 2n)^2 + 5p(3m - 2n)$
31 $(2u - 3v)(3m - 4n) - (2u - 3v)(m + 2n)$
32 $a(x + 2y) + (x + 2y)^2$
33 $3u(2x + y) - (2x + y)^2$
34 $(f - g)4e - (f - g)^2$
35 $(a - 3b)(2u - v) + (a - 3b)(u + 7v)$
36 $(5m + 2n)(6a + b) - (5m + 2n)(a - 4b)$
37 $(x + 3y)(m - n) + x + 3y$
38 $(2a - 3b)(c + d) - 2a + 3b$
39 $7u - 2v + (7u - 2v)^2$
40 $(2u - 7v)^2 + 7v - 2u$

Factorisation by grouping

Example 10
Factorise $cx + cy + 2dx + 2dy$.

The terms cx and cy have c in common.
The terms $2dx$ and $2dy$ have $2d$ in common.
Grouping in pairs in this way

$cx + cy + 2dx + 2dy = (cx + cy) + (2dx + 2dy)$
$= c(x + y) + 2d(x + y)$

The two products now have $(x + y)$ in common.

$$c(x + y) + 2d(x + y) = (x + y)(c + 2d)$$

Hence $cx + cy + 2dx + 2dy = (x + y)(c + 2d)$

Example 11
Factorise $3a - 6b + ax - 2bx$.

$3a - 6b + ax - 2bx = 3(a - 2b) + x(a - 2b)$
$= (a - 2b)(3 + x)$

Notice that to factorise in this way, the same bracket must occur twice in the first line of the working. If the given expression is to be factorised, there must be a repeated bracket. For this reason, it is often easiest to write this bracket down again immediately, as soon as it has been found. This is shown in Example 12.

Example 12
Factorise $2x^2 - 3x + 2x - 3$.

$2x^2 - 3x + 2x - 3 = x(2x - 3) \ldots (2x - 3)$
The terms $+2x - 3$ in the given expression are obtained by multiplying $(2x - 3)$ by $+1$. Thus,
$2x^2 - 3x + 2x - 3 = x(2x - 3) + 1(2x - 3)$
$= (2x - 3)(x + 1)$.

Exercise 13f

Factorise the following by grouping in pairs.

1 $ax + ay + 3bx + 3by$
2 $7a + 14b + ax + 2bx$
3 $x^2 + 5x + 2x + 10$
4 $pq + qr + ps + rs$
5 $a^2 - 9a + 3a - 27$
6 $8m - 2 + 4mn - n$
7 $5x^2 - 10x + 3x - 6$
8 $ab - bc + ad - cd$
9 $2ab - 5a + 2b - 5$
10 $3m - 1 + 6m^2 + 2m$

In many cases it is only possible to get a repeated second bracket if a negative common factor is taken. This is shown in Example 13.

Example 13

Factorise $2am - 2m^2 - 3ab + 3bm$.

$$2am - 2m^2 - 3ab + 3bm$$
$$= 2m(a - m) \ldots (a - m)$$

The terms $-3ab$ and $+3bm$ in the given expression are obtained by multiplying $(a - m)$ by $-3b$. Hence,

$$2am - 2m^2 - 3ab + 3bm$$
$$= 2m(a - m) - 3b(a - m)$$
$$= (a - m)(2m - 3b)$$

Exercise 13g

Factorise the following.

1 $ab + bc - am - cm$
2 $9x + 6 - 3x^2 - 2x$
3 $2ax - 2ay - 3bx + 3by$
4 $x^2 - 7x - 2x + 14$
5 $5a - 5b - ac + bc$
6 $3pq + 12pr - qy - 4ry$
7 $a^2 - 3a - 3a + 9$
8 $2ps + 5pt - 2rs - 5rt$
9 $x^2 - 6x - x + 6$
10 $3k + 1 - 3hk - h$

Sometimes the first attempt at grouping the terms does not give a common factor. In these cases, re-group the terms and try again. This is shown in Example 14.

Example 14

Factorise $cd - de + d^2 - ce$.

$$cd - de + d^2 - ce = d(c - e) \ldots (\quad)$$

d^2 and ce have no common factors. Re-group the given terms.

Either:

$$cd - de + d^2 - ce = cd + d^2 - ce - de$$
$$= d(c + d) - e(c + d)$$
$$= (c + d)(d - e)$$

or:

$$cd - de + d^2 - ce = cd - ce + d^2 - de$$
$$= c(d - e) + d(d - e)$$
$$= (d - e)(c + d)$$

Four terms can be grouped in pairs in three ways. If the expression factorises, two of these ways will give the required result and one will not.

Exercise 13h

Re-group then factorise the following.

1 $6a + bm + 6b + am$
2 $pr + qs + qr + ps$
3 $15 - xy + 5y - 3x$
4 $ac - bd - bc + ad$
5 $ax - xy + x^2 - ay$
6 $ad - cm - am + cd$
7 $x^2 + 15 - 3x - 5x$
8 $8a + 15by + 12y + 10ab$
9 $3a - cb - 3b + ac$
10 $t + 6sz + 3s + 2tz$

If *all* the terms contain a common factor, it should be taken out *first*. This is shown in Example 5. *This should always be the rule when factorising any type of expression.*

Example 15

Factorise $2sru + 6tru - 4srv - 12trv$.

$2r$ is a factor of every term in the given expression.

$$2sru + 6tru - 4srv - 12trv$$
$$= 2r\{su + 3tu - 2sv - 6tv\}$$
$$= 2r\{u(s + 3t) - 2v(s + 3t)\}$$
$$= 2r(s + 3t)(u - 2v)$$

Exercise 13i

Factorise the following where possible. If there are no factors, say so.

1 $mx + nx + my + ny$
2 $ax - ay + bx - by$
3 $hu + hv - ku - kv$
4 $au - bu - av + bv$
5 $am + 2bm + 2bn + an$
6 $cx - dx + 2cy - 2dy$
7 $2ce + 4df - de - 2cf$
8 $ab + 4xy - 2bx - 2ay$
9 $am - an + m - n$
10 $u + v - dv + du$
11 $a^3 + a^2 + a + 1$
12 $2mh - 3nh - 3nk + 2mk$
13 $3sx - 5ty + 5tx - 3sy$
14 $abx^2 + bxy + axy + y^2$
15 $hk - 2km + 3hn - 6mn$
16 $mn - 6xy - 3nx + 3my$
17 $2gk - 3gl + 2hk - 3hl$
18 $2fh + 4gh - fk - 2gk$
19 $3eg - 4eh - 6fg + 2fh$
20 $hl + 2kl - 3hm - 6km$
21 $3ce + 4df - 2de - 6cf$
22 $xy - 2ny - 6n^2 + 3nx$
23 $ab + 2b^2 - 2ac - 4bc$
24 $cd - ce + d^2 + de$
25 $8uv - 2v^2 + 12uw - 3vw$
26 $mn - 6pn + 3pm - 2n^2$
27 $3xy - 2ay - 3ax + 2y^2$
28 $3ab - 3bu + 3av - 3uv$
29 $ab + 6mn - 2bm - 3bn$
30 $8ce + 12de - 2cf - 3df$
31 $nuv - muv + mnu^2 - v^2$
32 $5mx - 5nx - 5my + 5ny$
33 $3ab + 3cd - bc - 9ad$
34 $6ab - 15bc - 10cd + 4ad$
35 $2amu + 2anu - 2amv - 2anv$
36 $abm^2 + 2bm - 3am - 6$
37 $4ax + 2bx + 8ay + 4by$
38 $21mn - xy - 3nx + 7my$
39 $3ax - 2a - 6bx + 2b$
40 $2am - 3m^2 + 4an - 6mn$
41 $10uv + 5u - 2v - 1$
42 $a^2m + am^2 - mn - an$
43 $2x^2y - xy^2 + 2ax - ay$
44 $1 + 3x - 5a - 15ax$
45 $2d^2x + 4dx^2y - 3dy - 6xy^2$

mx + nx + my + ny
(mx + nx) + (my + ny)
x(m + n) + y(m + n)

ax - ay + bx - by
(ax + bx) - (ay - by)
x(a + b) - y(a - b)

am + 2bm + 2bn + an
(am + 2bm) + (2bn + an)
m(a + 2b) + n(2b + a)
m(a + 2b) + (n a + 2b)

Chapter 14

Everyday arithmetic (2) Consumer arithmetic

Personal arithmetic

Interest

Bankers want people to save money. They give extra payments to encourage saving. The extra money is called **interest.** For example, a saver keeps \$100 in a bank for a year. If the **interest rate** is 8% per annum (i.e. 8% per year), there will be \$108 at the end of the year. The saver now has \$100 plus \$8 interest from the bank. Interest which is paid like this is called **simple interest.**

Example 1
Find the simple interest on \$600 *for 5 years at* 9% *per annum.*

Yearly interest = 9% of \$600 = $\frac{9}{100} \times \$600$
= \$54
Interest for 5 years = \$54 × 5 = \$270

Exercise 14a
Find the simple interest on the following.
1 \$400 for 1 year at 5% per annum
2 \$700 for 1 year at 4% per annum
3 \$100 for 3 years at 6%per annum
4 \$100 for 2 years at 4% per annum
5 \$100 for 4 years at $4\frac{1}{2}$% per annum
6 \$100 for $3\frac{1}{2}$ years at 4% per annum
7 \$300 for 2 years at 5% per annum
8 \$200 for 4 years at 6% per annum
9 \$700 for 3 years at 5% per annum
10 \$600 for $2\frac{1}{2}$ years at 5% per annum
11 \$350 for 4 years at 3% per annum
12 \$250 for 2 years at 6% per annum
13 \$150 for 3 years at 4% per annum
14 \$550 for 4 years at 6% per annum
15 \$250 for 3 years at 5% per annum

Sometimes people have to borrow money. When someone borrows money that person has to pay interest to the lender.

Example 2
A man borrows \$16 000 *to buy a house. He is charged interest at a rate of* 11% *per annum. In the first year he paid the interest on the loan. He also paid back* \$1 000 *of the money he borrowed. How much did he pay back altogether? If he paid this amount by monthly instalments, how much did he pay per month?*

Interest on \$16 000
for 1 year = 11% of \$16 000
= $\frac{11}{100}$ × \$16 000
= 11 × \$160
= \$1 760
Total money paid in
1st year = \$1 760 + \$1 000
= \$2 760
Monthly payments = \$2 760 ÷ 12 = \$230
(The man now owes \$15 000. Interest will be paid on this new amount in the second year.)

Exercise 14b
1 Find the total amount to be paid back (i.e. loan + interest) on the following loans.
(a) \$5 for 2 weeks at \$1 interest per week;
(b) \$20 for 3 weeks at 10 cents in the dollar interest per week;
(c) \$1 000 for 1 year at 9% simple interest per annum;
(d) \$10 000 for 15 years at 8% simple interest per annum;
(e) \$600 for 3 years at $7\frac{1}{2}$% simple interest per annum;
(f) \$860 for $2\frac{1}{2}$ years at $8\frac{1}{2}$% simple interest per annum.

2 A man borrows \$40 on a short term loan to help him buy some new clothes. He is charged interest of \$1 on each \$10 per week. How much does he pay back altogether if he borrows the money (a) for 1 week, (b) for 3 weeks, (c) for 10 weeks?

3 A woman borrows \$20 for 4 weeks. She agrees to pay \$25 back at the end of the 4 weeks.
   (a) How much interest does she pay over the 4 weeks?
   (b) How much interest does she pay per week?
   (c) Find the percentage rate of interest per week, that she pays.

4 A man gets a \$18 000 loan to buy some land. He pays interest at a rate of 9% per annum. In the first year he paid the interest on the loan. He also paid back \$1 400 of the money he borrowed.
   (a) How much did he pay in the first year altogether?
   (b) If he paid this amount in monthly instalments, how much did he pay each month?

5 A woman borrows \$3 000 to help to pay for a car. She agrees to pay the money back over 2 years, paying simple interest at 9% per annum.
   (a) Calculate the simple interest payable on a loan of \$3 000 at 9% per annum for 2 years.
   (b) Hence find the total amount she must pay back.
   (c) If the total money the woman owes is paid back in monthly instalments over 2 years, how much will she pay each month?

Discount buying

A **discount** is a reduction in price. Discounts are often given on items when the customer is able to pay in cash.

Example 3

A television set costs \$540. A $12\frac{1}{2}$% discount is given for cash. What is the cash price?

Either,

$$\text{discount} = 12\tfrac{1}{2}\% \text{ of } \$540 = \frac{12\frac{1}{2}}{100} \times \$540$$
$$= \tfrac{1}{8} \times \$540$$
$$= \$67{,}50$$

cash price = \$540 − \$67,50 = \$472,50

or,

$$\text{cash price} = (100\% - 12\tfrac{1}{2}\%) \text{ of } \$540$$
$$= 87\tfrac{1}{2}\% \text{ of } \$540$$
$$= \frac{87\frac{1}{2}}{100} \times \$540 = \tfrac{7}{8} \times \$540$$
$$= \$472{,}50$$

Discounts are often given for buying in bulk.

Example 4

A trader sells ballpoint pens at 14c each or 4 for 44c How much is saved by buying 4 pens at once instead of 4 pens separately?

Normal cost of 4 pens = 4 × 14c = 56c
Discount price of 4 pens = 44c
Saving = 56c − 44c
= 12c

Exercise 14c

1 Find the discount price if a discount of
   (a) 10% is given on a cost price of \$430;
   (b) $12\frac{1}{2}$% is given on a cost price of \$280;
   (c) 8% is given on a cost price of \$1 080;
   (d) 25% is given on a marked price of \$92;
   (e) 20% is given on a marked price of \$29,95.

2 The selling price of a table is \$140. The trader gives a 25% discount for cash. What is the cash price?

3 During a sale a shop takes 12c in the dollar off all marked prices. What would be the sale price of a bicycle marked \$144,50?

4 A trader sells eggs at 20c each or \$1,10 for 6. How much is saved by buying 3 dozen eggs in sixes instead of separately?

5 A 250 g bag of salt cost 18c. A 20 kg sack of salt costs \$7,80. Calculate the saving per kg by buying the 20 kg sack of salt.

6 A market trader asks \$15 for some cloth. A woman offers \$8. After bargaining, they agree a price half-way between the two starting prices. How much does the woman pay? What discount did she get by bargaining?

Instalment buying

An **instalment** is a part payment. Most people find it easier to buy expensive items by paying instalments.

Example 5

The cost of a hi-fi is either $680 in cash or a deposit of $80 and 12 monthly payments of $55. Find the difference between the instalment price and cash price.

Instalment price = deposit + instalments
= $80 + 12 × $55
= $80 + $660
= $740
Price difference = $740 − $680 = $60

Buying by instalment is called **hire purchase**. The buyer hires the use of an item before paying for it completely. It costs money to hire an item. This is why hire purchase is more costly than paying in cash.

Exercise 14d

1 The hire purchase price of a motor bike is $1 242. This is spread over 12 equal monthly instalments. Calculate each instalment.

2 The hire purchase price of a table and chairs is $840. 25% is paid as a deposit. The rest is spread over 12 equal monthly instalments.
(a) Calculate the amount of the deposit.
(b) Calculate the remainder to be paid.
(c) Find the amount of each monthly instalment.

3 To buy a suit, a man can either pay $112,50 cash or he can pay 16 weekly instalments of $8,05.
(a) Find the cost of the suit when paying by instalments.
(b) Find the difference between the cash and hire purchase prices.

4 A colour television set costs either $988 cash or 52 weekly payments of $22,85. How much more does the television set cost when paid for weekly?

5 The cash price of a car is $8 970,60. The hire purchase payments require a 10% deposit and 36 monthly payments of $244,20. Calculate the saving when paying in cash.

6 A refrigerator costs $835. A 5% discount is given for a cash payment. Alternatively, it can be paid by hire purchase. In this case the price is raised by 11%. Calculate the difference between paying cash and paying by hire purchase.

Income tax

Most people pay part of their income to the Government. The part they pay is called **income tax.** The Government uses income tax in the same way that it uses sales tax.

There are many ways of calculating income tax. The following method is a simplified version of that used in Zimbabwe.

1 Tax is paid each year on taxable income. The rates of taxable income for married and single people are given in Table 14.1.

Table 14.1

taxable income	rate of tax	
	married	single
first $1 000	10%	14%
second $1 000	12%	16%
third $1 000	14%	18%
fourth $1 000	16%	20%
fifth $1 000	18%	22%
sixth $1 000	20%	24%
seventh $1 000	22%	26%
:	increasing in steps of 2% as far as ...	
fifteenth $1 000	38%	42,5%
sixteenth $1 000	40%	45%
seventeenth $1 000	42,5%	45%
over $17 000	45%	45%

2 Abatements are given to help to meet the cost of personal and family commitments. Abatements include the following:

personal { single : $1 800
married : $3 000
children : $600 for each child
dependants : $400

A person's total abatements are called the **abateable amount.** The maximum abateable amounts are \$6 600 for a married person and \$3 800 for a single person.

3 Tax is calculated as follows:
Chargeable income tax
= tax on gross income − tax on abateable amount
Total tax payable
= chargeable income tax + 15% surcharge.

Example 6
A man has a total income of \$9 500. He has a wife and 3 children. He claims an abatement for a dependant. Calculate the amount of tax he pays.

first:
Taxable income = \$9 500

second:
Abatements:

married allowance	\$3 000
3 children	\$1 800
dependants	\$400
abateable amount =	\$5 200

third:
Gross income tax on \$9 500
= \$100 + \$120 + \$140 + \$160 + \$180 + \$200 + \$220 + \$240 + \$260 + 28% of \$500
= \$1 760
Tax on abateable amount of \$5 200
= \$100 + \$120 + \$140 + \$160 + \$180 + 20% of \$200
= \$740
Chargeable income tax = \$1 760 − \$740
= \$1 020

fourth:

Chargeable income tax:	\$1 020
Add 15% surcharge:	\$153
Total tax paid	= \$1 173

Exercise 14e
Use the tax system described above.

1 Calculate the gross income tax on each of the following salaries when earned by (i) married, (ii) single people.
(a) \$4 000 (b) \$7 000 (c) \$5 500
(d) \$12 000 (e) \$3 800 (f) \$8 422

2 A single woman earns a salary of \$6 500 and claims \$400 for her mother. Calculate the amount of tax she pays.

3 A married man earns \$9 100 per annum. He has 5 children.
(a) Calculate his abateable amount.
(b) Calculate his chargeable income tax (i.e. tax before surcharge).
(c) Calculate the tax he pays.
(d) Calculate his income after tax is paid.

4 A married man has 4 children and a dependent mother. His salary is \$7 500 per annum.
(a) Calculate his abateable amount.
(b) Calculate the total tax he pays.
(c) He is paid monthly and tax is taken from his pay in equal monthly instalments. Calculate his monthly 'take home' pay.

5 A husband and wife are both teachers. They each earn \$10 400. They have 3 children. Calculate how much tax they pay altogether. *Note:* Add the two salaries together but allow only one set of abatements.

Commercial arithmetic

Profit and loss

Example 7
A trader buys a book for \$8 and sells it at a profit of 15%. Find his actual profit and the selling price.

Profit = 15% of \$8 = $\frac{15}{100}$ × \$8 = \$$\frac{120}{100}$ = \$1,20
Selling price = \$8 + \$1,20 = \$9,20

Example 8
A hat is bought for \$2,50 and sold for \$2,20. What is the loss per cent?

Actual loss = \$2,50 − \$2,20 = 30c
The ratio, loss: cost price = 30c:\$2,50
= 30:250 = $\frac{30}{250}$
Thus the loss is $\frac{30}{250}$ of the cost price.

Percentage loss = $\frac{30}{250}$ × 100%
= 12%

Notice that the loss (or gain) is calculated as a percentage of the cost price.

Exercise 14f

1 Find (i) the profit or loss, and (ii) the selling price for the following cost prices.
   (a) $4; profit 20%
   (b) $10; profit 15%
   (c) $3,75; loss 8%
   (d) $1,44; loss $12\frac{1}{2}$%
   (e) 75c; profit 60%

2 Find (i) the actual, (ii) the percentage profit or loss for the following cost and selling prices.

	cost price	selling price
(a)	$3	$3,45
(b)	$1,80	$2,25
(c)	$360	$324
(d)	$96	$132
(e)	$4,20	$3,57

3 A farmer buys a cow for $400 and sells it for $330. What is the percentage loss?

4 A trader bought some hats for $1,90 each. She sold them at a 30% profit. What was the selling price?

5 A man bought some wood for 90c. He used the wood to make a box which he sold for $3. What percentage profit was this?

6 Bongani bought a hat for $2,50. He sold it to Nda at a 20% profit. What did Nda pay?

7 Nda (in question 6) was short of money. He sold the hat to Daniel at a loss of 20%. What did Daniel pay?

8 A car which cost $3 360 was sold at a loss of $17\frac{1}{2}$%. What was the selling price?

9 A woman buys a car for $3 500 and sells it for $3 745. Her son buys a bicycle for $80 and sells it for $86. Which one makes the greater profit per cent?

10 An article is bought for $2,25 and sold for $2,52. Find the profit per cent. Find the price at which it should be sold to make a profit of 16%.

11 A woman sells an article for $21,75 and makes a profit of 16%. What did the article cost? Find how much she should have sold it for to make a profit of 28%.

12 A trader buys pens at $4,40 a dozen and sells them at 44c each. Find her % profit.

13 A farmer paid $380 for 11 goats. He sold them at a profit of 32%. What was the selling price of each goat?

14 A bicycle can either be bought in cash for $247,00 or by paying 52 weekly payments of $5,70. By what percentage is the instalment cost greater than the cash price?

15 By selling goods for $5,35 a trader makes a profit of 7%. She reduces her price to $5,15. What is her percentage profit now?

Commission

Commission is payment for selling an item. For example, insurance agents get commission for selling insurance. The more insurance they sell, the more commission they get. Factories often employ sales representatives to sell their goods to shops and traders. The sales representatives often receive a proportion of the value of the goods they sell. This proportion is their commission.

Example 9

A sales representative works for an electric fan company. She gets a commission of 14c *in the* dollar. *In one week she sells* 4 *large fans at* $105 *each and* 9 *small fans at* $54 *each. Calculate her commission.*

Total sales = 4 × $105 + 9 × $54
= $420 + $486
= $906

She gets 14c for every dollar

Commission = 906 × 14c
= 12 684c
= $126,84

Exercise 14g

1 An estate agent gets 2% commission for selling a house. How much money does she get for selling a house for $59 900?

2 A car salesman gets 1c in the dollar commission. Calculate his commission if he sells $52 380 worth of cars in a month.

3 A man sells tickets for a pop concert. He gets $1 for every 5 tickets he sells. How much will he get for selling 285 tickets?

4 A woman is an agent for a mail order company. She gets 10% commission on all monthly payments. How much commission does she get for a monthly payment of $253,80?

5 An insurance agent sells $2 840 worth of insurance. His commission is 20%. How much money does he get?

6 A rent collector's commission is $4\frac{1}{2}\%$ of his takings. In one month he collects $8 428 in rent. How much money does he get?

7 A cinema manager gets 24c commission on every ticket he sells. How much money does he get during a week when he sells 1 318 tickets?

8 An electrical goods salesman gets 14c in the dollar commission. How much commission does he get if he sells 20 radios at $48,00 each and 2 television sets at $615,00 each?

9 A history book costs $4,20. The writers of the book get 10% of the price of each book that is sold. How much will they get if it sells 15 628 copies in one year?

10 A furniture salesman gets an 8% commission. How much will he get for selling 44 chairs at $53 each, 11 tables at $203 each and 5 beds at $154 each?

Sales tax

A proportion of the money paid for goods is given to the Government. The part which is given to the Government is called **sales tax.** The Government uses this tax and other taxes to pay for services such as defence, education, health and transport. At the time of going to press, sales tax was 12,5% of the selling price of consumable goods and 20% of the selling price of durable goods.

Example 10

An advertisement for a table says that its price is '$89 plus 20% sales tax'. *How much does the customer pay?*

Amount paid by customer

$= 120\%$ of $89

$= \$89 \times \frac{120}{100}$

$= \$106,80$

Note: The difference between $106,80 and $89 is $17,80. The Government receives $17,80 sales tax.

Example 11

A bed costs $224,20 *including tax at 20%. How much tax does the Government receive?*

If $224,20 includes tax at 20%, then $224,20 is 120% of the basic cost of the bed. The sales tax is 20% of the basic cost.

$$120\% \text{ of basic cost} = \$224,20$$

$$\Leftrightarrow \quad 1\% \text{ of basic cost} = \$\frac{224,20}{120}$$

Sales tax

$$= 20\% \text{ of basic cost} = \$\frac{224,20}{120} \times 20$$
$$= \$37,37$$

The government receives $37,37 tax.

Exercise 14h

1 Find out how much customers pay for each item in the following advertisement.

ZIMWE FASHIONS	
(a) Shirts	$8 + 12,5% sales tax
(b) Dresses	$24 + 12,5% sales tax
(c) Jackets	$29 + 12,5% sales tax
(d) Trousers	$19 + 12,5% sales tax
(e) Shoes	$22 + 12,5% sales tax

Fig. 14.1

2 Find the amount of tax that the Government receives on each item in the following advertisement.

MASLAND FURNISHINGS	
(a) Coffee tables	$59
(b) Mattresses	$88,50
(c) Beds	$103,25
(d) 3-piece suites	$198
(e) Kitchen tables	$69
• All prices include sales tax of 20%	

Fig. 14.2

Bills

People who receive public services such as water, electricity and maintenance of amenities also receive bills to pay for them. The following give some typical charges for household services. Remember, however, that rates and methods of payment for services vary from time to time and from place to place.

Electricity and water charges

Electricity and water charges are sometimes shown on the same bill. Fig. 14.3 shows a typical bill for these charges issued by the City of Harare and Zimbabwe Electricity Supply Authority (ZESA).

Although the charges are shown on the same bill, they are calculated differently as follows:

Electricity charges:

Fixed monthly charge	\$7,30
Consumption rate	6,65c per kWh
Surcharge	10% of total bill

The 10% surcharge is a handling charge and goes to Harare City Council. The rest goes to ZESA.

The **kilowatt-hour** is often shortened to kWh. This is the amount of electricity that is used when 1 000 watts of electricity are consumed in 1 hour. The kWh is the basic 'unit' of electrical consumption.

City of Harare/ZESA

P.O. Box 1680 TELEPHONE 707501

ELECTRICITY AND/OR WATER CHARGES

Previous Account	Last Payment	Credits	Credits To	Balance
42.52	29 AUG	42.52	12 SEP	0.00

Code	This Reading	Last Reading	Units	
ZESA CHARGES				
E A FIXED MONTHLY CHARGE				7.30
E A	22712	21958	814	54.13
CITY OF HARARE CHARGES				
W 1	6071	6038	33	14.44
10PC SURCHARGE ON ELEC ACC				6.14

Property Reference	Last Reading Date	Due and Payable	
49 19 123	05 AUG	27 SEP 91	82.01

Address:

Please remit this account with payment to the City Treasurer.

If this account remains unpaid after due date, supplies may be discontinued without further notice.

A RECEIPT ON THIS FORM IS NOT VALID UNLESS MACHINE PRINTED

Fig. 14.3

Water charges:

WATER TARIFF		
Please note that the Water Tariffs have been amended in respect of water consumed after the normal meter reading date in October 1990. Details of the amended Tariffs are given below.		
MUNICIPAL AREA		
Type	Scale	Rate per month per cubic metre
Single Family Dwelling Units	W1	first 13 m^3 at 36,5 c/m^3 next 26 m^3 at 48,5 c/m^3 next 31 m^3 at 60,0 c/m^3 over 70 m^3 at 71,5 c/m^3
Commercial and Industrial	W2	48,5 c/m^3
OUTSIDE MUNICIPAL AREA		
All consumers	W3	57,5 c/m^3
N.B. As water accounts are calculated in metric units all consumptions shown are cubic metres unless otherwise stated.		

Fig. 14.4

The unit of water is the cubic metre (m^3). (Note that this is equivalent to 1 000 litres, or 1 kl.)

Example 12

Check the bill shown in Fig. 14.3.

Electricity charges:

No of units used = 22 712 − 21 958
= 814
Charge for units = 6,65 c × 814
= \$54,13
Fixed monthly charge = \$7,30
Total for electricity = \$61,43
10% surcharge = 10% of \$61,43
= \$6,14 (to nearest cent)

Water charges:

No of units used = 6 071 − 6 038
= 33
33 = 13 + 20
Charge for units = (36,5c × 13) + (48,5c × 20)
= \$4,745 + \$9,70
= \$14,445 = \$14,44

Grand total: = \$61,43 + \$6,14 + \$14,44
= \$82,01

Example 13

A consumer used 580 units of electricity in a month. What will be the bill for electricity?

Cost for units used = 6,65c × 580 = 3 857 c
= \$38,57
Fixed monthly charge = \$7,30

Subtotal = \$45,87
10% surcharge = 10% of \$45,87
= \$4,59
Total bill = \$50,46
(i.e. \$45,87 + \$4,59)

Example 14

Find the bill for 51 m^3 *of water in a month when calculated by (a) Scale W1, (b) Scale W2, (c) Scale W3.*

(a) *Scale W1*

51 = 13 + 26 + 12

Water bill
= (36,5c × 13) + (48,5c × 26) + (60,0c × 12)
= 474,5c + 1261c + 720c = 2455,5c
= \$24,55 to nearest cent*

(b) *Scale W2*
Water bill = 48,5c × 51 = 2473,5c
= \$24,73 to nearest cent*

(c) *Scale W3*
Water bill = 57,5c × 51 = 2932,5c
= \$29,32 to nearest cent*

* In practice the 0,5c is rounded down.

Owner's charges (Household rates)

Property owners are charged for various services which are supplied to their houses and families. Such services include road construction and maintenance, refuse removal and the supply of public amenities such as civic centres, sports stadiums and parks. Bills for this are called **owner's charges** but are more commonly called **household rates**, or simply just **rates**. Fig. 14.5 on the next page shows a typical 6-monthly rates bill for a property in the Mount Pleasant area of Harare.

City of Harare

P.O. Box 1680 HARARE

Business Hours	
Monday, Tuesday	8.00 – 4.45
Wednesday	8.00 – 3.45
Thursday, Friday	8.00 – 4.45

IN RESPECT OF STAND 1903 MOUNT PLEASANT

Please quote Rates Code in communication
Rates Code 044968
Reflects payments to 19 SEP

ITEM	DATE FROM	DATE TO	VALUATION $	RATE cents per $	
LAND	1 JUL 90	31 DEC 90	6000	1,014	60,84
IMPROVEMENTS	1 JUL 90	31 DEC 90	10400	0,711	73,94
REFUSE REMOVAL	1 JUL 90	31 DEC 90			18,54

Due date 31 OCT 90 Unpaid balance after 30 NOV 90 will attract interest 10,75 PER CENT FROM DUE DATE — 153,32

Fig. 14.5

Note that the bill contains three components:

1 LAND
The plot of land (known as a *stand*) is evaluated and charged at a rate of 1,014 cents per $.

2 IMPROVEMENTS
Any improvements to the stand are evaluated and charged at a rate of 0,711 cents per $. Note that such improvements normally include those to the main house and outbuildings.

3 REFUSE REMOVAL
A fixed charge of $18,54 for the regular disposal of unwanted rubbish.

Example 15

Check that the rates bill shown in Fig. 14.5 is correct.

Charge for
land = 6 000 × 1,014c = $60,84
improvements = 10 400 × 0,711c = $73,94
refuse removal (fixed charge) = $18,54

Total owner's charges = $153,32

Exercise 14i

Unless told otherwise, use the water charges, electricity charges and household rates as given above when doing this exercise.

1 The electricity bill for someone who uses 696 units is $58,94, including the fixed monthly charge and the 10% surcharge. Check that this bill is correct.

2 What would be the bill for someone who used half as many units as the person in question 1?

3 The reading on an electricity meter changes from 10 819 to 11 127 in one month. Calculate the bill.

4 The water bill for a house charged on the W1 Scale was $20,35 when 44 m^3 of water was used. Check that this bill is correct.

5 What would be the bill for the house in question 4 if, next month, its owner used half as much water?

6 At the beginning and end of a month the readings on a water meter are 898 m^3 and 963 m^3 respectively. Calculate the bill if charges are made on the W1 Scale.

7 If 60 m^3 of water are used in a month, calculate the bill when charged on (a) Scale W1, (b) Scale W2, (c) Scale W3.

8 A property is given the following valuations:

LAND	\$4 000
IMPROVEMENTS	\$12 500

Including REFUSE REMOVAL, Calculate the 6-monthly rates bill for the property.

9 The rates for the property in question 8 are increased to the following:

LAND	1,205c per \$
IMPROVEMENTS	0,822c per \$
REFUSE REMOVAL	\$24,64 (fixed)

Assuming that the valuations remain as before, calculate the new 6-monthly rates bill for the property.

10 A house in the Mount Pleasant area of Harare has a LAND valuation of \$8 000, an IMPROVEMENTS valuation of \$15 000 and is subject to Scale W1 water charges.

On 31 December, readings of the water and electricity meters show that in the month of December the household consumed 64 m^3 and 905 kWh respectively.

Find the total amount that the bills will come to (i.e. the water and electricity bills for December and the 6-monthly rates bill for 1 July to 31 December).

Chapter 15

Geometrical constructions (2) Ruler and compasses

Drawing accurately

Remember the following points:

1 All constructions should be made with a pencil. Use a hard pencil with a sharp point. This will give thin lines which will be more accurate.

2 Check that your ruler has an undamaged straight edge. A damaged ruler is useless for construction work.

3 Check that your compasses are working properly and are not too loose. Loose compasses can be tightened with a small screwdriver.

4 All construction lines must be seen. Do not rub out anything which leads to the final result.

5 Always take great care, especially when drawing a line through a point.

6 Where possible, arrange that the angles of intersection between lines and arcs are about 90°.

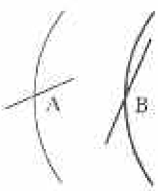


Fig. 15.1

In Fig. 15.1, there is a clear point of intersection at A. At B there is a large 'area of intersection'; this is because the lines are too thick and the angle between them is too small.

7 It is often helpful to draw lines which are longer than the required segments, using compasses to mark off the required points as in Fig. 15.2(a).

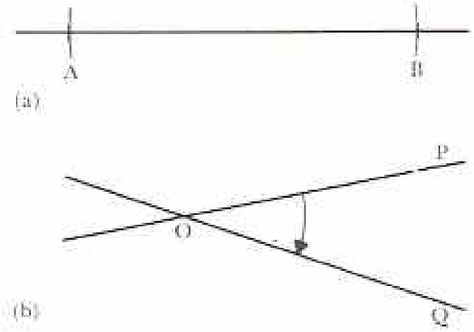


Fig. 15.2

Similarly, when drawing an angle, extend its arms beyond its vertex as in Fig. 15.2 (b). This will improve the accuracy of the drawing.

Abigail Ndudzo
Mutare girls
High

To bisect a straight line segment

Fig. 15.3

In Fig. 15.3, the **line segment** AB is the part of the line between A and B, including the points A and B.

To **bisect** the line segment AB means to divide it into two equal parts.

(a) Open a pair of compasses so that the radius is about $\frac{3}{4}$ of the length of AB.

(b) Place the sharp point of the compasses on A. Draw two arcs, one above, the other below the middle of AB, as in Fig. 15.4.

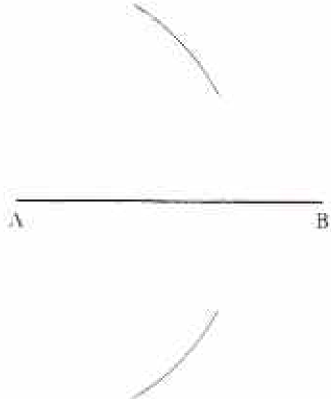


Fig. 15.4

(c) *Keep the same radius* and place the sharp point of the compasses on B. Draw two arcs so that they cut the first arcs at P and Q as in Fig. 15.5.

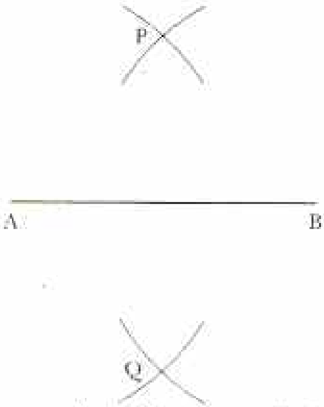


Fig. 15.5

(d) Draw a straight line through P and Q so that it cuts AB at M.

M is the mid-point of AB. PQ meets AB perpendicularly. PQ is the **perpendicular bisector** of AB. Use a ruler and protractor to check that AM = MB and $A\hat{M}P = B\hat{M}P = 90°$ in Fig. 15.6.

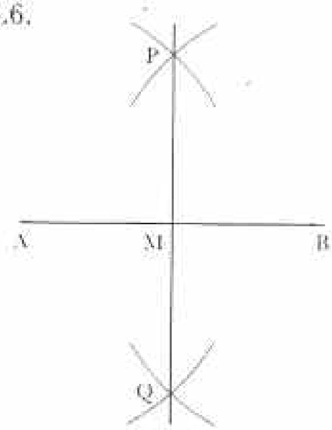


Fig. 15.6

Exercise 15a

Use ruler and compasses **only** in this exercise.

1 Draw any line segment AB. Use the above method to find the mid-point of AB. Check by measurement that your answer is correct.

2 Fig. 15.7 represents a paper triangle, ABC.

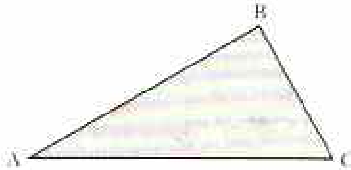


Fig. 15.7

△ABC is folded so that A meets C. This gives a fold line PM as shown in Fig. 15.8.

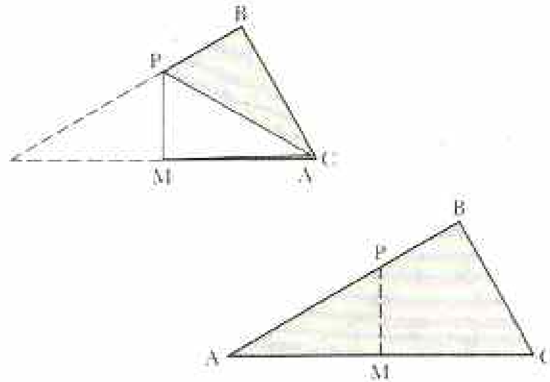


Fig. 15.8

(a) What can be said about the point M?
(b) What can be said about line PM?

3 (a) Cut out a large paper triangle, ABC (as in Fig. 15.7). △ABC should be scalene and acute-angled.
(b) Fold △ABC so that A meets C.
(c) Open out △ABC, then make a second fold so that B meets C.
(d) In the same way, make a third fold so that A meets B.
(e) What do you notice about the three folds?
(f) What can be said about each fold?

4 Draw any triangle ABC.
(a) Construct the perpendicular bisector of each side.
(b) What do you notice?

5 Draw any circle and any two chords AB and XY. (Neither chord should be a diameter.)
   (a) Construct the perpendicular bisectors of AB and XY.
   (b) What do you notice?
6 Draw any circle and any diameter AB.
   (a) Construct the perpendicular bisector of AB and extend it if necessary to cut the circumference at P and Q.
   (b) What kind of chord is PQ?
   (c) Join AP, PB, BQ, QA. What kind of quadrilateral is APBQ?
7 Draw any triangle ABC.
   (a) Construct the mid-point, M, of AB.
   (b) Construct the mid-point, N, of BC.
   (c) Measure MN and AC.
   (d) What do you notice?
8 (a) Draw a line 10 cm long.
   (b) Construct a square with this line as diagonal.
   (c) Measure a side of the square.

To bisect a given angle

Given any angle ABC as in Fig. 15.9:

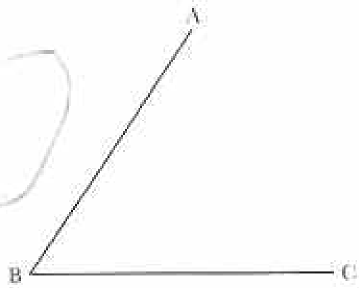


Fig 15.9

(a) With centre B and any radius (i.e. open a pair of compasses to any radius and place the sharp point at B) draw an arc to cut the arms BA, BC at P, Q. See Fig. 15.10.

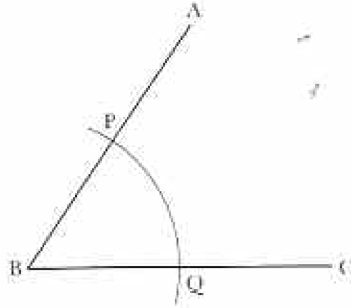


Fig. 15.10

(b) With centres P, Q and equal radii, draw arcs to cut each other at R.

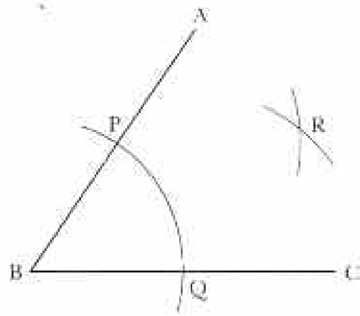


Fig. 15.11

(c) Join BR.

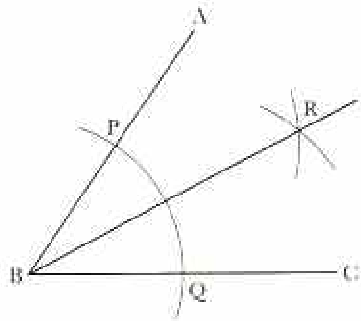


Fig. 15.12

BR bisects $A\hat{B}C$. BR is the **bisector** of $A\hat{B}C$. Use a protractor to check that $A\hat{B}R = C\hat{B}R$ in Fig. 15.12.

Exercise 15b

Use ruler and compasses *only* in this exercise.

1 Draw any angle ABC. Use the above method to construct the bisector of $A\hat{B}C$. Use a protractor to check your result.

2 A paper triangle like that of Fig. 15.7 is folded so that AB lies along AC. This gives a fold line AR as shown in Fig. 15.13.

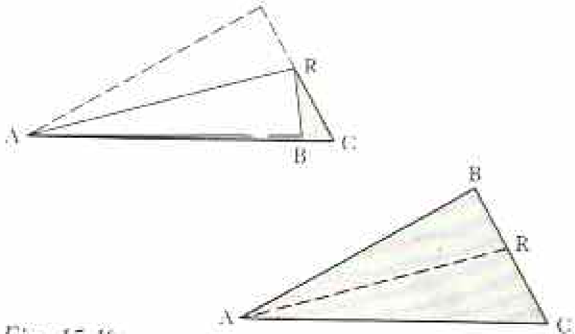


Fig. 15.13

(a) What can be said about angles BAR and CAR?
(b) What is the correct name for the line AR?

3 (a) Cut out a large paper triangle, ABC, such as that of Fig. 15.7.
(b) Fold △ABC so that AB lies along AC.
(c) Open out △ABC, then make a second fold so that BC lies along BA.
(d) In the same way, make a third fold so that CB lies along CA.
(e) What do you notice about the three folds?

4 (a) Draw a scalene triangle PQR such that $\hat{Q}$ is obtuse.
(b) Construct the bisectors of $\hat{P}$, $\hat{Q}$ and $\hat{R}$.
(c) If necessary, produce each bisector so that it cuts the other two.
(d) What do you notice about the three bisectors?

5 (a) Construct an isosceles triangle XYZ such that XY = YZ = 8 cm.
(b) Construct the bisector of $\hat{Y}$.
(c) Construct the perpendicular bisector of side XZ.
(d) What do you notice?

6 (a) Draw any obtuse angle ABC.
(b) RB is the bisector of $A\hat{B}C$. Construct RB.
(c) SB is the bisector of $R\hat{B}C$. Construct SB.
(d) TB is the bisector of $S\hat{B}C$. Construct TB.
(e) What fraction of $A\hat{B}C$ is $T\hat{B}C$?

7 (a) Draw a triangle with sides 6, 8, 10 cm.
(b) Bisect the smallest angle.
(c) The bisector cuts the opposite side into two parts. Measure the lengths of the two parts.

8 (a) Draw a circle of radius 75 mm.
(b) Construct two diameters at right angles to each other.
(c) Construct two more diameters bisecting the angles between those drawn first.
(d) Join the ends of the diameters to form a regular polygon. What sort of polygon is it?
(e) Measure the length of one of the sides of the polygon.

To construct an angle of 90°

Given a point B on a straight line AC:

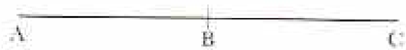


Fig. 15.14

It is required to construct a line BR through B such that $R\hat{B}A = R\hat{B}C = 90°$.

(a) With centre B and any radius draw arcs to cut AC at P and Q. See Fig. 15.15.

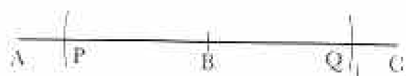


Fig. 15.15

(b) With centres P, Q and equal radii, draw arcs to cut each other at R.

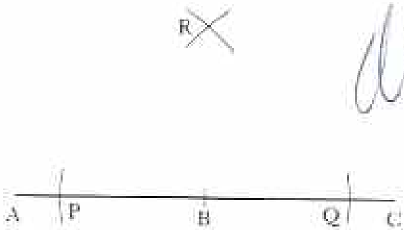


Fig. 15.16

(c) Join BR.

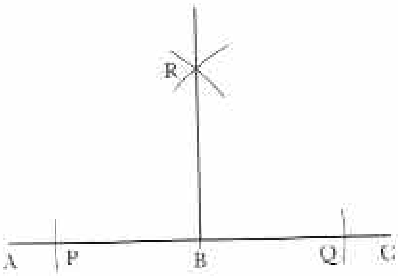


Fig. 15.17

BR is perpendicular to AC. Hence $R\hat{B}A = R\hat{B}C = 90°$. Use a protractor to check this result in Fig. 15.17. Notice that this method is equivalent to bisecting an angle of 180°.

The following method is useful if B is near the edge of the paper.

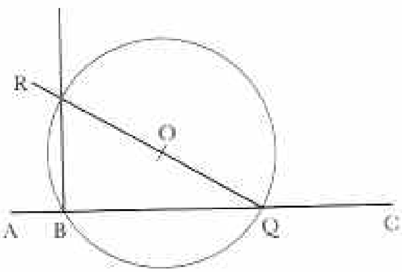


Fig. 15.18

In Fig. 15.18,

(a) Draw any circle to pass through B and cut AC and Q. Mark the centre of the circle, O.

(b) Join QO. Produce QO to cut the circle again at R.

(c) Join BR. $R\hat{B}C = 90°$.

Use a protractor to check that $R\hat{B}C = 90°$ in Fig. 15.18.

To construct an angle of 45°

$45° = \frac{1}{2}$ of 90°. To construct an angle of 45°, first construct an angle of 90° and then bisect it. This is shown in Fig. 15.19.

Use a protractor to check the data in Fig. 15.19.

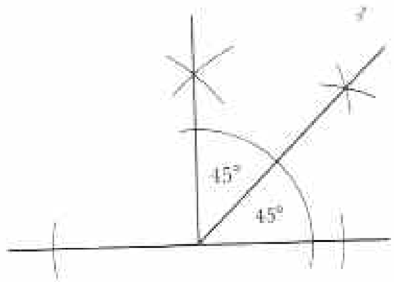


Fig. 15.19

Exercise 15c

Use ruler and compass *only* in this exercise.

1 Construct angles of 90° and 45°.

2 Construct an angle of 135°. (135° = 90° + 45°)

3 (a) Construct a square of side 83 mm.
(b) Measure the length of a diagonal.

4 (a) Construct a rectangle measuring 7,4 cm by 10,3 cm.
(b) Measure the length of a diagonal.

5 (a) Construct $\triangle PQR$ such that $\hat{Q} = 90°$, PQ = 5 cm and QR = 6 cm.
(b) Measure the length of hypotenuse PR.

6 (a) Draw any circle and any chord AB. (AB should *not* be a diameter.)
(b) Construct another chord BC such that $A\hat{B}C = 90°$.
(c) Join AC. What do you notice about the line AC?

7 (a) Construct $\triangle XYZ$ such that $\hat{X} = \hat{Z} = 45°$ and XZ = 8 cm.
(b) Measure either of the sides XY or YZ.

8 (a) Construct an isosceles triangle with the equal sides 9 cm long and the angle between them 45°.
(b) Measure the third side.

To construct an angle of 60°

Given a straight line BC:

Fig. 15.20

To construct a point such that $A\hat{B}C = 60°$,

(a) With centre B and any radius, draw an arc to cut AB at X. Notice in Fig. 15.21, the arc is extended well above BC.

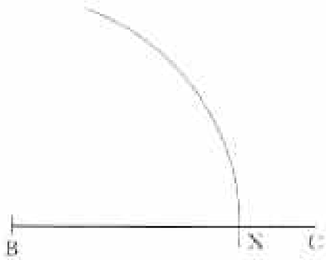


Fig. 15.21

(b) With centre X and the *same* radius, draw an arc to cut the first arc at A.

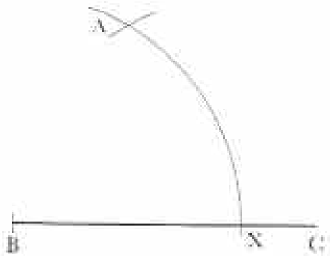


Fig. 15.22

(c) Join AB.

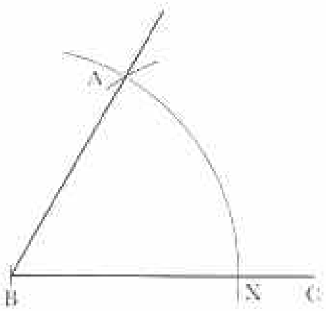


Fig. 15.23

$A\hat{B}C = 60°$. Use a protractor to check that $A\hat{B}C = 60°$ in Fig. 15.23 Notice that the points A, B and X form an equilateral triangle.

To construct an angle of 30°

$30° = \frac{1}{2}$ of 60°. To construct an angle of 30°, first construct an angle of 60° and then bisect it. This is shown in Fig. 15.24.

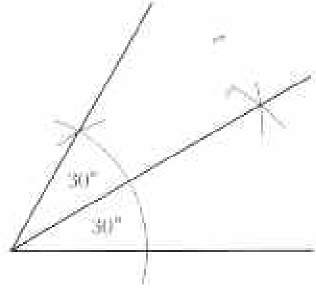


Fig. 15.24

Use a protractor to check the data in Fig. 15.24.

Exercise 15d

Use ruler and compasses *only* in this exercise.

1 Construct angles of 60°, 30°, 15°.

2 Construct angles of 120°, 105°. (105° = 90° + 15°)

3 Construct an equilateral triangle with sides of length 7,2 cm.

4 (a) Draw a circle of radius 5 cm.
(b) Construct radii at 60° intervals in the circle.
(c) Hence construct a regular hexagon.
(d) How long are the sides of the hexagon?

5 Construct a parallelogram with sides 6 cm and 9 cm, the angle between these sides being 60°. Measure the diagonals.

6 (a) Construct a rhombus with sides 6 cm such that one of its acute angles is 75°.
(b) Measure the diagonals of the rhombus.

7 (a) Construct $\triangle LMN$ in which LM = 105 mm, $\hat{M} = 60°$ and $\hat{N} = 90°$.
(b) Measure LN and MN.

8 (a) Construct a kite ABCD in which AB = 5 cm, AD = 8 cm and $\hat{A} = \hat{C} = 105°$.
(b) Measure the diagonal AC.

Exercise 15e (Further practice)

In this exercise draw a rough figure first. This will give you an idea of the shape of the final drawing. All construction lines should be left in your work.

1 (a) Construct an isosceles $\triangle ABC$ so that AB = AC, BC = 75 mm and the length of the perpendicular from A to BC is 60 mm.
(b) Measure AB.

2 (a) Construct △ABC with sides 7 cm, 8 cm, 9 cm.
(b) Draw the perpendicular bisectors of all three sides. These should meet at one point O.
(c) With centre O and radius OA, draw a circle.
(d) What is the radius of the circle? Does the circle also pass through B and C?

3 (a) Draw a triangle PQR with sides 69 mm, 102 mm, 135 mm.
(b) Use the method of question 2 to construct the circle passing through P, Q and R.
(c) Measure the radius of the circle.

4 (a) Construct a △ with sides 6 cm, 8 cm, 9 cm.
(b) Use ruler and compasses to find the mid-point of each side.
(c) Join each vertex to the mid-point of the opposite side. (These three lines are called **medians**.)
(d) Do the three medians meet at a point?
(e) By careful measurement, find the ratio in which this point divides the length of each median.

5 (a) Use the method of question 4 with a triangle of any size.
(b) Do the three medians behave in the same way as before?

6 (a) Construct △XYZ in which XY = 8.3 cm, YZ = 11,9 cm and $X\hat{Y}Z$ = 60°.
(b) Construct M, the mid-point of XZ.
(c) Measure YM.

7 (a) Construct △ABC with AB = 68 mm, AC = 102 mm and $\hat{B}$ = 120°.
(b) Construct the perpendicular bisectors of AB and BD and let them meet at O.
(c) Measure OA, OB, OC.

8 (a) Construct △ABC in which AB = 9 cm, BC = 12 cm, $A\hat{B}C$ = 60°.
(b) Construct the bisector of $\hat{A}$ and let it meet BC at D.
(c) Measure DC.

9 (a) Construct a triangle ABC in which AB = 99 mm, BC = 114 mm, CA = 126 mm.
(b) Use ruler and compasses to find the position of M, the mid-point of BC.
(c) Through M, construct lines parallel to AC, AB to meet AB, AC in H, K respectively.
(d) Measure HK.

10 (a) Construct a parallelogram ABCD with BD = 104 mm, DC = 48 mm and $B\hat{D}C$ = 30°.
(b) Measure AC.

11 (a) Construct a trapezium PQRS in which PQ is parallel to SR, PQ = 6 cm, PS = 5 cm, SR = 11 cm and QS = 9 cm.
(b) Measure QR.

12 (a) The diagonals of a parallelogram bisect each other. Construct a parallelogram with one side 10 cm long, and diagonals 15 cm and 10 cm long. (Draw a sketch first.)
(b) Measure the side of the parallelogram which is not given.

Chapter 16

Scale drawing (2) Angles of elevation and depression

Horizontal and vertical

Any surface which is parallel to the surface of the earth is said to be **horizontal.** For example, the surface of liquid in a container is always horizontal, even if the container is held at an angle as shown in Fig. 16.1.

Fig. 16.1

The floor of a room is usually horizontal. Any line drawn on a horizontal surface will also be horizontal. Any line or surface which is perpendicular to a horizontal surface is said to be **vertical.** The walls of your classroom are vertical. The thread on a plumb-line hangs vertically. A plumb-line is a mass which hangs freely on a thread (Fig. 16.2).

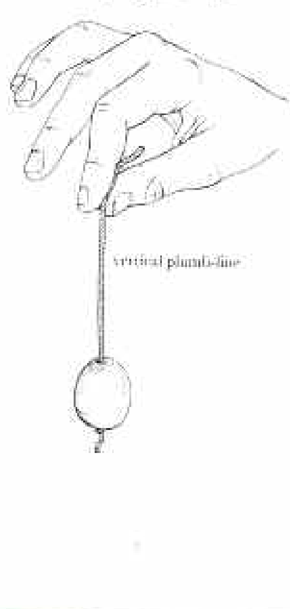


Fig. 16.2

Exercise 16a (Oral)

Fig. 16.3 shows a corner of a student's study. Use the picture to answer the questions in this exercise.

1 Say whether the following are horizontal or vertical, or neither.
- (a) the table top
- (b) the door
- (c) the pictures
- (d) the floor boards
- (e) the back of the chair
- (f) the table legs
- (g) the ruler (on the table)
- (h) the line where the walls meet
- (i) the brush handle
- (j) the top edge of the wall

2 Name a further 5 things in Fig. 16.3 which are (a) horizontal, (b) vertical, (c) neither horizontal nor vertical.

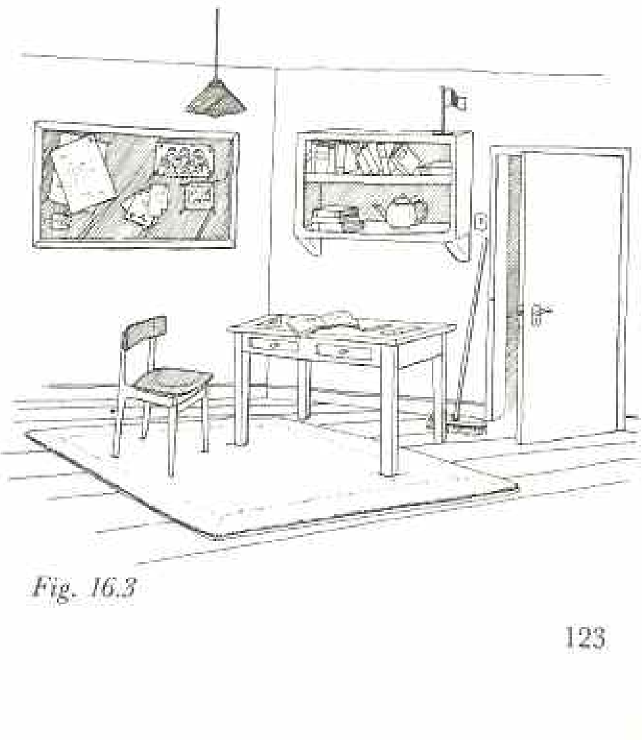


Fig. 16.3

Angle of elevation

In Fig. 16.4 (b) the boy B is looking at the top of the tree T. To do this he has to raise his line of sight through an angle $e°$ from the horizontal. The angle $e°$ in Fig. 19.4 is called the **angle of elevation** of T from B.

Fig. 16.4

Angle of depression

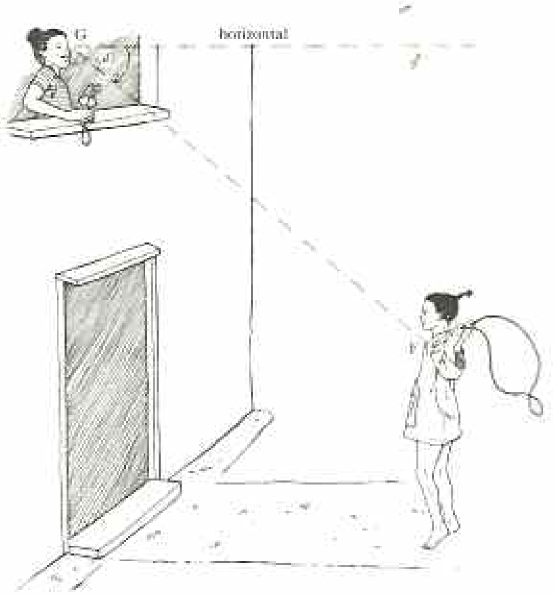


Fig. 16.5

In Fig. 16.5, the girl in the window at G is looking down at her friend at F. To do this she has to lower her line of sight from the horizontal through an angle $d°$. The angle $d°$ in Fig. 16.5 is called the **angle of depression** of F from G.

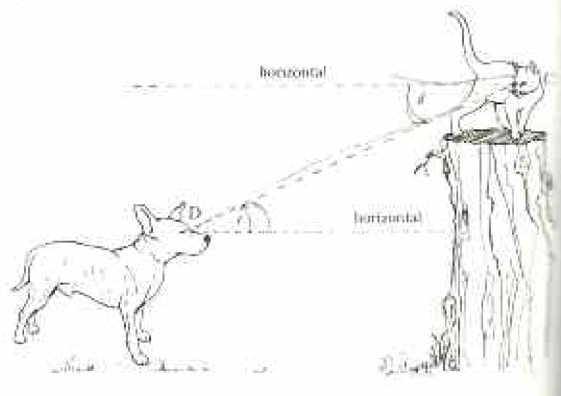


Fig. 16.6

Fig. 16.6 shows that there is a connecti between angles of elevation and depression.

The angle of elevation of the cat, C, from th dog, D, is equal in size to the angle of depressi of D from C. They are alternate angles.

Exercise 16b

You will need a protractor for this exercise.

1 Assume that Figs 16.4, 16.5 and 16.6 are all scale drawings. Measure the following:
  - (a) the angle of elevation, $e°$, in Fig. 16.4;
  - (b) the angle of depression, $d°$, in Fig. 16.5;
  - (c) the angle of elevation of C from D in Fig. 16.6;
  - (d) the angle of depression of D from C in Fig. 16.6.

2 (a) Measure the angle of elevation of A from C in Fig. 16.7.
  - (b) Hence state the angle of depression of C from A.

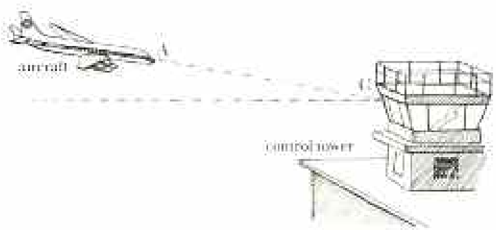


Fig. 16.7

3 (a) Measure the angle of depression of man B from man A in Fig. 16.8.
  - (b) Hence state the angle of elevation of A from B.

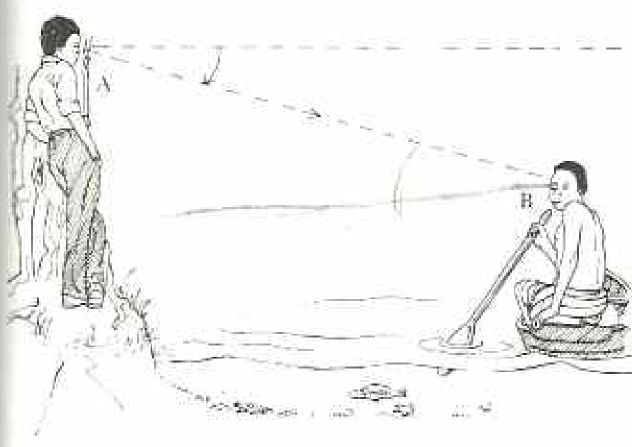


Fig. 16.8

4 (a) Measure the angle of elevation of the light bulb from student P in Fig. 16.9.
  - (b) Measure the angle of elevation of the light bulb from student Q.

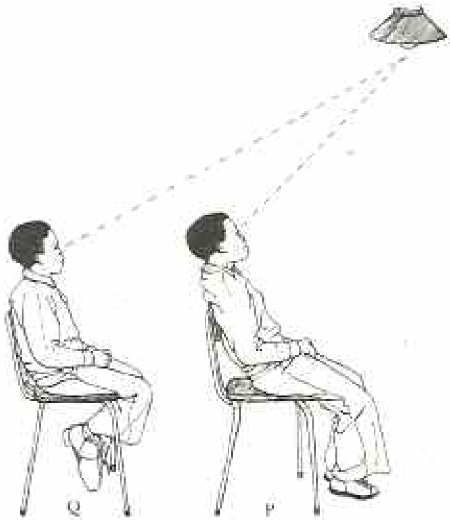


Fig. 16.9

5 In Fig. 16.10, measure the angle of depression of the coin from the man.

Fig. 16.10

Measuring angles of elevation and depression

Angles of elevation and depression can be measured with a simple instrument called a **clinometer.** Fig. 16.11 overleaf shows a clinometer made from a blackboard protractor.

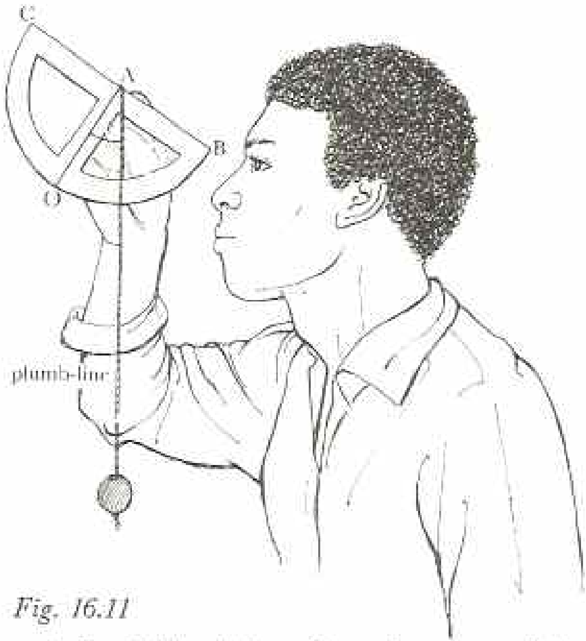


Fig. 16.11

A plumb-line is hung from the centre of the protractor at A. The observer sights an object along the line BAC. The angle of elevation, $\hat{e}$°, is the angle between AO and the plumb-line. The size of $\hat{e}$° can be read from the scale. Notice that $\hat{e}$ increases from 0° at O to 90° at B. If a blackboard protractor is used it may be helpful if the angle markings are changed.

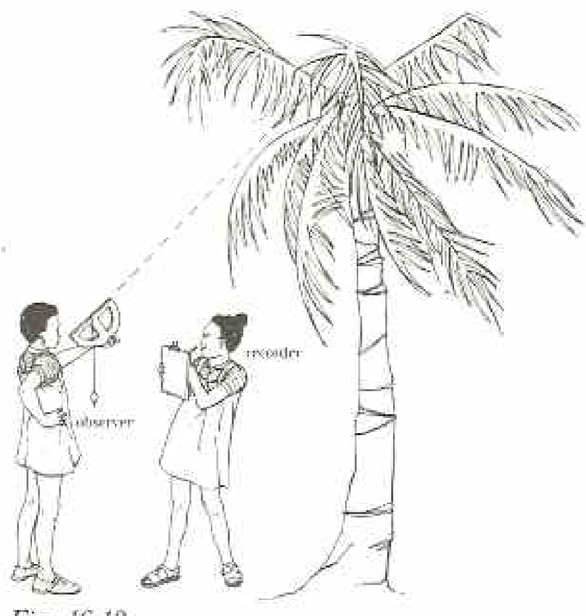


Fig. 16.12

Fig. 16.12 shows how a clinometer is used to find the angle of elevation of the top of a tree. An observer uses the clinometer and a recorder takes down the reading of $\hat{e}$.

Example 1

Two girls use a clinometer to find the angle of elevation of the top of a tree as in Fig. 16.12. They also measure the distance of the tree from the observer and the height of the observer's eye above the ground. Their results are shown in the sketch in Fig. 16.13.

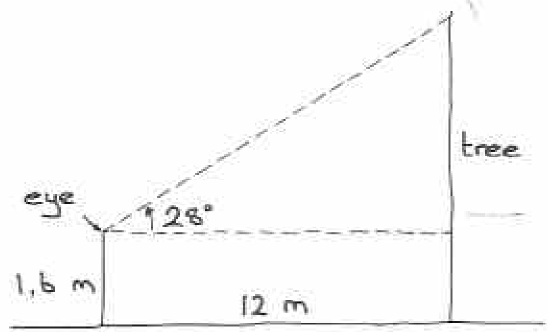


Fig. 16.13

Use the data of the sketch to make an accurate drawing. Hence find the height of the tree.

Fig. 16.14 is a scale drawing using a scale 1 cm to 2 m. In Fig. 16.14, HT represents the full height of the tree.

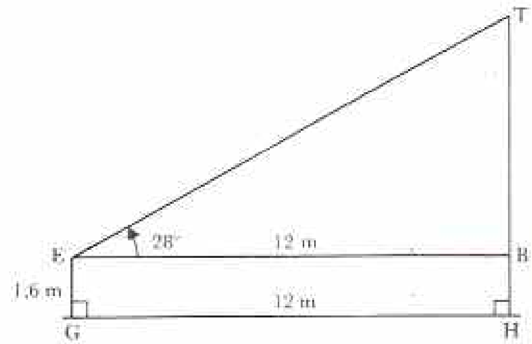


Fig. 16.14

HT = 3,95 cm
Height of tree = 3,95 × 2 m
= 7,9 m.

There are likely to be small errors in measurements and drawings. It is more sensible to say that the height of the tree is about 8 m to the nearest metre.

Notice that EG is the height of the observer's eye above the ground. Another way to find the height of the tree is to construct triangle TEB. The length BT can be found. The height of the tree will be BT + EG.

Exercise 16c

You will need a clinometer and a tape measure for this exercise. A clinometer can be made from a blackboard protractor.

Use the method of Example 1 to find the heights of some objects in your school.

Exercise 16d

You need a protractor, set square and a ruler. All questions can be answered by making a scale drawing. Choose a suitable scale in each case.

1

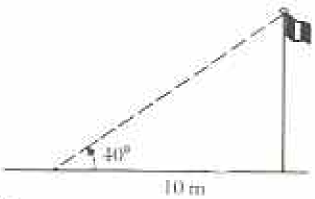


Fig. 16.15

Find the height of the flagpole to the nearest ½ m.

2

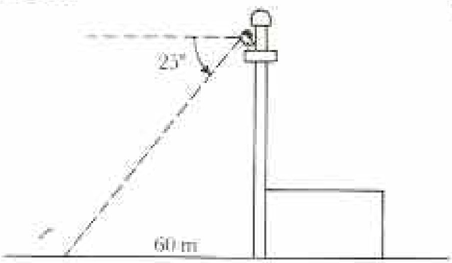


Fig. 16.16

Find the height of the roof R above the ground to the nearest metre.

3

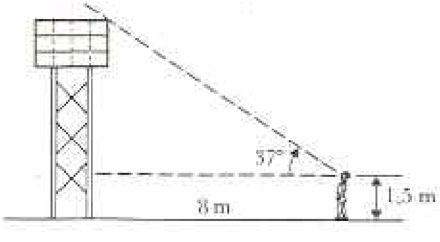


Fig. 16.17

Find the height of the water-tower to the nearest ½ m.

4

Fig. 16.18

Find the width of the river to the nearest metre.

5 The angle of elevation of the top of a tower from a point 42 m away from its base is 36° Find the height of the tower.

6 From the top of a building 50 m high, the angle of depression of a car is 55°. Find its distance from the foot of the building.

7 The angle of elevation of the sun is 45°. A tree has a shadow 12 m long. Find the height of the tree.

8 The angle of elevation of the sun is 27°. A man is 180 cm tall. How long is his shadow? Give your answer to the nearest 10 cm.

9 The angle of elevation of the top of a radio mast from a point 53 m from its base on level ground is 61°. Find the height of the mast to the nearest 5 m.

10 Fig. 16.19 shows the angles of elevation of an aircraft from two points 1 100 m apart.

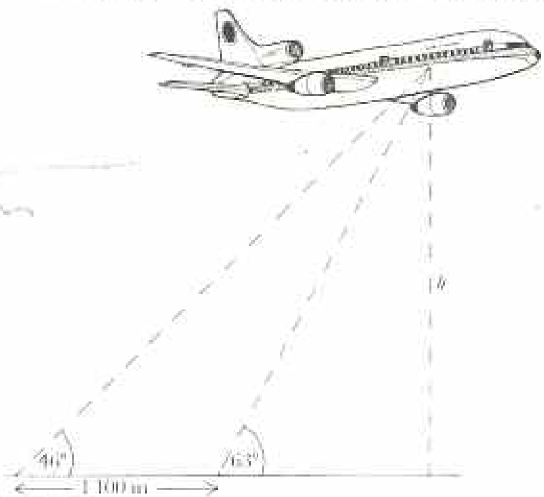


Fig. 16.19

Find the height of the aircraft above the ground to the nearest 100 m.

Chapters 9–16

Revision exercise 5 (Chapters 9, 13)

1 Theresa has \$50. She buys x blouses and y skirts. A blouse costs \$9 and a skirt costs \$13. If she gets change, write down an inequality in x and y.

2 On a cartesian plane sketch the region which represents the following set of points.
(a) $\{(x; y) : x < 7\}$ (b) $\{(x; y) : y \geqslant -1\}$
(c) $\{(x; y) : x > -2\} \cap \{(x; y) : y \leqslant 0\}$

3 Find the range of values of x for which
(a) $x - 3 < 2$
(b) $2 - x > 5$
(c) $2x - 2 < \frac{x + 2}{2}$
(d) $\frac{x + 2}{5} \geqslant \frac{x - 3}{3} + 1$

4 Factorise the following.
(a) $9a - 27$ (b) $3r - 8rt$
(c) $42x^2 - 28xy$ (d) $42a^2b - 51ab^2$

5 Factorise the following by grouping in pairs.
(a) $an + am - 3m - 3n$
(b) $a^2 - 7a + 3a - 21$
(c) $3xy - 6xz - 5ay + 10az$

6 In each of the following, re-group the given terms then factorise.
(a) $3ay - 2bx + 2xy - 3ab$
(b) $ds + rt - dt - rs$

7 If $n = 37^2 + 37 \times 63$ use factorisation to find the value of n.

8 Use number lines to draw graphs of the solutions of the following.
(a) $x - 2 \leqslant 0$ (b) $2x + 5 \leqslant 3$

9 Find the solution sets of the following, given that x is an integer.
(a) $5 - x \geqslant 7$ (b) $3 - 7x \leqslant 59$
(c) $\frac{2x + 3}{3} > 5$ (d) $\frac{3x + 2}{5} < \frac{x - 8}{3}$

10 A rectangle is of length x cm and breadth 5 cm. Its perimeter is p cm where $14 \leqslant p \leqslant 32$. Find the corresponding range of values of x.

Revision test 5 (Chapters 9, 13)

1 If x is an integer, what is the highest value of x in the range $-6\frac{3}{4} < x < 2\frac{2}{3}$?
A 3 B 2 C 1 D -6 E -7

2 A boy has more than \$9. He spends \$3 and has \$$m$ left. Which one of the following is the correct inequality in m?
A $m > 5$ B $m < 6$ C $m > 6$
D $m < 12$ E $m > 12$

3 If $4 \geqslant 25 - 3y$, what is the lowest possible value of y?
A 3 B 4 C 6 D 7 E 8

4 Which of the lines in Fig. R16 has the equation $y = -2$?

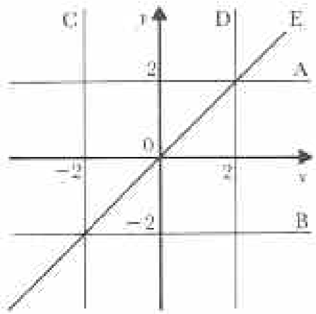


Fig. R16

5 The highest common factor of $10a^2b$ and $8ab$ is
A $80a^2b$ B $2ab$ C $5a$ D $1\frac{1}{4}a$ E 4

6 If $x \in$ {integers}, find the solution sets of the following.
(a) $x - 7 < -4$ (b) $3 - 2x \leqslant 15$
(c) $\frac{4 - 7x}{8} \geqslant -3$ (d) $13 - 10x > 1 - 4x$

7 n is an integer. 3 times n is subtracted from 38. The result is less than 20.
(a) Make an inequality in n.
(b) Find the four lowest values of n.

8 Factorise the following.
(a) $7x - 28$ (b) $5m + 8mn$
(c) $27ab + 36b^2$ (d) $35p^2q - 14pq^2$

9 Factorise the following, simplifying where possible.
(a) $3a^2 + a(2a + b)$
(b) $(5x - 2y)(a - b) - (2x - y)(a - b)$
(c) $pq - pr + 8q - 8r$
(d) $5x + ky + 5y + kx$

10 (a) Factorise $\pi r^2 + 2\pi rh$. (b) Hence find the value of the expression when $\pi = \frac{22}{7}$, $r = 4$ and $h = 5$.

Revision exercise 6 (Chapters 10, 12)

1 A flag on a public building is 4 m long by 2,4 m broad. A small similar flag is 36 cm long. How broad is the small flag?

2 In Fig. R17 △ABC is similar to △PQR. From the data in the figure, calculate the length of (a) QR, (b) PR.

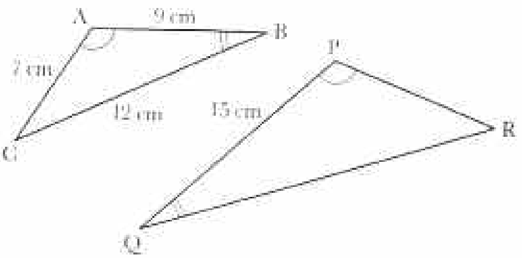


Fig. R17

3 With the data as given in Fig. R18 calculate the length of YZ.

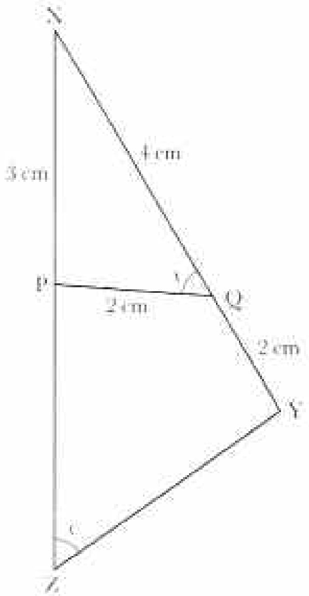


Fig. R18

4 The adjacent sides of a parallelogram are in the ratio 2:5. A similar parallelogram has one side of length 10 cm. Find two possible values for the length of its adjacent side.

5 In Fig. R19, △ APQ is an enlargement of △AXY.

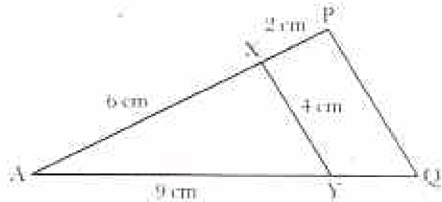
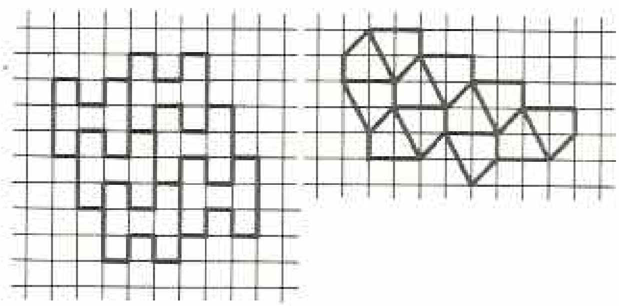


Fig. R19

(a) What is the scale factor of the enlargement?
(b) Calculate the length of PQ.

6 Copy patterns (a) and (b) of Fig. R20 on to graph paper. Extend each pattern by repeating the sequence of basic shapes.



(a) (b)

Fig. R20

In Fig. R21, AB = AC and AM bisects BÂC. Use Fig. R21 to answer questions 7, 8 and 9.

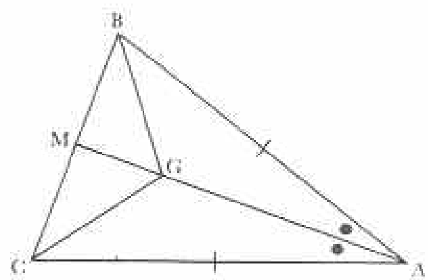


Fig. R21

7 In Fig. R21,
(a) name the triangle which is congruent to $\triangle ACG$;
(b) state the condition of congruency;
(c) identify two other pairs of congruent triangles.

8 In Fig. R21, if $B\hat{A}C = 38°$ and $M\hat{B}G = 38°$, calculate (a) $A\hat{B}C$, (b) $M\hat{G}B$, (c) $A\hat{G}C$.

9 In Fig. R21, if $B\hat{A}C = 42°$ and MB = MG, calculate $A\hat{C}G$.

10 ABCD is a parallelogram. CX is drawn parallel to diagonal DB to meet AB produced at X. Prove that B is the mid-point of AX.

Revision test 6 (Chapters 10, 12)

1 Two similar triangles are such that AB and CB in the first correspond to RT and ST in the second. Which of the following angles is the same size as $A\hat{B}C$?
A $R\hat{S}T$ B $T\hat{S}R$ C $S\hat{R}T$
D $T\hat{R}S$ E $R\hat{T}S$

2 Given Fig. R22, find the length of XQ.

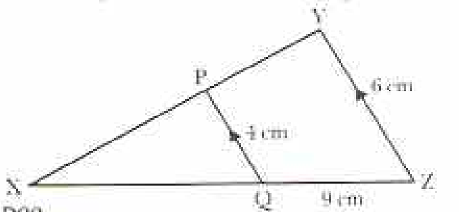


Fig. R22

A 6 cm B 7 cm C 11 cm
D 15 cm E 18 cm

3 With the data in Fig. R23, calculate YQ.

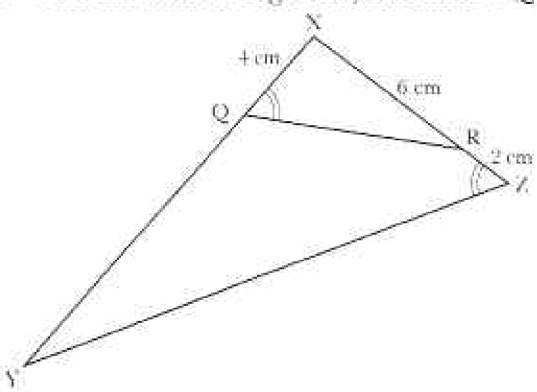


Fig. R23

A 6 cm B 8 cm C 10 cm
D 12 cm E 16 cm

4 A line 12 cm long is enlarged by a scale factor of $-\frac{2}{3}$. What is the length of the enlarged line?
A 4 cm B 8 cm C 9 cm
D 16 cm E 20 cm

5 KLMN is a quadrilateral such that $K\hat{L}M = K\hat{N}M = 90°$ and KN = LM. Which one of the following angles must be equal to $L\hat{K}M$?
A $K\hat{M}L$ B $K\hat{M}N$ C $K\hat{N}L$
D $M\hat{K}N$ E $\hat{M}LN$

6 In Fig. R24 name the triangle which is similar to $\triangle OAB$. If OA = 3 cm, OX = 7,5 cm and AB = 4 cm, calculate XY.

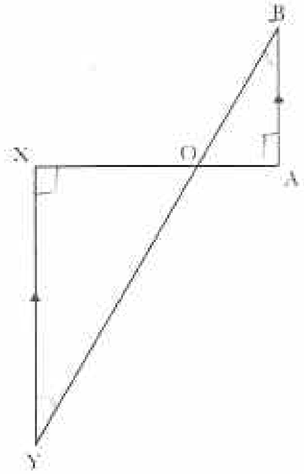


Fig. R24

7 PQR is an equilateral triangle. X is a point on QR such that $P\hat{X}R = 109°$. What is the size of $X\hat{P}R$?

8 In quadrilateral WXYZ, XZ bisects $W\hat{X}Y$ and $W\hat{Z}Y$. Prove that $X\hat{W}Y = X\hat{Y}Z$.

9 In Fig. R25, $\triangle ABC$ is equilateral and D and E lie on AC such that $A\hat{D}B = 97°$ and $C\hat{B}E = 23°$.

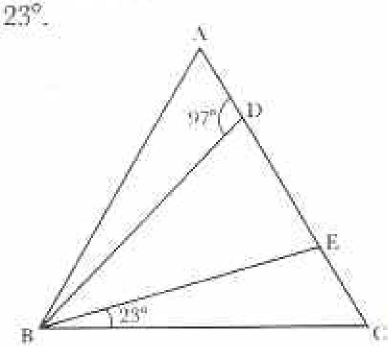


Fig. R25

Prove that $\triangle DBE$ is isosceles.

10 If, in Fig. R25 above, $\triangle ABC$ is equilateral, D and E lie on AC such that $\triangle DBE$ is isosceles and $A\hat{D}B = x^0$, express $D\hat{B}E$ in terms of x.

Revision exercise 7 (Chapters 11, 14)

1 A client borrowed $125 at simple interest. After 8 months she paid back $130 as repayment of her debt plus interest on it. Calculate the percentage rate of interest per annum.

2 A car cost $7 600. A 9% discount is given for paying in cash. The car can also be bought by paying 24 monthly instalments of $364. Find the cost of the car (a) when cash is paid, (b) when it is paid for by instalments. (c) Find the difference between paying in cash and paying by instalments.

Use the following data in questions 3, 4, 5, 6. Table R6 shows the distribution of ages of a group of police cadets.

Table R6

ages in years	21	22	23	24
frequency	5	10	6	9

3 Which age is the mode of the above data?

4 (a) How many cadets were in the group?
(b) Find the median age of the cadets.

5 Calculate the mean age of the cadets.

6 Draw a bar chart of the data in Table R6.

7 The mean of eight numbers is 9. The mean of seven of the numbers is 10. What is the eighth number?

8 In a test the marks of four boys were 23, 18, 24, 27 and the marks of three girls were 21, 16, 29. Find the mean mark of (a) the boys, (b) the girls, (c) all seven students.

9 A bookseller sells $4 812 worth of books in a month. His commission is 4c in the $. How much money does he get?

10 The midday temperatures for a week in a town were 27, 29, 29, 33, 28, 24, 26°C.
(a) What is the modal temperature?
(b) What is the median temperature?
(c) Calculate the mean temperature.
(d) What is the greatest deviation from the mean (i) above, (ii) below?

Revision test 7 (Chapters 11, 14)

1 The selling price of a chair is $53,10 when a sales tax of 20% is included. What is the actual sales tax on the chair?
A $4,38 B $5,31 C $6,93
D $8,85 E $10,62

2 After five games a football team's goal average is 2,8. After one more game the goal average is 3. The number of goals scored in the 6th game was
A 3 B 4 C 5 D 6 E 7

Use the following set of numbers in questions 3, 4 and 5. 2; 2; 2; 5; 5; 8; 9; 19; 11

3 The mode of the above set of numbers is
A 2 B 3 C 5 D 6 E 9

4 The median of the above set of numbers is
A 2 B 3 C 5 D 6 E 9

5 The mean of the above set of numbers is
A 2 B 3 C 5 D 6 E 9

6 The hire purchase price of a motor bike is $2 040. $12\frac{1}{2}$% is paid as a deposit. The remainder is spread over 12 equal monthly instalments.
(a) Calculate the amount of the deposit.
(b) Calculate the remainder to be paid.
(c) Find the amount of each monthly instalment to the nearest cent.

7 A dealer sells a gold ring for $194,40 and makes a profit of 8%. Find the selling price if the dealer is to make a profit of 17%.

8 During a week, the midday temperatures measured in a school were 28, 29, 29, 33, 28, 24, 25°C. Calculate the mean midday temperature.

9 The ages of 14 secondary school students in years and months are: 15.5; 15.0; 15.3; 14.5; 14.7; 15.4; 15.10; 14.9; 15.2; 13.11; 15.1; 16.1; 15.5; 14.11
(a) Find the average of the highest and lowest ages and use this as an assumed mean to find the mean age of all the students.
(b) What is the greatest deviation from the mean (i) above, (ii) below?

10 After 12 games a basketball player had a points average of 18,5. How many points must he score in the next game to raise his average to 20?

Revision exercise 8 (Chapters 15, 16)

1 Using ruler and compasses only, construct angles of (a) 45°, (b) 60°, (c) 150°, (d) 75°.
Note: 150° = 180° − 30°, 75° = 45° + 30°

2 Draw any quadrilateral so that its 4 sides are of different lengths. Use ruler and compasses to find the mid-point of each side. Join the mid-points to form a new quadrilateral. What kind of quadrilateral is it?

3 (a) Construct △XYZ such that $\hat{Y} = 90°$, XY = 8 cm and YZ = 5 cm.
(b) Measure the length of the hypotenuse XZ.
(c) Check your result by calculation.

4 Construct a rhombus such that its angles are 60°, 120°, 60°, 120° and the longer of its diagonals is 12 cm. Measure the length of the shorter diagonal.
Hint: make a sketch first.

5 Draw a circle of radius 7 cm. Construct a chord AB such that AB = 10 cm. Construct a chord BC such that $A\hat{B}C = 90°$. Join AC. What kind of chord is AC? Check your answer by measuring AC.

6 (a) Make a scale drawing of the information in Fig. R26.

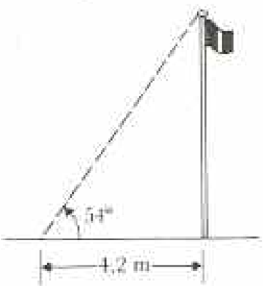


Fig. R26

(b) Hence find the height of the flag pole to the nearest 0,1 m.

7 From the top of a tower 14 m high, the angle of depression of a woman is 32°. Make a scale drawing and find the distance of the woman from the foot of the tower to the nearest ½ m.

8 A radio aerial 16 m long is tightly stretched between a pole and a tree. If the aerial is inclined at 30° to the horizontal, make a scale drawing to find the horizontal distance between the pole and the tree.

9 An aeroplane, flying at 5 000 m, begins to descend to an airport when it is 25 km away horizontally. By scale drawing, find the angle of depression of the airport from the aeroplane when it begins its descent.

10 When the elevation of the sun is 33°, a man has a shadow 2,3 m long. Make a scale drawing and hence find the height of the man to the nearest 5 cm.

Revision test 8 (Chapters 15, 16)

1 Fig. R27 shows a right square-based pyramid resting so that its base is horizontal.

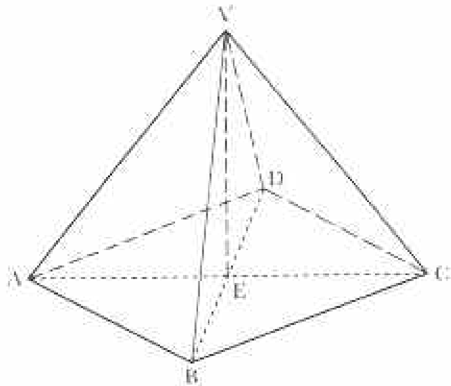


Fig. R27

Which of the points A, B, C, D, E, is vertically below V?

2 In Fig. R27 if the angle of elevation of V from A is 38°, what is the angle of depression of D from V?
A 38°   B 45°   C 52°   D 90°
E impossible to say

3 Which one of the following angles can be constructed using ruler and compasses only?
A 115°   B 125°   C 135°
D 145°   E 155°

4 A vertical fence post has a shadow 1 m long when the angle of elevation of the sun is 45°. The height of the fence post is
A 0,5 m   B 0,7 m   C 1 m
D 1,4 m   E 2 m

5 The angle of elevation of P from Q is 63°. The angle of depression of Q from P is
A 27°   B 63°   C 117°   D 153°   E 243°

6 Draw a line AB 6 cm long. Construct the perpendicular bisector of AB. Hence construct an isosceles triangle ACB such that CA = CB = 8 cm. Measure $\hat{C}$.

7. Construct △ABC such that BC = 6,5 cm, $\hat{B} = 45°$ and BA = 7,5 cm. Measure AC.

8 The angle of elevation of the top of a tower from a point 23 m from its base on level ground is 50°. Make a scale drawing to find the height of the tower to the nearest metre.

9 A road slopes uniformly downwards at an angle of 12° to the horizontal. A ball rolls down the road for 130 m. Make an accurate drawing to find how far vertically the ball drops.

10 From the top of a tree, the angle of depression of a stone on horizontal ground is 55°. If the stone is 8 m from the foot of the tree, find, by scale drawing, the height of the tree.

General revision test B (Chapters 9–16)

1 The mean of 3; 5; 4; 8; 6; 4; 6; 2; 3; 6 is
A 4,5 B 4,7 C 5 D 6 E 10

2 A car dealer gains $600 on a sale which is equivalent to a profit of 8%. What was the cost price?
A $8 100 B $7 500 C $4 800
D $75 E $48

3 Two angles of a triangle are $x°$ and $(x - 90)°$. In terms of x, the third angle of the triangle is
A $(2x - 90)°$ B $(90 - 2x)°$
C $(2x + 90)°$ D $(270 - 2x)°$
E $(270 + 2x)°$

4 Which of the following are factors of $6a^2 - 2ab - 3ab + b^2$?
I $(3a - b)$ II $(2a + b)$ III $(2a - b)$
A I only B I and II only
C I and III only D II and III only
E all of them

5 The average mass of 6 people is 58 kg. If the lightest person has a body mass of 43 kg what is the average mass of the other 5 people?
A 58 kg B 59 kg C 61 kg
D 64 kg E 68 kg

6 In Fig. R28 calculate the length of BX.

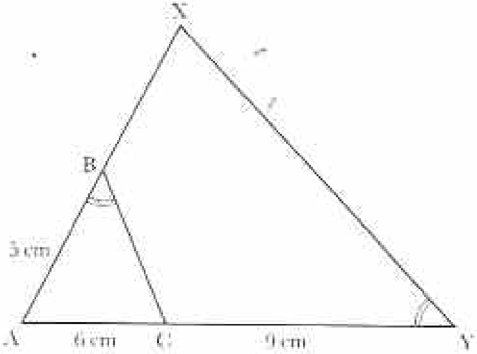


Fig. R28

A 7,5 cm B 8 cm C 10,8 cm
D 13 cm E 18 cm

Fig. R29 shows a cuboid resting so that the base shown shaded is horizontal. Use Fig. R29 to answer questions 7 and 8.

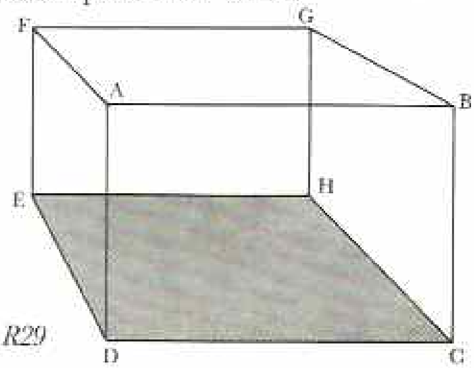


Fig. R29

7 All except one of the points A, B, C, D, E lie in the same vertical plane. Which one is the exception?

8 If the angle of elevation of A from C is 40°, what is the angle of elevation of A from H?
A 20° B 40° C 45° D 50°
E More information is needed

9 The range of values of a for which $11 - 2a \geqslant 1$ is
A $a \geqslant 5$ B $a \geqslant -5$ C $a = 5$
D $a \leqslant 5$ E $a \leqslant -5$

10 The mean of three numbers is 6. The mode of the numbers is 7. The lowest of the three numbers is
A 2 B 3 C 4 D 6 E 7

11 A salesman gets a commission of $8\frac{1}{3}\%$ of the value of the things he sells. Find his commission for selling 2 guitars at $118 each, 5 tennis rackets at $33 each and 14 books at $8,50 each.

12 From a point on level ground 60 m away, the angle of elevation of the top of a tree is $24\frac{1}{2}°$. Calculate the height of the tree to the nearest metre.

13 Use number lines to draw graphs of the solutions of the following.
(a) $x + 2\frac{1}{2} > 0$ (b) $9 \geqslant 1 - 2x$

14 Factorise the following, simplifying brackets where necessary.
(a) $52x - 8x^2$ (b) $5a^2 + a(2b - 3a)$
(c) $x^2 + 5x - 9x - 45$
(d) $6ax - 2by + 4ay - 3bx$

15 A small factory employs 10 workers, 2 supervisors and 1 manager. The workers get \$35 per week, the supervisors get \$55 per week and the manager gets \$86 per week. (a) Calculate the mean weekly wage at the factory. (b) Compare this result with the modal wage. Which average is most representative of the weekly wages?

16 A cuboid is 12 cm long, 9 cm wide and 5 cm high. Calculate the width and height of a similar cuboid of length 15 cm.

17 Fig. R30 is a sketch of a simple bridge. M is the mid-point of the bridge.

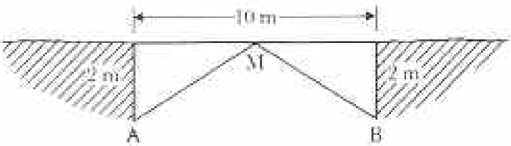


Fig. R30

(a) Use a scale of 1 cm to 1 m to draw an accurate scale drawing of the bridge.
(b) Find the length of the support AM.

18 Find the solution sets for the following inequalities, given that x is an integer.
(a) $2x + 4 > 7$ (b) $30 - 5x < 2x + 9$
(c) $\frac{3}{2}x - \frac{7}{6} \leqslant \frac{5}{2} - \frac{1}{3}x$ (d) $-\frac{x}{5} > -\frac{13}{10}$

19 Using ruler and compasses only, construct $\triangle$PQR such that PQ = 8,4 cm, $\hat{Q} = 60°$ and QR = 4,2 cm. Measure $\hat{P}$ and find the length of PR.

20 Fig. F31 shows notes that a secondary school student made of a survey she did of a tree.

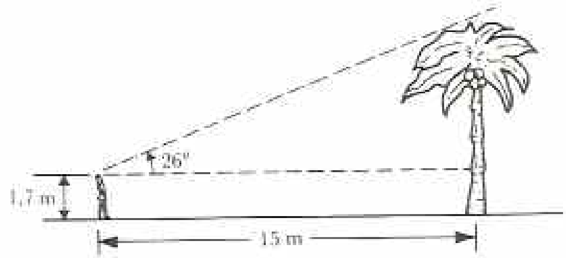


Fig. R31

Make a scale drawing of Fig. R31 and hence find the height of the tree to the nearest $\frac{1}{2}$ metre.

Chapter 17

Everyday arithmetic (3) Money transactions

Budgeting

In daily life it is important to keep accurate accounts of income and expenditure This is true whether at a personal, business, cooperative or state level. Accurate accounts enable individuals, groups and countries to plan their spending. The planning of expenditure is called **budgeting.**

Budgeting is greatly assisted by keeping **cash accounts** of transactions. All businesses and cooperatives are required by law to keep cash accounts which show income and expenditure. Such accounts show the cash which comes in and the cash which is paid out. Table 17.1 is a typical cash account showing one month's trading figures for a small cooperative.

Table 17.1

Eastside Coop Trading: Accounts for May 1990

CASH RECEIVED			CASH SPENT		
Date	**Details**	**$**	**Date**	**Details**	**$**
01/05/91	Brought f'ward	998,68	02/05/91	Stock purchase	500,00
05/05/91	Weekly sales	173,56	04/05/91	Transport	29,80
12/05/91	Weekly sales	245,48	04/05/91	Wages	60,00
19/05/91	Weekly sales	107,70	06/05/91	Repairs	77,95
26/05/91	Weekly sales	148,30	11/05/91	Wages	60,00
			18/05/91	Wages	60,00
			25/05/91	Wages	60,00
			31/05/91	Rent	50,00
			31/05/91	Balance c/f	775,97
		1673,72			1673,72
01/06/91	Brought f'ward	775,97			

Notice the following:

1 The cash account is in two parts: *cash received* (income) on the left, *cash spent* (expenditure) on the right.

2 The total in the left-hand side gives the total cash received.

3 The total in the right-hand side must balance the total in the left-hand side. To do this, (a) the actual expenditure is added, (b) the resulting sub-total is subtracted from the total cash received to give a *balance c/f* (carried forward). In this case:

   (a) \$(500,00 + 29,80 + 60,00 + 77,95 + 60,00 + 60,00 + 60,00 + 50,00) = \$897,75

   (b) \$1 673,72 − \$897,75 = \$775,97

4 The balance is brought forward as cash received into the cash account for the next period.

Example 1

Using Table 17.1, calculate Eastside Coop Trading's profit or loss for the month of May 1990.

Profit = balance carried forward at end of May − cash brought forward at start of May
= \$775,97 − \$998,68
= − \$222,71

The profit is a negative amount. Thus Eastside Coop Trading made a *loss* of \$222,71 in May 1990.

(This loss was caused by the cost of stock purchase and repairs, not by fixed costs such as wages and rent.)

Exercise 17a

1 Refer to Table 17.1.

   (a) Find the total income from sales during May.

   (b) Find the total amount of fixed costs (i.e. wages and rent).

   (c) Find the difference between sales and fixed costs for the month.

2 Refer to Table 17.1.

   By 30 June, Eastside Coop Trading's sales totalled \$704,30 for the month. Against this there were only fixed costs of 4 weeks' wages and rent (as in May). Prepare a cash account for the cooperative for June 1990. Hence (a) state the balance in hand at the end of June, (b) find the profit or loss made in June.

3 The following account gives the cash received and spent by a community school for the month of October.

   The school started the month with \$4 159,60 brought forward from September. On 5 October parents donated \$2 025 towards the cost of a new laboratory. On 15 October students raised \$431,80 from a sponsored walk. On 18 October ticket sales from a school concert amounted to \$330,20 and on the same day an ex-student left \$3 000 to the school in his will.

   On 3 October the school settled bills of \$701,80 and \$954 for stationery and textbooks respectively. On 14 October work began on the laboratory. This involved the purchase of building materials totalling \$3 640 and an advance to the local building cooperative of \$2 200. On 30 October the school settled an electricity bill of \$651,60.

   Make up a cash account for the school for the month of October. Hence find the balance in hand at the end of the month.

Table 17.2 on the facing page is a simplified version of the Central Government's Budget Account during the late 1980s.

Use Table 17.2 to answer questions 4–12.

4 How much money was obtained from miscellaneous sources in 1986/87?

5 In which year did investments provide \$321 million?

6 Which item always provides the greatest source of revenue?

7 Which item caused the greatest source of expenditure in (a) 1985/86, (b) 1987/88?

8 Is it true to say that expenditure on Goods & Services has nearly doubled during the four years?

9 What was the total expenditure on capital expenses during the four year period?

Table 17.2

Central Government Budget Account, 1985–89

	1985/86	1986/87	1987/88	1988/89
Revenue (Income)	**$ million**			
Taxes	2 248	2 637	3 109	3 577
Investments	115	139	321	191
International aid	100	102	124	160
Miscellaneous	157	177	231	284
Total Revenue	2 620	3 055	3 785	4 212
Expenditure	**$ million**			
Goods & Services	1 392	1 758	2 327	2 614
Government grants	1 534	1 758	1 611	1 880
Capital expenses	200	307	357	521
Total expenditure	3 126	3 823	4 295	5 015

10 Is it true to say that income from international aid constitutes between 3% and 4% of the total revenue?

11 In 1988/89, what percentage of the total revenue was obtained from taxes? (Answer to nearest whole percent.)

12 **Budget deficit** is the amount by which expenditure exceeds revenue. Thus the budget deficit for 1985/86 was $3 126 million − $2 620 million = $506 million. Find out which year had (a) the greatest (b) the smallest budget deficit.

[*Note:* It is quite common for governments to have budget deficits. The money required to make up the deficit is usually borrowed from international agencies such as The World Bank.]

Bank statements

Fig. 17.1 (page 138) shows a typical page from a bank statement of money paid into and taken out of a current account.

Note the following:

1 Payments are made *from* the account. Receipts are paid *into* the account.

2 Each transaction is recorded on the day that it reaches the bank. Thus the final column gives the balance of account on any given day.

3 Banks give varying amounts of detail. The bank in Fig. 17.1 overleaf gives the cheque number, where appropriate.

The presentation of bank statements varies. Fig. 17.2 overleaf shows a page from a building society savings account book.

Note the following:

1 'Investment' is money deposited into the account.

2 'Withdrawal' is money taken out of the account.

3 Since this is a savings account, interest is added from time to time. This is done automatically by the bank.

ZZZZ BANK PTY LTD			STATEMENT OF ACCOUNT	
			CURRENT ACCOUNT NO: 60614912	
DETAILS	PAYMENTS	RECEIPTS	DATE	BALANCE
			1991	
BALANCE FORWARD			1 SEP	125,67
CH 100547	47,29		1 SEP	78,38
CASH	30,00		1 SEP	48,38
COUNTER CREDIT		82,50	8 SEP	130,88
CH 100549	2,50		18 SEP	128,38
CASH	40,00		19 SEP	88,38
CH 100548	13,29		20 SEP	75,09
CH 100550	4,00		25 SEP	71,09
CASH	30,00		26 SEP	41,09
STO BSOC	45,80		28 SEP	4,71 DR
COUNTER CREDIT		209,02	30 SEP	204,31

Abbreviations CH Cheque STO Standing Order DR Overdrawn Balance

Fig. 17.1

CASTLE BUILDING SOCIETY			Account No: Cb3037564Mac	
Date	Details	Investment	Withdrawal	Balance
8 JUL 90	BT FORWARD			867,95
28 OCT 90	CHEQUE	17,4'		885,38
19 DEC 90	CHEQUE		84,70	800,68
31 DEC 90	INTEREST	31,61		832,29
13 JAN 91	CHEQUE	119,97		952,26
10 FEB 91	CHEQUE	50,00		1 002,26
28 JUN 91	CHEQUE	32,73		1 034,99
30 JUN 91	INTEREST	51,03		1 086,02
7 SEP 91	CHEQUE	365,16		1 451,18
12 SEP 91	CHEQUE	7,81		1 458,99
20 OCT 91	CASH		100,00	1 358,99
21 OCT 91	CHEQUE		12,87	1 346,12
30 NOV 91	CHEQUE	20,12		1 366,24
6 DEC 91	CASH		150,00	1 216,24
31 DEC 91	INTEREST	68,84		1 285,08

Fig. 17.2

Exercise 17b

Refer to Fig. 17.1 above when answering questions 1–6.

1 How much money was Cheque No 100549 for?

2 On what date was the account credited with $82,50?

3 How much money was taken out of the account *as cash* during September?

4 How much money was taken out of the account altogether on 1 September?

5 Explain the entry which is given as '4,71DR'.

6 (a) Add together the two 'Receipts' and the initial balance brought forward.
(b) Find the sum of all the amounts in the 'Payments' column.
(c) Subtract your result in part (b) from your result in part (a). What do you notice?

Refer to Fig. 17.2 on page 138 when answering questions 7–12.

7 What was the biggest cheque and on which date was it invested?

8 What was the biggest cash withdrawal and when was it withdrawn?

9 Interest is added to the savings at 6-monthly intervals, on which dates?

10 How much money was withdrawn during October 1991?

11 What was the total amount of interest paid into the account during the period 8 July 1990 to 31 December 1991?

12 By how much did the balance of the account increase during 1991?

Foreign exchange ('forex')

When money is moved from one country to another, it is necessary to exchange the currency of the first country for that of the second. The various currencies of the world are linked together in agreed ratios or **foreign exchange** rates. This makes it possible to transfer money between countries and to buy goods which are available only in other countries. Foreign exchange is often abbreviated to 'forex'.

Table 17.3 gives the currency units of some countries and some example forex rates for Z$1 (Zimbabwe dollar).

Exchange rates vary from day to day. You can find a table of updated exchange rates on page 199. In the following examples and exercise, unless other rates are given, values are taken from Table 17.3.

Table 17.3 Foreign Exchange Rates

Country	Currency	Units to Z$1
Botswana	Pula (P)	P1,00
France	Franc (F)	F3,20
Italy	Lira (L)	680,00L
Japan	Yen	66,00 yen
Kenya	Shilling (Sh)	Sh9,60
Mozambique	Metical (Mc)	Mc350,00
Nigeria	Naira (₦)	₦3,70
RSA	Rand (R)	R1,30
UK	Pound (£)	£0,30
USA	Dollar ($)	$0,50
Russia	Rouble (R)	R2,30
West Germany	Deutsch Mark (DM)	DM0,90
Zambia	Kwacha (K)	K5,40

Example 2

A Zimbabwean visits her relative in London, UK. She changes Z$800 to pounds (£). How many pounds does she get?

Z$1 = £0,30
Z$800 = £0,30 × 800 = £240

Example 3

A traveller arrives in Zimbabwe from Italy with a 1 million lira note. How much is this in dollars?

680 lira = Z$1

$$1 \text{ lira} = Z\$\frac{1}{680}$$

$$1 \text{ million lira} = 1\,000\,000 \times Z\$\frac{1}{680}$$
$$= Z\$1\,470{,}59 \text{ (to 2 d.p.)}$$

Note that money calculations should be rounded to 2 decimal places.

Example 4

How many francs (France) are equivalent to 1 000 kwacha?

From Table 17.3,
K5,40 = F3,20 (both = Z$1)

$$K1{,}00 = F\,\frac{3{,}20}{5{,}40}$$

$$K1\,000 = F\,\frac{3{,}20}{5{,}40} \times 1000$$
$$= F593 \text{ (to the nearest franc)}$$

Notice that Examples 2, 3 and 4 are all solved using a unitary method. In Examples 2, 3 and 4 and in Exercise 17c which follows, the symbol Z$ is used to denote the Zimbabwe dollar. This is to distinguish it from the US dollar (US$).

Exercise 17c

1 Exchange Z$20 into the following currencies.
(a) Pula (b) Rands (c) Metical
(d) Yen (e) Pounds (f) US dollars

2 Exchange the following amounts into Z$.
(a) Me5 250 (b) DM756 (c) 17 000 L
(d) ₦148 (e) £24 (f) US$200

3 Find, to the nearest cent, the Z$ value of £1.

4 What, to the nearest cent, is the Z$ value of Sh1?

5 How many Deutsch marks are equivalent to 1 000 naira?

6 How many pula can be bought with 5 000 yen?

7 How many Kenyan shillings can an Italian tourist buy with 1 million lira?

8 A British tourist leaves the UK for a holiday in Zimbabwe. She takes traveller's cheques worth £900 with her which she cashes into Z$. Altogether she spends Z$2 466 in Zimbabwe. How much money has she left (a) in Z$, (b) in £?

9 How much does the Z$ value of a US$100 note rise or fall, if the exchange rate changes from US$1,30 to US$1,28 to the Z$? (Answer to nearest Zimbabwe cent.)

10 Banks normally operate *two* forex rates:
1 a *buying rate*, for customers who wish to buy foreign currency from the bank;
2 a *selling rate*, for customers who wish to sell foreign currency to the bank.

Buying and selling rates are regularly published in the newspapers. One day the buying and selling rates for Swedish kroner are respectively Kr3,10 and Kr3,15 to the Z$. In the morning a Swedish traveller sells Kr1 000 to the bank. Later that day a businessman buys Kr1 000 from the bank.
(a) How many Z$ does the Swedish traveller get?
(b) How many Z$ does the businessman pay?
(c) What is the bank's profit on the transaction?

Chapter 18

Further mensuration (1) Trapezium, everyday problems

Area of basic shapes (revision)

The formulae for the areas of some basic shapes are given in Fig. 18.1. These were previously found in Book 1.

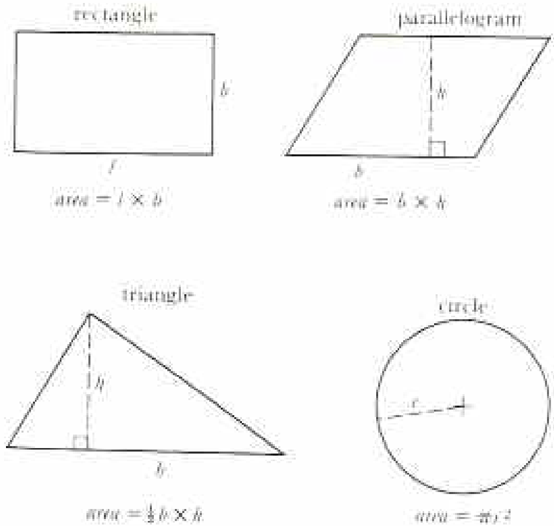
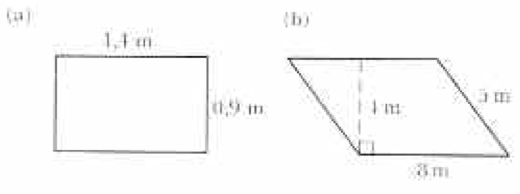


Fig. 18.1

Exercise 18a (Revision)
In this exercise, use the value $\frac{22}{7}$ for π unless told otherwise.

1 Calculate the areas of the shapes in Fig. 18.2.

2 Calculate the areas of the parallelograms in Fig. 18.3.



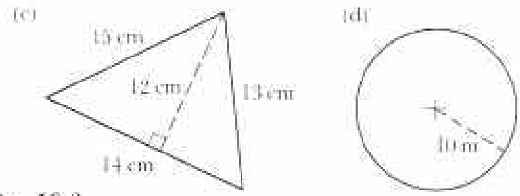


Fig. 18.2

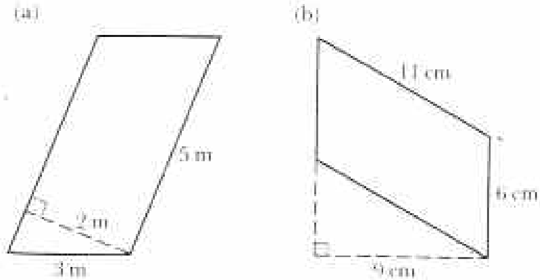


Fig. 18.3

3 Calculate the areas of the shapes in Fig. 18.4.

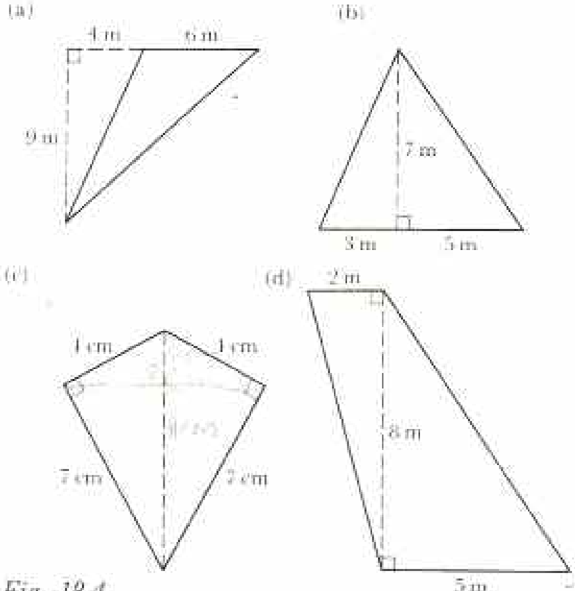


Fig. 18.4

4 Calculate the shaded areas in Fig. 18.5. Use the value 3,1 for π. All dimensions are in cm.

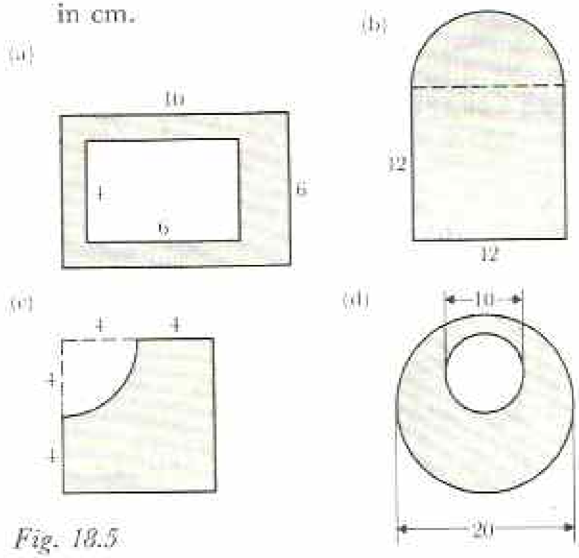


Fig. 18.5

5 A wooden door is 2,1 m high and 0,8 m wide. Find (a) the area of the door, (b) the cost of the door if the wood costs \$15 per m^2.

6 A page in a photograph album measures 30 cm by 20 cm. It contains six square photos each of side 6 cm. Calculate the area of the page which is *not* covered by the photos.

7 The seconds hand on a watch is 14 mm long. What area does it sweep through in 30 seconds?

8 A goat is tied by a rope $2\frac{1}{2}$ m long to a peg in the ground. The goat eats 1 m^2 of grass in 28 min. How long will it take to eat all that it can reach?

9 A gold disc 10 cm in diameter costs \$33. What is the cost per m^2?

10 Discs of diameter 6 cm are cut from a sheet 130 cm long and 70 cm wide as shown in Fig. 18.6.

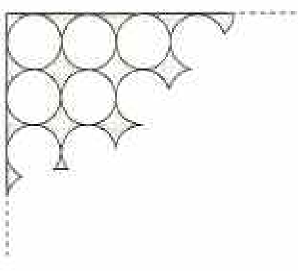


Fig. 18.6

(a) How many discs can be cut in this way?
(b) What area of the sheet is wasted?

Area of a trapezium

In Fig. 18.7, ABCD is a trapezium with AB || DC.

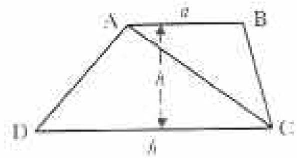


Fig. 18.7

Let the lengths of AB and DC be a and b respectively. Let their perpendicular distance apart be h. Join AC.

Area of ABCD
$= \text{area of } \triangle ABC + \text{area of } \triangle ACD$
$= \frac{1}{2}ah + \frac{1}{2}bh$
$= \frac{1}{2}h(a + b) \text{ or } \frac{1}{2}(a + b)h$

The area of a trapezium is the product of the average length of its parallel sides and the perpendicular distance between them.

Example 1

Find the area of the trapezium in Fig. 18.8.

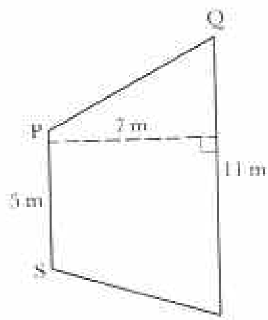


Fig. 18.8

In Fig. 18.8, SP || RQ.

Area of PQRS $= \frac{1}{2}(5 + 11) \times 7 \text{ m}^2$
$= \frac{1}{2} \times 16 \times 7 \text{ m}^2$
$= 8 \times 7 \text{ m}^2 = 56 \text{ m}^2$

Example 2

If the area of the trapezium in Fig. 18.9 is $40\frac{1}{2}$ cm^2, *find the value of x.*

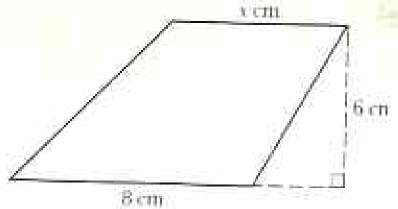


Fig. 18.9

$$\text{Area of trapezium} = \tfrac{1}{2}(x + 8) \times 6\ \text{cm}^2$$
$$= 3(x + 8)\ \text{cm}^2$$

Thus, $3(x + 8) = 40\tfrac{1}{2}$

$$x + 8 = 40\tfrac{1}{2} \div 3 = 13\tfrac{1}{2}$$
$$x = 13\tfrac{1}{2} - 8 = 5\tfrac{1}{2}$$

Exercise 18b

1 Find the areas of the trapeziums in Fig. 18.10. All dimensions are in cm.

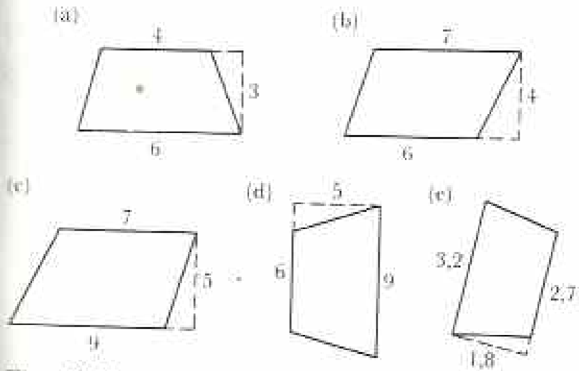


Fig. 18.10

2 In each of the trapeziums in Fig. 18.11, find the value of x.

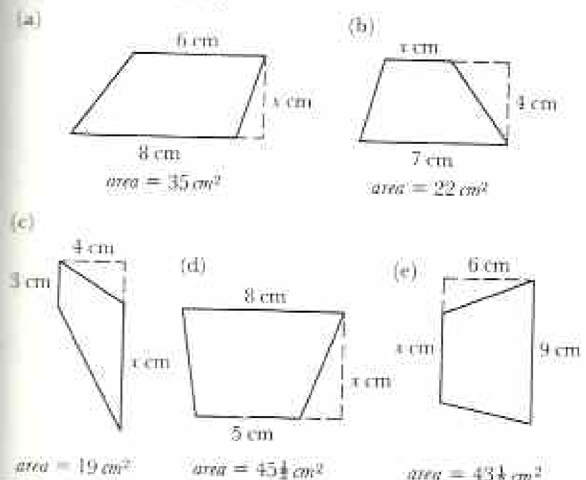


Fig. 18.11

Everyday problems with area

Tiles are often used to cover the floor of a room. The number of tiles needed can be calculated from the dimensions of the room. There is usually some wastage since tiles are sold in whole numbers.

Example 3

Square tiles, 30 cm × 30 cm, *are used to cover a floor. How many tiles are needed for a floor* 4,4 m *long and* 3,8 m *wide?*

Length of room = 4,4 m = 440 cm

Number of tiles $= \dfrac{440}{30} = 14\tfrac{2}{3}$

Thus 15 tiles are needed along each length of the room. (The last tile will be cut.)

Width of room = 3,8 m = 380 cm

Number of tiles $= \dfrac{380}{30} = 12\tfrac{2}{3}$

Thus 13 tiles are needed across each width of the room.

Total number of tiles needed = 15 × 13
= 195

Exercise 18c

1 How many tiles, each 30 cm by 30 cm, will be needed for floors with the following dimensions?

(a) 6 m by 4,2 m (b) 3,6 m by 3 m
(c) 5 m by 4,2 m (d) 9 m by 6,2 m
(e) 10 m by 8,4 m (f) 5,2 m by 4,1 m
(g) 2,9 m by 3,4 m (h) 5,83 m by 3,44 m

2 Square polystyrene tiles, 50 cm by 50 cm, are used to cover the ceiling of a classroom measuring 7,4 m by 4,5 m. (a) Find the number of tiles that are needed. (b) Find the cost at 65c per tile.

3 The walls of a bathroom are to be covered with wall tiles 15 cm by 15 cm. How many tiles are needed for a bathroom 2,7 m long, 2,25 m wide and 3 m high? (Do not allow for doors and windows.)

4 An open rectangular box, 1 m long, 70 cm wide and 50 cm deep is painted inside and outside. Find the cost at 45c per m^2.

5 How many paving stones, each 1 m long and 80 cm wide, are needed to cover an area 13,6 m long and 11 m wide?

6 Fig. 18.12 is a sketch of a building with a corrugated roof.

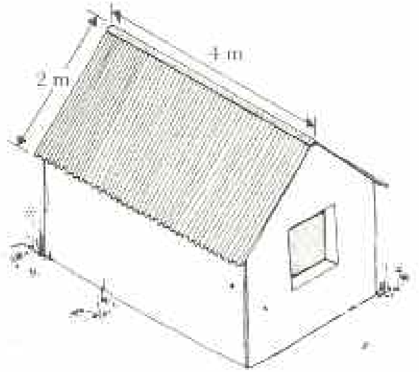


Fig. 18.12

If corrugated iron is sold in sheets measuring 2 m by 60 cm, find the number of sheets that are needed for the building.

7 The walls of a bathroom 2,5 m long, 2,05 m wide and 3 m high are to be covered with tiles, each 15 cm by 15 cm. If a saving of 108 tiles is made on doors and windows, how many tiles will be needed altogether? (Note that some tiles will have to be cut.)

8 A rectangular area, 8,55 m long by 5,89 m wide, is to be paved with the largest possible square tiles which will fit in exactly. How many tiles will there be? (*Hint*: express 855 and 589 as products of prime numbers.)

9 A room 5 m long, 4 m wide and 2,5 m high is to have its walls covered with plywood panels. The plywood is sold in sheets 3 m long and 1,5 m wide. If there are no horizontal joins in the panels between floor and ceiling, how many sheets will be needed? Allow a saving of 1 sheet for doors and windows which are not panelled.

10 A room 4,38 m long, 3,74 m wide and 2,36 m high has two doorways, each 76 cm by 198 cm, and three windows, each 88 cm by 106 cm. Find the cost, to the nearest cent, of painting the walls of this room at 37c per m^2.

Circles, rings, sectors

Example 4

What is the diameter of a circle of area 3 850 m^2?

$$\text{Area} = \pi r^2 = 3\,850 \text{ m}^2$$
$$\text{Thus } \tfrac{22}{7} r^2 = 3\,850$$
$$r^2 = 3\,850 \times \tfrac{7}{22} = 175 \times 7 = 5^2 \times 7^2$$
$$r = 5 \times 7 = 35$$
$$\text{diameter} = 2 \times 35 \text{ m} = 70 \text{ m}$$

Example 5

What is the area of a flat washer 4,8 cm *in outside diameter, the hole being of diameter* 2,2 cm?

The required area is shaded in Fig. 18.13.

Fig. 18.13

$$\text{Area} = \pi(2{,}4)^2 - \pi(1{,}1)^2 \text{ cm}^2$$
$$= \pi(5{,}76 - 1{,}21) \text{ cm}^2 = \tfrac{22}{7} \times 4{,}55 \text{ cm}^2$$
$$= 14{,}3 \text{ cm}^2$$

Note: Use of the difference of two squares (Chapter 22) will simplify the arithmetic in this kind of problem. See Example 21 on page 172.

Example 6

The sector of a circle of radius 7 cm *has an angle of* 108° *at its centre. Calculate* (*a*) *the length of the arc of the sector,* (*b*) *the area of the sector.*

Fig. 18.14 shows the sector of the circle.

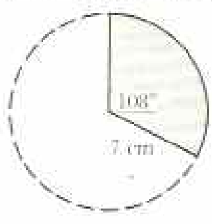


Fig. 18.14

(a) Length of arc $= \frac{108}{360}$ of $2\pi 7$ cm

$$= \frac{108}{360} \times 2 \times \frac{22}{7} \times 7 \text{ cm} = \frac{3}{10} \times 44 \text{ cm}$$

$$= 13{,}2 \text{ cm}$$

(b) Area of sector $= \frac{108}{360}$ of $\pi \times 7^2$ cm²

$$= \frac{108}{360} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= \frac{3}{10} \times 22 \times 7 \text{ cm}^2 = 46{,}2 \text{ cm}^2$$

Example 7

Find the shaded area for a quadrant radius 14 cm.

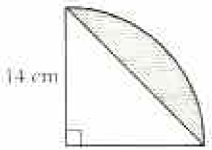


Fig. 18.15

A **quadrant** is a sector of a circle with an angle of 90°.

$$\begin{aligned} \text{Quadrant area} &= \tfrac{1}{4} \times \pi \times 14^2 \\ &= \tfrac{1}{4} \times \tfrac{22}{7} \times 14 \times 14 \text{ cm}^2 \\ &= 11 \times 14 = 154 \text{ cm}^2 \\ \text{Triangle area} &= \tfrac{1}{2} \times 14 \times 14 \text{ cm}^2 = 98 \text{ cm}^2 \\ \text{Shaded area} &= 154 \text{ cm}^2 - 98 \text{ cm}^2 = 56 \text{ cm}^2 \end{aligned}$$

Exercise 18d

Throughout this exercise, take π to be $\frac{22}{7}$.

1 Find the area of each of the rings whose outside and inside diameters are as follows.
(a) 8 m and 6 m (b) 22 cm and 20 cm
(c) 15 m and 6 m (d) 8,6 cm and 8,2 cm

2 Complete Table 18.1 for sectors of circles. Make a rough sketch in each case.

Table 18.1

	radius	angle at centre	length of arc	area of sector
(a)	7 cm	90°		
(b)	35 m	72°		
(c)	4,2 cm	120°		
(d)	5,6 cm	135°		
(e)	14 m	300°		

3 Find the area of the shaded part of each part of Fig. 18.16. All dimensions are in cm.

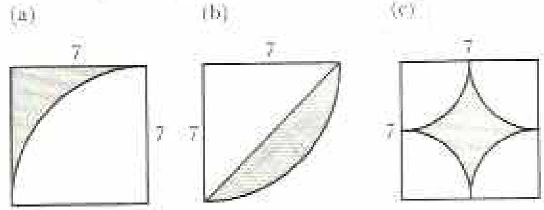


Fig. 18.16

4 Find the radii of circles with the following areas: (a) 154 cm², (b) 1386 cm², (c) $86\frac{5}{8}$ m², (d) 6,16 ha.

5 Two circular bronze discs of radii 3 cm and 4 cm are melted down and cast into a single disc of the same thickness as before. What is the radius of the new disc?

6 The disc brake in a car is a flat metal ring 22 cm in diameter with a 6-cm diameter hole in the middle. Calculate the area of the metal.

7 The friction pad in a motorcycle shock absorber is a flat ring of fibre 10 cm in diameter with a 3-cm diameter hole in the middle. What is the area of the fibre?

8 The cloth for a wedding dress is cut in the form of a 210° sector of a 3 m radius circle. What is the area of the cloth used?

9 Find the cross-sectional area of a round metal pipe if its outside diameter is 13,5 cm and the metal is 0,25 cm thick.

10 The windscreen wiper of a car sweeps through an angle of 150°. The blade of the wiper is 21 cm long and the radius of the unswept sector is 6 cm. See Fig. 18.17.

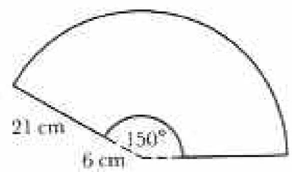


Fig. 18.17

What area of the windscreen is swept clean?

Chapter 19

Formulae: substitution, change of subject

Formulae

A **formula** is an equation with letters which stand for quantities. For example,

$$c = 2\pi r$$

is the formula which gives the circumference, c, of a circle of radius r.
In science,

$$I = \frac{V}{R}$$

is the formula which shows the relation between the current, I amps, voltage, V volts, and resistance, R ohms, in an electrical circuit. In arithmetic,

$$I = \frac{PRT}{100}$$

is the formula which gives the interest, I, gained on a principal, P, invested at R% per annum for T years.

Notice in the above formulae that sometimes the same letter can stand for different quantities in different formulae. For example, I stands for current in the science formula and I stands for interest in the arithmetic formula. **Formulae** is the plural of formula.

Substitution in formulae

Example 1

A gas at a temperature of θ °C *has an absolute temperature of* T K, *where* $T = \theta + 273$.
(*a*) *Find the absolute temperature of a gas at a temperature of* 68 °C.
(*b*) *If the absolute temperature of a gas is* 380 K, *find its temperature in* °C.

(a) $T = \theta + 273$
When $\theta = 68$,
$T = 68 + 273 = 341$

The absolute temperature is 341 K.

(b) $T = \theta + 273$
When $T = 380$,
$380 = \theta + 273$
Subtract 273 from both sides
$380 - 273 = \theta$
$107 = \theta$
The temperature of the gas is 107 °C.

Example 2

The formula $W = VI$ *gives the power,* W watts, *used by an electrical item when a current of* I amps *flows through a circuit of* V volts.
(*a*) *An air conditioner on maximum power needs a current of* 25 amps *in a* 120 volt *circuit. Find the power being used.*
(*b*) *An electric light bulb is marked* 100 watts, 240 volts. *Find the current required to light the bulb.*

(a) $W = VI$
When $V = 120$ and $I = 25$,
$W = 120 \times 25$
$= 3\ 000$

The maximum power is 3 000 watts.

(b) $W = VI$
When $W = 100$ and $V = 240$,
$100 = 240I$

Divide both sides by 240,

$$\frac{100}{240} = I$$

$$I = \frac{10}{24} = \frac{5}{12}$$

The current required is $\frac{5}{12}$ amp.

Exercise 19a

1 A gas at a temperature of θ °C has an absolute temperature of T K, where $T = \theta + 273$.
   (a) Find the absolute temperature of a gas at a temperature of 36 °C.
   (b) If the absolute temperature of a gas is 400 K, find its temperature in °C.

2 The perimeter of a rhombus of side d cm is p cm, where $p = 4d$.
   (a) Find the perimeter of a rhombus of side 3, 2 cm.
   (b) Find the length of a side of a rhombus of perimeter 14 cm.

3 Two quantities x and y are connected by the formula, $y = 7 - 9x$.
   (a) Find the value of y when $x = 0$.
   (b) Find the value of x when $y = 0$.

4 A rectangle l units long and b units wide has an area of A square units, where $A = lb$.
   (a) Find the floor-area of a room 5 m long and 3 m wide.
   (b) Find the width of a postcard of area 112 cm^2, the length being 14 cm.
   (c) Find the length of a rectangular piece of plastic of area 171 cm^2 and width $9\frac{1}{2}$ cm.
   (d) The area of a picture is 3,125 m^2 and its width is 1,25 m. Find its length.

5 The simple interest formula

$$I = \frac{PRT}{100}$$

gives the interest I on a principal P invested at a rate of R% per annum for T years.
   (a) Find the interest when $1500 is invested at 5% per annum for 4 years.
   (b) Find the principal that gains an interest of $161 in 5 years at 7% per annum.

6 A circuit of voltage V volts and resistance R ohms has a current of I amps, where

$$I = \frac{V}{R}$$

   (a) Find the current when the voltage is 240 volts and the resistance is 80 ohms.
   (b) Find the voltage when the current is 0,6 amps and the resistance is 5 ohms.
   (c) Find the resistance when the current is 0,1 amp and the voltage is 9 volts.

7 A rectangular room l m long and b m wide has a perimeter p, where $p = 2l + 2b$.
   (a) Find the perimeter of a room which is 3,5 m long and 2 m wide.
   (b) Find the length of a room of perimeter 20 m and width 3 m.

8 The mass of water in a rectangular tank l m long, b m wide and h m deep is M kg, where $M = 1\,000\, lbh$.
   (a) What is the mass of water in a tank 5 m long, 4 m wide and 3 cm deep?
   (b) How deep is the water in a tank 4 m long and 3 m wide if its mass is 24 000 kg?
   (c) How wide is a tank 3 m long and 0,5 m deep if it holds exactly one tonne of water?

9 The circumference, C units, of a circle of radius r units is given by the formula $C = 2\pi r$, where $\pi = \frac{22}{7}$.
   (a) What is the circumference of a circle of radius 7 cm?
   (b) What is the radius of a circle whose circumference is 22 m?
   (c) What is the circumference of a circle of radius one metre?
   (d) What is the radius of a circle of circumference 2,75 m?

10 The speed s km/h of a certain car t seconds after starting is given by the formula $s = 12t$.
   (a) Find the speed of the car 5 seconds after starting.
   (b) How long does it take the car to reach a speed of 75 km/h?

Example 3

If $y = 3 - x$, find the values of y when $x = 1; 2; 3; 4; 5$.

When $x = 1$, $y = 3 - 1 = 2$
When $x = 2$, $y = 3 - 2 = 1$
When $x = 3$, $y = 3 - 3 = 0$
When $x = 4$, $y = 3 - 4 = -1$
When $x = 5$, $y = 3 - 5 = -2$

The working and results in Example 3 can be set out more neatly in a **table of values** as follows.

Table 19.1 $y = 3 - x$

x	1	2	3	4	5
$3 - x$	$3 - 1$	$3 - 2$	$3 - 3$	$3 - 4$	$3 - 5$
$y = 3 - x$	2	1	0	-1	-2

Example 4

If $y = 2x - 5$, *make a table of values of* y *for* $x = -1$; 0; 1; 2; 3.

Table 19.2 $y = 2x - 5$

x	-1	0	1	2	3
$2x - 5$	$-2 - 5$	$0 - 5$	$2 - 5$	$4 - 5$	$6 - 5$
y	-7	-5	-3	-1	1

Example 5

The monthly cost, d dollars, *of running a household of* n *people is given by the formula* $d = 12n + 35$.
(*a*) *Find the monthly cost for 5 people.*
(*b*) *How many people are there if the monthly cost is* \$119?

(a) There are 5 people, thus $n = 5$.

$d = 12n + 35$

When $n = 5$,

$d = 12 \times 5 + 35 = 60 + 35$

$= 95$

The monthly cost is \$95.

(b) The monthly cost is \$119, thus $d = 119$.

$d = 12n + 35$

When $d = 119$,

$119 = 12n + 35$

Subtract 35 from both sides

$84 = 12n$

Divide both sides by 12

$7 = n$

There are 7 people.

Exercise 19b

1 If $y = 5 - x$, find the values of y when $x = 1$; 2; 3; 4; 5.

2 If $d = c + 3$, find the values of d when $c = -2$; -1; 0; 1; 2.

3 If $y = 2x + 1$, find the values of y when $x = 0$; 1; 2; 3; 4.

4 Given that $y = 3x + 2$, copy and complete Table 19.3 to show values of y for $x = -1$; 0; 1; 2; 3.

Table 19.3 $y = 3x + 2$

x	-1	0	1	2	3
$3x + 2$	$+2$	$+2$	$+2$	$+2$	$+2$
y					

5 If $y = 17 - 6x$, make a table of values of y for $x = 0$; 1; 2; 3; 4; 5.

6 The cost, c cents, of hiring a car for a journey of d km is given by the formula

$c = 5d + 850$.

(a) Find the cost of hiring a car for a journey of 420 km.

(b) How long is a journey if the cost of car hire is \$15,80?

7 The time, t min, to cook meat is given by the formula $t = 40m + 25$ where m is the mass of the meat in kg.

(a) Find how long it takes to cook a piece of meat of mass 1,2 kg.

(b) Find the mass of a piece of meat which takes 2 h 9 min to cook.

8 On a certain island the tax, \$$T$, paid on an income of \$$I$ is given by the formula $T = 0{,}2I - 50$. How much tax is paid on an income of (a) \$1 000, (b) \$6 225? (c) What income would have a tax of \$450?

9 A car starts a journey with a full petrol tank. The amount of petrol, p litres, left in the tank after travelling for t hours is given by the formula $p = 63 - 10t$.

(a) Find the amount of petrol left after travelling for $2\frac{1}{2}$ hours.

(b) If there are 18 litres of petrol left, how long has the car been travelling?

(c) How long will it take the car to run out of petrol? (i.e. find t when $p = 0$).

10 A closed cylinder of height h cm and base radius r cm has a surface area, A cm^2, where $A = 2\pi r^2 + 2\pi rh$. Use the value $\frac{22}{7}$ for π to find

(a) the surface area of a closed cylinder of height 9 cm and base radius 5 cm;
(b) the height of a closed cylinder of base radius 7 cm and surface area 1 012 cm^2.

Example 6

If $y = 5x^2 - 1$, find (a) the value of y when $x = -3$, (b) the values of x when $y = 79$.

(a) $y = 5x^2 - 1$

When $x = -3$

$y = 5 \times (-3)^2 - 1$
$= 5 \times (+9) - 1$
$= 45 - 1$
$= 44$

(b) $y = 5x^2 - 1$

When $y = 79$

$79 = 5x^2 - 1$

Add 1 to both sides

$80 = 5x^2$

Divide both sides by 5

$16 = x^2$

Take the square root of both sides

$\sqrt{16} = x$

$x = +4$ or -4

Notice that there are two possible values for x. We can shorten this to $x = \pm 4$ where $\pm$ is short for '+ or −'.

Example 7

From a height of h metres *above sea level it is possible to see a distance of approximately d* kilometres, *where d and h are connected by the formula $2d^2 = 25h$.*
(a) From what height is it possible to see a distance of 10 km?
(b) What distance can be seen from a height of 18 m?

(a) $2d^2 = 25h$

The distance is 10 km. Thus, when $d = 10$,

$2 \times 10^2 = 25h$
$2 \times 100 = 25h$
$200 = 25h$

Divide both sides by 25

$8 = h$

The height is 8 metres.

(b) The height is 18 m. Thus, when $h = 18$,

$2d^2 = 25h$

becomes $2d^2 = 25 \times 18$

$2d^2 = 450$

Divide both sides by 2

$d^2 = 225$

Take the square root of both sides

$d = \sqrt{225}$
$= \pm 15$

In this example the value $d = -15$ would not be sensible; the distance that can be seen is 15 km.

Exercise 19c

1 If $y = 40x^2$, find (a) y when $x = 0$; 1; 2; 3; 4; 5, (b) x when $y = 10$; 360; 1 000; 4 000.

2 If $y = 16 - x^2$, find (a) y when $x = -4$; -2; 0; 2; 4, (b) x when $y = 0$; 7; 12; 15.

3 If $m = \dfrac{100}{n^2}$, find (a) m when $n = 1$; 5; 10; 20, (b) n when $m = 1$; 4; 9; 25.

4 The area, A square units, of a circle of radius r units, is given by the formula $A = \pi r^2$, where $\pi = \frac{22}{7}$.
(a) What is the area of a circle of radius 7 m?
(b) What is the radius of a circle of area 616 cm^2?
(c) What is the radius of a circle of area 38,5 m^2?

5 The volume, V cm^3, of a cylinder of base radius r cm and height h cm is given by the formula $V = \pi r^2 h$. Use the value $\frac{22}{7}$ for π to find
(a) the volume of a cylinder of base radius $3\frac{1}{2}$ cm and height 8 cm;
(b) the height of a cylinder of volume 1 100 cm^3 and radius 5 cm;
(c) the base radius of a cylinder of volume 770 cm^3 and height 45 cm.

6 If a stone is dropped, the distance, d m, which it falls in t seconds is given by the formula $d = 4{,}9t^2$.
(a) How far does it fall in 3 seconds?
(b) How far does it fall in $1\frac{4}{7}$ seconds?
(c) How long does it take to fall 490 m?
(d) How long does it take to fall $122\frac{1}{2}$ m?
(e) How far does it fall in the fifth second?

7 The time, t minutes, taken over a commitee meeting, is given by the formula $t = 5n^2 + 15$ when n people are present.
(a) How long does the meeting take if there are 4 people?
(b) How many people are present if the meeting takes 2 h 20 min?

8 The visible distance, D km, of the horizon from a height of h m is given by the formula $h = \frac{2}{25}D^2$.
(a) How high must a cliff be if a ship $12\frac{1}{2}$ km away is visible from it?
(b) How high is an observation tower on the top of the cliff in (a) if a ship 15 km away is visible from it?
(c) What is the distance of the visible horizon from the top of a wireless mast 200 m high?

9

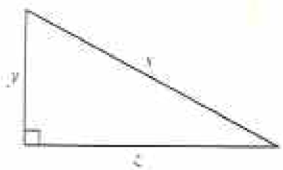


Fig. 19.1

In the right-angled triangle in Fig. 19.1, the length of the hypotenuse, x cm, is given by the formula, $x = \sqrt{y^2 + z^2}$.
(a) Find the value of x when $y = 2\frac{1}{2}$ cm and $z = 6$ cm.
(b) Find the value of y when $x = 16$ and $z^2 = 60$.

10 If there are n numbers in a uniformly increasing series (like 2; 5; 8; 11;...), starting with a and ending with l, the sum of the numbers is S, where $S = \frac{1}{2}n(a + l)$.
(a) What is the sum of all whole numbers from 1 to 50 inclusive?
(b) How many numbers are there in the series 2; ...; 20 if the sum is 143?
(c) What is the last term of a series of 10 numbers beginning with 7 if their sum is 85?

Change of subject of formulae

The letter I is the **subject** of the formula

$$I = \frac{PRT}{100}$$

The subject stands on its own. Its value can be found directly by substituting the values of the other letters in the formula.

It is often necessary to **change the subject** of a formula. This means to rearrange the order of the letters in the formula so that one of the other letters becomes the subject.

Example 8

Make P, R and T in turn the subject of the formula $I = \dfrac{PRT}{100}$.

$$I = \frac{PRT}{100}$$

Multiply both sides by 100

$100I = PRT$ (*a*)

Dividing both sides of equation (*a*) by RT gives

$$P = \frac{100I}{RT} \qquad (b)$$

Dividing both sides of equation (*a*) by PT gives

$$R = \frac{100I}{PT} \qquad (c)$$

Dividing both sides of equation (*a*) by PR gives

$$T = \frac{100I}{PR} \qquad (d)$$

Equations (*b*), (*c*) and (*d*) show P, R and T respectively as subjects of the given formula.

Example 9

Make x the subject of the following.

(*a*) $y = x - 9$ (*b*) $N = 7x$ (*c*) $\dfrac{x}{a} = 8$

(*d*) $\dfrac{h}{x} = k$ (*e*) $y = 2x + 1$

(a) $y = x - 9$

Add 9 to both sides,

$y + 9 = x$

$\Leftrightarrow \quad x = y + 9$

(b) $N = 7x$

Divide both sides by 7

$$\frac{N}{7} = x \text{ or } x = \frac{N}{7}$$

(c) $\frac{x}{a} = 8$

Multiply both sides by a

$x = 8a$

(d) $\frac{h}{x} = k$

Multiply both sides by x

$h = kx$

Divide both sides by k

$$\frac{h}{k} = x \text{ or } x = \frac{h}{k}$$

(e) $y = 2x + 1$

Subtract 1 from both sides

$y - 1 = 2x$

Divide both sides by 2

$$\frac{y-1}{2} = x$$

$$\Leftrightarrow x = \frac{y-1}{2}$$

To change the subject of a formula:

1 treat the formula as an algebraic equation;

2 solve the equation for the letter which is to be the subject of the formula.

Exercise 19d

In each question a formula is given. A letter is printed in heavy type after it. Make that letter the subject of the formula. If more than one letter is given, make each the subject in turn.

1	$y = x + 8$	**x**
2	$y = x - 3$	**x**
3	$b = a + c$	**a, c**
4	$y = 3x$	**x**
5	$y = \frac{x}{4}$	**x**
6	$b = ac$	**a, c**
7	$n = 5ax$	**a, x**
8	$\frac{x}{y} = 9$	**x**
9	$\frac{y}{x} = 2$	**x**
10	$\frac{m}{n} = p$	**m, n**
11	$y = 6x + 11$	**x**
12	$y = 7x - 2$	**x**
13	$b = 5a - c$	**a**
14	$x + y = 13$	**x, y**
15	$2p - q = 0$	**q, p**
16	$2x - y = d$	**x, y**
17	$p = 4d$	**d**
18	$c = 2\pi r$	**r**
19	$T = \theta + 273$	**θ**
20	$W = VI$	**V, I**
21	$A = \pi rl$	**r, l**
22	$V = lbh$	**l, b, h**
23	$A = 2\pi rh$	**r, h**
24	$s = \frac{1}{2}vt$	**v, t**
25	$A = \frac{1}{2}bh$	**b, h**
26	$V = \frac{1}{3}lbh$	**l, b, h**
27	$I = \frac{PRT}{100}$	**R**
28	$I = \frac{V}{R}$	**V, R**
29	$s = 2l + 2b$	**l, b**
30	$v = u + at$	**u, a, t**

Example 10

Given $3x - 2y = 8$, (a) express x in terms of y and find x when $y = 11$, (b) obtain a formula for y and find y when $x = 2$.

(a) 'express x in terms of y' means 'make x the subject of the formula'.

$3x - 2y = 8$

Add $2y$ to both sides

$3x = 8 + 2y$

Divide both sides by 3

$$x = \frac{8 + 2y}{3}$$

When $y = 11$

$$x = \frac{8 + 2 \times 11}{3} = \frac{8 + 22}{3} = \frac{30}{3} = 10$$

(b) 'obtain a formula for y' means 'make y the subject of the formula'.

$3x - 2y = 8$

Add $2y$ to both sides

$3x = 8 + 2y$

Subtract 8 from both sides

$3x - 8 = 2y$

Divide both sides by 2

$\frac{3x - 8}{2} = y$

When $x = 2$

$$y = \frac{3 \times 2 - 8}{2}$$
$$= \frac{6 - 8}{2}$$
$$= \frac{-2}{2}$$
$$= -1$$

Exercise 19e

1 If $y = 2x - 9$, (a) express x in terms of y, (b) find x when $y = 5$.

2 If $3x + y = d$, (a) express x in terms of y and d, (b) find x when $d = 1$ and $y = 13$.

3 A cylinder of radius r cm and height h cm has a curved surface area A cm^2, where $A = 2\pi rh$. (a) Obtain a formula for h, and (b) find the value of h when $A = 93$, $r = 2{,}5$ and $\pi = 3{,}1$.

4 A triangle of base b cm and height h cm has an area A cm^2, where $A = \frac{1}{2}bh$. (a) Express b in terms of A and h and (b) hence find the value of b when $A = 135$ and $h = 18$.

5 The wage, w dollars, of someone who works r hours of overtime is given by the formula $w = 2r + 59$. (a) Make r the subject of this formula and (b) hence find the number of hours of overtime worked by someone whose total wage is \$82.

6 (a) Make P the subject of the simple interest formula $I = \frac{PRT}{100}$

(b) Hence find the principal which makes an interest of \$297,50 in 7 years at a rate of 5% per annum.

7 (a) Make T the subject of the simple interest formula $I = \frac{PRT}{100}$.

(b) Hence find how long it takes for a principal of \$250 to make an interest of \$26,25 at 3% per annum.

8 (a) Make R the subject of the simple interest formula $I = \frac{PRT}{100}$.

(b) Hence find the rate of interest if \$162 makes an interest of \$21,60 in 4 years.

9 If $p = \frac{c}{d}$, (a) express d in terms of p and c and (b) find d when $p = 3$ and $c = 5{,}7$.

10 In an electrical circuit, the current, I amps, the voltage, V volts and the resistance, R ohms, are connected by the formula $I = \frac{V}{R}$.

(a) Make V the subject of the formula and find the voltage when $I = 2$ and $R = 6$.

(b) Express R in terms of I and V and find the resistance when $V = 240$ and $I = 0{,}1$.

Chapter 20

Scale drawing (3)
Bearings and distances

The magnetic compass

Fig. 20.1 is a photograph of a **magnetic compass**.

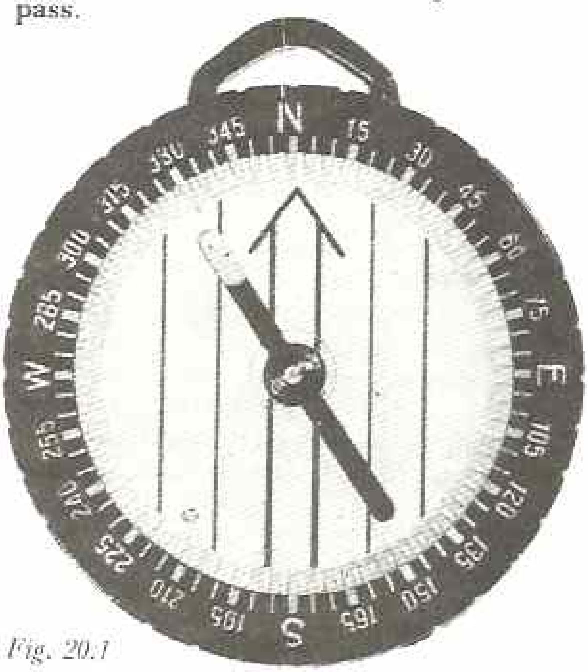


Fig. 20.1

The magnetic compass is used for finding **direction**. It has a magnetic needle which always points in the direction north.

Points of the compass

Fig. 20.2 shows the main **points** of the compass:

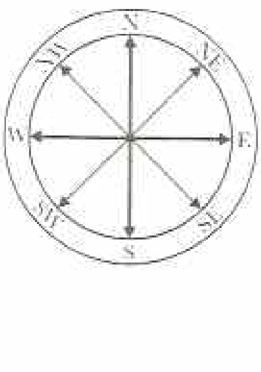


Fig. 20.2

There are 4 main points, or directions: north (N), south (S), east (E) and west (W). There are 4 secondary directions: north-east (NE), south-east (SE), south-west (SW) and north-west (NW). The angle between the directions N and E is 90°. NE is the direction mid-way between N and E. Thus the angle between N and NE is 45°.

Bearings

Fig. 20.3 shows two compasses placed at points A and B.

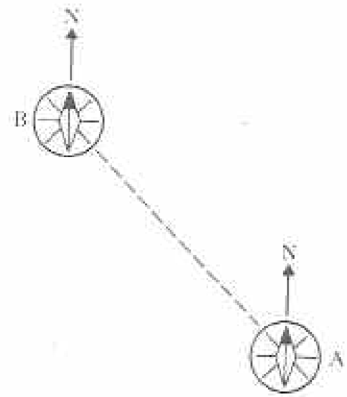


Fig. 20.3

The pointers of both compasses point northwards (N). The compass at B is in a direction NW of A. We say that the **bearing of B from A** is NW. Similarly, the compass at A is in a direction SE of B. The bearing of A from B is SE. In this sense, bearing simply means direction.

Exercise 20a (Oral)

In each of the diagrams overleaf, state (a) the bearing of B from A, (b) the bearing of A from B.

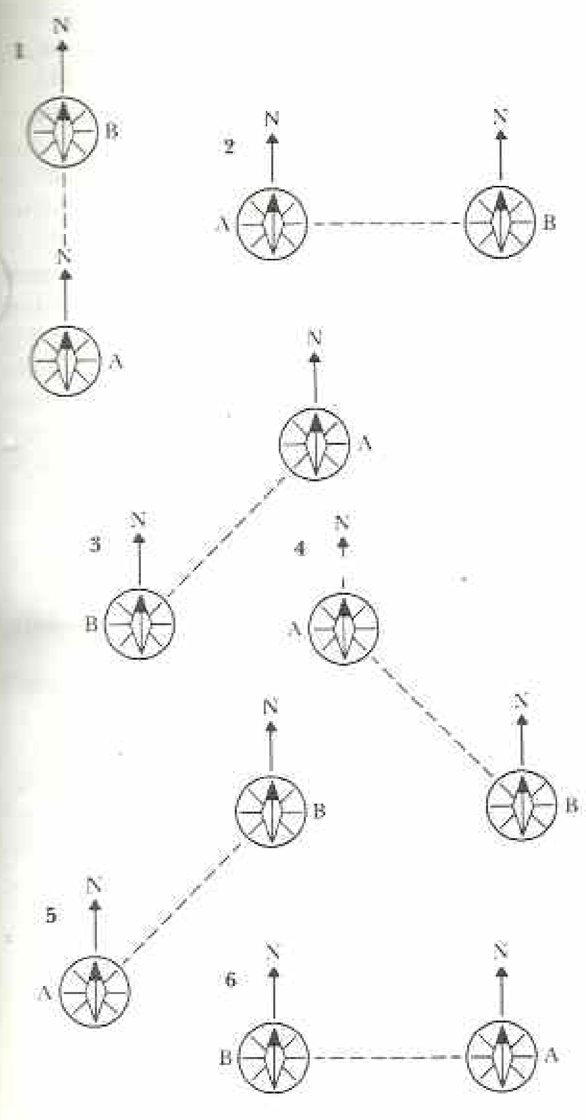


Fig. 20.4

Three-figure bearings

Fig. 20.5 shows the plan of a tree, a round house, a borehole and a flag-pole. Imagine you are standing at A with a compass. The compass bearing of the tree from A is N. The compass bearing of the borehole from A is NE. It is not possible to give an exact bearing of the round house or the flag-pole in terms of points of the compass.

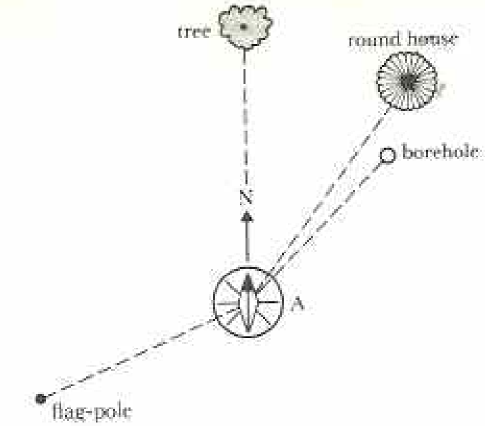


Fig. 20.5

Fig. 20.6 shows that the direction of the round house from A makes an angle of 37° with north.

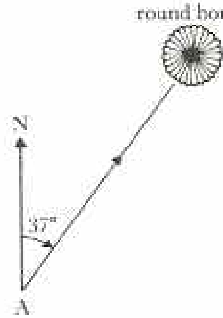


Fig. 20.6

Instead of using the directions N, NE, E, etc., we can give the bearing of the round house as a **three-figure bearing**, measured as the number of degrees from north, in a clockwise direction. The three-figure bearing of the round house from A is 037°.

Fig. 20.7 shows that the direction of the flag-pole from A makes an angle of 246° with north, measured clockwise.

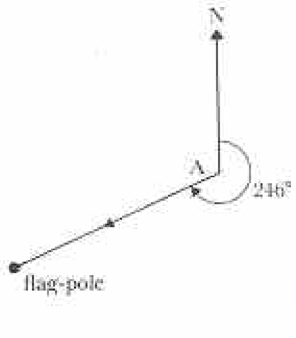


Fig. 20.7

The three-figure bearing of the flag-pole is 246°.

Any direction can be given as a three-figure bearing. Three digits are always given. For angles less than 100°, zeros must be written in front of the digits. For example, the direction east is given as 090°. North is 000° or 360°. Three-figure bearings are often called **true bearings**.

Compass bearings

Bearings are also given in terms of acute angles referred to the four main points of the compass. For example 037° can also be given as N37°E. Think of this as 'face north then turn 37° towards east'. Similarly 246° can be given as S66°W, 'face south then turn 66° towards west'. A bearing in the form $N\alpha°E$, $N\beta°W$, $S\theta°E$ or $S\phi°W$ is called a **compass bearing**. Note that it is conventional to give either the north (N) or south (S) direction *before* the east (E) and west (W) directions.

Throughout this course both the three-figure bearing method and the compass bearing method will be used.

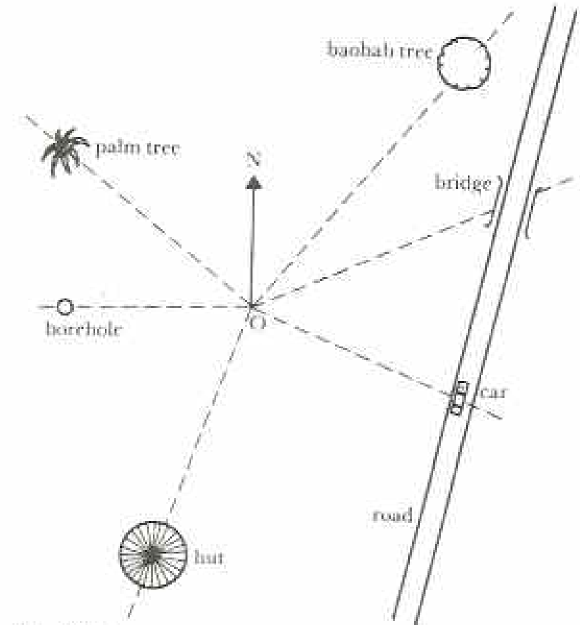


Fig. 20.8

Exercise 20b (Practical assignments)

1 Use a protractor to find the bearings of the objects in the map of Fig. 20.8 from O.

2 (a) Mark a point O on a piece of cardboard. With O as centre, draw round a protractor. If necessary, move the protractor so as to make a complete circle with O as centre. Use the protractor to mark off the circle in 30° intervals as shown in Fig. 20.9. Cut out the circle. The circle is a compass face.

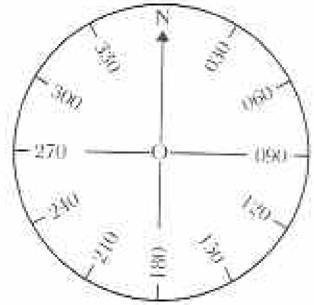


Fig. 20.9

(b) Cut a pointer from a thin strip of cardboard. Push a drawing pin through the centre of the compass and the pointer as shown in Fig. 20.10. This gives a model compass. The model compass can be used for estimating the sizes of bearings.

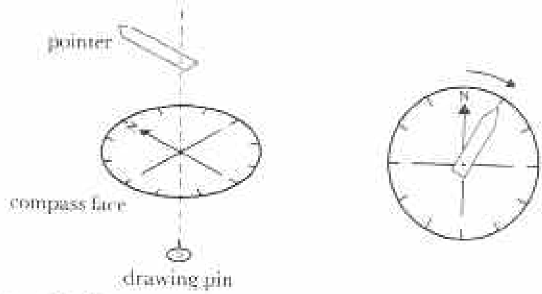


Fig. 20.10

3 For this exercise, take the front of your classroom to be north. Place your model compass so that N points towards the front of your classroom.

(a) Turn the pointer and estimate the bearings of the four corners of the room to the nearest 10°. See Fig. 20.11 overleaf.

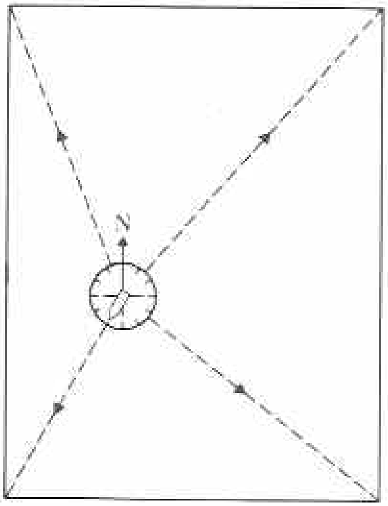


Fig. 20.11

(b) Estimate the bearings of the following.
   (i) the centre of the door;
   (ii) the centre of each window;
   (iii) the teacher's chair;
   (iv) the friend who is closest to you.

4 Take your desk outside. Place your compass on your desk so that it points northwards (*to find north without a compass:* face the direction the sun rises (east); make a quarter turn to the left; you are now facing north). Estimate the bearings of some things in your school compound, for example: a flag-pole, the Principal's office, the corner of a classroom block, some big trees, the school gate.

Calculating bearings

Example 1

In Fig. 20.12, find (i) the three-figure bearings, (ii) the compass bearings of A, B, C *and* D *from* X.

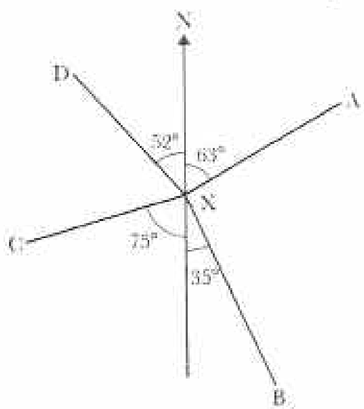


Fig. 20.12

The arrow N shows the direction north.
$N\hat{X}A = 63°$
The bearing of A from X is
(i) 063° or (ii) N63°E.
$N\hat{X}B = 180° - 35° = 145°$
The bearing of B from X is (i) 145° or (ii) S35°E.
NXC clockwise $= 180° + 75° = 255°$
The bearing of C from X is
(i) 255° or (ii) S75°W.
$N\hat{X}D$ clockwise $= 360° - 52° = 308°$
The bearing of D from X is
(i) 308° or (ii) N52°W.

Example 2

If the bearing of X *from* Y *is* 247°, *find the bearing of* Y *from* X.

The question gives the bearing of X *from* Y. Start by drawing a point Y. Draw a line, YN_1, pointing north from Y.

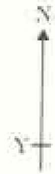


Fig. 20.13

X is on a bearing 247° from Y. Sketch a line YX such that $N_1\hat{Y}X$ is 247° clockwise from the line YN_1. Mark a point X on this line. From X, draw a line, XN_2, pointing north. See the sketch in Fig. 20.14. The two lines pointing north are parallel. Angle N_2XY is the bearing of Y from X.

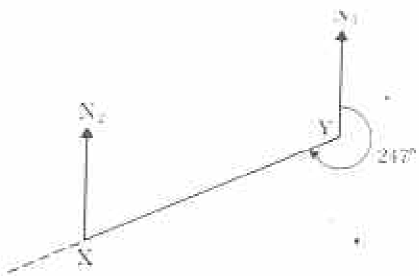


Fig. 20.14

There are many ways of finding the angle. Fig. 20.15 shows two ways. The bearing of Y from X is 067°.

Notice that when making sketches, it is usual to take the top of the page as north.

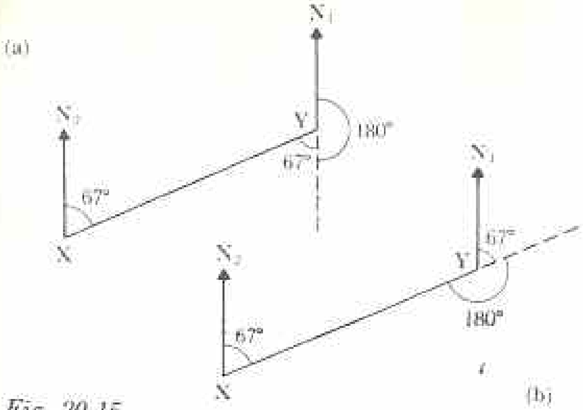


Fig. 20.15

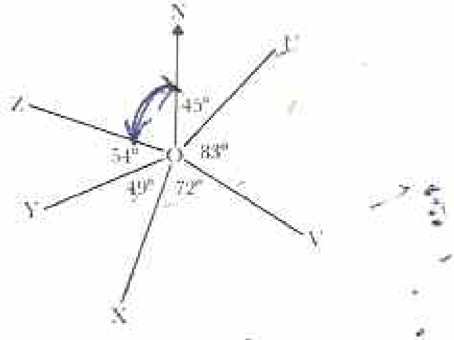


Fig. 20.18

Exercise 20c

1 For each sketch in Fig. 20.16, state (i) the three-figure bearing, (ii) the compass bearing of B from A.

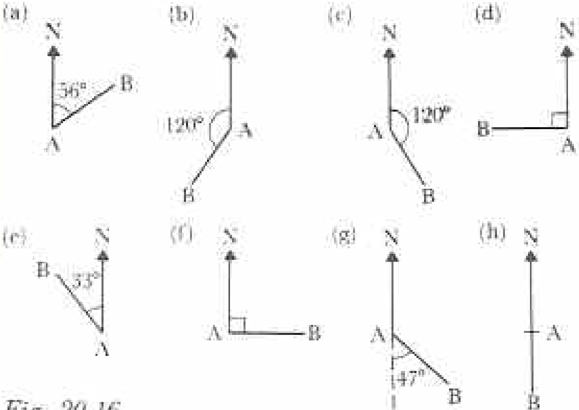


Fig. 20.16

2 In Fig. 20.17, find the bearings of A, B, C and D from X.

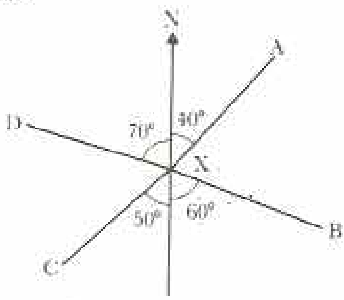


Fig. 20.17

3 In Fig. 20.18, find the bearings of U, V, X, Y, Z from O.

4 Make a sketch of the following bearings. Each sketch should show all the data and must contain a line pointing north.
(a) The bearing of B from A is 040°.
(b) X is on a bearing S20°E from Y.
(c) P is on bearing 320° from Q.
(d) L is on a bearing S20°W from K.
(e) The bearing of G from H is 180°.

5 In each diagram in Fig. 20.19, calculate, (i) the bearing of A from B, (ii) the bearing of B from A.

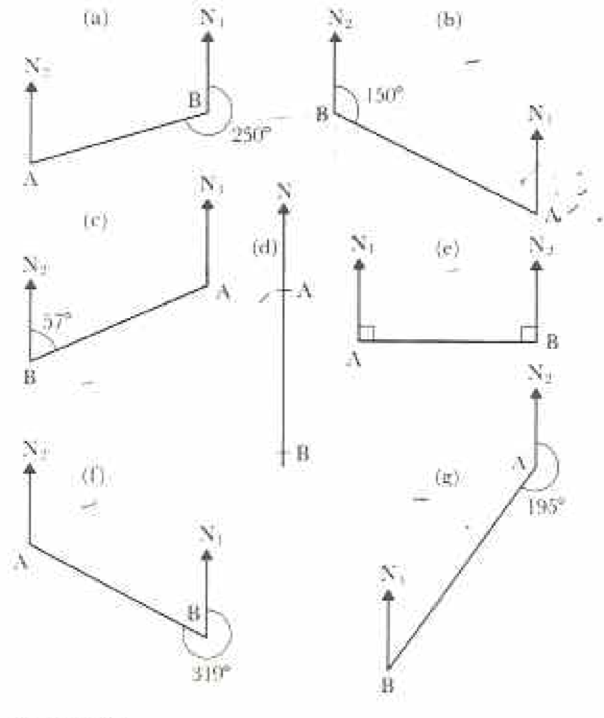


Fig. 20.19

Surveying

To **survey** an area means to take measurements so that a scale drawing of the area can be made. Fig. 20.20 shows a sketch that a student made while surveying a classroom block and a tree.

The student has measured the bearings and distances of the tree and two corners of the classroom block from a point P. For the tree, (118°, 15 m) means that the tree is on a bearing of 118° and a distance 15 m from P. The width of the classroom block has also been measured.

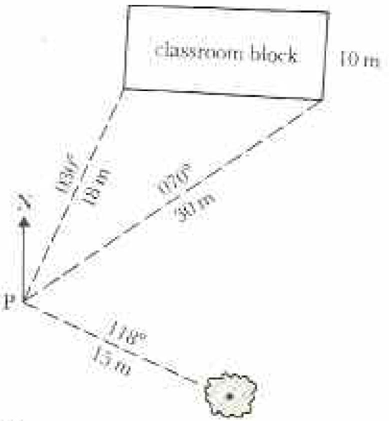


Fig. 20.20

Fig. 20.21 is a scale drawing of the tree and the classroom block.

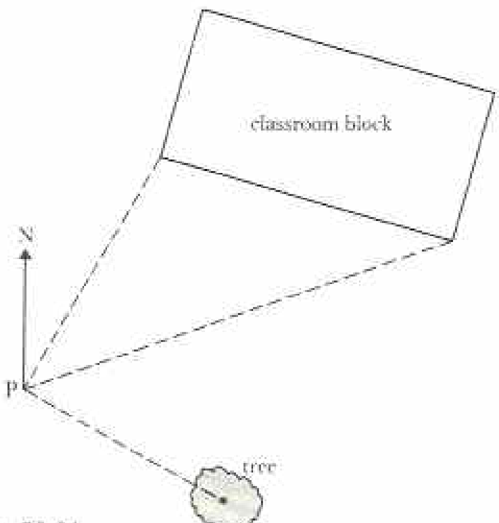


Fig. 20.21

Method: P is any point. A line pointing north is drawn from P. Lines are drawn on bearings 030°, 070° and 118° from P. These are shown dotted in Fig. 20.21. Distances of 18 m, 30 m and 15 m are marked on these lines respectively. These give the positions of the front corners of the classroom block and the centre of the tree. The rest of the classroom block is drawn, using the fact that it is 10 m wide.

Note: In this case, three-figure bearings are more convenient.

Exercise 20d

1 By taking measurements on Fig. 20.21, find, (a) the length of the classroom block, (b) the perpendicular distance of the tree from the classroom block.

2 Fig. 20.22 is a sketch and notes from a survey of three trees, A, B and C. Choose a suitable scale and draw an accurate plan of the three trees. Hence find the distance and bearing of A from C.

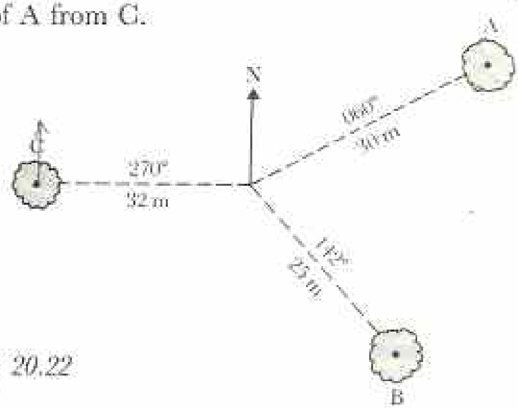


Fig. 20.22

3 Fig. 20.23 is a sketch of part of a river. A and B are 100 m apart. The bearing of the tree from A is 000°. The bearing of the tree from B is 290°. The edges of the river are roughly straight and parallel.

Make a scale drawing of points A and B and the tree. Hence find the approximate width of the river.

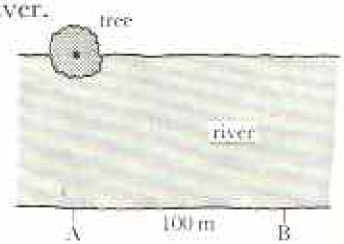


Fig. 20.23

4 Fig. 20.24 is a sketch and notes from a survey of a road. The road has two straight parts, AB and BC. It is 5 m wide.

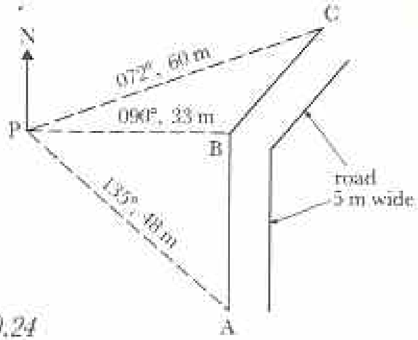


Fig. 20.24

Draw an accurate plan of the road to a suitable scale. Hence find the bearing of B from A and the bearing of C from B.

5 Fig. 20.25 is a sketch and notes from a survey of a straight length of railway line.

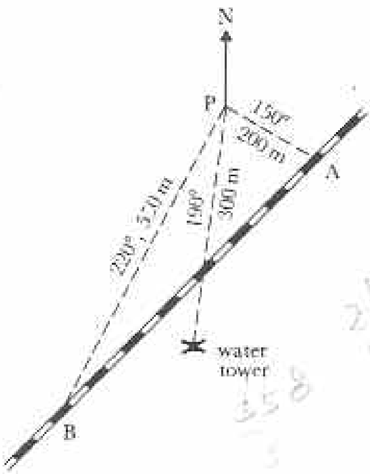


Fig. 20.25

Draw an accurate plan to show the position of the railway line and the water-tower. Find the bearing of A from B. Find the distance, to the nearest 10 m, of the water-tower from the railway line.

Example 3

A boy starts from A *and walks* 3 km *east to* B. *He then walks* 5 km *on a bearing* 152° *from* B. *He reaches a point* C. *Find the distance and bearing of* C *from* A.

First make a sketch of the information. See Fig. 20.26.

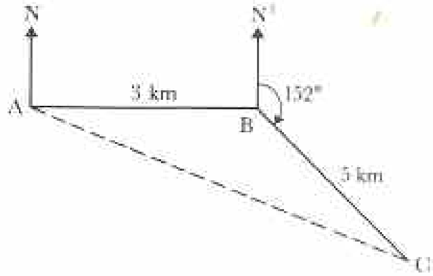


Fig. 20.26

Then make the scale drawing. See Fig. 20.27.

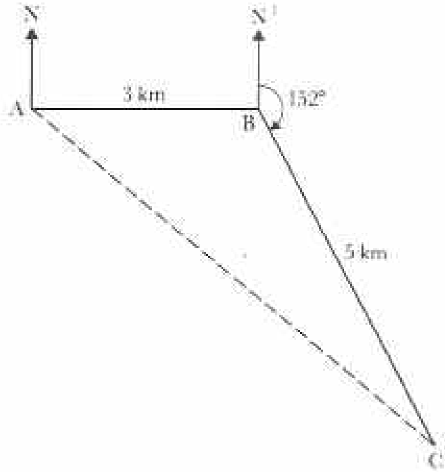


Fig. 20.27

Join AC.

By measurement, AC $= 6{,}9$ cm

Distance of C from A $= 6{,}9 \times 1$ km

$= 6{,}9$ km

By measurement, $N\hat{A}C = 130°$

Bearing of C from A is 130°.

Example 4

Gweru is 152 km *on a bearing of* 061° *from Bulawayo. How far is Gweru north of Bulawayo? How far west of Gweru is Bulawayo?*

Make a scale drawing of the data as in Fig. 20.28 overleaf.

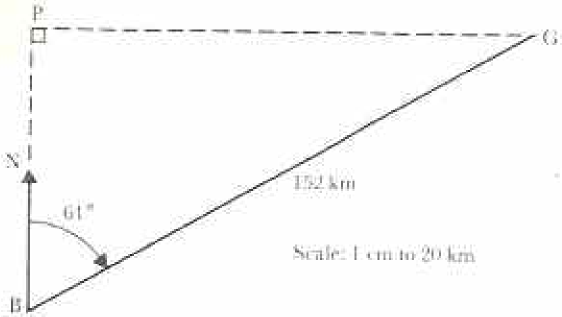


Fig. 20.28

P is the point that is due north of Bulawayo and due west of Gweru. BP represents the distance that Gweru is north of Bulawayo.
By measurement, BP ≃ 3,7 cm
The true distance BP ≃ 3,7 × 20 km
= 74 km.
Similarly GP represents the distance that Bulawayo is west of Gweru.
By measurement, GP ≃ 6,65 cm
The true distance GP ≃ 6,65 × 20 km
= 133 km.
Hence Gweru is approximately 74 km north of Bulawayo and Bulawayo is approximately 133 km west of Gweru.

Exercise 20e

Answer each question by making a scale drawing. Always make a rough sketch first.

1 A boy starts at A and walks 3 km east to B. He then walks 4 km north to C. Find the distance and bearing of C from A.

2 A girl starts at A and walks 2 km south to B. She then walks 3 km west to C. Find the distance and bearing of C from A.

3 An aeroplane flies 400 km west then 100 km north. Find its distance and bearing from its starting point.

4 A boy cycles 14 km east and then 10 km south-east. Find his distance and bearing from his starting point.

5 A road starts at a college and goes due north for 2 000 m. It then goes 2 000 m on a bearing 040° and ends at a market. How far is the market from the college? What is the bearing of the market from the college?

6 Mutare is 310 km from Zvishavane on a bearing 062°. Find (a) how far Zvishavane is south of Mutare, (b) how far Mutare is east of Zvishavane.

7 Maputo is approximately 850 km from Harare. The bearing of Maputo from Harare is 170°.
(a) How far north of Maputo is Harare?
(b) How far west of Maputo is Harare?

8 Bindura is 60 km on a bearing of 310° from Murewa. Dorowa is 150 km due south of Murewa. Find the bearing and distance of Bindura from Dorowa.

9 A ship is 3 km due east of a harbour. Another ship is also 3 km from the harbour but on a bearing 042° from it.
(a) Find the distance between the two ships.
(b) Find the bearing of the second ship from the first.

10 A plane flies 200 km on a bearing 032°. It then flies 350 km on a bearing 275°.
(a) Find the bearing and distance of the plane from its starting point.
(b) Find how far north and how far west the plane is from its starting point.

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Chapter 21

Simultaneous equations (1) Linear

The graph of an equation

$y = 3x - 4$ is called an **equation in x and y**. For any value of x there is a corresponding value of y. For example, if $x = 1$, then $y = -1$ and if $x = 3$, $y = 5$.

Before drawing a graph, it is usually necessary to make a table of values. Points on the graph can be plotted from values in the table. In the equation $y = 3x - 4$, x and y are often called the **variables**. The equation gives a connection between the variables. A table of values can be calculated from the equation. This is shown in Example 1 below.

Example 1

Draw the graph of $y = 3x - 4$ for values of x from -3 to $+3$. Read off (a) the value of y when $x = 2{,}5$, (b) the value of x when $y = -2$, (c) the coordinates of the points where the line cuts the axes.

Begin by making a table of values. Calculate values of y which correspond to whole-number values of x within the given range.

When $x = -3$, $y = 3(-3) - 4 = -9 - 4$
$= -13$,
when $x = -2$, $y = 3(-2) - 4 = -6 - 4$
$= -10$,

and so on. Table 21.1 gives the corresponding values of x and y for values of x in whole numbers from -3 to $+3$.

Table 21.1

x	-3	-2	-1	0	1	2	3
y	-13	-10	-7	-4	-1	2	5

The table gives seven ordered pairs: $(-3; -13)$, $(-2; -10)$, ..., $(3; 5)$. The pairs can be plotted as points on a cartesian plane.

x is called the **independent** variable. Val of x are given on the horizontal axis. y is cal the **dependent** variable. Values of y are gi on the vertical axis.

Fig. 21.1 shows the graph of the equat $y = 3x - 4$.

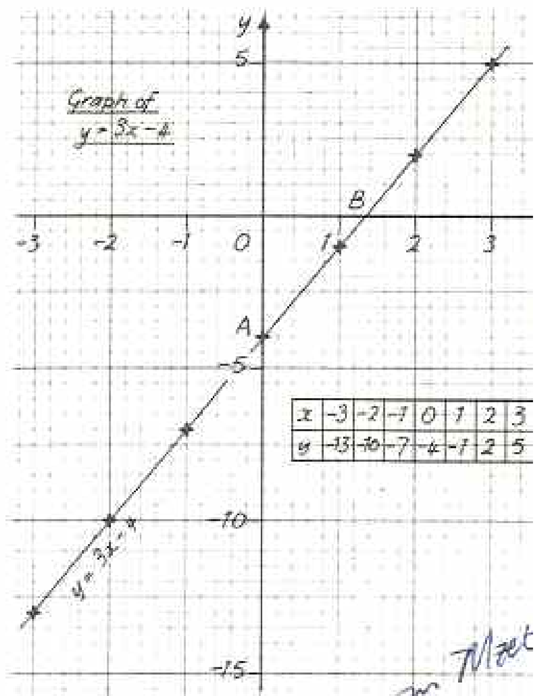


Fig. 21.1

Notice that the seven points can be joined us a ruler to give a straight line. This line r resents the set of all points $(x; y)$ such t $y = 3x - 4$, i.e. $\{(x; y): y = 3x - 4\}$.

(a) From the graph, $y = 3{,}5$ when $x = 2{,}5$
(b) Similarly, $x = 0{,}7$ when $y = -2$.
(c) The line cuts the axes at $A(0; -4)$ a $B(1{,}3; 0)$.

The line in Fig. 21.1 is the **graph** of the equation $y = 3x - 4$. $y = 3x - 4$ is called the equation of the line.

$y = 3x - 4$ is a **linear** equation in x and y. The variables in a linear equation are always separate and have a power of 1 (i.e. there are no terms such as xy, x^2, y^3, etc.). The graph of a linear equation is always a straight line. Thus it is sufficient to plot only two points to be able to draw the line. In practice it is better to plot *three* points. If the three points lie in a straight line, the working is probably correct.

Exercise 21a

1 Table 21.2 gives corresponding values of x and y for the equation $y = x + 2$.

Table 21.2

x	-4	0	$+4$
y	-2	2	$+6$

(a) Using a scale of 2 cm to 1 unit on both axes, draw the graph of $y = x + 2$. (b) Find the value of y when $x = 3$. (c) Find the value of x when $y = 1$. (d) Write down the coordinates of the points where the line cuts the axes.

2 Using a scale of 1 cm to 1 unit on both axes, draw the graphs of the following equations for values of x from -2 to $+2$.
(a) $y = x + 3$ (b) $y = 2x - 1$
(c) $y = 5x$ (d) $y = 3x - 2$

3 (a) Draw the graph of $y = 2x - 3$ for values of x from -1 to $+3$. Use a scale of 2 cm to 1 unit on both axes. (b) On the *same axes*, draw the lines $y = 2x$ and $y = 2x + 1$. (c) What do you notice about the three lines you have drawn?

4 (a) Draw the line $2y = 3x + 1$ for values of x from -2 to $+3$. Use $y = \frac{1}{2}(3x + 1)$ to make a table of values. (b) Write down the coordinates of the points where the line crosses the axes.

5 (a) Draw the graphs of $y = x + 1$ and $y = 3x - 2$ on the *same axes* for values of x from 0 to $+3$. (b) Write down the coordinates of the points where the two lines cross.

6 (a) Within the *same axes*, draw lines which represent the sets $\{(x; y) : y = 2 - x\}$ and $\{(x; y) : 2y = x + 5\}$. (b) Find the coordinates of the point where the two lines cross.

7 (a) On the *same axes*, draw the graphs of $y + 2x = 0$ and $2y = x + 5$ for values of x from 0 to 3. (b) Extend the lines until they cross each other. Find the coordinates of the point where they cross. (c) What is the angle between the lines?

8 Find the coordinates of the points where the following pairs of lines cross.
(a) $y = x - 3$, $3y = 1 - 2x$ (b) $y = x - 2$, $y = 7 - 2x$
(c) $3x - 4y = 0$, $5x - 2y = 7$ (d) $2x + y = -1$, $3x - y = -10$

Simultaneous linear equations

Consider the equation $2x + y = 7$.

For any value of x there is a corresponding value of y. If $x = 0$, $y = 7$; if $x = 1$, $y = 5$; and so on. Table 21.3 gives some of these pairs of values.

Table 21.3

x	0	1	2	3	4	5	...
y	7	5	3	1	-1	-3	...

Similarly, consider the equation $x - y = 2$.

Table 21.4 sets out pairs of values for this equation in the same way.

Table 21.4

x	0	1	2	3	4	5	...
y	-2	-1	0	1	2	3	...

Look at the pairs of values in Tables 21.3 and 21.4. Notice that the pair $x = 3, y = 1$ appears in *both* tables. This is the *only* pair that appears in both tables. This means that this pair of values satisfies both equations *simultaneously* (i.e. at the same time). Hence the solution of the **simultaneous equations** $2x + y = 7$ and $x - y = 2$ is $x = 3$ and $y = 1$.

This result can be shown by drawing the graphs of the two lines as in Fig. 21.2.

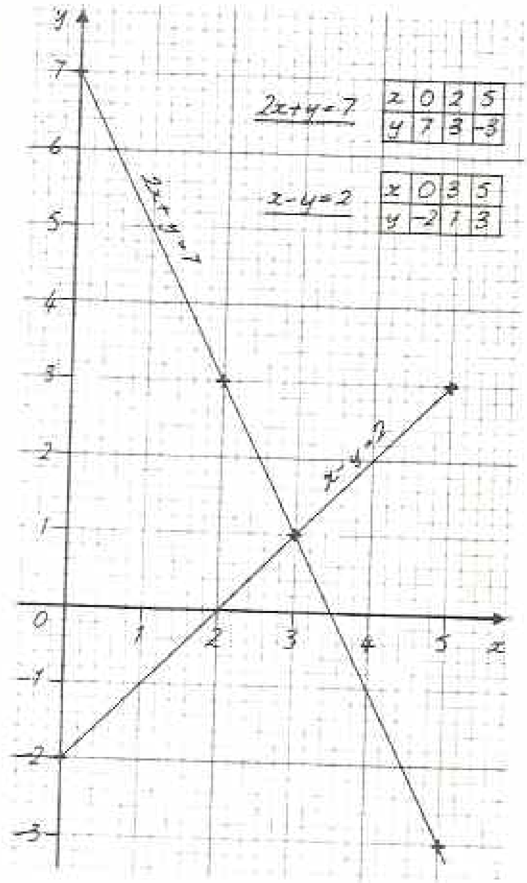


Fig. 21.2

The two lines *intersect* (i.e. cut each other) at the point (3; 1). This is the only point which is on both lines. The coordinates of the point of intersection give the solution of the simultaneous equations.

In set language:

$$\{(x, y) : 2x + y = 7\} \cap \{(x, y) : x - y = 2\} = \{(3;1)\}$$

Example 2

Solve graphically the simultaneous equations $2x - y = -1$, $x - 2y = 4$.

1st step: Make tables of values for each equation. Three pairs of values are sufficient for each:

$2x - y = -1$

x	0	1	2
y	1	3	5

$x - 2y = 4$

x	0	1	2
y	-2	$-1\frac{1}{2}$	-1

2nd step: Choose a suitable scale and plot the points. Draw both lines. Extend the lines if necessary so that they intersect. Fig. 21.3 shows the graphs of the two lines.

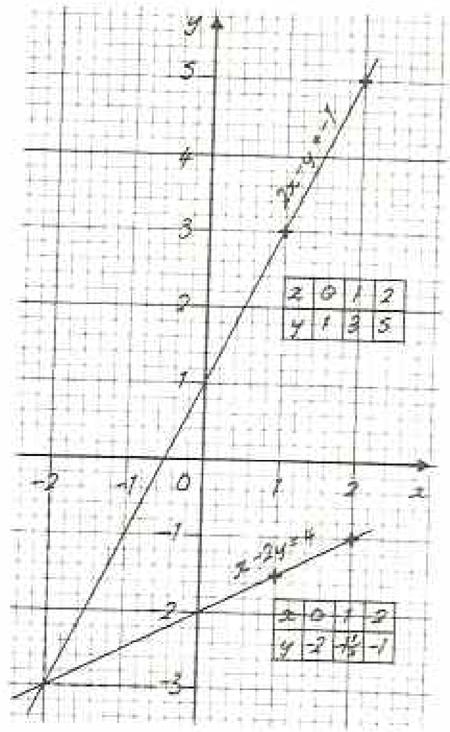


Fig. 21.3

Notice in Fig. 21.3 that it was necessary to extend both lines to find the point of intersection.

3rd step: Find the coordinates of the point of intersection. From the graphs, the lines intersect at $(-2; -3)$.

The solution of the simultaneous equations is $x = -2$ and $y = -3$.

Exercise 21b

Solve graphically the following pairs of simultaneous equations.

1 $x + y = 3$ $3x - y = 1$	**2** $x - y = 1$ $x + 2y = 7$
3 $x - 2y = 1$ $2x + y = 2$	**4** $y = 2x + 2$ $3x + 2y = 4$
5 $3y = 2x + 8$ $x + y = 1$	**6** $x + 3y = 0$ $x - 3y = 6$
7 $x - y = 0$ $3x - y + 2 = 0$	**8** $4x - 2y = 7$ $x + 3y = 7$
9 $x + y = 3$ $5x - 5y = 1$	**10** $2x - 2y = 5$ $2x + 3y + 1 = 0$
11 $3x + 7y = 11$ $x - y + 4 = 0$	**12** $5x - 2y = 1$ $4x + 3y = -10{,}7$

Method of substitution

The graphical method is time consuming and can often be inaccurate. It is quicker and more accurate to use an algebraic method to solve simultaneous equations. Read Example 3 carefully.

Example 3

Solve the equations $2x + y = 7$, $x - y = 2$.

Note: When asked to solve two equations with two unknowns, assume that they are simultaneous equations.

Write out the equations, one below the other. Label the equations (1) and (2).

$2x + y = 7 \quad (1)$

$x - y = 2 \quad (2)$

From equation (2),

$x = 2 + y \quad (3)$

Substitute $(2 + y)$ for x in equation (1)

$2(2 + y) + y = 7$

Clear brackets and collect terms

$4 + 2y + y = 7$

$3y = 3$

$y = 1$

Substitute the value 1 for y in equation (3)

$x = 2 + 1 = 3$

Thus $x = 3$ and $y = 1$.

Check: Substitute 3 for x and 1 for y in (1) and (2).

(1) $2x + y = 2 \times 3 + 1 = 6 + 1 = 7 = \text{RHS}$

(2) $x - y = 3 - 1 = 2 = \text{RHS}$

Use the method of **substitution** when the coefficient of one of the unknowns in the given equations is 1.

Example 4

Solve the equations $3a + b = 10$, $2a + 4b = 0$.

$3a + b = 10 \quad (1)$

$2a + 4b = 0 \quad (2)$

From (1),

$b = 10 - 3a \quad (3)$

Substitute $(10 - 3a)$ for b in (2)

$2a + 4(10 - 3a) = 0$

Clear brackets and collect terms

$2a + 40 - 12a = 0$

$-10a = -40$

$a = 4$

Substitute 4 for a in (3)

$b = 10 - 3 \times 4 = 10 - 12$

$b = -2$

Thus $a = 4$ and $b = -2$.

Check: Substitute 4 for a and -2 for b in (1) and (2).

(1) $3a + b = 3 \times 4 + (-2) = 12 - 2 = 10 = \text{RHS}$

(2) $2a + 4b = 2 \times 4 + 4(-2) = 8 - 8 = 0 = \text{RHS}$

Always check the accuracy of the answers by substituting the values into the original equations.

Exercise 21c

Use the method of substitution to solve the following pairs of simultaneous equations.

1 $y = x + 1$ $x + y = 3$	**2** $y = 2x - 4$ $3x + y = 11$
3 $a = 5 - 2b$ $5a + 2b = 1$	**4** $2m + n = 0$ $m + 2n = 3$
5 $x + y = 4$ $2x - y = 5$	**6** $y - 2x = 1$ $3x - 4y = 1$
7 $a - 2b = 9$ $2a + 3b = 4$	**8** $3x + 2y = 10$ $4x - y = 6$
9 $x + 2y = 7$ $3x - 2y = -3$	**10** $2a + b = 19$ $3a - 2b = 11$
11 $4x - 3y = 1$ $x - 2y = 4$	**12** $4x = y + 7$ $3x + 4y + 9 = 0$

Method of elimination

When none of the coefficients of the unknowns is 1, use the method of **elimination**. The aim of this method is to get rid of one of the unknowns by making its coefficient the same in both equations. The equations are then added or subtracted as necessary. Read Example 5 carefully.

Example 5

Solve the equations $3x + 2y = 12$ *and* $5x - 3y = 1$.

$3x + 2y = 12$ (1)
$5x - 3y = 1$ (2)

The coefficients of y can be made the same if (1) is multiplied by 3 and (2) is multiplied by 2.*

(1) × 3: $9x + 6y = 36$
(2) × 2: $10x - 6y = 2$
Adding: $19x = 38$

Thus $x = 2$

Substitute 2 for x in (1)

$3 \times 2 + 2y = 12$
$2y = 12 - 6 = 6$
$y = 3$

Thus $x = 2$ and $y = 3$.

Check: Substitute 2 for x and 3 for y in (1) and (2)
(1) $3x + 2y = 3 \times 2 + 2 \times 3 = 6 + 6 = 12 = \text{RHS}$
(2) $5x - 3y = 5 \times 2 - 3 \times 3 = 10 - 9 = 1 = \text{RHS}$

*Remember that an equation remains true if every term is multiplied or divided by the same number.

In the above example, instead of substituting 2 for x to find y, it may be simpler to start again with the original equations and eliminate x to find y. For example:

(1) × 5: $15x + 10y = 60$
(2) × 3: $15x - 9y = 3$
Subtracting: $19y = 57$
$y = 3$

This method can be very useful when the first value found is a fraction, since fractions often give difficult working when substituted.

Example 6

Solve the equations $3f = 4 - 4e$ and $2e = 5f + 15$.

$3f = 4 - 4e$ (1)
$2e = 5f + 15$ (2)

Arrange the equations so that unknowns are in alphabetical order on the LHS and numbers are on the RHS.

$4e + 3f = 4$ (3)
$2e - 5f = 15$ (4)

Multiply (4) by 2

$4e - 10f = 30$ (5)

Subtract equation (5) from equation (3) to eliminate terms in e

$13f = -26$
$f = -2$

Substitute -2 for f in (2)

$2e = 5(-2) + 15$
$2e = -10 + 15 = 5$
$e = 2\frac{1}{2}$

The check is left as an exercise.

Where necessary, arrange the given equations so that the unknowns are in alphabetical order on the LHS and the numbers are on the RHS.

Exercise 21d

Use the method of elimination as shown in Examples 5 and 6 to solve the following pairs of simultaneous equations.

1 $5a - 2b = 14$, $2a + 2b = 14$
2 $4p + 3q = 9$, $2p + 3q = 3$
3 $2x + 5y = 4$, $2x - 2y = 18$
4 $5x + 2y = 2$, $2x + 3y = -8$
5 $5x + 3y = 1$, $2x + 3y = -5$
6 $4x + 3y = 9$, $2x + 5y = 15$
7 $4a = 5b + 5$, $2a = 3b + 2$
8 $2x + 5y = 0$, $3x - 2y = 19$
9 $3x - 2y = 4$, $2x + 3y = -6$
10 $6h = 2k + 9$, $3h + 4k = 12$
11 $2p - 5q = 8$, $3p - 7q = 11$
12 $2r + 3s = 29$, $3r + 2s = 16$
13 $2x - 5y = -6$, $4x - 3y = -12$
14 $2x + 5y + 1 = 0$, $3x + 7y = 1$
15 $3a = 2b + 1$, $3b = 5a - 3$
16 $5v = 11 + 3u$, $2u + 7v = 3$
17 $5d = 2e - 14$, $5e = d + 12$
18 $6x - 5y = -7$, $3x + 4y = 16$
19 $3f - 4g = 1$, $6f - 6g = 5$
20 $8y + 4z = 7$, $6y - 8z = 41$

Word problems

Example 7

4 pens and 6 pencils cost \$1,36. *6 pens and 5 pencils cost* \$1,64. *Find the cost of one pen and one pencil.*

Let one pen cost x cents and one pencil cost y cents. Then 4 pens cost $4x$ cents

6 pencils cost $6y$ cents

and \$1,36 = 136 cents

Thus, $4x + 6y = 136$ (1)
(1st sentence in question)

Similarly, $6x + 5y = 164$ (2)
(2nd sentence in question)

(1) × 3: $12x + 18y = 408$
(2) × 2: $12x + 10y = 328$

Subtracting: $8y = 80$
$y = 10$

Substitute 10 for y in (1)

$4x + 60 = 136$
$4x = 136 - 60 = 76$
$x = 19$

A pen costs 19c and a pencil costs 10c.

Check: 4 pens cost \$0,76
6 pencils cost \$0,60
\$1,36

6 pens cost \$1,14
5 pencils cost \$0,50
\$1,64

Example 8

Kudzai's age and Rudo's age add up to 24 years. Six years ago, Kudzai was three times as old as Rudo. What are their ages?

Let Kudzai's age be a years and Rudo's age be b years. Then $a + b = 24$ (1) (1st sentence)
Six years ago, Kudzai was $(a - 6)$ and Rudo was $(b - 6)$. Hence $(a - 6) = 3(b - 6)$ (2nd sentence)

Clear brackets and collect terms

$a - 6 = 3b - 18$
$a - 3b = -12$ (2)

Subtract (2) from (1)

$a + b = 24$ (1)
$a - 3b = -12$ (2)

Subtracting: $4b = 36$
$b = 9$

Substitute 9 for b in (1)

$a + 9 = 24$
$a = 15$

Kudzai is 15 and Rudo is 9.

Check: Sum of ages = 15 + 9 = 24 years;
6 years ago, Kudzai was 9 and Rudo was 3;
$9 = 3 \times 3$.

Note: In both of these examples, the given facts are checked and *not* the equations.

Exercise 21e

1 The sum of two numbers is 19. Their difference is 5. Find the numbers.

2 A father is 25 years older than his son. The sum of their ages is 53 years. Find their ages.

3 The sum of two numbers is 17. The difference between twice the larger number and three times the smaller is 4. Find the numbers.

4 Maria and Teurai have 60c between them. Teurai has 24c more than Maria. How much does each have?

5 A newspaper and a magazine cost 55c together. The newspaper costs 35c less than the magazine. Find the cost of each.

6 A pencil and a rubber cost 27c together. 4 pencils cost the same as 5 rubbers. Find the cost of each.

7 I have x 5-c coins and y 10-c coins. There are 8 coins altogether and their total value is 55 cents. How many of each coin do I have?

8 Tanya's age and Ruby's age add up to 25 years. Eight years ago, Tanya was twice as old as Ruby. How old are they now?

9 The sides of the rectangle in Fig. 21.4 are given in cm.

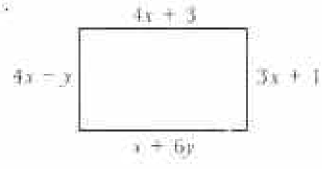


Fig. 21.4

Find x and y and the area of the rectangle.

10 A boy travels for x hours at 5 km/h and for y hours at 10 km/h. He travels 35 km altogether and his average speed is 7 km/h. Find x and y.

Chapter 22

Quadratic expressions (1)

Expanding algebraic expressions

The expression $(x + 2)(x - 5)$ means $(x + 2) \times (x - 5)$. The product of $(x + 2)$ and $(x - 5)$ is found by multiplying each term in the first bracket by each term in the second bracket. Read the following examples carefully.

Example 1
Find the product of $(x + 2)$ and $(x - 5)$.

$$\begin{aligned}(x + 2)(x - 5) &= x(x + 2) - 5(x + 2)\\ &= x^2 + 2x - 5x - 10\\ &= x^2 - 3x - 10\end{aligned}$$

We say that $(x + 2)(x - 5)$ is **expanded** to $x^2 - 3x - 10$. Notice that the terms in x, $+2x$ and $-5x$, are added together in the final line of working in Example 1.

Example 2
Expand $(2c - 3m)(c - 4m)$.

$$\begin{aligned}(2c - 3m)(c - 4m) &= c(2c - 3m) - 4m(2c - 3m)\\ &= 2c^2 - 3cm - 8cm + 12m^2\\ &= 2c^2 - 11cm + 12m^2\end{aligned}$$

Example 3
Expand $(3a + 2)^2$.

$$\begin{aligned}(3a + 2)^2 &= (3a + 2)(3a + 2)\\ &= 3a(3a + 2) + 2(3a + 2)\\ &= 9a^2 + 6a + 6a + 4\\ &= 9a^2 + 12a + 4\end{aligned}$$

Exercise 22a
Expand each expression. Arrange the working as in Examples 1, 2 or 3.

1 $(a + 2)(a + 3)$
2 $(c + 6)(c - 1)$
3 $(e - 3)(e + 2)$
4 $(d - 6)(d + 3)$
5 $(x - 1)(x - 2)$
6 $(a + 3)^2$
7 $(b - 5)^2$
8 $(m + 4)(m - 4)$
9 $(n + 5)(n - 4)$
10 $(d + 3)(d - 7)$
11 $(b - 5)(b + 6)$
12 $(p - 3)(p - 5)$
13 $(q - 3)(q + 3)$
14 $(u - 9)(u + 5)$
15 $(v - 4)(v - 9)$
16 $(2a + 1)(a + 3)$
17 $(b + 4)(3b + 2)$
18 $(2c - 5)(c - 3)$
19 $(d - 9)(2d + 3)$
20 $(2x + 1)^2$
21 $(5x + 2)(2x - 3)$
22 $(3y - 5)(2y + 1)$
23 $(m + 4n)^2$
24 $(u + 2v)(u + 3v)$
25 $(3d - 2e)(3d + 2e)$
26 $(3b + 2c)(2b - c)$
27 $(2s - 5t)(3s + t)$
28 $(2c - 3d)^2$
29 $(4m - n)(3m - 3n)$
30 $(2c - 9e)(4c + 5e)$

Direct expansion of products

With practice, it is not necessary to write down all the working as in Exercise 22a. The product of an expansion can be written down directly.

For example, in the product

$$(c - 3)(c - 4) = c^2 - 7c + 12$$

notice that

1 the first term in the expansion is the product of the first two terms in the brackets:

$$(c - 3)(c - 4) = \mathbf{c^2} - 7c + 12$$

2 the last term in the expansion is the product of the last two terms in the brackets:

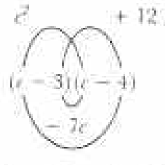
$$(c - 3)(c - 4) = c^2 - 7c + \mathbf{12}$$

3 the middle term in the expansion is found by adding the products of the inner and outer pairs of terms:

$$(c - 3)(c - 4) = c^2 - \mathbf{7c} + 12$$

$$(-3c) + (-4c) = -7c$$

Fig. 22.1 shows these steps in one diagram:



$(c-3)(c-4) = c^2 - 7c + 12$

Fig. 22.1

Example 4
Expand $(d-2)(d+5)$ *directly.*

$(d-2)(d+5) = d^2 + 3d - 10$
In the answer,
d^2 is the product of d and d,
$+3d$ is the result of adding $-2 \times d = -2d$ to $d \times +5 = +5d$,
-10 is the product of -2 and $+5$.

Example 5
Expand $(x+7)^2$.

$$(x+7)^2 = (x+7)(x+7) = x^2 + 14x + 49$$

Example 6
Expand $(10+x)(10-x)$ *without showing any working.*

$(10+x)(10-x) = 100 - x^2$
Notice in this example that the middle term reduces to zero:
$+10x + (-10x) = 0$.

Exercise 22b
Expand the following without showing any working.

1 $(a+1)(a+2)$ **2** $(a+2)(a+3)$
3 $(a+3)(a+4)$ **4** $(b+1)(b-2)$
5 $(b+2)(b-3)$ **6** $(b+3)(b-4)$
7 $(c-3)(c-4)$ **8** $(d+7)(d+1)$
9 $(e+2)(e+9)$ **10** $(f-5)(f-4)$
11 $(x-7)(x-1)$ **12** $(y-2)(y-9)$
13 $(h+6)^2$ **14** $(k-5)^2$
15 $(z+2)(z-9)$ **16** $(a+4)(a+6)$
17 $(a-4)(a-6)$ **18** $(a-4)(a+6)$
19 $(a+4)(a-6)$ **20** $(b+6)(b-3)$
21 $(c-1)(c-2)$ **22** $(m-1)^2$
23 $(n+1)^2$ **24** $(f+9)(f+11)$
25 $(e-3)(e-5)$ **26** $(d-2)(d+10)$
27 $(h+3)(h-8)$ **28** $(a+3)^2$
29 $(a-3)^2$ **30** $(a+3)(a-3)$
31 $(b-5)(b+5)$ **32** $(c+7)(c-7)$

Coefficients of terms

The **coefficient** of an algebraic term is the number which multiplies the unknown. For example, in $3x^2$, the coefficient of x^2 is 3
in, $-2y$, the coefficient of y is -2
in $\frac{2}{3}d$, the coefficient of d is $\frac{2}{3}$

Example 7
Find the coefficient of x *in the expansion of* $(x+9)(x+3)$.

It is not necessary to expand the expression fully. The middle term is the term in x.
Middle term $= (+9x) + (+3x) = +12x$
The coefficient of x in the expansion is $+12$.

Example 8
Find the coefficient of ab *in the expansion of* $(5a+2b)(4a-3b)$.

The terms containing ab are
$2b \times 4a = 8ab$
and $5a \times (-3b) = -15ab$.
These add to give $-7ab$. The coefficient of ab in the expansion is -7.

With practice, it should be possible to find coefficients without any written work.

Exercise 22c

1 Find the coefficient of d in the expansion of
(a) $(d+2)(d+7)$ (b) $(d-4)(d+6)$
(c) $(d-3)(d-1)$ (d) $(d-8)(d+3)$
(e) $(d+7)^2$

2 Find the coefficient of x in the expansion of
(a) $(x-5)(x+4)$ (b) $(x+8)(x-11)$
(c) $(x-3)(x+5)$ (d) $(x-4)(x+4)$
(e) $(x-5)^2$

3 Find the coefficient of u in the expansion of
(a) $(u+2)(2u+3)$ (b) $(u-4)(3u+5)$
(c) $(2u-5)(3u+5)$
(d) $(4u-5)(2u-7)$
(e) $(3u-4)^2$

4 Find the coefficient of ab in the expansion of
(a) $(3a + b)(a + 2b)$
(b) $(a - b)(3a - 2b)$
(c) $(4a + 3b)(5a - 3b)$
(d) $(5a + 2b)(5a - 2b)$
(e) $(a - 3b)^2$

Factorisation of quadratic expressions

A **quadratic** expression is one in which 2 is the highest power of the unknown(s) in the expression. For example, $x^2 - 4x - 12$, $16 - a^2$, $3x^2 + 17xy + 10y^2$ are quadratic expressions. Since $(x + 2)(x - 6) = x^2 - 4x - 12$
$(x + 2)$ and $(x - 6)$ are the **factors** of $x^2 - 4x - 12$. Just as in arithmetic, $5 \times 7 = 35$ where 5 and 7 are the factors of 35.

A quadratic expression may *not* have factors. In arithmetic, 13 is prime since it has no factors other than itself and 1. Similarly $x^2 + 2x - 6$ has no factors (other than itself and 1).

To **factorise** a quadratic expression is to express it as a product of its factors. Thus $x^2 - 4x - 12$ factorises to become $(x + 2)(x - 6)$.

Example 9 shows the steps to be followed when factorising quadratic expressions.

Example 9

Factorise the quadratic expression $x^2 + 7x + 10$.

The problem is to fill the brackets in the statement $x^2 + 7x + 10 = (\quad)(\quad)$.

1st step: Look at the *first* term in the given expression, x^2. From work done in expanding brackets (as in Exercise 22b), when the first term in the expansion is x^2, x appears first in each bracket:

$x^2 + 7x + 10 = (x \quad)(x \quad)$.

2nd step: Look at the *last* term in the given expression, + 10. The *product* of the last terms in the two brackets must be + 10. Number pairs which have a product of + 10 are:

(a) + 10 and + 1 (b) + 5 and + 2
(c) − 10 and − 1 (d) − 5 and − 2

These give four possible answers:
(a) $(x + 10)(x + 1)$
(b) $(x + 5)(x + 2)$
(c) $(x - 10)(x - 1)$
(d) $(x - 5)(x - 2)$

3rd step: Look at the coefficient of the *middle* term in the given expression, + 7. The *sum* of the last terms in the two brackets must be + 7. Adding the number pairs in turn:
(a) $(+10) + (+1) = +11$
(b) $(+5) + (+2) = +7$
(c) $(-10) + (-1) = -11$
(d) $(-5) + (-2) = -7$
Of these, only (b) gives + 7. It follows that:
$x^2 + 7x + 10 = (x + 5)(x + 2)$

Note: **1** The answer can be checked by expanding the brackets.
2 The order of the brackets is not important.
$(x + 5)(x + 2) = (x + 2)(x + 5)$

In Example 9, both the last term and the coefficient of x were positive. It was not really necessary to consider the possibility of having negative factors of + 10. Example 10 shows how the method may be shortened.

Example 10

Factorise $d^2 + 11d + 18$.

1st step: $d^2 + 11d + 18 = (d \quad)(d \quad)$.
2nd step: Find two numbers such that their *product* is + 18 and their sum is + 11. Since the 18 is positive *and* the 11 is positive, consider positive factors only.

	factors of + 18	*sum of factors*
(a)	+ 1 and + 18	+ 19
(b)	+ 2 and + 9	+ 11
(c)	+ 3 and + 6	+ 9

Of these, only (b) gives the required result. Thus,
$d^2 + 11d + 18 = (d + 2)(d + 9)$

The method of Examples 9 and 10 is called a **trial and error** method. It is necessary to try various number pairs in turn, until the correct pair is found. To begin with, this will take time. With practice, it will be possible to factorise quadratic expressions quite quickly.

Exercise 22d

Factorise the following quadratic expressions.

1 $x^2 + 6x + 5$ **2** $x^2 + 12x + 11$
3 $a^2 + 14a + 13$ **4** $b^2 + 8b + 7$
5 $y^2 + 9y + 8$ **6** $z^2 + 6z + 8$
7 $c^2 + 8c + 15$ **8** $d^2 + 13d + 22$
9 $n^2 + 8n + 12$ **10** $r^2 + 9r + 20$
11 $s^2 + 10s + 16$ **12** $t^2 + 8t + 16$

Example 11

Factorise $x^2 - 9x + 8$.

1st step: $x^2 - 9x + 8 = (x \quad)(x \quad)$.
2nd step: Find two numbers such that their product is $+8$ and their sum is -9. List the possible pairs and find their sums:

	factors of $+8$	*sum of factors*
(a)	$+8$ and $+1$	$+9$
(b)	$+4$ and $+2$	$+6$
(c)	-8 and -1	-9
(d)	-4 and -2	-6

Of these, only (c) gives the required result. Thus,

$x^2 - 9x + 8 = (x - 8)(x - 1)$

In Example 11, the last term is positive and the coefficient of x is negative. This gives a negative sign in both brackets.

Example 12

Factorise $t^2 - 10t + 24$.

1st step: $t^2 - 10t + 24 = (t \quad)(t \quad)$.
2nd step: Find two numbers such that their product is $+24$ and their sum is -10. Since the 24 is positive and the 10 is negative, consider negative factors only.

	factors of $+24$	*sum of factors*
(a)	-1 and -24	-25
(b)	-2 and -12	-14
(c)	-3 and -8	-11
(d)	-4 and -6	-10

Of these, only (d) gives the required result.
Thus, $t^2 - 10t + 24 = (t - 4)(t - 6)$

Exercise 22e

Factorise the following.

1 $x^2 - 4x + 3$ **2** $y^2 - 3y + 2$
3 $z^2 - 18z + 17$ **4** $a^2 - 8a + 7$
5 $b^2 - 5b + 6$ **6** $c^2 - 7c + 6$
7 $d^2 - 9d + 14$ **8** $n^2 - 7n + 10$
9 $p^2 - 11p + 24$ **10** $q^2 - 10q + 21$
11 $f^2 - 16f + 28$ **12** $x^2 - 10x + 25$

So far, the given quadratic expressions have all contained a positive last term. Examples 13 and 14 show what happens when the last term is negative.

Example 13

Factorise the expression $x^2 + 2x - 15$.

1st step: $x^2 + 2x - 15 = (x \quad)(x \quad)$.
2nd step: Find two numbers such that their product is -15 and their sum is $+2$. List the possible pairs and find their sums.

	factors of -15	*sum of factors*
(a)	-15 and $+1$	-14
(b)	$+15$ and -1	$+14$
(c)	-5 and $+3$	-2
(d)	$+5$ and -3	$+2$

Of these, only (d) gives the correct result.
$x^2 + 2x - 15 = (x + 5)(x - 3)$

Example 14

Factorise $x^2 - 4x - 12$.

Find two numbers such that their product is -12 and their sum is -4.

	factors of -12	*sum of factors*
(a)	-12 and $+1$	-11
(b)	$+12$ and -1	$+11$
(c)	-6 and $+2$	-4
(d)	$+6$ and -2	$+4$
(e)	-4 and $+3$	-1
(f)	$+4$ and -3	$+1$

Of these, only (c) gives the required result.
$x^2 - 4x - 12 = (x - 6)(x + 2)$

Notice that if the last term in the given expression is negative, the signs inside the brackets are different; one positive and one negative.

Exercise 22f

Factorise the following.

1 $x^2 + 4x - 5$ **2** $a^2 - 4a - 5$
3 $x^2 + 6x - 7$ **4** $b^2 - 6b - 7$
5 $n^2 + n - 2$ **6** $r^2 - 2r - 3$

7 $x^2 - 10x - 11$ **8** $y^2 + 12y - 13$
9 $x^2 - 2x - 15$ **10** $x^2 + 14x - 15$
11 $s^2 + 5s - 6$ **12** $t^2 - 5t - 6$
13 $u^2 - u - 6$ **14** $v^2 + v - 6$
15 $z^2 + z - 20$ **16** $c^2 - 8c - 20$
17 $x^2 - 49$ **18** $x^2 - 4$

Perfect squares

$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$

$$(a + b)^2 = a^2 + 2ab + ab^2$$

$(a - b)^2 = (a - b)(a - b) = a^2 - ab - ab + b^2$

$$(a - b)^2 = a^2 - 2ab + b^2$$

These results are very important and should be remembered.

Fig. 22.2 gives a geometrical representation of $(a + b)^2 = a^2 + 2ab + b^2$.

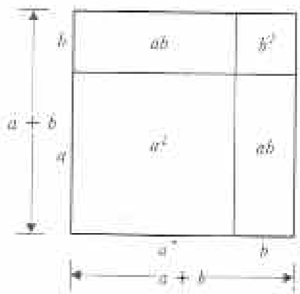


Fig. 22.2

Example 15

Expand the following.
(a) $(3m + 7n)^2$ (b) $(4u - 5v)^2$.

(a) $(3m + 7n)^2$
$= (3m)^2 + 2 \times 3m \times 7n + (7n)^2$
$= 9m^2 + 42mn + 49n^2$

(b) $(4u - 5v)^2$
$= (4u)^2 - 2 \times 4u \times 5v + (5v)^2$
$= 16u^2 - 40uv + 25v^2$

Notice that the squared terms, are *always positive*.

Exercise 22g (Oral or written)

Expand the following.

1 $(a + 4)^2$ **2** $(b - 3)^2$ **3** $(5 + c)^2$
4 $(2 - d)^2$ **5** $(1 + m)^2$ **6** $(2n + 1)^2$
7 $(3x + y)^2$ **8** $(u - 2v)^2$ **9** $(5h - k)^2$
10 $(p + 4q)^2$ **11** $(2a + 3d)^2$ **12** $(3b - 5c)^2$
13 $(7e - 2f)^2$ **14** $(10x - 1)^2$ **15** $(1 + 12y)^2$
16 $(3a + 7b)^2$ **17** $(c - 8d)^2$ **18** $(9u + v)^2$

The expansion of a perfect square can sometimes be used to shorten the working when squaring numbers.

Example 16

Find the value of (a) 104^2, (b) 97^2

(a) $104^2 = (100 + 4)^2$
$= 100^2 + 2 \times 100 \times 4 + 4^2$
$= 10\,000 + 800 + 16$
$= 10\,816$

(b) $97^2 = (100 - 3)^2$
$= 100^2 - 2 \times 100 \times 3 + 3^2$
$= 10\,000 - 600 + 9$
$= 9\,409$

Exercise 22h

Find the squares of the following numbers.

1 101   **2** 99   **3** 103
4 98   **5** 1 001   **6** 999
7 1 005   **8** 996   **9** 995
10 72   **11** 83   **12** 79

Example 17

Factorise the following.
(a) $h^2 + 12h + 36$ (b) $25h^2 - 30hk + 9k^2$

(a) Notice that h^2 is the square of h, 36 is the square of 6 and $12h$ is twice the product of h and 6.
$h^2 + 12h + 36 = (h + 6)(h + 6)$
$= (h + 6)^2$

(b) $25h^2$ is the square of $5h$
$9k^2$ is the square of $3k$
$30hk$ is twice the product of $5h$ and $3k$
$25h^2 - 30hk + 9k^2 = (5h - 3k)^2$

Exercise 22i (Oral or written)

Give the following as the square of an expression in brackets.

1 $a^2 + 10a + 25$ **2** $b^2 + 8b + 16$
3 $c^2 + 6c + 9$ **4** $d^2 + 20d + 100$
5 $m^2 - 6m + 9$ **6** $n^2 - 12n + 36$
7 $x^2 - 4x + 4$ **8** $y^2 - 2y + 1$

9 $z^2 + 16z + 64$ **10** $k^2 - 14k + 49$
11 $4 - 4b + b^2$ **12** $81 + 18d + d^2$
13 $x^2 + 6xy + 9y^2$ **14** $4u^2 - 12u + 9$
15 $1 - 2a + a^2$ **16** $25n^2 - 30nv + 9v^2$
17 $9a^2 - 24ab + 16b^2$
18 $121 - 22y + y^2$

Difference of two squares

$(a + b)(a - b) = a^2 + ab - an - b^2 = a^2 - b^2$
Hence $\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$
Fig. 22.3 shows how a cardboard model can be made to demonstrate that
$a^2 - b^2 = (a + b)(a - b)$.

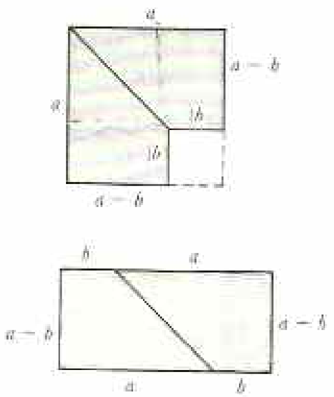


Fig. 22.3

Example 18

Factorise the following.
(*a*) $y^2 - 4$ (*b*) $36 - 9a^2$ (*c*) $25m^2 - 16n^2$

(a) $y^2 - 4 = (y)^2 - (2)^2$
$= (y + 2)(y - 2)$
(b) $36 - 9a^2 = (6)^2 - (3a)^2$
$= (6 + 3a)(6 - 3a)$
(c) $25m^2 - 16n^2 = (5m)^2 - (4n)^2$
$= (5m + 4n)(5m - 4n)$

Example 19

Factorise $5a^2 - 45$.

The two terms have the factor 5 in common.

Take this out first:
$5a^2 - 45 = 5(a^2 - 9)$
$= 5(a^2 - 3^2)$
$= 5(a + 3)(a - 3)$

Exercise 22j (Oral or written)

Factorise the following.

1 $x^2 - 1$	**2** $1 - y^2$
3 $4m^2 - n^2$	**4** $u^2 - 16v^2$
5 $1 - a^2b^2$	**6** $9 - 4c^2$
7 $4d^2 - 9e^2$	**8** $3 - 3f^2$
9 $4g^2 - 4$	**10** $4h^2 - 25$
11 $25k^2 - 16$	**12** $49m^2 - n^2$
13 $p^2q^2 - 9$	**14** $25 - u^2v^2$
15 $81 - w^2$	**16** $100x^2 - 1$
17 $16y^2 - 4z^2$	**18** $16h^2 - k^2$
19 $4c^2 - 49d^2$	**20** $e^2 - 4f^2$
21 $36a^2 - 49b^2$	**22** $5c^2 - 45d^2$
23 $x^2y^2 - z^2$	**24** $100 - w^2$

The difference of two squares can sometimes be used to shorten calculations.

Example 20

Find the value of $173^2 - 127^2$.

$173^2 - 127^2 = (173 + 127)(173 - 127)$
$= 300 \times 46$
$= 13\,800$

Example 21

Fig. 22.4 shows a circular metal washer. If the diameters of the washer and its hole are 3 cm and 1 cm respectively, find the area of the washer. Use the value 3,14 for π.

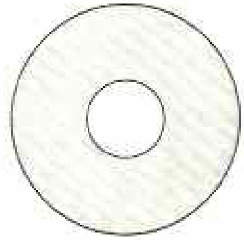


Fig. 22.4

Let the outer and inner radii of the washer be R and r respectively.

Area of washer $= \pi R^2 - \pi r^2$
$= \pi(R^2 - r^2)$
$= \pi(R + r)(R - r)$

But $R = 1\frac{1}{2}$ cm and $r = \frac{1}{2}$ cm.

Area of washer $= \pi(1\frac{1}{2} + \frac{1}{2})(1\frac{1}{2} - \frac{1}{2})$ cm^2
$= \pi \times 2 \times 1$ cm^2
$= 2\pi$ cm^2
$= 2 \times 3{,}14$ cm^2
$= 6{,}28$ cm^2

Exercise 22k

In questions 1–10, use the difference of two squares to find the values of the given numerical expressions.

1 $96^2 - 4^2$ **2** $118^2 - 18^2$ **3** $73^2 - 71^2$
4 $98^2 - 4$ **5** $103^2 - 9$ **6** $52^2 - 48^2$
7 $63^2 - 37^2$ **8** $57^2 - 55^2$ **9** $1\,004^2 - 16$
10 $997^2 - 9$

11 A metal washer has an outer diameter of 14 mm and an inner diameter of 6 mm. Use the value 3,14 for π to find the area of the metal washer in mm^2.

12 A cylindrical clay pipe has inner and outer radii of 3,7 cm and 2,3 cm respectively. Use the value $\frac{22}{7}$ for π to find the area of clay in a cross-section of the pipe.

Quadratic equations

If the product of two numbers is 0, then one of the numbers (or possibly both of them) must be zero. For example,

$3 \times 0 = 0$, $0 \times -5 = 0$ and $0 \times 0 = 0$

In general, if $a \times b = 0$
then *either* $a = 0$
or $b = 0$

Example 22

Solve the equation $(x - 2)(x + 7) = 0$.

If $(x - 2)(x + 7) = 0$
then *either* $x - 2 = 0$ or $x + 7 = 0$
$\Leftrightarrow$ $x = 2$ or $x = -7$

Example 23

Solve $a(a + 3) = 0$

If $a(a + 3) = 0$
then either $a = 0$ or $a + 3 = 0$
$\Leftrightarrow$ $a = 0$ or -3

Example 24

Solve the equation $(m - 5)^2 = 0$.

If $(m - 5)^2 = 0$
then $(m - 5)(m - 5) = 0$
Hence $m - 5 = 0$ (twice)
$\Leftrightarrow$ $m = 5$ (twice)

The solutions of the equations in Examples 22, 23, 24 are called the **roots** of the equations. Notice that Example 24 contains a repeated root.

Exercise 22*l*

Solve the following equations.

1 $(a - 3)(a + 5) = 0$
2 $(b - 2)(b - 1) = 0$
3 $(x + 2)(x + 6) = 0$
4 $(y - 5)y = 0$
5 $(m + 3)(m - 4) = 0$
6 $(n - 5)(n + 3) = 0$
7 $u(u + 1) = 0$
8 $(v + 5)(v + 3) = 0$
9 $x(3 + x) = 0$
10 $y(4 - y) = 0$
11 $5(a + 2)(a - 4) = 0$
12 $4b(b + 6) = 0$
13 $3d(d - 7) = 0$
14 $6m(m + 3) = 0$
15 $(6 - n)(4 + n) = 0$
16 $0 = (5 + u)(3 - u)$
17 $(v - 2)(v + 2) = 0$
18 $(x + 5)(x - 5) = 0$
19 $(9 - f)^2 = 0$
20 $(1 + z)^2 = 0$

Example 25

Solve the equation $(3a + 2)(2a - 7) = 0$.

If $(3a + 2)(2a - 7) = 0$
then either $3a + 2 = 0$ or $2a - 7 = 0$
$\Leftrightarrow$ $3a = -2$ or $2a = 7$
$\Leftrightarrow$ $a = -\frac{2}{3}$ or $a = \frac{7}{2}$
$\Leftrightarrow$ $a = -\frac{2}{3}$ or $3\frac{1}{2}$

Check: By substitution,
if $a = -\frac{2}{3}$, $(3a + 2)(2a - 7)$
$= (-2 + 2)(-1\frac{1}{3} - 7)$
$= 0 \times (-8\frac{1}{3}) = 0$
if $a = 3\frac{1}{2}$, $(3a + 2)(2a - 7)$
$= (10\frac{1}{2} + 2)(7 - 7)$
$= 12\frac{1}{2} \times 0 = 0$

Exercise 22m

Solve the following equations. Check the results by substitution.

1 $(d-5)(3d-2)=0$
2 $(2m-1)(m+4)=0$
3 $(a+3)(5a+2)=0$
4 $(4x+3)(3x+1)=0$
5 $(2y-7)(y+2)=0$
6 $(4b-12)(b-5)=0$
7 $(4h-1)(2h+3)=0$
8 $(5-d)(5-2d)=0$
9 $(5+3m)(2-5m)=0$
10 $(3n+7)(4n-1)=0$
11 $(3a+10)(3a-12)=0$
12 $(4b-3)^2=0$
13 $(2c+1)^2=0$
14 $(1-2d)(2+3d)=0$
15 $(3e+5)^2=0$ **16** $5m(3m-4)=0$
17 $(9+3n)(5-7n)=0$
18 $(11-4x)^2=0$
19 $3y(2y+9)=0$ **20** $5a(15+4a)=0$

A **quadratic equation** is one in which 2 is the highest power of the letter (or letters) in the equation. We say that it is an equation of the **second degree**. For example, $m^2-5m-14=0$ is equation of the second degree, or a quadratic equation. In this example, the LH side of the equation can be factorised: $(m+2)(m-7)=0$. The equation is now like many of those in Exercises 22*l* and 22m.

Example 26

Solve the equation $y^2+4y-21=0$.

$y^2+4y-21=0$
$\Leftrightarrow (y+7)(y-3)=0$
Either $y+7=0$ or $y-3=0$
$\Leftrightarrow \quad y=-7$ or $y=3$

Example 27

Find the roots of $a^2-3a=0$.

$a^2-3a=0$
$\Leftrightarrow a(a-3)=0$
Either $a=0$ or $a-3=0$
$a=0$ or 3
The roots of the equation are 0 and 3

Example 28

Solve the equation $m^2=16$.

First rearrange the equation:
$m^2=16$
$\Leftrightarrow m^2-16=0$
Factorise (difference of two squares):
$(m-4)(m+4)=0$
Either $m-4=0$ or $m+4=0$
$\Leftrightarrow \quad m=4$ or $m=-4$
$\Leftrightarrow \quad m=\pm 4$

Example 29

Solve the equation $x^2=4x+5$.

Rearrange the equation to give a quadratic expression on the LHS and 0 (zero) on the RHS:

$x^2=4x+5$
$\Leftrightarrow \quad x^2-4x-5=0$
$\Leftrightarrow (x-5)(x+1)=0$
Either $x-5=0$ or $x+1=0$
$\Leftrightarrow x=5$ or $x=-1$

Notice in Examples 28 and 29, that where necessary the given equation should be arranged to give a quadratic expression on the left-hand side and zero (0) on the other side.

Exercise 22n

Solve the following quadratic equations.

1 $a^2-3a+2=0$ **2** $b^2+5b+6=0$
3 $c^2-c-2=0$ **4** $d^2+2d-3=0$
5 $e^2-7e+10=0$ **6** $m^2-4m=0$
7 $n^2+5n=0$ **8** $p^2+7p+12=0$
9 $q^2+2q-8=0$ **10** $x^2-2x+1=0$
11 $y^2-5y+4=0$ **12** $a^2-9a=0$
13 $b^2-9=0$ **14** $c^2=25$
15 $u^2-8u-9=0$ **16** $v^2+2v-35=0$
17 $x^2-6x+9=0$ **18** $y^2+8y+16=0$
19 $z^2-4z=0$ **20** $z^2-4=0$
21 $h^2-15h+54=0$
22 $k^2-15k-54=0$
23 $m^2+4m=0$ **24** $m^2+4m+3=0$
25 $m^2+4m=32$ **26** $4r^2=49$
27 $a^2+a=90$ **28** $b^2-b=72$
29 $k^2=10k+39$ **30** $z^2=66-19z$

Everyday arithmetic (4) Ready reckoners, tables

Ready reckoners

A **ready reckoner** is a book containing tables which make calculations easier. Ready reckoners are often used in shops and offices.

Cost of articles

Fig. 23.1 shows a page from a ready reckoner. It is a table which gives the cost of from 1 to 140 articles at 72 cents each.

72 cents							
1	0,72	36	25,92	71	51,12	106	76,32
2	1,44	37	26,64	72	51,84	107	77,04
3	2,16	38	27,36	73	52,56	108	77,76
4	2,88	39	28,08	74	53,28	109	78,48
5	3,60	40	28,80	75	54,00	110	79,20
6	4,32	41	29,52	76	54,72	111	79,92
7	5,04	42	30,24	77	55,14	112	80,64
8	5,76	43	30,96	78	56,16	113	81,36
9	6,48	44	31,68	79	56,88	114	82,08
10	7,20	45	32,40	80	57,60	115	82,80
11	7,92	46	33,12	81	58,32	116	83,52
12	8,64	47	33,84	82	59,04	117	84,24
13	9,36	48	34,56	83	59,76	118	84,96
14	10,08	49	35,28	84	60,48	119	85,68
15	10,80	50	36,00	85	61,20	120	86,40
16	11,52	51	36,72	86	61,92	121	87,12
17	12,24	52	37,44	87	62,64	122	87,84
18	12,96	53	38,16	88	63,36	123	88,56
19	13,68	54	38,88	89	64,08	124	89,28
20	14,40	55	39,60	90	64,80	125	90,00
21	15,12	56	40,32	91	65,52	126	90,72
22	15,84	57	51,04	92	66,24	127	91,44
23	16,56	58	41,76	93	66,96	128	92,16
24	17,28	59	42,48	94	67,68	129	92,88
25	18,00	60	43,20	95	68,40	130	93,60
26	18,72	61	43,92	96	69,12	131	94,32
27	19,44	62	44,64	97	69,84	132	95,04
28	20,16	63	45,36	98	70,56	133	95,76
29	20,88	64	46,08	99	71,28	134	96,48
30	21,60	65	46,80	100	72,00	135	97,20
31	22,32	66	47,52	101	72,72	136	97,92
32	23,04	67	48,24	102	73,44	137	98,64
33	23,76	68	48,96	103	74,16	138	99,36
34	24,48	69	49,68	104	74,88	139	100,08
35	25,20	70	50,40	105	75,60	140	100,80

Fig. 23.1

The complete ready reckoner contains tables for prices from $\frac{1}{2}$ cent to 99 cents. Cents can be replaced by any other decimal currency. Find out how Fig. 23.1 gives the following results.

1 12 notebooks at 72c each cost $8,64
2 65 tickets at 72c each cost $46,80
3 87 bottles at 72c each cost $62,64
4 128 packets at 72c each cost $92,16

Exercise 23a (Oral)

Use the ready reckoner in Fig. 23.1.

1 Find the cost of the following articles.
(a) 9 at 72c each (b) 24 at 72c each
(c) 31 at 72c each (d) 50 at 72c each
(e) 63 at 72c each (f) 72 at 72c each
(g) 86 at 72c each (h) 97 at 72c each
(i) 125 at 72c each (j) 138 at 72c each

2 A worker gets 72c per hour for a 42-hour week. Calculate his basic weekly wage.

3 A trader bought 60 packets of tea for $35. She sold them at 72c per packet. Calculate her profit.

4 A large store bought 125 packets of biscuits at 72c per packet. It sold them at $1 per packet. Calculate the total profit.

5 Find the cost of 24 m of cloth at $5,72 per metre. (*Hint:* notice that $5,72 = $5 + 72c)

Since currencies are decimal, Fig. 23.1 can be used for decimal fractions and multiples of 72. For example,

19 articles at 72c cost $13,68
19 articles at $7,20 cost $136,80
19 articles at 7,2c cost $1,368
= $1,37 (to the nearest cent)

The ready reckoner can also be used to find quantities which are not given in the table. For example,

19 articles at 72c cost $13,68
190 articles at 72c cost $136,80

Example 1

Find the cost of 365 articles at 72 cents each.

365 = 300 + 65

At 72c each, 300 articles cost $216,00

65 articles cost $46,80

Adding, 365 articles cost $262,80

Example 2

Calculate the cost of 298 units of electricity at 7,2c per unit.

First, find the cost at 72c per unit.

At 72c, 200 units cost $144,00

98 units cost $70,56

298 units cost $214,56

Divide this result by 10 to find the cost at 7,2c per unit.

Cost of electricity = $214,56 ÷ 10 = $21,456

= $21,46 to the nearest cent.

Exercise 23b

Use the ready reckoner in Fig. 23.1 in this exercise.

1 Find the cost of the following articles.
(a) 6 at $7,20 each (b) 20 at $7,20 each
(c) 8 at $720 each (d) 32 at $7,20 each
(e) 75 at 7,2c each (f) 112 at 7,2c each
(g) 97 at 7,2c each (h) 57 at $72 each
(i) 33 at 7,2c each (j) 70 at $7,20 each

2 A large store buys in a stock of toothpaste. It pays 72c per tube. Find the cost of buying the following numbers of tubes.
(a) 200 (b) 300 (c) 500
(d) 800 (e) 1 000 (f) 150
(g) 250 (h) 340 (i) 180
(j) 144 (k) 165 (l) 216

3 A small hotel charges $7,20 a day. Find the cost of staying at the hotel for the months of October, November and December.

4 A trader bought 16 chickens for $80. She sold them at $7,20 each. Find the profit she makes.

5 A shoe shop buys 88 pairs of shoes at $17,20 per pair. How much does this cost altogether? (*Hint*: $17,20 = $10 + $7,20)

6 Calculate the cost of 415 units of electricity at 7,2c per unit.

7 Calculate the cost of 388 units of electricity at 10,72c per unit. (*Hint*: 10,72 = 10 + 0,72)

8 During one month, a factory makes 916 television sets at a total cost of $535 860. The factory sells them all at $720 each. Find the profit that the factory makes.

9 Fig. 23.2 shows part of a ready reckoner for calculating the cost of articles costing 1, 2, 3, 4 and 5 units. Copy and complete Fig. 23.2.

Number of articles	cost per article (units)				
	1,00	2,00	3,00	4,00	5,00
1	1	2	3		
2	2	4	6		
3	3	6	9		
4	4	8	12		
5	5	10	15		
6	6	12	18		
7	7	14	21		
8	8	16	24		
9	9	18	27		
10	10	20			
20	20	40			
30	30	60			
40	40	80			
50	50	100			
60	60	120			
70	70	140			
80	80	160			
90	90	180			
100	100	200			

Fig. 23.2

10 Use your ready reckoner (Fig. 23.2) to find the cost of the following articles.
(a) 87 at $2 each (b) 66 at $2 each
(c) 18 at $3 each (d) 72 at $4 each
(e) 81 at $5 each (f) 55 at $3 each
(g) 120 at $4 each (h) 234 at $5 each

11 Use your ready reckoner (Fig. 23.2) and Fig. 23.1 to find the cost of these articles.
(a) 30 at $2,72 each
(b) 40 at $5,72 each
(c) 7 at $1,72 each
(d) 90 at $4,72 each

(e) 32 at $3,72 each
(f) 25 at $1,72 each
(g) 64 at $4,72 each
(h) 83 at $3,72 each
(i) 140 at $5,72 each
(j) 616 at $2,72 each

12 A shop buys 58 records at $5,25 each. It sells them at $8,97 each. (a) Find the profit on 1 record. (b) Use the ready reckoner to find the total profit.

Profit and discount

Fig. 23.3 is another ready reckoner. It is a table which gives the profit/discount at a rate of 9% on prices from 1 unit to 95 units.

9% Profit and Discount c = cost in *Units*

c	Discount	Cost + Profit	Cost − Discount	c	Discount	Cost + Profit	Cost − Discount
1	0,090	1,090	0,910	34	3,060	37,060	30,940
2	0,180	2,180	1,820	35	3,150	38,150	31,850
3	0,270	3,270	2,730	36	3,240	39,240	32,760
4	0,360	4,360	3,640	37	3,330	40,330	33,670
5	0,450	5,450	4,550	38	3,420	41,420	34,580
6	0,540	6,540	5,460	39	3,510	42,510	35,490
7	0,630	7,630	6,370	40	3,600	43,600	36,400
8	0,720	8,720	7,280	41	3,690	44,690	37,310
9	0,810	9,810	8,190	42	3,780	45,780	38,220
10	0,900	10,900	9,100	43	3,870	46,870	39,130
11	0,990	11,990	10,010	44	3,960	47,960	40,040
12	1,080	13,080	10,920	45	4,050	49,050	40,950
13	1,170	14,170	11,830	46	4,140	50,140	41,860
14	1,260	15,260	12,740	47	4,230	51,230	42,770
15	1,350	16,350	13,650	48	4,320	52,320	43,680
16	1,440	17,440	14,560	49	4,410	53,410	44,590
17	1,530	18,530	15,470	50	4,500	54,500	45,500
18	1,620	19,620	16,380	51	4,590	55,590	46,410
19	1,710	20,710	17,290	53	4,770	57,770	48,230
20	1,800	21,800	18,200	55	4,950	59,950	50,050
21	1,890	22,890	19,110	57	5,130	62,130	51,870
22	1,980	23,980	20,020	59	5,310	64,310	53,690
23	2,070	25,070	20,930	61	5,490	66,490	55,510
24	2,160	26,160	21,840	63	5,670	68,670	57,330
25	2,250	27,250	22,750	65	5,850	70,850	59,150
26	2,340	28,340	23,660	67	6,030	73,030	60,970
27	2,430	29,430	24,570	69	6,210	75,210	62,790
28	2,520	30,520	25,480	71	6,390	77,390	64,610
29	2,610	31,610	26,390	73	6,570	79,570	66,430
30	2,700	32,700	27,300	75	6,750	81,750	68,250
31	2,790	33,790	28,210	85	7,650	92,650	77,350
32	2,880	34,880	29,120	90	8,100	98,100	81,900
33	2,970	35,970	30,030	95	8,550	103,550	86,450

Fig. 23.3

The complete ready reckoner contains tables of rates from ½% to 95%. The money units can be in $ or any other currency. Example 3 shows how the profit/discount ready reckoner is used.

Example 3

The selling price of a shirt is $28.
(a) If the price is raised by 9%, find the new selling price.
(b) If the trader gives a 9% discount, find the new selling price.

Look opposite number 28 in the ready reckoner (Fig. 23.4).

c	Discount	Cost + Profit	Cost − Discount
28	2,520	30,520	25,480

Fig. 23.4

The number under 'Discount' is 2,520. This means that 2,52 is 9% of 28. $2,52 can be added to $28 to give a 9% profit or it can be subtracted from $28 to give a 9% discount.

(a) The number under 'Cost + Profit' is 30,520. Thus the selling price to make a 9% profit is $30,52.
(Notice that $30,52 = $28 + $2,52)

(b) The number under 'Cost − Discount' is 25,480. Thus the selling price to give a 9% discount is $25,48.
(Notice that $25,48 = $28 − $2,52)

Example 4

The cost price of a guitar is $79,95. Find the price if a 9% discount is given for cash.

Notice that values for $79 and 95 cents are *not* given in the table. However,

$79 = $75 + $4

Discount price for $75 = $68,25 (1)
Discount price for $4 = $3,64 (2)
95 cents = $95 ÷ 100
Discount price for $95 = $86,45

Discount price for 95c = 86,45c (3)
Adding (1), (2) and (3),
Discount price for $79,95
= $68,25 + $3,64 + 86c
= $72,75

Exercise 23c

Use the ready reckoner in Fig. 23.3 for this exercise.

1 Find 9% of
(a) 13 (b) 73 (c) 35
(d) $25 (e) $40 (f) $61
(g) $31 (h) $95 (i) $57
(j) $69 (k) $85 (l) $48

2 Find the selling price if a 9% profit is made on a cost price of
(a) $9 (b) $18 (c) $22
(d) $37 (e) $51 (f) $75
(g) $28 (h) $46 (i) $38
(j) $90 (k) $16 (l) $59

3 Find the selling price if a 9% discount is given on a cost price of
(a) $43 (b) $36 (c) $50
(d) $21 (e) $53 (f) $29
(g) $71 (h) $55 (i) $8
(j) $32 (k) $95 (l) $48

4 Find 9% of the following.
(a) $60 (b) $80 (c) $100

7% Interest 1–6 months

Units	1	2	3	4	5	6
1	0,006	0,012	0,018	0,023	0,029	0,035
2	0,012	0,023	0,035	0,047	0,058	0,070
3	0,018	0,035	0,053	0,070	0,088	0,105
4	0,023	0,047	0,070	0,093	0,117	0,140
5	0,029	0,058	0,088	0,117	0,146	0,175
6	0,035	0,070	0,105	0,140	0,175	0,210
7	0,041	0,082	0,123	0,163	0,204	0,245
8	0,047	0,093	0,140	0,187	0,233	0,280
9	0,053	0,105	0,158	0,210	0,263	0,315
10	0,058	0,117	0,175	0,233	0,292	0,350
11	0,064	0,128	0,193	0,257	0,321	0,385
12	0,070	0,140	0,210	0,280	0,350	0,420
13	0,076	0,152	0,228	0,303	0,379	0,455
14	0,082	0,163	0,245	0,327	0,408	0,490
15	0,088	0,175	0,263	0,350	0,438	0,525
20	0,117	0,233	0,350	0,467	0,583	0,700
25	0,146	0,292	0,438	0,583	0,729	0,875
30	0,175	0,350	0,525	0,700	0,875	1,050
40	0,233	0,467	0,700	0,933	1,167	1,400
50	0,292	0,583	0,875	1,167	1,458	1,750
60	0,350	0,700	1,050	1,400	1,750	2,100
70	0,408	0,817	1,225	1,633	2,042	2,450
80	0,467	0,933	1,400	1,867	2,333	2,800
90	0,525	1,050	1,575	2,100	2,625	3,150
100	0,583	1,167	1,750	2,333	2,917	3,500
200	1,167	2,333	3,500	4,667	5,833	7,000
300	1,750	3,500	5,250	7,000	8,750	10,500
400	2,333	4,667	7,000	9,333	11,667	14,000
500	2,917	5,833	8,750	11,667	14,583	17,500
600	3,500	7,000	10,500	14,000	17,500	21,000
700	4,083	8,167	12,250	16,333	20,417	24,500
800	4,667	9,333	14,000	18,667	23,333	28,000
900	5,250	10,500	15,750	21,000	26,250	31,500
1000	5,833	11,667	17,500	23,333	29,167	35,000

7–12 months **Interest 7%**

Units	7	8	9	10	11	12
1	0,041	0,047	0,053	0,058	0,064	0,070
2	0,082	0,093	0,105	0,117	0,128	0,140
3	0,123	0,140	0,158	0,175	0,193	0,210
4	0,163	0,187	0,210	0,233	0,257	0,280
5	0,204	0,233	0,263	0,292	0,321	0,350
6	0,245	0,280	0,315	0,350	0,385	0,420
7	0,286	0,327	0,368	0,408	0,449	0,490
8	0,327	0,373	0,420	0,467	0,513	0,560
9	0,368	0,420	0,473	0,525	0,578	0,630
10	0,408	0,467	0,525	0,583	0,642	0,700
11	0,449	0,513	0,578	0,642	0,706	0,770
12	0,490	0,560	0,630	0,700	0,770	0,840
13	0,531	0,607	0,683	0,758	0,834	0,910
14	0,572	0,653	0,735	0,817	0,898	0,980
15	0,613	0,700	0,788	0,875	0,963	1,050
20	0,817	0,933	1,050	1,167	1,283	1,400
25	1,021	1,167	1,313	1,458	1,604	1,750
30	1,225	1,400	1,575	1,750	1,925	2,100
40	1,633	1,867	2,100	2,333	2,567	2,800
50	2,042	2,333	2,625	2,917	3,208	3,500
60	2,450	2,800	3,150	3,500	3,850	4,200
70	2,858	3,267	3,675	4,083	4,492	4,900
80	3,267	3,733	4,200	4,667	5,133	5,600
90	3,675	4,200	4,725	5,250	5,775	6,300
100	4,083	4,667	5,250	5,833	6,417	7,000
200	8,167	9,333	10,500	11,667	12,833	14,000
300	12,250	14,000	15,750	17,500	19,250	21,000
400	16,333	18,667	21,000	23,333	25,667	28,000
500	20,417	23,333	26,250	29,167	32,083	35,000
600	24,500	28,000	31,500	35,000	38,500	42,000
700	28,583	32,667	36,750	40,833	44,917	49,000
800	32,667	37,333	42,000	46,667	51,333	56,000
900	36,750	42,000	47,250	52,500	57,750	63,000
1000	40,833	46,667	52,500	58,333	64,167	70,00

Fig. 23.5

(d) \$77 (e) \$54 (f) \$93
(g) 39c (h) 51c (i) 67c
(j) 33c (k) 66c (l) 88c

5 Find the selling price if a 9% profit is made on a cost price of
(a) \$52 (b) \$70 (c) \$78
(d) \$74 (e) \$99 (f) \$82

6 Find the selling price if a 9% discount is given on a cost price of
(a) \$92 (b) \$88 (c) \$64
(d) \$58 (e) \$68 (f) \$76

7 The cost price of an electric fan is \$117,60. Find the price if a 9% discount is given for cash.

8 A sales tax of 9% is added to the price of a car which costs \$7 420 before tax. Find the selling price of the car.

9 The original price of an article is \$330. The price is raised by 9%. Find the new price. Find the selling price if the trader then gives a 9% discount.

10 There is a 90% import tax on refrigerators. Find the amount of tax to be paid on a refrigerator which has a pre-tax value of \$950. (*Hint:* 90% = 10 × 9%)

Simple interest

Fig. 23.5 is a ready reckoner which gives the simple interest at 7% per annum on various amounts of money over a period from 1 to 12 months.

Example 5

Find the simple interest if \$345 is saved for 11 months at 7% per annum.

Units of money are given in the left-hand column on each page of the table in Fig. 23.5. The column giving rates for 11 months is on the right-hand page of the table.

From the table in Fig. 23.5, at 7% per annum for 11 months,

simple interest on \$300 = \$19,250
simple interest on \$40 = \$2,567
simple interest on \$5 = \$0,321

Adding, simple interest on \$345 = \$22,138

The interest is \$22,14 to the nearest cent.

Exercise 23d

1 Find the interest on the following, given that the interest rate is 7% per annum. Round all answers to the nearest cent.
(a) \$70 for 3 months
(b) \$15 for 6 months
(c) \$200 for 7 months
(d) \$40 for 8 months
(e) \$80 for 11 months
(f) \$5 for 1 month
(g) \$90 for 5 months
(h) \$700 for 2 months
(i) \$8 for 10 months
(j) \$14 for 4 months
(k) \$1 000 for 9 months
(l) \$800 for 1 year

2 Use the method of Example 5 to find the interest on the following at 7% per annum. Round each answer to the nearest cent.
(a) \$680 for 8 months
(b) \$74 for 5 months
(c) \$18 for 11 months
(d) \$990 for 10 months
(e) \$408 for 6 months
(f) \$814 for 7 months
(g) \$285 for 3 months
(h) \$527 for 4 months
(i) \$1 620 for 9 months
(j) \$752 for 2 months
(k) \$1 980 for 1 year
(l) \$386 for 1 month

3 A man borrows \$12 500 to buy a house. He repays the loan with interest at a rate of 7% per annum. How much interest does he pay in his first year?

4 Find the simple interest on \$4 650 which is saved for 2 years 8 months at 7% per annum.

5 A woman saves \$900 for 1 year at 7% interest per annum. (a) Find the total amount of money she has at the end of the year. (b) She leaves this money in the bank for a further 5 months. Find the total amount of money she has by then. (Assume that the total money at the end of the first year gets interest at 7% for 5 months.)

Tables

Using tables of squares

The table of squares on page 203 can be used to convert 3-digit numbers to squares of those numbers.

Example 6

Use the table of squares to find $4{,}16^2$.

The digits 4,1 appear in the left-hand column of the table of squares. 6 is the third digit. Look for the column headed 6. Find the number which is across from 4,1 and under 6. See Fig. 23.6.

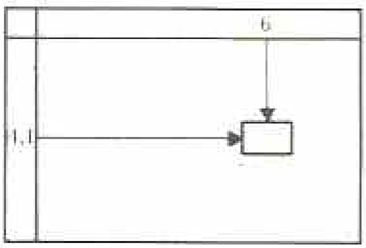


Fig. 23.6

The number is 17,3.
Hence $4{,}16^2 = 17{,}3$.
This result is correct to 3 s.f.

Example 7

Use the table of squares to find (a) 19^2, *(b)* 190^2.

(a)
$$\begin{aligned} 19 &= 1{,}9 \times 10 \\ 19^2 &= (1{,}9 \times 10)^2 \\ &= 1{,}9^2 \times 10^2 \end{aligned}$$

From the table of squares,
$$1{,}9^2 = 3{,}61$$
Hence
$$\begin{aligned} 19^2 &= 3{,}61 \times 100 \\ &= 361 \end{aligned}$$

(b)
$$\begin{aligned} 190 &= 1{,}9 \times 100 \\ 190^2 &= (1{,}9 \times 100)^2 \\ &= 1{,}9^2 \times 100^2 \\ &= 3{,}61 \times 10\,000 \\ &= 36\,100 \end{aligned}$$

In Example 7, notice that
$$\begin{aligned} 1{,}9^2 &= 3{,}61 \\ 19^2 &= 361 \\ 190^2 &= 36\,100 \end{aligned}$$

When a number is multiplied by increasing powers of 10, its square is multiplied by increasing powers of 100.

Exercise 23e

Use the table of squares on page 203 in this exercise.

1 Find the values of the following.

(a) $1{,}4^2$ (b) $2{,}3^2$ (c) $6{,}8^2$
(d) $7{,}2^2$ (e) $4{,}9^2$ (f) $8{,}6^2$
(g) $5{,}63^2$ (h) $9{,}08^2$ (i) $3{,}15^2$
(j) $1{,}88^2$ (k) $5{,}71^2$ (l) $4{,}54^2$

2 Find the values of the following.

(a) 18^2 (b) 31^2 (c) 32^2
(d) 15^2 (e) 29^2 (f) 44^2
(g) $70{,}5^2$ (h) $20{,}6^2$ (i) $62{,}7^2$
(j) $59{,}8^2$ (k) $81{,}3^2$ (l) $90{,}9^2$

3 Round off the following to 3 s.f. and then find the approximate square of each number.

(a) 1,733 (b) 2,808 (c) 78,65
(d) 52,14 (e) 96,47 (f) 49,57
(g) 632,6 (h) 805,3 (i) 303,6

4 Find the values of the following.

(a) 130^2 (b) 410^2 (c) 870^2
(d) 504^2 (e) $2\,700^2$ (f) $8\,350^2$

5 Look at the following pattern:

$1{,}5^2 = 2{,}25 = 1 \times 2 + 0{,}25$
$2{,}5^2 = 6{,}25 = 2 \times 3 + 0{,}25$
$3{,}5^2 = 12{,}25 = 3 \times 4 + 0{,}25$

Find out if the pattern continues in the same way.

Square root tables

Tables of square roots are given on pages 204 and 205. Notice that there are *two* tables.

Example 8

Use square root tables to find (a) $\sqrt{5{,}7}$, *(b)* $\sqrt{57}$.

(a) 5,7 lies between 1 and 9,99. Use the first table (page 204).

$\sqrt{5{,}7} = 2{,}39$

(b) 57 lies between 10 and 99,9. Use the second table (page 205).

$\sqrt{57} = 7{,}55$

The square root tables give results rounded to 3 s.f. For example $\sqrt{5,7} = 2,39$ to 3 s.f. However, if $2,39^2$ is worked out, the result is 5,712 1, *not* 5,7. Nevertheless, 3 significant figures are accurate enough for most purposes.

Example 9

Use square root tables to find (a) $\sqrt{875}$, *(b)* $\sqrt{3\,827}$.

(a) $875 = 8,75 \times 100$

$\sqrt{875} = \sqrt{8,75 \times 100}$

$= \sqrt{8,75} \times \sqrt{100}$

$= \sqrt{8,75} \times 10$

From the first table $\sqrt{8,75} = 2,96$

Hence $\sqrt{875} = 2,96 \times 10 = 29,6$ to 3 s.f.

(b) $\sqrt{3\,827} = 3\,830$ to 3 s.f. $= 38,3 \times 100$

$\sqrt{3\,827} = \sqrt{38,3 \times 100}$

$= \sqrt{38,3} \times \sqrt{100}$

$= \sqrt{38,3} \times 10$

From the second table $\sqrt{38,3} = 6,19$

Hence $\sqrt{3\,827} = 6,19 \times 10 = 61,9$ to 3 s.f.

Notice again that the final results are not exact. For example, $61,9^2 = 3\,831,61$, *not* 3 827.

Exercise 23f

Use the square root tables on pages 204 and 205 in this exercise.

1 Find the square roots of the following.
(a) 9 (b) 90 (c) 2,8 (d) 28
(e) 4,7 (f) 47 (g) 5,04 (h) 50,4
(i) 36,2 (j) 3,62 (k) 25,7 (l) 2,57

2 Find the square roots of the following.
(a) 7 (b) 70 (c) 700 (d) 7 000
(e) 2,9 (f) 29 (g) 290 (h) 2 900
(i) 38,2 (j) 382 (k) 3 820 (l) 38 200
(m) 10 (n) 100 (o) 1 000 (p) 10 000
(q) 2 (r) 439 (s) 8 450 (t) 72 100

3 Round off the following to 3 s.f. Then find their approximate square roots.
(a) 9,286 (b) 78,23 (c) 463,8
(d) 8,455 (e) 61,27 (f) 612,7
(g) 59,03 (h) 5,806 (i) 5 003
(j) 500,3 (k) 63 945 (l) 1 982

4 Find out if $\sqrt{10}$ is a good approximation for π.

5 (a) Use square root tables to find m if $m = \sqrt{40}$.
(b) Using the value of m found in part (a), find the value of m^2 from the table of squares.
(c) What do you notice? Explain.

Chapter 24

Calculator skills

Know your calculator

There are many kinds of calculator. Some have more buttons or keys than others. Some work in different ways from others. So learn about *your* calculator. Find out what it can do. If you use it well, it will do tedious arithmetic for you. It will save your brain for more important things.

Fig. 24.1 shows the main parts of a four-function calculator which has percentage and square root keys as well as a memory.

Power
A calculator gets its power either from small batteries or, if it is fitted with solar cells, from any light source (e.g. daylight or even candle-light). If your calculator has batteries, switch it off when not in use. Get into the habit of saving your batteries.

Display
The display shows the answers. The digits in the display are a little unusual. They are made of small line segments.

Keyboard
The keyboard has four main sets of keys or buttons:

Electronic calculator

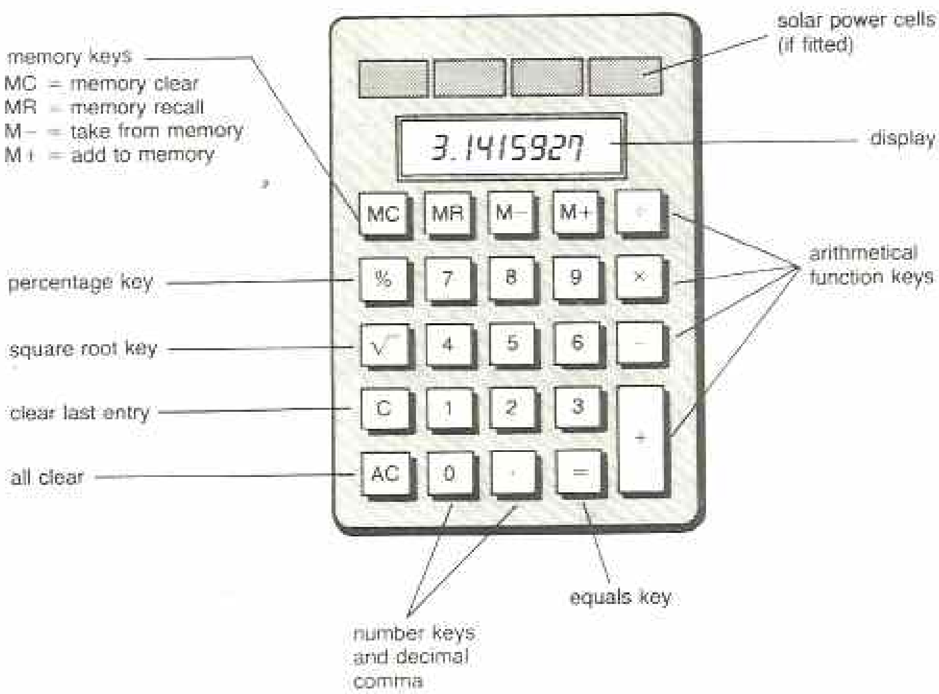


Fig. 24.1

1 *Number keys*
Press these keys: [0], [1], [2], [3], [4], [5], [6], [7], [8], [9] and the decimal comma key (usually shown as a dot [·]) to enter numbers into the calculator.

2 *Function keys*
Press these keys: [+], [−], [×], [÷], [%] and [√] to operate on the numbers your have entered

3 *Clearing keys*
The [C] key clears the last number you entered. Press [C] if you enter a wrong number by mistake. On some calculators [CE] is written instead of [C]. The [AC] button clears the whole calculation that you are working on. Use this if you want to start from the beginning again. Also use it before starting any calculation. Often the [AC] button is linked to the calculator's switch and is written as [ON/AC].

4 *Memory keys*
Memory keys are used to store numbers. They keep numbers in store while the calculator is busy working with other numbers. Numbers can be added to the memory using [M+] or subtracted using [M−]. Press [MR] to display the number in store. Press [MC] to clear the memory. If there is a number in the memory, the calculator usually shows a small M in one corner of the display. Numbers in the memory are not cleared by the [C] or [AC] keys.

Exercise 24a

1 *This question may be done orally in class.*
Copy and complete Table 24.1. For each key sequence first guess what you think the outcome will be. Then use your calculator to get a result. If anything unexpected happens, make a note in the right hand column.

Table 24.1

	key sequence	guess	result on calculator	notes
(a)	[2][3][+][4][5][=]			
	[1][2][−][5][=]			
	[6][×][4][=]			
	[3][6][÷][9][=]			
(b)	[5][+][2][=][=][=]			
	[1][9][−][3][=][=][=]			
	[5][×][2][=][=][=]			
	[3][2][÷][2][=][=][=]			
(c)	[1][÷][4][×][4][=]			
	[1][÷][3][×][3][=]			
	[2][÷][9][×][9][=]			
(d)	[5][×][×][=]			
	[5][×][×][=][=]			
	[7][+][+][=]			
	[7][+][+][=][=][=]			
(e)	[49][√]			
	[81][√][√]			
	[7][+][36][√][=]			
	[49][√][−][5][=]			
(f)	[2][÷][5][%]			
	[12][×][25][%]			
	[12][+][25][%]			
	[12][−][25][%]			
(g)	[2][+][5][×][3][=]			
	[5][+][3][×][2][=]			
	[3][+][5][÷][2][=]			
	[2][+][3][÷][5][=]			
	[2][×][5][+][3][=]			
	[5][×][3][+][2][=]			
	[5][÷][2][+][3][=]			
	[3][÷][5][+][2][=]			

	key sequence	guess	result on calculator	notes
(h)	6 + 3 c 4 =			
	2 × 8 c 7 =			
	8 ÷ 5 c 4 =			
	9 + 5 − 3 c 2 =			
	7 − 2 × 8 c 9 =			
(i)	2 × 8 AC 3 =			
	8 ÷ 5 AC 8 − 5 =			
(j)	7 − − 3 =			
	7 − + 3 =			
	7 − × 3 =			
	7 + − 3 =			
(k)	MC 2 M+ 8 M+ 3 M+ MR			
	MC 29 M+ 8 M− MR			
	MC 25 M+ 4 M+ MC MR			
	MC 2 × 7 M+ 5 × 3 M+ MR			
	MC 8 × × M+ 6 × × M+ MR √			

2 Display on your calculator (a) the highest possible number, (b) the lowest positive number.

3 To find what a snail lives in:
(a) calculate 5 × 31 × 499;
(b) turn your calculator upside down and read the display.

4 To find out what plants grow in:
(a) calculate $\sqrt{50\ 481\ 025}$;
(b) turn your calculator upside down and read the display.

5 (a) Use your calculator to complete Table 24.2.

Table 24.2

Powers of 7	value
7^1	7
7^2	49
7^3	343
7^4	
7^5	
7^6	
7^7	
7^8	

(b) Look at the final digits of the values displayed in Table 24.2. Is there a pattern? If so, what is it?
(c) Is there a recognizable pattern in the final two digits?
(d) Try the above with a different starting number, e.g. 3, 6, 11, or 13. Are there any patterns?

6 '100 up' is a game for one person.
To start: Enter any 2-digit prime number into your calculator.
Aim: To get the calculator to display a number in the form 100, ***** where * may be any digit.
Rule: You must *multiply* the number shown in the calculator display by any number of your choice.
Scoring: Record each multiplication as a trial. Try to achieve your aim in as few trials as possible, i.e. you should keep your score as *low as possible*.
Here is a sample game:

	display	*press keys*	*trial no (score)*
START	29	× 3	1
	87	× 1,2	2
	104,4	× 0,9	3
	93,96	× 1,05	4
	98,658	× 1,02	5
FINISH	100,63116		

The score for this game is 5. Starting with 29, can you do better? Try to beat 5, then play some games starting with other prime numbers.

Addition and subtraction

Use the [+], [−] keys to add and subtract and [=] to display the result.

Example 1

Calculate 356 + 717.

Keystrokes:

[AC] [3] [5] [6] [+] [7] [1] [7] [=]

Display:

0 3 35 356 356 7 71 717 *1073* (answer)

Rough check: 400 + 700 = 1100

It is a good idea to start any new calculation by pressing the [AC] key. This clears any previous calculation or data which the calculator may contain. When using a calculator it is possible to make keying-in mistakes. So make a habit of doing a rough check. Do the check mentally.

Example 2

Calculate 89 − 54 − 17

Keystrokes:

[AC] [8] [9] [−] [5] [4] [−] [1] [7] [=]

Display

0 8 89 89 5 54 *35* 1 17 *18* (answer)

Rough check: 90 − 50 − 20 = 20

Notice the value 35 in the display. This is an intermediate result (89 − 54 = 35). It appears when the second operation is entered.

Example 3

Calculate 9 − 16 + 18.

Keystrokes:

[AC] [9] [−] [1] [6] [+] [1] [8] [=]

Display:

0 9 9 1 16 −7 1 8 *11* (answer)

Rough check: 10 − 20 + 20 = 10

Notice that the calculator gives a *negative* outcome if it is programmed to subtract a larger number from a smaller number. Thus 9 − 16 gives − 7 as an intermediate result during the above calculation.

Exercise 24b

1 Do the following on your calculator. Write down what appears in the display when you press each key and underline the final answer.

(a) 7 + 2 (b) 9 − 5
(c) 57 − 29 (d) 38 + 48
(e) 94 − 38 − 26 (f) 18 + 37 + 42
(g) 123 + 456 − 543 (h) 38 − 82 + 71
(i) 32,7 − 8,4
(j) 3,4 + 7,8 + 4,3

2 Look at the following. Six of them are incorrect.

(i) 6 + 7 = 14 (ii) 48 + 19 = 912
(iii) 22 − 12 = 10 (iv) 950 − 42 = 53
(v) 235 + 680 = 3 015
(vi) 8,9 + 4,5 = 13,4
(vii) 87 − 59 = 82 (viii) 36 + 48 = − 12

(a) Decide which ones appear to be incorrect.
(b) Use your calculator to correct them.

3 Look at the following before doing them. What kind of answer do you expect? Do the calculations.

(a) 2 − 7 (b) 5 − 15
(c) 16 − 49 (d) 36 − 73
(e) 8 − 75 (f) 44 − 260
(g) 256 − 911 (h) 56 − 46 − 66

4 An athlete buys some clothes. The bill is shown in Fig. 24.2. Check that the shop assistant has added up everything correctly.

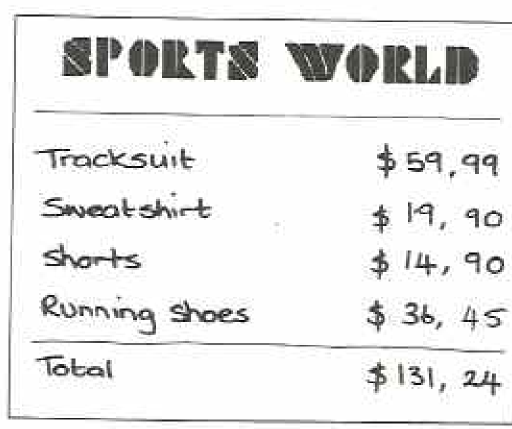


Fig. 24.2

5 What will the shopping in Fig. 24.3 cost?

Bottle orange juice	$4.29
Jar coffee	$5.82
Packet of tea	$1.95
Sugar	$1.80
Margarine	$2.99
Jar peanut butter	$2.65
Oranges	$3.89
Packet of bacon	$4.09
Shampoo	$4.05
Bag of meal	$3.76
Chicken	$14.50

Fig. 24.3

Multiplication and division

Use the [×] and [÷] buttons to multiply and divide numbers. Given a multiplication (or a division) in the form $a \times b$ (or $a \div b$), press the keys [*a*] [×] [*b*] (or [*a*] [÷] [*b*]) then either [+], [−], [×], [÷] or [=] will display the answer.

Example 4
Calculate 68 × 29.
Keystrokes:

[AC] [6] [8] [×] [2] [9] [=]

Display:

0 6 68 68 2 29 *1972* (answer)

Rough check: 70 × 30 = 2 100

Example 5
Calculate 725 ÷ 25 × 14.

Keystrokes:

[AC] [7] [2] [5] [÷] [2] [5] [×] [1] [4] [=]

Display:

0 7 72 725 725 2 25 *29* 1 14 *406* (answer)

Rough check: 700 ÷ 20 × 10 = 350

Notice that 725 ÷ 25 = 29, 29 appears in the display at an intermediate stage of the calculation.

Calculators have a limited number of spaces in the display (usually eight spaces). Because of this there is a limit to the size of answer they can display. Try the calculation 68 000 × 29 000 on a calculator. Fig. 24.4 shows what could result on some eight-digit calculators:

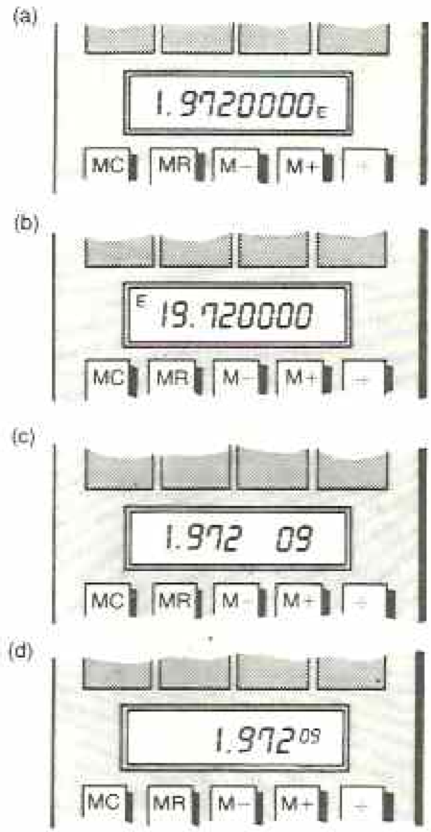


Fig. 24.4

Note that 68 000 × 29 000 = 1 972 000 000 (requiring ten digits).

The calculators in (a) and (b) show the digits 1972 but are unable to display the full number properly. So they print a small E (error) to warn the user.

The calculators in (c) and (d) group the display into two parts: 1.972 and 09. This is short for 1.972 × 1 000 000 000. Note the nine zeros in the second number. They correspond to the 09. This type of calculator is called a *scientific calculator*. Scientific calculators can cope with very large numbers.

Example 6
How many seconds are there in a 31-day month?

Number of seconds
= 60 × 60 × 24 × 31
= 2 678 400 [calculator]

Exercise 24c

1 Do the following on your calculator. Write down what appears in the display as you press each button. Underline your final answer.
(a) 67 × 88 (b) 4 234 ÷ 58
(c) 513 ÷ 19 (d) 46^2
(e) 49 × 67 × 13
(f) 7 938 ÷ 81 ÷ 14
(g) 102 × 104 ÷ 78
(h) 495 ÷ 33 × 41

2 The following calculations all involve decimal commas. Give each answer (i) as displayed on the calculator, (ii) rounded off to 2 decimal places.
(a) 6,74 × 9,08 (b) 51,73 × 24,79
(c) 28 341 ÷ 85 (d) 74,184 ÷ 40,08
(e) 4 ÷ 3 × 6 (f) 7 ÷ 11 × 44
(g) 773 ÷ 4,17 × 5,308
(h) 3,142 × 4,5 × 4,5

3 Look at the following. Six of them are incorrect.
(i) 5 × 9 = 30
(ii) 100 000 ÷ 100 = 1 000
(iii) 67 × 84 = 3 216
(iv) 690 ÷ 15 = 45
(v) 1 000 ÷ 30 = 33
(vi) 1,7 × 1,5, × 1,3 = 33,15
(vii) 360 ÷ 18 ÷ 5 = 4
(viii) 4 123 ÷ 814 = 5
(a) Decide which ones appear to be incorrect.
(b) Use your calculator to correct them.

4 Multiply 30 000 by 50 000 on your calculator. What is displayed?

5 (a) Calculate 10 000 000 ÷ 0,9.
(b) Calculate 10 000 000 ÷ 0,9 ÷ 0,9.

6 How many seconds are there in a 365-day year?

7 (a) Write your age to the nearest year.
(b) Calculate how many days you have lived (assume 365 days in a year).
(c) Calculate how many hours you have lived.
(d) Calculate how many minutes, you have lived.
(e) Is it possible for your calculator to calculate the number of seconds you have lived?

8 A health inspector gets a salary of $17 324 per annum. How much does this represent (a) per month, (b) per day? Give answers to the nearest cent.

9 The 42 members of a club hired a bus to visit Great Zimbabwe. If the bus company charged $350, how much did each member have to pay?

10 An aeroplane travels 550 km in 1 hour.
(a) How many km does it travel in one minute?
(b) How many metres does it travel in one minute?
(c) How many metres does it travel in one second?

Mixed operations, brackets

Look back to Exercise 24a, question 1, part (g). In some cases calculators appear to give two answers to the same problem. According to the rules of precedence in arithmetic,
2 + 5 × 3 = 2 + 15 = 17
and 5 × 3 + 2 = 15 + 2 = 17.

However, the calculator gives on the one hand:
Keystrokes: [2] [+] [5] [×] [3] [=]
Display: 2 2 5 7 3 *21* (answer)
and on the other:
Keystrokes: [3] [×] [5] [+] [2] [=]
Display: 3 3 5 15 2 *17* (answer)
This is because the calculator follows the operations in the order it receives them.

Example 7
Calculate 34 + 8 × 52.

There are no brackets, but multiplication is done before addition. Rearrange the numbers as follows:
34 + 8 × 52 = 8 × 52 + 34
= (8 × 52) + 34
= 450 [calculator]

Example 8
Calculate $2,3 \times (8,9 - 2,1)$.

The brackets show that the subtraction is to be done first. Rearrange the numbers so that the subtraction comes before the multiplication:

$$\begin{aligned} &2,3 \times (8,9 - 2,1) \\ &\quad = (8,9 - 2,1) \times 2,3 \\ &\quad = 15,64 \text{ [calculator]} \end{aligned}$$

In the above examples it is possible to 'turn the calculation round' because in general $a \times b = b \times a$. However, with division this is not possible. Read the following example carefully.

Example 9 (optional)
Calculate $84 \div (37 - 23)$.

The subtraction in the brackets must be done first. The outcome is held in memory, to be recalled when it is needed. The sequence of working is as follows:

37 [−] 23 [M+] 84 [÷] [MR] [=] 6

Check the above sequence on your own calculator and note the changes in display.

It is essential to enter numbers and operations in an order which will enable the calculator to give correct results. This means doing calculations in brackets first and storing them if necessary. Thereafter, do multiplications and divisions before additions and subtractions.

Exercise 24d

1 In six of the following cases, calculators will give incorrect results if the numbers and operations are entered in the given order. In those cases rearrange the numbers so that calculators will compute correct results.

(a) $89 \times 6 - 231$ (b) $45 + 68 \div 17$
(c) $22 + 42 \div 3$ (d) $63 + 18 \times 5$
(e) $18 \times (17 - 15)$ (f) $(19 + 9) \div 7$
(g) $487 \times (6 + 3)$ (h) $100 \times (31 - 14)$

2 Calculate the following, rearranging the order *where necessary*.

(a) $95 \times 7 - 436$ (b) $101 + 51 \times 9$
(c) $55 + 75 \div 5$ (d) $666 \div 36 + 2,5$
(e) $49 \times (19 - 3)$ (f) $(434 - 343) \div 13$
(g) $8,438 + 36,2 \div 2,6$
(h) $8,8 \times (6,12 - 3,47)$

3 *Optional*
All of the following require part of the calculation to be stored (either on paper or in memory).

(a) $68 - 14 \times 3$ (b) $216 \div (25 - 7)$
(c) $708 \div (28 + 31)$ (d) $444 - 261 \div 29$
(e) $46,7 \div (15,28 - 3,59)$
(f) $381,04 - 12,6 \times 7,8$

Junior Certificate Practice Examination

This section contains a full scale practice examination at Junior Certificate level. If the following papers are to be effective, they should be done under examination conditions, i.e. the time allowed for each paper should be observed and neither the textbook nor notes should be referred to.

General advice:

1 Be sure that you read and understand the examination **rubric** (instructions). Typical rubric is given before each paper. Note, however, that the authorities may change the rubric at any time for any reason if they wish. So, always check the rubric.

2 Work out how much time you can afford to spend on each question. This can be as little as 2 to $2\frac{1}{2}$ minutes per question on Paper 1. Allow time for reading the questions and for checking your answers at the end.

3 If any question involves drawing, make a rough sketch first. This helps you to position your final answer on the paper, whether it is a graph or a scale drawing.

4 Check your answers to see if they are sensible in terms of the given question. For example, a car costing $22 or a walking speed of 5 600 km/h are sure signs that a mistake has been made in an earlier part of the working.

5 Show all of your working in the body of the question that you are answering. Do not be ashamed of your rough working; examiners know what to look for and may be able to give credit for such working – but only if they see it on the paper.

Mathematics

Paper I

(Time: 2 hours)

Answer **all** *questions.*
In each question, choose **one** *of the letters A, B, C, D, E which corresponds to the correct answer.*
If **more** *than* **one** *letter is selected, that answer will be regarded as wrong.*
All *questions carry equal marks and no deductions will be made for wrong answers.*

1 What is the value of the digit 5 in the number 624,95?
A 5 hundreds B 5 tens
C 5 units D 5 tenths
E 5 hundredths

2 Which **one** of the following statements is **not** true?
A 2 is a natural number
B 2 is a negative number
C 2 is an even number
D 2 is a prime number
E 2 is a rational number

3 The number of perfect squares which lie between 15 and 65 is
A 3 B 4 C 5 D 6 E 8

4 If 0,68 is expressed as a fraction in its lowest terms, its denominator will be
A 17 B 25 C 34 D 50 E 68

5 490 ÷ 10 000 =
A 0,000 049 B 0,000 49 C 0,004 9
D 0,049 E 0,49

6 The HCF (highest common factor) of 36, 72 and 90 is
A 9 B 18 C 36 D 90 E 360

7 Solve $\frac{2x-1}{5} + 2x = 19$. $x =$
A $1\frac{2}{3}$ B $2\frac{1}{2}$ C 5 D 8 E 24

8 In Fig. R32 PQRS is a parallelogram and STUP is a square.

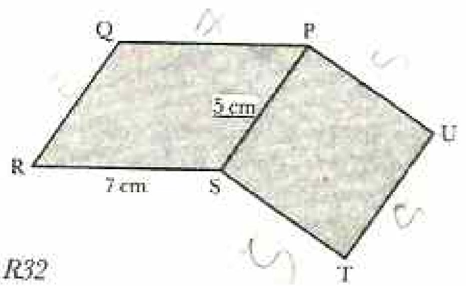


Fig. R32

If PS = 5 cm and RS = 7 cm the perimeter of PQRSTU is
A 24 cm B 29 cm C 34 cm
D 36 cm E 39 cm

9 When $y = 9$, the value of $3y - 5$ is
A 7 B 12 C 22 D 32 E 34

10 The number which is 3 greater than n is
A $n + 3$ B $3n$ C $n - 3$
D $\frac{1}{3}n$ E $3 - n$

11 Expressed in tonnes, 486 kg is
A 48,6 t B 4,86 t C 0,486 t
D 0,048 6 t E 0,004 86 t

12 How much simple interest does $600 make in three years at 7% per annum?
A $7 B $18 C $21 D $42 E $126

13 When 27 people share a sack of meal they each get 4 kg. When 12 people share a similar sack of meal they each get
A 3 kg B 8 kg C 9 kg
D 10 kg E 12 kg

14 In Fig. R33, A contains 15 elements, B contains 12 elements and A ∩ B contains 8 elements.

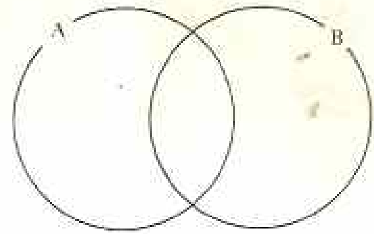


Fig. R33

How many elements are in A ∪ B?
A 19 B 23 C 27 D 31 E 35

15 What is the value of $\frac{a - 3b}{a}$ when $a = 2$ and $b = -8$?
A -18 B -11 C $12\frac{1}{2}$
D 13 E 24

16 Which of the points in Fig. R34 has co-ordinates $(-4; 1)$?

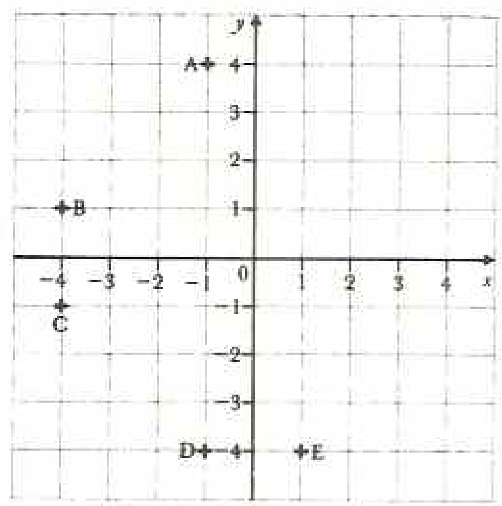


Fig. R34

17 Which of the following statements about diagonals is/are true for **all** rectangles? I they are equal in length, II they cross at right angles, III they bisect each other
A I only B I and II only
C II only D I and III only
E III only

18 Round off all the numbers in the given expression to the nearest whole number.

$$\left(\frac{19{,}926}{4{,}997}\right)^{1{,}976}$$

Hence find the approximate value of the expression.
A 5 B 8 C 10 D 16 E 25

19 $8 - 9 - (-5) =$
A -6 B -4 C 4 D 6 E 12

20 Given that $245 \times n = m$, find the smallest value of n such that m is a perfect square.
A 5 B 7 C 20 D 25 E 49

21 If M = {multiples of 11} and E = {2; 4; 6; …; 20} write down the set $M \cap E$.
A ∅ B {11}
C {22} D {11; 22}
E {2; 4; 6; 8; 10; 11; 12; 14; 16; 18; 20}

22 What are the coordinates of the point P in Fig. R35?

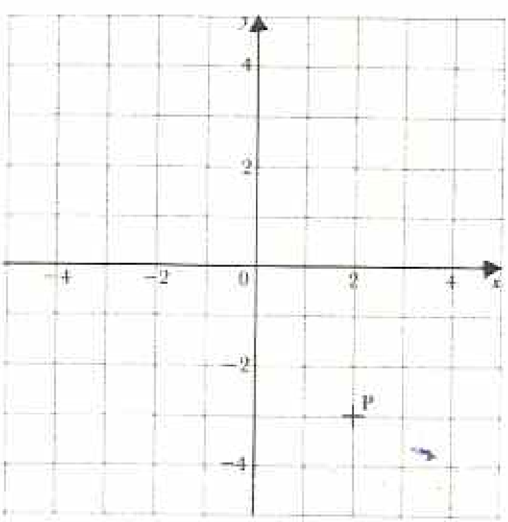


Fig. R35

A $(-3; 2)$ B $(2; -3)$ C $(4; -4)$
D $(3; -2)$ E $(-2; 3)$

23 The first digit of the square root of 79 is
A 2 B 4 C 7 D 8 E 9

24 A map is drawn to a scale 2 cm to 100 km. On the map the distance between two towns is 2,5 cm. What is their true distance apart?
A 40 km B 80 km C 125 km
D 250 km E 500 km

The cost of hiring a car includes a charge for distance travelled. Fig. R36 is a graph which is used to work out this part of the cost.

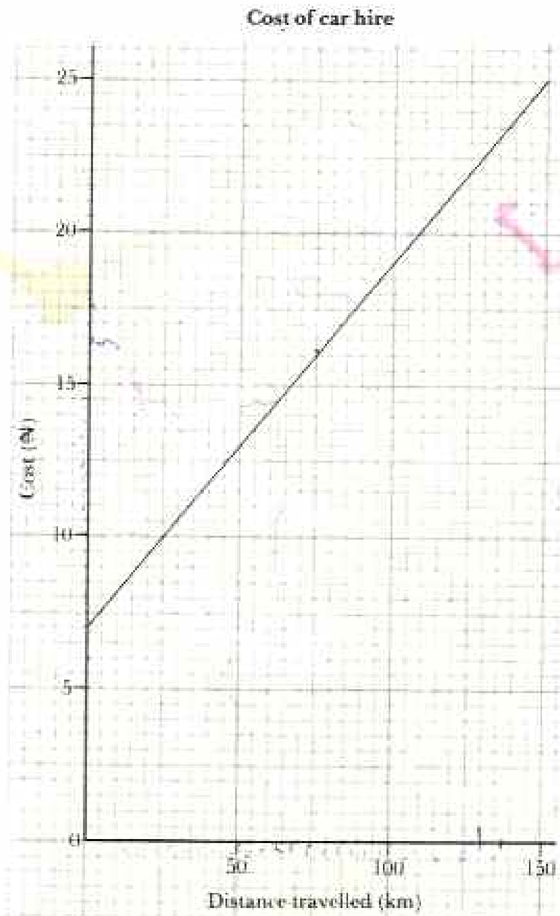


Fig. R36

Use Fig. R36 to answer questions 25, 26 and 27.

25 How much is the charge if the car travels 75 km?
A \$15 B \$16 C \$17
D \$17,50 E \$18

26 Approximately how far has the car travelled if the distance charge is \$23,50?
A 123 km B 125 km C 138 km
D 145 km E 175 km

27 What is the distance charge if no distance is travelled?
A \$0 B \$5 C \$7
D \$7,50 E \$9

28 There are 180 girls in a mixed school. If the ratio of girls to boys is 4 : 3, the total number of students in the school is
A 225 B 315 C 360
D 405 E 420

29 Fig. R37 is the graph of a set of numbers, S.

Fig. R37

Which one of the following defines S?
A $\{x: -1 < x < 2\}$
B $\{x: -1 \leq x \leq 2\}$
C $\{x: x = -1 \text{ or } 2\}$
D $\{x: -1 < x \leq 2\}$
E $\{x: -1 \leq x < 2\}$

30 Given Fig. R38, calculate the length of XQ.

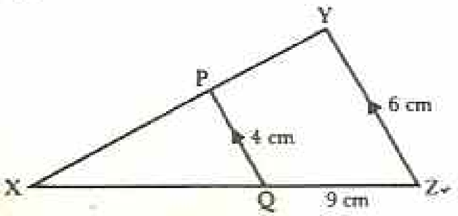


Fig. R38

A 6 cm B 7 cm C 11 cm
D 15 cm E 18 cm

Use the following set of numbers to answer questions 31, 32 and 33.

8, 9, 5, 6, 2, 4, 8, 0

31 The mode of the above numbers is
A $4\frac{1}{2}$ B $5\frac{1}{4}$ C $5\frac{1}{2}$ D 7 E 8

32 The median of the above numbers is
A $4\frac{1}{2}$ B $5\frac{1}{4}$ C $5\frac{1}{2}$ D 7 E 8

33 The mean of the above numbers is
A $4\frac{1}{2}$ B $5\frac{1}{4}$ C $5\frac{1}{2}$ D 7 E 8

34 Which of the following is/are factor(s) of $15pr - 10qs - 30qr + 5ps$?
I 5 II $(3r - s)$ III $(p - 2q)$
A I only B I and II only
C II only D I and III only
E III only

35 A West German visits Harare on a day when the exchange rate is DM0,80 for Z\$1. How many Z\$ would she get for DM500?
A Z\$400 B Z\$500 C Z\$580
D Z\$625 E Z\$660

36 Which of the sketches in Fig. R39 show(s) how to construct a perpendicular at P on line XY?

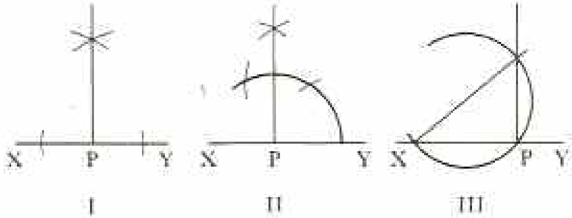


Fig. R39

A all three B I only
C III only D I and III only
E II only

37 A thread is wound 200 times round a reel of diameter 5 cm. Use the value 3 for π to find an approximate value for the length of the thread in metres.
A 15 B 30 C $37\frac{1}{2}$ D 60 E 75

38 Four lines meet at a point. The sum of three of the angles at the point is 267°. The size of the other angle is
A 87° B 89° C 90° D 91° E 93°

39 If $I = \dfrac{E}{X + Y}$, express X in terms of I, E and Y. X =

A $\dfrac{E}{I + Y}$ B $\dfrac{Y - E}{IY}$

C $\dfrac{E - IY}{I}$ D $\dfrac{E - I}{IY}$

E $\dfrac{EI - Y}{I}$

40 If $f = \dfrac{uv}{u + v}$ find f when $u = 20$ and $v = -30$.

A -60 B -12 C -1
D 12 E 60

41 An equilateral triangle of side 16 cm has the same perimeter as a square. The area of the square in cm^2 is

A 48 B 64 C 96 D 144 E 256

42 The bearing of X from Y is 148°. The bearing of Y from X is

A 032° B 058° C 148°
D 212° E 328°

43 If $3x - y = 14$ and $7x - y = 46$, then $x =$

A 6 B 8 C 15 D 22 E 28

44 An athlete ran a 400 metre race in 72 seconds. Express her speed in kilometres per hour.

A 0,2 km/h B 0,5 km/h
C 2 km/h D 20 km/h
E 200 km/h

45 Factorise $x^2 - 7x - 30$.

A $(x + 1)(x - 30)$ B $(x + 2)(x - 15)$
C $(x + 3)(x - 10)$ D $(x + 5)(x - 6)$
E $(x + 6)(x - 5)$

46 In Fig. R40, $x =$

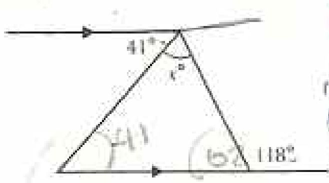


Fig. R40

A 41 B 49 C 62 D 77 E 139

47 Solve $x^2 = 4x$ completely. $x =$

A 0 B 2 C 4
D $+2$ or -2 E 0 or 4

48 Express $\dfrac{a}{3} + \dfrac{4}{b}$ as a single fraction.

A $\dfrac{3a + 4b}{3b}$ B $\dfrac{a + 4}{3+b}$

C $\dfrac{ab + 12}{3b}$ D $\dfrac{ab + 12}{3 + b}$

E $\dfrac{a + 4}{3b}$

49 250 g of coffee costs $14,95 at shop X. 100 g of coffee costs $5,95. The difference in cost per kg is

A 0 c B 3 c C 30 c
D $4,50 E $9

50 In an exam 35 out of 125 students failed. What percentage passed?

A 28% B 35% C 65%
D 72% E 90%

Mathematics
Paper II

(Time: 2 hours)

Answer **all ten** *questions in Section A and* **all five** *questions in Section B. To obtain full marks for any question* **all working** *must be shown. Do* **not** *measure from given diagrams.*

SECTION A

Answer **all** *questions in this section.*

1 (a) Evaluate 14,56 ÷ 0,52.
(b) Express 165 g as a percentage of 1 kg.

2 (a) In a cafe a cup of tea cost t cents. How many cups can a customer buy with n dollars?
(b) Find y if $10y + 8 = y - 19$.

3 X = {f; o; r; m}, Y = {t; w; o}, Z = {t; e; a; m}

(a) Draw a Venn diagram showing sets X, Y, Z and their elements.

(b) Hence, or otherwise, list the following sets.

(i) X ∩ Y (ii) Y ∪ Z (iii) (X ∩ Z) ∪ Y

4 Fig. R41 is a sketch showing Y 400 m north of X and Z 800 m east of Y.

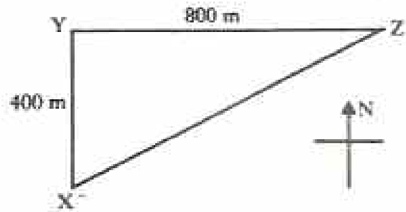


Fig. R41

(a) Using a scale of 1 cm to 100 m, make an accurate scale drawing of X, Y and Z.

(b) Measure the distance and bearing of X from Z.

5 In Fig. R42, PQRS is a field with dimensions as shown.

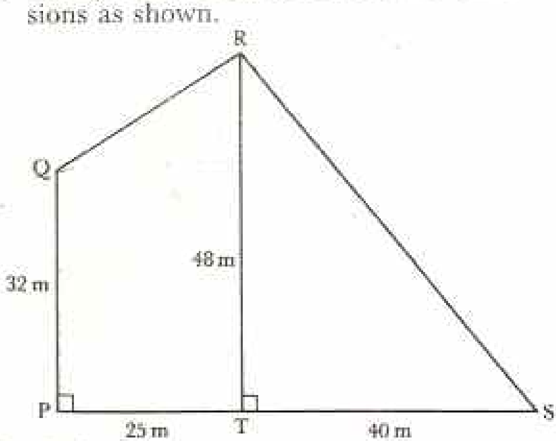


Fig. R42

Calculate the area of the field.

6 A metal is made of copper and zinc in the ratio 3 : 2 by volume. The densities of copper and zinc are 8,9 g/cm^3, and 7,1 g/cm^3. Find the mass of 100 cm^3 of the metal.

7 (a) Factorise $16 - h^2$.

(b) Simplify completely $5(a - 3b) - 2(2a - 7b)$.

(c) Expand $(3x - 4)(2x + 11)$.

8 (a) Solve the inequality $1 - 3x \leq 13$ and draw a line graph of the solution.

(b) Use set notation to describe the set of points represented by the *unshaded* region in Fig. R43.

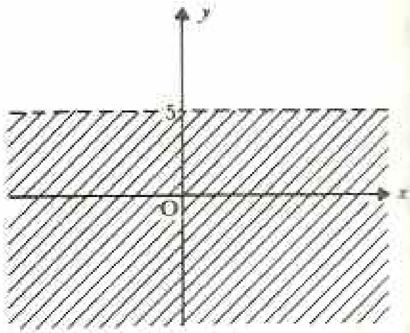


Fig. R43

9 In Fig. R44 the regular pentagon ABCDE and the square ABPQ have a common side AB as shown.

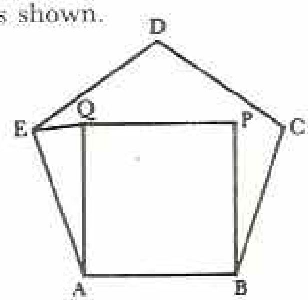


Fig. R44

Calculate the angles of △AEQ.

10 Fig. R45 shows part of an 'Owners Charges' bill for rates and refuse removal for a period of six months.

(a) Find the total Owners Charges to be paid (i.e. find the value of x).

(b) The last two digits in the Land Rate are illegible. Use the bill to calculate what they should be.

(c) The owner pays the bill late, on 3? December 1991. If the unpaid balance rate given on the bill is 'per annum' what percentage of the bill will actually be payable as interest?

City of Harare OWNERS CHARGES

ITEM	VALUATION $	RATE cents per $	
LAND	8000	1,0[illegible]	81,92
IMPROVEMENTS	13000	0,811	105,43
REFUSE REMOVAL			18,54
Due date 31 OCT 91. Unpaid balance after 30 NOV 91 will attract interest at 11,4 PER CENT from due date			$x

Last amount in this column is the sum due

Fig. R45

SECTION B

Answer **all** *questions in this section.*

11 (a) A car is advertised as costing '\$8 950 plus sales tax of 20%'. To pay by hire purchase requires a deposit of 40% of the cost price and 36 monthly payments of \$208. Calculate (i) the cost price (including sales tax); (ii) the total cost when paying by hire purchase.

(b) Village A has 200 families with an average of 3,9 children each. Village B has 300 families with an average of 4,4 children each. Find (i) the total number of children in the two villages; (ii) the average number of children per family in the two villages.

12 A vertical pole stands on level ground. A student whose eye, E, is 1,5 m vertically above her feet, F, stands 8 m from the pole. The angle of elevation of the top of the pole from E is 33°.

(a) Using a scale of 2 cm to 1 m, make an accurate scale drawing to show this information. (Use a straight line EF to represent the student.)

(b) Use your scale drawing to find to the nearest 0,1 m or degree (i) the distance of E from the top of the pole; (ii) the height of the pole; (iii) the angle of depression of F from the top of the pole.

13 The coordinates of quadrilateral ABCD are A(− 1; 0), B(− 3; 4), C(1; 6), D(3; 2).

(a) Using a scale of 1 cm to 1 unit on both axes, draw ABCD on graph paper.

(b) What kind of quadrilateral is ABCD?

(c) How many lines of bilateral symmetry has ABCD?

(d) State the coordinates of the point through which all these lines of symmetry pass.

(e) X is a point such that △DAX is right-angled at A and is equal in area to quadrilateral ABCD. State the coordinates of the two possible positions of X.

14 (a) Solve the simultaneous equations

$2a - 3b + 2 = 0$

$3a + 2b - 23 = 0.$

(b) Solve $x^2 - 2x - 15 = 0$.

(c) A knife has a mass of m grammes. It is 20 g heavier than a spoon. (i) Write down an expression in m for the mass of the spoon. (ii) The total mass of four of the spoons and one knife is 330 g. Make an equation in m and solve it to find the mass of a knife.

15 (a) Table R7 gives the ages of a group of students.

Table R7

age (yr)	12	13	14	15	16	17	18
frequency	2	0	1	3	4	3	2

(i) How many students are there?
(ii) What is the modal age of the group?
(iii) What is median age of the group?
(iv) Calculate the group's mean age.

(b) At 5.05 p.m. Dandi leaves her office to cycle home. She rides steadily for 10 minutes and travels 2 km. She then stops and talks to a friend for 20 min. She cycles the remaining 3 km at a steady rate and arrives home at 5.45 p.m.

(i) Represent the above information on a distance–time graph. Use scales of 2 cm to 1 km and 2 cm to 10 min.
(ii) How far was Dandi from home at 5.40 p.m.?
(iii) Find Dandi's average speed for the whole journey.

Mensuration tables and formulae, three-figure tables

SI units

Length

The metre is the basic unit of length.

unit	abbreviation	basic units
1 kilometre	1 km	1 000 m
1 hectometre	1 hm	100 m
1 decametre	1 dam	10 m
1 metre	1 m	1 m
1 decimetre	1 dm	0,1 m
1 centimetre	1 cm	0,01 m
1 millimetre	1 mm	0,001 m

The most common measures are the millimetre, the metre and the kilometre.

1 m = 1 000 mm

1 km = 1 000 m = 1 000 000 mm

Mass

The **gramme** is the basic unit of mass.

unit	abbreviation	basic units
1 kilogramme	1 kg	1 000 g
1 hectogramme	1 hg	100 g
1 decagramme	1 dag	10 g
1 gramme	1 g	1 g
1 decigramme	1 dg	0,1 g
1 centigramme	1 cg	0,01 g
1 milligramme	1 mg	0,001 g

The **tonne** (t) is used for large masses. The most common measures of mass are the milligramme, the gramme, the kilogramme and the tonne.

1 g = 1 000 mg

1 kg = 1 000 g = 1 000 000 mg

1 t = 1 000 kg = 1 000 000 g

Time

The **second** is the basic unit of time.

unit	abbreviation	basic units
1 second	1 s	1 s
1 minute	1 min	60 s
1 hour	1 h	3 600 s

Area

The **square metre** is the basic unit of area. Units of area are derived from units of length.

unit	abbreviation	relation to other units of area
square millimetre	mm^2	
square centimetre	cm^2	$1\ cm^2 = 100\ mm^2$
square metre	m^2	$1\ m^2 = 10\ 000\ cm^2$
square kilometre	km^2	$1\ km^2 = 1\ 000\ 000\ m^2$
hectare (for land measure)	ha	$1\ ha = 10\ 000\ m^2$

Volume

The **cubic metre** is the basic unit of volume. Units of volume are derived from units of length.

unit	abbreviation	relation to other units of volume
cubic millimetre	mm^3	
cubic centimetre	cm^3	$1\ cm^3 = 1\ 000\ mm^3$
cubic metre	m^3	$1\ m^3 = 1\ 000\ 000\ cm^3$

Capacity

The **litre** is the basic unit of capacity. 1 litre takes up the same space as $1\ 000\ cm^3$.

unit	abbreviation	relation to other units of capacity	relation to units of volume
millilitre	mℓ		$1\ m\ell = 1\ cm^3$
litre	ℓ	1 ℓ = 1 000 mℓ	$1\ \ell = 1\ 000\ cm^3$
kilolitre	kℓ	1 kℓ = 1 000 ℓ	$1\ k\ell = 1\ m^3$

Money

Some African currencies

Zimbabwe	100 cents (c)	= 1 dollar ($)
Botswana	100 thebe (t)	= 1 pula (P)
Kenya	100 cents (c)	= 1 shilling (Sh)
Malawi	100 tambala (t)	= 1 kwacha (K)
Mozambique	100 centavos (c)	= 1 metical (M)
Nigeria	100 kobo (k)	= 1 naira (₦)
Zambia	100 ngwee (n)	= 1 kwacha (K)

Other currencies

Britain	100 pence (p)	= 1 pound (£)
USA	100 cents (c)	= 1 dollar ($)

Exchange rates

At the time of going to press, $1 (Zimbabwe) was approximately equivalent to the following.

US dollar	$0,40
UK sterling	£0,20
Botswana	P0,80
Kenya	Sh9
Mozambique	M370
Zambia	K16

Note: Exchange rates change from day to day. The above rates may only be taken as approximate.

The calendar

Remember this poem:

Thirty days have September,
April, June and November.
All the rest have thirty-one,
Excepting February alone;
This has twenty-eight days clear,
And twenty-nine in each Leap Year.

For a Leap Year, the year date must be divisible by 4.
Thus 1984 was a Leap Year.
Century year dates, such as 1900 and 2000, are Leap Years only if they are divisible by 400.
Thus 1900 was not a Leap Year but 2000 will be a Leap Year.

Multiplication table

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Divisibility tests

Any whole number is exactly divisible by

2	if its last digit is even
3	if the sum of its digits is divisible by 3
4	if its last two digits form a number divisible by 4
5	if its last digit is 5 or 0
6	if its last digit is even and the sum of its digits is divisible by 3
8	if its last three digits form a number divisible by 8
9	if the sum of its digits is divisible by 9
10	if its last digit is 0

Angle and length

In an n-sided polygon,
sum of angles $= (n - 2) \times 180°$

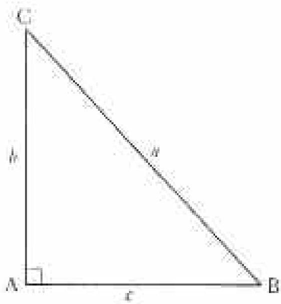


Fig. T1

In the right-angled triangle shown in Fig. T1,

$$a^2 = b^2 + c^2 \text{ (Pythagoras' theorem)}$$

$\tan B = \dfrac{b}{c}$ $\quad \tan C = \dfrac{c}{b}$

$\sin B = \dfrac{b}{a}$ $\quad \sin C = \dfrac{c}{a}$

$\cos B = \dfrac{c}{a}$ $\quad \cos C = \dfrac{b}{a}$

Mensuration formulae

	perimeter	area	volume
square side s	$4s$	s^2	
rectangle length l, breadth b	$2(l + b)$	lb	
circle radius r	$2\pi r$	πr^2	
trapezium height h, parallels of length a and b		$\frac{1}{2}(a + b)h$	
triangle base b, height h		$\frac{1}{2}bh$	
parallelogram base b, height h		bh	
cube edge s		$6s^2$	s^3
cuboid length l, breadth b, height h			lbh
right-triangular prism length l, breadth b, height h			$\frac{1}{2}lbh$
cylinder base radius r, height h		$2\pi rh + 2\pi r^2$	πr^2h

Reciprocals

$x \rightarrow \frac{1}{x}$

x	0	1	2	3	4	5	6	7	8	9
1,0	1,000	0,990	0,980	0,971	0,962	0,952	0,943	0,935	0,926	0,917
1,1	0,909	0,901	0,893	0,885	0,877	0,870	0,862	0,855	0,847	0,840
1,2	0,833	0,826	0,820	0,813	0,806	0,800	0,794	0,787	0,781	0,775
1,3	0,769	0,763	0,758	0,752	0,746	0,741	0,735	0,730	0,725	0,719
1,4	0,714	0,709	0,704	0,699	0,694	0,690	0,685	0,680	0,676	0,671
1,5	0,667	0,662	0,658	0,654	0,649	0,645	0,641	0,637	0,633	0,629
1,6	0,625	0,621	0,617	0,613	0,610	0,606	0,602	0,599	0,595	0,592
1,7	0,588	0,585	0,581	0,578	0,575	0,571	0,568	0,565	0,562	0,559
1,8	0,556	0,552	0,549	0,546	0,543	0,541	0,538	0,535	0,532	0,529
1,9	0,526	0,524	0,521	0,518	0,515	0,513	0,510	0,508	0,505	0,503
2,0	0,500	0,498	0,495	0,493	0,490	0,488	0,485	0,483	0,481	0,478
2,1	0,476	0,474	0,472	0,469	0,467	0,465	0,463	0,461	0,459	0,457
2,2	0,455	0,452	0,450	0,448	0,446	0,444	0,442	0,441	0,439	0,437
2,3	0,435	0,433	0,431	0,429	0,427	0,426	0,424	0,422	0,420	0,418
2,4	0,417	0,415	0,413	0,412	0,410	0,408	0,407	0,405	0,403	0,402
2,5	0,400	0,398	0,397	0,395	0,394	0,392	0,391	0,389	0,388	0,386
2,6	0,385	0,383	0,382	0,380	0,379	0,377	0,376	0,375	0,373	0,372
2,7	0,370	0,369	0,368	0,366	0,365	0,364	0,362	0,361	0,360	0,358
2,8	0,357	0,356	0,355	0,353	0,352	0,351	0,350	0,348	0,347	0,346
2,9	0,345	0,344	0,342	0,341	0,340	0,339	0,338	0,337	0,336	0,334
3,0	0,333	0,332	0,331	0,330	0,329	0,328	0,327	0,326	0,325	0,324
3,1	0,323	0,322	0,321	0,319	0,318	0,317	0,316	0,315	0,314	0,313
3,2	0,313	0,312	0,311	0,310	0,309	0,308	0,307	0,306	0,305	0,304
3,3	0,303	0,302	0,301	0,300	0,299	0,299	0,298	0,297	0,296	0,295
3,4	0,294	0,293	0,292	0,292	0,291	0,290	0,289	0,288	0,287	0,287
3,5	0,286	0,285	0,284	0,284	0,282	0,282	0,281	0,280	0,279	0,279
3,6	0,278	0,277	0,276	0,275	0,275	0,274	0,273	0,272	0,272	0,271
3,7	0,270	0,270	0,269	0,268	0,267	0,267	0,266	0,265	0,265	0,264
3,8	0,263	0,262	0,262	0,261	0,260	0,260	0,259	0,258	0,258	0,257
3,9	0,256	0,256	0,255	0,254	0,254	0,253	0,253	0,252	0,251	0,251
4,0	0,250	0,249	0,249	0,248	0,248	0,247	0,246	0,246	0,245	0,244
4,1	0,244	0,243	0,243	0,242	0,242	0,241	0,240	0,240	0,239	0,239
4,2	0,238	0,238	0,237	0,236	0,236	0,235	0,235	0,234	0,234	0,233
4,3	0,233	0,232	0,231	0,231	0,230	0,230	0,229	0,229	0,228	0,228
4,4	0,227	0,227	0,226	0,226	0,225	0,225	0,224	0,224	0,223	0,223
4,5	0,222	0,222	0,221	0,221	0,220	0,220	0,219	0,219	0,218	0,218
4,6	0,217	0,217	0,216	0,216	0,216	0,215	0,215	0,214	0,214	0,213
4,7	0,213	0,212	0,212	0,211	0,211	0,211	0,210	0,210	0,209	0,209
4,8	0,208	0,208	0,207	0,207	0,207	0,206	0,206	0,205	0,205	0,204
4,9	0,204	0,204	0,203	0,203	0,202	0,202	0,202	0,201	0,201	0,200
5,0	0,200	0,200	0,199	0,199	0,198	0,198	0,198	0,197	0,197	0,196
5,1	0,196	0,196	0,195	0,195	0,195	0,194	0,194	0,193	0,193	0,193
5,2	0,192	0,192	0,192	0,191	0,191	0,190	0,190	0,190	0,189	0,189
5,3	0,189	0,188	0,188	0,188	0,187	0,187	0,187	0,186	0,186	0,186
5,4	0,185	0,185	0,185	0,184	0,184	0,183	0,183	0,183	0,182	0,182

x	0	1	2	3	4	5	6	7	8	9
5,5	0,182	0,181	0,181	0,181	0,181	0,180	0,180	0,180	0,179	0,179
5,6	0,179	0,178	0,178	0,178	0,177	0,177	0,177	0,176	0,176	0,176
5,7	0,175	0,175	0,175	0,175	0,174	0,174	0,174	0,173	0,173	0,173
5,8	0,172	0,172	0,172	0,172	0,171	0,171	0,171	0,170	0,170	0,170
5,9	0,169	0,169	0,169	0,169	0,168	0,168	0,168	0,168	0,167	0,167
6,0	0,167	0,166	0,166	0,166	0,166	0,165	0,165	0,165	0,164	0,164
6,1	0,164	0,164	0,163	0,163	0,163	0,163	0,162	0,162	0,162	0,162
6,2	0,161	0,161	0,161	0,161	0,160	0,160	0,160	0,159	0,159	0,159
6,3	0,159	0,158	0,158	0,158	0,158	0,157	0,157	0,157	0,157	0,156
6,4	0,156	0,156	0,156	0,156	0,155	0,155	0,155	0,155	0,154	0,154
6,5	0,154	0,154	0,153	0,153	0,153	0,153	0,152	0,152	0,152	0,152
6,6	0,152	0,151	0,151	0,151	0,151	0,150	0,150	0,150	0,150	0,149
6,7	0,149	0,149	0,149	0,149	0,148	0,148	0,148	0,148	0,147	0,147
6,8	0,147	0,147	0,147	0,146	0,146	0,146	0,146	0,146	0,145	0,145
6,9	0,145	0,145	0,145	0,144	0,144	0,144	0,144	0,143	0,143	0,143
7,0	0,143	0,143	0,142	0,142	0,142	0,142	0,142	0,141	0,141	0,141
7,1	0,141	0,141	0,140	0,140	0,140	0,140	0,140	0,139	0,139	0,139
7,2	0,139	0,139	0,139	0,138	0,138	0,138	0,138	0,138	0,137	0,137
7,3	0,137	0,137	0,137	0,136	0,136	0,136	0,136	0,136	0,136	0,135
7,4	0,135	0,135	0,135	0,135	0,134	0,134	0,134	0,134	0,134	0,134
7,5	0,133	0,133	0,133	0,133	0,133	0,132	0,132	0,132	0,132	0,132
7,6	0,132	0,131	0,131	0,131	0,131	0,131	0,131	0,130	0,130	0,130
7,7	0,130	0,130	0,130	0,129	0,129	0,129	0,129	0,129	0,129	0,128
7,8	0,128	0,128	0,128	0,128	0,128	0,127	0,127	0,127	0,127	0,127
7,9	0,127	0,126	0,126	0,126	0,126	0,126	0,126	0,125	0,125	0,125
8,0	0,125	0,125	0,125	0,125	0,124	0,124	0,124	0,124	0,124	0,124
8,1	0,123	0,123	0,123	0,123	0,123	0,123	0,123	0,122	0,122	0,122
8,2	0,122	0,122	0,122	0,122	0,121	0,121	0,121	0,121	0,121	0,121
8,3	0,120	0,120	0,120	0,120	0,120	0,120	0,120	0,119	0,119	0,119
8,4	0,119	0,119	0,119	0,119	0,118	0,118	0,118	0,118	0,118	0,118
8,5	0,118	0,118	0,117	0,117	0,117	0,117	0,117	0,117	0,117	0,116
8,6	0,116	0,116	0,116	0,116	0,116	0,116	0,115	0,115	0,115	0,115
8,7	0,115	0,115	0,115	0,115	0,114	0,114	0,114	0,114	0,114	0,114
8,8	0,114	0,114	0,113	0,113	0,113	0,113	0,113	0,113	0,113	0,112
8,9	0,112	0,112	0,112	0,112	0,112	0,112	0,112	0,111	0,111	0,111
9,0	0,111	0,111	0,111	0,111	0,111	0,110	0,110	0,110	0,110	0,110
9,1	0,110	0,110	0,110	0,110	0,109	0,109	0,109	0,109	0,109	0,109
9,2	0,109	0,109	0,108	0,108	0,108	0,108	0,108	0,108	0,108	0,108
9,3	0,108	0,107	0,107	0,107	0,107	0,107	0,107	0,107	0,107	0,106
9,4	0,106	0,106	0,106	0,106	0,106	0,106	0,106	0,106	0,105	0,105
9,5	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,104	0,104	0,104
9,6	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,103	0,103	0,103
9,7	0,103	0,103	0,103	0,103	0,103	0,103	0,102	0,102	0,102	0,102
9,8	0,102	0,102	0,102	0,102	0,102	0,102	0,101	0,101	0,101	0,101
9,9	0,101	0,101	0,101	0,101	0,101	0,101	0,100	0,100	0,100	0,100

Squares

$x \longrightarrow x^2$

x	0	1	2	3	4	5	6	7	8	9
1,0	1,00	1,02	1,04	1,06	1,08	1,10	1,12	1,14	1,17	1,19
1,1	1,21	1,23	1,25	1,28	1,30	1,32	1,35	1,37	1,39	1,42
1,2	1,44	1,46	1,49	1,51	1,54	1,56	1,59	1,61	1,64	1,66
1,3	1,69	1,72	1,74	1,77	1,80	1,82	1,85	1,88	1,90	1,93
1,4	1,96	1,99	2,02	2,04	2,07	2,10	2,13	2,16	2,19	2,22
1,5	2,25	2,28	2,31	2,34	2,37	2,40	2,43	2,46	2,50	2,53
1,6	2,56	2,59	2,62	2,66	2,69	2,72	2,76	2,79	2,82	2,86
1,7	2,89	2,92	2,96	2,99	3,03	3,06	3,10	3,13	3,17	3,20
1,8	3,24	3,28	3,31	3,35	3,39	3,42	3,46	3,50	3,53	3,57
1,9	3,61	3,65	3,69	3,72	3,76	3,80	3,84	3,88	3,92	3,96
2,0	4,00	4,04	4,08	4,12	4,16	4,20	4,24	4,28	4,33	4,37
2,1	4,41	4,45	4,49	4,54	4,58	4,62	4,67	4,71	4,75	4,80
2,2	4,84	4,88	4,93	4,97	5,02	5,06	5,11	5,15	5,20	5,24
2,3	5,29	5,34	5,38	5,43	5,48	5,52	5,57	5,62	5,66	5,71
2,4	5,76	5,81	5,86	5,90	5,95	6,00	6,05	6,10	6,15	6,20
2,5	6,25	6,30	6,35	6,40	6,45	6,50	6,55	6,60	6,66	6,71
2,6	6,76	6,81	6,86	6,92	6,97	7,02	7,08	7,13	7,18	7,24
2,7	7,29	7,34	7,40	7,45	7,51	7,56	7,62	7,67	7,73	7,78
2,8	7,84	7,90	7,95	8,01	8,07	8,12	8,18	8,24	8,29	8,35
2,9	8,41	8,47	8,53	8,58	8,64	8,70	8,76	8,82	8,88	8,94
3,0	9,00	9,06	9,12	9,18	9,24	9,30	9,36	9,42	9,49	9,55
3,1	9,61	9,67	9,73	9,80	9,86	9,92	9,99	10,0	10,1	10,2
3,2	10,2	10,3	10,4	10,4	10,5	10,6	10,6	10,7	10,8	10,8
3,3	10,9	11,0	11,0	11,1	11,2	11,2	11,3	11,4	11,4	11,5
3,4	11,6	11,6	11,7	11,8	11,8	11,9	12,0	12,0	12,1	12,2
3,5	12,3	12,3	12,4	12,5	12,5	12,6	12,7	12,7	12,8	12,9
3,6	13,0	13,0	13,1	13,2	13,2	13,3	13,4	13,5	13,5	13,6
3,7	13,7	13,8	13,8	13,9	14,0	14,1	14,1	14,2	14,3	14,4
3,8	14,4	14,5	14,6	14,7	14,7	14,8	14,9	15,0	15,1	15,1
3,9	15,2	15,3	15,4	15,4	15,5	15,6	15,7	15,8	15,8	15,9
4,0	16,0	16,1	16,2	16,2	16,3	16,4	16,5	16,6	16,6	16,7
4,1	16,8	16,9	17,0	17,1	17,1	17,2	17,3	17,4	17,5	17,6
4,2	17,6	17,7	17,8	17,9	18,0	18,1	18,1	18,2	18,3	18,4
4,3	18,5	18,6	18,7	18,7	18,8	18,9	19,0	19,1	19,2	19,3
4,4	19,4	19,4	19,5	19,6	19,7	19,8	19,9	20,0	20,1	20,2
4,5	20,3	20,3	20,4	20,5	20,6	20,7	20,8	20,9	21,0	21,1
4,6	21,2	21,3	21,3	21,4	21,5	21,6	21,7	21,8	21,9	22,0
4,7	22,1	22,2	22,3	22,4	22,5	22,6	22,7	22,8	22,8	22,9
4,8	23,0	23,1	23,2	23,3	23,4	23,5	23,6	23,7	23,8	23,9
4,9	24,0	24,1	24,2	24,3	24,4	24,5	24,6	24,7	24,8	24,9
5,0	25,0	25,1	25,2	25,3	25,4	25,5	25,6	25,7	25,8	25,9
5,1	26,0	26,1	26,2	26,3	26,4	26,5	26,6	26,7	26,8	26,9
5,2	27,0	27,1	27,2	27,4	27,5	27,6	27,7	27,8	27,9	28,0
5,3	28,1	28,2	28,3	28,4	28,5	28,6	28,7	28,8	28,9	29,1
5,4	29,2	29,3	29,4	29,5	29,6	29,7	29,8	29,9	30,0	30,1
5,5	30,3	30,4	30,5	30,6	30,7	30,8	30,9	31,0	31,1	31,2
5,6	31,4	31,5	31,6	31,7	31,8	31,9	32,0	32,1	32,3	32,4
5,7	32,5	32,6	32,7	32,8	32,9	33,1	33,2	33,3	33,4	33,5
5,8	33,6	33,8	33,9	34,0	34,1	34,2	34,3	34,5	34,6	34,7
5,9	34,8	34,9	35,0	35,2	35,3	35,4	35,5	35,6	35,8	35,9
6,0	36,0	36,1	36,2	36,4	36,5	36,6	36,7	36,8	37,0	37,1
6,1	37,2	37,3	37,5	37,6	37,7	37,8	37,9	38,1	38,2	38,3
6,2	38,4	38,6	38,7	38,8	38,9	39,1	39,2	39,3	39,4	39,6
6,3	39,7	39,8	39,9	40,1	40,2	40,3	40,4	40,6	40,7	40,8
6,4	41,0	41,1	41,2	41,3	41,5	41,6	41,7	41,9	42,0	42,1
6,5	42,3	42,4	42,5	42,6	42,8	42,9	43,0	43,2	43,3	43,4
6,6	43,6	43,7	43,8	44,0	44,1	44,2	44,4	44,5	44,6	44,8
6,7	44,9	45,0	45,2	45,3	45,4	45,6	45,7	45,8	46,0	46,1
6,8	46,2	46,4	46,5	46,6	46,8	46,9	47,1	47,2	47,3	47,5
6,9	47,6	47,7	47,9	48,0	48,2	48,3	48,4	48,6	48,7	48,9
7,0	49,0	49,1	49,3	49,4	49,6	49,7	49,8	50,0	50,1	50,3
7,1	50,4	50,6	50,7	50,8	51,0	51,1	51,3	51,4	51,6	51,7
7,2	51,8	52,0	52,1	52,3	52,4	52,6	52,7	52,9	53,0	53,1
7,3	53,3	53,4	53,6	53,7	53,9	54,0	54,2	54,3	54,5	54,6
7,4	54,8	54,9	55,1	55,2	55,4	55,5	55,7	55,8	56,0	56,1
7,5	56,3	56,4	56,6	56,7	56,9	57,0	57,2	57,3	57,5	57,6
7,6	57,8	57,9	58,1	58,2	58,4	58,5	58,7	58,8	59,0	59,1
7,7	59,3	59,4	59,6	59,8	59,9	60,1	60,2	60,4	60,5	60,7
7,8	60,8	61,0	61,2	61,3	61,5	61,6	61,8	61,9	62,1	62,3
7,9	62,4	62,6	62,7	62,9	63,0	63,2	63,4	63,5	63,7	63,8
8,0	64,0	64,2	64,3	64,5	64,6	64,8	65,0	65,1	65,3	65,4
8,1	65,6	65,8	65,9	66,1	66,3	66,4	66,6	66,7	66,9	67,1
8,2	67,2	67,4	67,6	67,7	67,9	68,1	68,2	68,4	68,6	68,7
8,3	68,9	69,1	69,2	69,4	69,6	69,7	69,9	70,1	70,2	70,4
8,4	70,6	70,7	70,9	71,1	71,2	71,4	71,6	71,7	71,9	72,1
8,5	72,3	72,4	72,6	72,8	72,9	73,1	73,3	73,4	73,6	73,8
8,6	74,0	74,1	74,3	74,5	74,6	74,8	75,0	75,2	75,3	75,5
8,7	75,7	75,9	76,0	76,2	76,4	76,6	76,7	76,9	77,1	77,3
8,8	77,4	77,6	77,8	78,0	78,1	78,3	78,5	78,7	78,9	79,0
8,9	79,2	79,4	79,6	79,7	79,9	80,1	80,3	80,5	80,6	80,8
9,0	81,0	81,2	81,4	81,5	81,7	81,9	82,1	82,3	82,4	82,6
9,1	82,8	83,0	83,2	83,4	83,5	83,7	83,9	84,1	84,3	84,5
9,2	84,6	84,8	85,0	85,2	85,4	85,6	85,7	85,9	86,1	86,3
9,3	86,5	86,7	86,9	87,0	87,2	87,4	87,6	87,8	88,0	88,2
9,4	88,4	88,5	88,7	88,9	89,1	89,3	89,5	89,7	89,9	90,1
9,5	90,3	90,4	90,6	90,8	91,0	91,2	91,4	91,6	91,8	92,0
9,6	92,2	92,4	92,5	92,7	92,9	93,1	93,3	93,5	93,7	93,9
9,7	94,1	94,3	94,5	94,7	94,9	95,1	95,3	95,5	95,6	95,8
9,8	96,0	96,2	96,4	96,6	96,8	97,0	97,2	97,4	97,6	97,8
9,9	98,0	98,2	98,4	98,6	98,8	99,0	99,2	99,4	99,6	99,8

Square roots from 1 to 9,99

$x \to x\sqrt{x}$

x	0	1	2	3	4	5	6	7	8	9
1,0	1,00	1,00	1,01	1,01	1,02	1,02	1,03	1,03	1,04	1,04
1,1	1,05	1,05	1,06	1,06	1,07	1,07	1,08	1,08	1,09	1,09
1,2	1,10	1,10	1,10	1,11	1,11	1,12	1,12	1,13	1,13	1,14
1,3	1,14	1,14	1,15	1,15	1,16	1,16	1,17	1,17	1,17	1,18
1,4	1,18	1,19	1,19	1,20	1,20	1,20	1,21	1,21	1,22	1,22
1,5	1,22	1,23	1,23	1,24	1,24	1,24	1,25	1,25	1,26	1,26
1,6	1,26	1,27	1,27	1,28	1,28	1,28	1,29	1,29	1,30	1,30
1,7	1,30	1,31	1,31	1,32	1,32	1,32	1,33	1,33	1,33	1,34
1,8	1,34	1,35	1,35	1,35	1,36	1,36	1,36	1,37	1,37	1,37
1,9	1,38	1,38	1,39	1,39	1,39	1,40	1,40	1,40	1,41	1,41
2,0	1,41	1,42	1,42	1,42	1,43	1,43	1,44	1,44	1,44	1,45
2,1	1,45	1,45	1,46	1,46	1,46	1,47	1,47	1,47	1,48	1,48
2,2	1,48	1,49	1,49	1,49	1,50	1,50	1,50	1,51	1,51	1,51
2,3	1,52	1,52	1,52	1,53	1,53	1,53	1,54	1,54	1,54	1,55
2,4	1,55	1,55	1,56	1,56	1,56	1,57	1,57	1,57	1,57	1,58
2,5	1,58	1,58	1,59	1,59	1,59	1,60	1,60	1,60	1,61	1,61
2,6	1,61	1,62	1,62	1,62	1,62	1,63	1,63	1,63	1,64	1,64
2,7	1,64	1,65	1,65	1,65	1,66	1,66	1,66	1,66	1,67	1,67
2,8	1,67	1,68	1,68	1,68	1,69	1,69	1,69	1,69	1,70	1,70
2,9	1,70	1,71	1,71	1,71	1,71	1,72	1,72	1,72	1,73	1,73
3,0	1,73	1,73	1,74	1,74	1,74	1,75	1,75	1,75	1,75	1,76
3,1	1,76	1,76	1,77	1,77	1,77	1,77	1,78	1,78	1,78	1,79
3,2	1,79	1,79	1,79	1,80	1,80	1,80	1,81	1,81	1,81	1,81
3,3	1,82	1,82	1,82	1,82	1,83	1,83	1,83	1,84	1,84	1,84
3,4	1,84	1,85	1,85	1,85	1,85	1,86	1,86	1,86	1,87	1,87
3,5	1,87	1,87	1,88	1,88	1,88	1,88	1,89	1,89	1,89	1,89
3,6	1,90	1,90	1,90	1,91	1,91	1,91	1,91	1,92	1,92	1,92
3,7	1,92	1,93	1,93	1,93	1,93	1,94	1,94	1,94	1,94	1,95
3,8	1,95	1,95	1,95	1,96	1,96	1,96	1,96	1,97	1,97	1,97
3,9	1,97	1,98	1,98	1,98	1,98	1,99	1,99	1,99	1,99	2,00
4,0	2,00	2,00	2,00	2,01	2,01	2,01	2,01	2,02	2,02	2,02
4,1	2,02	2,03	2,03	2,03	2,03	2,04	2,04	2,04	2,04	2,05
4,2	2,05	2,05	2,05	2,06	2,06	2,06	2,06	2,07	2,07	2,07
4,3	2,07	2,08	2,08	2,08	2,08	2,09	2,09	2,09	2,09	2,10
4,4	2,10	2,10	2,10	2,10	2,11	2,11	2,11	2,11	2,12	2,12
4,5	2,12	2,12	2,13	2,13	2,13	2,13	2,14	2,14	2,14	2,14
4,6	2,14	2,15	2,15	2,15	2,15	2,16	2,16	2,16	2,16	2,17
4,7	2,17	2,17	2,17	2,17	2,18	2,18	2,18	2,18	2,19	2,19
4,8	2,19	2,19	2,20	2,20	2,20	2,20	2,20	2,21	2,21	2,21
4,9	2,21	2,22	2,22	2,22	2,22	2,22	2,23	2,23	2,23	2,23
5,0	2,24	2,24	2,24	2,24	2,24	2,25	2,25	2,25	2,25	2,26
5,1	2,26	2,26	2,26	2,26	2,27	2,27	2,27	2,27	2,28	2,28
5,2	2,28	2,28	2,28	2,29	2,29	2,29	2,29	2,30	2,30	2,30
5,3	2,30	2,30	2,31	2,31	2,31	2,31	2,32	2,32	2,32	2,32
5,4	2,32	2,33	2,33	2,33	2,33	2,33	2,34	2,34	2,34	2,34

x	0	1	2	3	4	5	6	7	8	9
5,5	2,35	2,35	2,35	2,35	2,35	2,36	2,36	2,36	2,36	2,36
5,6	2,37	2,37	2,37	2,37	2,37	2,38	2,38	2,38	2,38	2,39
5,7	2,39	2,39	2,39	2,39	2,40	2,40	2,40	2,40	2,40	2,41
5,8	2,41	2,41	2,41	2,41	2,42	2,42	2,42	2,42	2,42	2,43
5,9	2,43	2,43	2,43	2,44	2,44	2,44	2,44	2,44	2,45	2,45
6,0	2,45	2,45	2,45	2,46	2,46	2,46	2,46	2,46	2,47	2,47
6,1	2,47	2,47	2,47	2,48	2,48	2,48	2,48	2,48	2,49	2,49
6,2	2,49	2,49	2,49	2,50	2,50	2,50	2,50	2,50	2,51	2,51
6,3	2,51	2,51	2,51	2,52	2,52	2,52	2,52	2,52	2,53	2,53
6,4	2,53	2,53	2,53	2,54	2,54	2,54	2,54	2,54	2,55	2,55
6,5	2,55	2,55	2,55	2,56	2,56	2,56	2,56	2,56	2,57	2,57
6,6	2,57	2,57	2,57	2,57	2,58	2,58	2,58	2,58	2,58	2,59
6,7	2,59	2,59	2,59	2,59	2,60	2,60	2,60	2,60	2,60	2,61
6,8	2,61	2,61	2,61	2,61	2,62	2,62	2,62	2,62	2,62	2,62
6,9	2,63	2,63	2,63	2,63	2,63	2,64	2,64	2,64	2,64	2,64
7,0	2,65	2,65	2,65	2,65	2,65	2,66	2,66	2,66	2,66	2,66
7,1	2,66	2,67	2,67	2,67	2,67	2,67	2,68	2,68	2,68	2,68
7,2	2,68	2,69	2,69	2,69	2,69	2,69	2,69	2,70	2,70	2,70
7,3	2,70	2,70	2,71	2,71	2,71	2,71	2,71	2,71	2,72	2,72
7,4	2,72	2,72	2,72	2,73	2,73	2,73	2,73	2,73	2,73	2,74
7,5	2,74	2,74	2,74	2,74	2,75	2,75	2,75	2,75	2,75	2,75
7,6	2,76	2,76	2,76	2,76	2,76	2,77	2,77	2,77	2,77	2,77
7,7	2,77	2,78	2,78	2,78	2,78	2,78	2,79	2,79	2,79	2,79
7,8	2,79	2,79	2,80	2,80	2,80	2,80	2,80	2,81	2,81	2,81
7,9	2,81	2,81	2,81	2,82	2,82	2,82	2,82	2,82	2,82	2,83
8,0	2,83	2,83	2,83	2,83	2,84	2,84	2,84	2,84	2,84	2,84
8,1	2,85	2,85	2,85	2,85	2,85	2,85	2,86	2,86	2,86	2,86
8,2	2,86	2,87	2,87	2,87	2,87	2,87	2,87	2,88	2,88	2,88
8,3	2,88	2,88	2,88	2,89	2,89	2,89	2,89	2,89	2,89	2,90
8,4	2,90	2,90	2,90	2,90	2,91	2,91	2,91	2,91	2,91	2,91
8,5	2,92	2,92	2,92	2,92	2,92	2,92	2,93	2,93	2,93	2,93
8,6	2,93	2,93	2,94	2,94	2,94	2,94	2,94	2,94	2,95	2,95
8,7	2,95	2,95	2,95	2,95	2,96	2,96	2,96	2,96	2,96	2,96
8,8	2,97	2,97	2,97	2,97	2,97	2,97	2,98	2,98	2,98	2,98
8,9	2,98	2,98	2,99	2,99	2,99	2,99	2,99	2,99	3,00	3,00
9,0	3,00	3,00	3,00	3,00	3,01	3,01	3,01	3,01	3,01	3,01
9,1	3,02	3,02	3,02	3,02	3,02	3,02	3,03	3,03	3,03	3,03
9,2	3,03	3,03	3,04	3,04	3,04	3,04	3,04	3,04	3,05	3,05
9,3	3,05	3,05	3,05	3,05	3,06	3,06	3,06	3,06	3,06	3,06
9,4	3,07	3,07	3,07	3,07	3,07	3,07	3,08	3,08	3,08	3,08
9,5	3,08	3,08	3,09	3,09	3,09	3,09	3,09	3,09	3,10	3,10
9,6	3,10	3,10	3,10	3,10	3,10	3,11	3,11	3,11	3,11	3,11
9,7	3,11	3,12	3,12	3,12	3,12	3,12	3,12	3,13	3,13	3,13
9,8	3,13	3,13	3,13	3,14	3,14	3,14	3,14	3,14	3,14	3,14
9,9	3,15	3,15	3,15	3,15	3,15	3,15	3,16	3,16	3,16	3,16

Square roots from 10 to 99,9

$x \rightarrow \sqrt{x}$

x	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
10	3,16	3,18	3,19	3,21	3,22	3,24	3,26	3,27	3,29	3,30
11	3,32	3,33	3,35	3,36	3,38	3,39	3,41	3,42	3,44	3,45
12	3,46	3,48	3,49	3,51	3,52	3,54	3,55	3,56	3,58	3,59
13	3,61	3,62	3,63	3,65	3,66	3,67	3,69	3,70	3,71	3,73
14	3,74	3,75	3,77	3,78	3,79	3,81	3,82	3,83	3,85	3,86
15	3,87	3,89	3,90	3,91	3,92	3,94	3,95	3,96	3,97	3,99
16	4,00	4,01	4,02	4,04	4,05	4,06	4,07	4,09	4,10	4,11
17	4,12	4,14	4,15	4,16	4,17	4,18	4,20	4,21	4,22	4,23
18	4,24	4,25	4,27	4,28	4,29	4,30	4,31	4,32	4,34	4,35
19	4,36	4,37	4,38	4,39	4,40	4,42	4,43	4,44	4,45	4,46
20	4,47	4,48	4,49	4,51	4,52	4,53	4,54	4,55	4,56	4,57
21	4,58	4,59	4,60	4,62	4,63	4,64	4,65	4,66	4,67	4,68
22	4,69	4,70	4,71	4,72	4,73	4,74	4,75	4,76	4,77	4,79
23	4,80	4,81	4,82	4,83	4,84	4,85	4,86	4,87	4,88	4,89
24	4,90	4,91	4,92	4,93	4,94	4,95	4,96	4,97	4,98	4,99
25	5,00	5,01	5,02	5,03	5,04	5,05	5,06	5,07	5,08	5,09
26	5,10	5,11	5,12	5,13	5,14	5,15	5,16	5,17	5,18	5,19
27	5,20	5,21	5,22	5,22	5,23	5,24	5,25	5,26	5,27	5,28
28	5,29	5,30	5,31	5,32	5,33	5,34	5,35	5,36	5,37	5,38
29	5,39	5,39	5,40	5,41	5,42	5,43	5,44	5,45	5,46	5,47
30	5,48	5,49	5,50	5,50	5,51	5,52	5,53	5,54	5,55	5,56
31	5,57	5,58	5,59	5,59	5,60	5,61	5,62	5,63	5,64	5,65
32	5,66	5,67	5,67	5,68	5,69	5,70	5,71	5,72	5,73	5,74
33	5,74	5,75	5,76	5,77	5,78	5,79	5,80	5,81	5,81	5,82
34	5,83	5,84	5,85	5,86	5,87	5,87	5,88	5,89	5,90	5,91
35	5,92	5,92	5,93	5,94	5,95	5,96	5,97	5,97	5,98	5,99
36	6,00	6,01	6,02	6,02	6,03	6,04	6,05	6,06	6,07	6,07
37	6,08	6,09	6,10	6,11	6,12	6,12	6,13	6,14	6,15	6,16
38	6,16	6,17	6,18	6,19	6,20	6,20	6,21	6,22	6,23	6,24
39	6,24	6,25	6,26	6,27	6,28	6,28	6,29	6,30	6,31	6,32
40	6,32	6,33	6,34	6,35	6,36	6,36	6,37	6,38	6,39	6,40
41	6,40	6,41	6,42	6,43	6,43	6,44	6,45	6,46	6,47	6,47
42	6,48	6,49	6,50	6,50	6,51	6,52	6,53	6,53	6,54	6,55
43	6,56	6,57	6,57	6,58	6,59	6,60	6,60	6,61	6,62	6,63
44	6,63	6,64	6,65	6,66	6,66	6,67	6,68	6,69	6,69	6,70
45	6,71	6,72	6,72	6,73	6,74	6,75	6,75	6,76	6,77	6,77
46	6,78	6,79	6,80	6,80	6,81	6,82	6,83	6,83	6,84	6,85
47	6,86	6,86	6,87	6,88	6,88	6,89	6,90	6,91	6,91	6,92
48	6,93	6,94	6,94	6,95	6,96	6,96	6,97	6,98	6,99	6,99
49	7,00	7,01	7,01	7,02	7,03	7,04	7,04	7,05	7,06	7,06
50	7,07	7,08	7,09	7,09	7,10	7,11	7,11	7,12	7,13	7,13
51	7,14	7,15	7,16	7,16	7,17	7,18	7,18	7,19	7,20	7,20
52	7,21	7,22	7,22	7,23	7,24	7,25	7,25	7,26	7,27	7,27
53	7,28	7,29	7,29	7,30	7,31	7,31	7,32	7,33	7,33	7,34
54	7,35	7,36	7,36	7,37	7,38	7,38	7,39	7,40	7,40	7,41

x	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
55	7,42	7,42	7,43	7,44	7,44	7,45	7,46	7,46	7,47	7,48
56	7,48	7,49	7,50	7,50	7,51	7,52	7,52	7,53	7,54	7,54
57	7,55	7,56	7,56	7,57	7,58	7,58	7,59	7,60	7,60	7,61
58	7,62	7,62	7,63	7,64	7,64	7,65	7,66	7,66	7,67	7,67
59	7,68	7,69	7,69	7,70	7,71	7,71	7,72	7,73	7,73	7,74
60	7,75	7,75	7,76	7,77	7,77	7,78	7,78	7,79	7,80	7,80
61	7,81	7,82	7,82	7,83	7,84	7,84	7,85	7,85	7,86	7,87
62	7,87	7,88	7,89	7,89	7,90	7,91	7,91	7,92	7,92	7,93
63	7,94	7,94	7,95	7,96	7,96	7,97	7,97	7,98	7,99	7,99
64	8,00	8,01	8,01	8,02	8,02	8,03	8,04	8,04	8,05	8,06
65	8,06	8,07	8,07	8,08	8,09	8,09	8,10	8,11	8,11	8,12
66	8,12	8,13	8,14	8,14	8,15	8,15	8,16	8,17	8,17	8,18
67	8,19	8,19	8,20	8,20	8,21	8,22	8,22	8,23	8,23	8,24
68	8,25	8,25	8,26	8,26	8,27	8,28	8,28	8,29	8,29	8,30
69	8,31	8,31	8,32	8,32	8,33	8,34	8,34	8,35	8,35	8,36
70	8,37	8,37	8,38	8,38	8,39	8,40	8,40	8,41	8,41	8,42
71	8,43	8,43	8,44	8,44	8,45	8,46	8,46	8,47	8,47	8,48
72	8,49	8,49	8,50	8,50	8,51	8,51	8,52	8,53	8,53	8,54
73	8,54	8,55	8,56	8,56	8,57	8,57	8,58	8,58	8,59	8,60
74	8,60	8,61	8,61	8,62	8,63	8,63	8,64	8,64	8,65	8,65
75	8,66	8,67	8,67	8,68	8,68	8,69	8,69	8,70	8,71	8,71
76	8,72	8,72	8,73	8,73	8,74	8,75	8,75	8,76	8,76	8,77
77	8,77	8,78	8,79	8,79	8,80	8,80	8,81	8,81	8,82	8,83
78	8,83	8,84	8,84	8,85	8,85	8,86	8,87	8,87	8,88	8,88
79	8,89	8,89	8,90	8,91	8,91	8,92	8,92	8,93	8,93	8,94
80	8,94	8,95	8,96	8,96	8,97	8,97	8,98	8,98	8,99	8,99
81	9,00	9,01	9,01	9,02	9,02	9,03	9,03	9,04	9,04	9,05
82	9,06	9,06	9,07	9,07	9,08	9,08	9,09	9,09	9,10	9,10
83	9,11	9,12	9,12	9,13	9,13	9,14	9,14	9,15	9,15	9,16
84	9,17	9,17	9,18	9,18	9,19	9,19	9,20	9,20	9,21	9,21
85	9,22	9,22	9,23	9,24	9,24	9,25	9,25	9,26	9,26	9,27
86	9,27	9,28	9,28	9,29	9,30	9,30	9,31	9,31	9,32	9,32
87	9,33	9,33	9,34	9,34	9,35	9,35	9,36	9,36	9,37	9,38
88	9,38	9,39	9,39	9,40	9,40	9,41	9,41	9,42	9,42	9,43
89	9,43	9,44	9,44	9,45	9,46	9,46	9,47	9,47	9,48	9,48
90	9,49	9,49	9,50	9,50	9,51	9,51	9,52	9,52	9,53	9,53
91	9,54	9,54	9,55	9,56	9,56	9,57	9,57	9,58	9,58	9,59
92	9,59	9,60	9,60	9,61	9,61	9,62	9,62	9,63	9,63	9,64
93	9,64	9,65	9,65	9,66	9,66	9,67	9,67	9,68	9,69	9,69
94	9,70	9,70	9,71	9,71	9,72	9,72	9,73	9,73	9,74	9,74
95	9,75	9,75	9,76	9,76	9,77	9,77	9,78	9,78	9,79	9,79
96	9,80	9,80	9,81	9,81	9,82	9,82	9,83	9,83	9,84	9,84
97	9,85	9,85	9,86	9,86	9,87	9,87	9,88	9,88	9,89	9,89
98	9,90	9,90	9,91	9,91	9,92	9,92	9,93	9,93	9,94	9,94
99	9,95	9,95	9,96	9,96	9,97	9,97	9,98	9,98	9,99	9,99

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Answers

Exercise 1a (p. 1)

1 (a) 1; 2; 3; 6; 9; 18 (b) 2; 3 (c) 2×3^2
2 (a) 1; 2; 4; 7; 14; 28 (b) 2; 7 (c) $2^2 \times 7$
3 (a) 1; 3; 11; 33 (b) 3; 11 (c) 3×11
4 (a) 1; 3; 5; 9; 15; 45 (b) 3; 5 (c) $3^2 \times 5$
5 (a) 1; 2; 4; 8; 16 (b) 2 (c) 2^4
6 (a) 1; 2; 11; 22 (b) 2; 11 (c) 2×11
7 (a) 1; 2; 3; 5; 6; 10; 15; 30 (b) 2; 3; 5 (c) $2 \times 3 \times 5$
8 (a) 1; 2; 3; 4; 6; 8; 12; 16; 24; 48 (b) 2; 3 (c) $2^4 \times 3$
9 (a) 1; 2; 3; 4; 6; 12 (b) 2; 3 (c) $2^2 \times 3$
10 (a) 1; 2; 3; 4; 6; 9; 12; 18; 36 (b) 2; 3 (c) $2^2 \times 3^2$
11 (a) 1; 3; 13; 39 (b) 3; 13 (c) 3×13
12 (a) 1; 2; 4; 7; 8; 14; 28; 56 (b) 2; 7 (c) $2^3 \times 7$
13 (a) 1; 2; 3; 6; 7; 14; 21; 42 (b) 2; 3; 7 (c) $2 \times 3 \times 7$
14 (a) 1; 2; 5; 10; 25; 50 (b) 2; 5 (c) 2×5^2
15 (a) 1; 3; 7; 9; 21; 63 (b) 3; 7 (c) $3^2 \times 7$
16 (a) 1; 2; 3; 4; 6; 8; 9; 12; 18; 24; 36; 72 (b) 2; 3 (c) $2^3 \times 3^2$

Exercise 1b (p. 1)

1 3^3 **2** $2^2 \times 11$
3 $2^2 \times 13$ **4** 3×5^2
5 2×7^2 **6** $2^3 \times 13$
7 $2^2 \times 29$ **8** $3^2 \times 13$
9 $2^3 \times 5^2$ **10** $3^2 \times 31$
11 $2^2 \times 7 \times 13$ **12** $2^2 \times 3 \times 37$

Exercise 1c (p. 2)

1 14 **2** 15 **3** 8 **4** 6
5 3 **6** 6 **7** 12 **8** 24
9 6 **10** 18 **11** 108 **12** 63

Exercise 1d (p. 2)

1 36 **2** 40 **3** 30 **4** 120
5 165 **6** 168 **7** 42 **8** 60
9 120 **10** 180 **11** 180 **12** 140

Exercise 1e (p. 3)

1 (a) 20; 24; 28; 32; 36; 40; 44; 48; 52; 56; 60; 64; 68; 72; 76; 80; 84; 88; 92; 96
(b) 30; 36; 42; 48; 54; 60; 66; 72; 78; 84; 90
(c) 40; 48; 56; 64; 72; 80; 88
(d) 45; 54; 63; 72; 81; 90

3 (a) 17; 20; 23; 26 (b) 26; 31; 36; 41
(c) 56; 67; 78; 89 (d) 5; 4; 3; 2
(e) 15; 21; 28; 36 (f) 32; 64; 128; 256
(g) 31; 43; 57; 73 (h) 26; 37; 50; 65
(i) 36; 49; 64; 81 (j) 34; 55; 89; 144

4

1	2	3	4	5	6	7	8
1	3	6	10	15	21	28	36

5 (b) if n is any whole number, n^2 = sum of the first n odd numbers
(c) the total column contains square numbers

Exercise 2a (p. 6)

1 (a) $\notin$ (b) $\in$ (c) $\notin$ (d) $\notin$
2 (a) 5 (b) 10 (c) 3 (d) 12 (e) 2 (f) 12
3 {even primes greater than 2}, {positive numbers less than 0}, {lines of symmetry in a scalene triangle} there are many more
4 {prime numbers}, {negative numbers}, {lines of symmetry in a circle} there are many more
5 (a) $\varnothing$, {3}, {4}, {5}, {3; 4}, {3; 5}, {4; 5}, {3; 4; 5}
(b) $\varnothing$, $\{x\}$, $\{y\}$, $\{x; y\}$ (c) $\varnothing$, {0}, {2}, {0; 2}
(d) $\varnothing$, $\{f\}$, $\{o\}$, $\{u\}$, $\{r\}$, $\{f; o\}$, $\{f; u\}$, $\{f; r\}$, $\{o; u\}$, $\{o; r\}$ $\{u; r\}$, $\{f; o; u\}$, $\{f; o; r\}$ $\{o; u; r\}$, $\{f; u; r\}$, $\{f; o; u; r\}$
6 (a) {2; 6} $\subset$ {factors of 18}
(b) {trees} $\not\subset$ {metal objects}
(c) {vehicles} $\supset$ {buses}
(d) {children} $\subset$ {humans}
8 (a) and (d) are disjoint
(b) {35; 70; 105; ...}
(c) {Zaire; Zambia}
9 (a) $M \cup L = \{2; 4; 6; 8; 10; 12; 18\}$
(b) $M \cap L = \{6; 12\}$
(c) $M \cup \mathscr{E} = \{2; 4; 6; \ldots; 20\} = \mathscr{E}$
(d) $\mathscr{E} \cap M = \{6; 12; 18\} = M$
(e) $\mathscr{E} \cap L = \{2; 4; 6; \ldots; 12\} = L$
(f) $L \cup \mathscr{E} = \{2; 4; 6; \ldots; 20\} = \mathscr{E}$
10 (b) (i) 7, (ii) 2

Exercise 2b (p. 7)

1 (a) {+ 4; 20} (b) {0; + 4; 20}
(c) {− 19; 0; + 4; 20} (d) set A

2 {− 19; 0} 3 {− 19}

4 all of set *A* except the elements 0; + 4; 20

5 {0; + 4; 20}

Exercise 2c (p. 8)

1 $x = 17$ 2 9

3 $x = 23$, n(ℰ) = 46 4 25

5 61% 6 9

7 11 8 15

9 90 10 5

Exercise 3a (p. 10)

1 R(3), S(− 1), T($2\frac{1}{2}$), U(− $1\frac{1}{2}$), V(0)

2 B(0,2), C(1,5), D(− 0,5), E(− 0,9), F(1,8), G(− 0,1)

3

Fig. A1

4

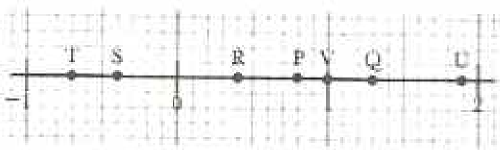

Fig. A2

Exercise 3c (p. 13)

1 A(1; 1), B(1; 3), C(3; 3), D(2; − 1), E(3; − 2), F(2; − 3), G(− 1; − 3), H(0; − 1), I(− 2; 0), J(− 3; 2)

2 (a) C (b) F (c) I
(d) K (e) D (f) A
(g) G (h) J (i) E
(j) L (k) H (l) B

3 T(− 2; 1), U(− 3; 1), V(0; 4), W(3; 1), X(2; 1), Y(2; − 1), Z(− 2; − 1)

4 (10; 7), (14; − 4), (16; − 2), (17; − 2), (15) − 5), (13; − 5), (11; 0), (9; − 2), (9; − 7), (6; − 7), (6; − 4), (− 2; − 4), (− 4; − 7), (− 7; − 7), (− 6; 5), (− 2; 8), (3; 8), (3; 9), (6; 9)

5 (a) (4; 7) (b) (5; 5)
(c) (5; 1) (d) (6; 1)
(e) (3; 2) (f) (1; 3)
(g) (2; 2) (h) (2; 7)
(i) (4; 4) (j) (5; 6)
for every point, the x-coordinate = 2

6 (a) (0; 2), (1; 2), (2; 2), (3; 2), (4; 2), (5; 2), (6; 2
for every point, the y-coordinate = 2
(b) (0; 1), (1; 2), (2; 3), (3; 4), (4; 5), (5; 6), (6; 7
for every point, the y-coordinate is 1 more than the x-coordinate

Exercise 3d (p. 16)

2 the points join to form a star shape (Fig. A3)

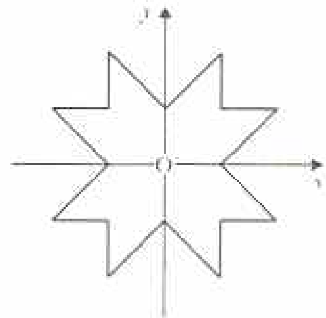

Fig. A3

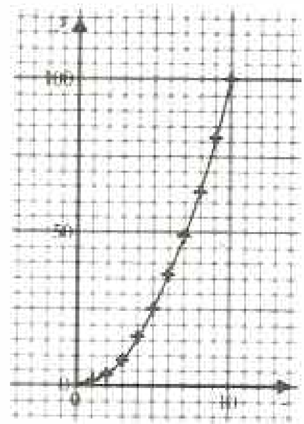

Fig. A4

3 a horse's head

4 (b) 10 unit², 9 unit², 6 unit², 9 unit²

5 (a) parallelogram, X(− 1; 0)
(b) B, X, D, E lie in a straight line
(c) square, Y($2\frac{1}{2}$; $\frac{1}{2}$)

6 (a) (6; 36), (7; 49), (8; 64), (9; 81), (10; 100)
(c) (d) see Fig. A4
(e) approximate values: 70,5; 42; 4,5; 9,5

Exercise 4a (p. 17)

1 (a) million (b) billion

2 (a) 40 000 000 cm² (b) 1 million

3 1 million

4 just over $11\frac{1}{2}$ days

5 (d) (there are over $31\frac{1}{2}$ million seconds in a yea

Exercise 4b (p. 18)

1 1 000 000 **2** 59 244
3 721 568 397 **4** 2 312 400
5 8 000 000 **6** 3 000 000 000
7 9 215 **8** 14 682 053
9 108 412 **10** 12 345
11 100 000 000 **12** 987 654

Exercise 4c (p. 18)

1 (a) \$2 000 000 (b) 150 000 000 km
(c) 3 000 000 000 (d) 5 500 000
(e) \$2 100 000 000 (f) 4 200 000 litres
(g) 400 000 000 (h) \$1 250 000
(i) 700 000 tonnes (j) \$750 000
(k) 450 000 (l) \$580 000 000

2 (a) 8 million tonnes
(b) \$6 million
(c) 2 billion
(d) \$3,7 billion
(e) \$7, 4 million
(f) \$1$\frac{3}{4}$ million
(g) 0,2 million litres
(h) $\frac{1}{2}$ billion or 500 million
(i) 0,3 million tonnes
(j) $\frac{1}{3}$ million
(k) 0,98 million barrels
(l) 0,49 billion or 490 million

Exercise 4d (p. 19)

1 0,06 **2** 0,004
3 0,9 **4** 0,000 008
5 0,000 4 **6** 0,000 06
7 0,003 **8** 0,000 09
9 0,000 7 **10** 0,16
11 0,034 **12** 0,002 6
13 0,000 28 **14** 0,084
15 0,075 6 **16** 2,7
17 0,65 **18** 0,402
19 0,2 **20** 0,24
21 0,7 **22** 0,006 2
23 0,003 3 **24** 0,040 2
25 0,9 **26** 0,9
27 0,03 **28** 0,72
29 0,072 **30** 0,0072

Exercise 4e (p. 20)

(a) x^5 (b) 10^7 (c) a^7
(d) 10^6 (e) n^6 (f) 10^9
(g) $24a^6$ (h) $20x^{10}$ (i) $6c^7$

(a) m^8 (b) a^{10} (c) c^{14}
(d) 10^9 (e) b^{15} (f) x^8
(g) $10e^{14}$ (h) 15×10^9 (i) $15y^8$

Exercise 4f (p. 20)

1 (a) a^4 (b) 10^3 (c) c^3
(d) 10^3 (e) d (f) 10^2
(g) $3x^4$ (h) $2a^2$ (i) x^5

2 (a) x^2 (b) b^3 (c) c^6
(d) a^2 (e) 10^2 (f) x^6
(g) $2x$ (h) $4x^3$ (i) 2×10^3

Exercise 4g (p. 21)

1 $\frac{1}{100}$ **2** $\frac{1}{10\,000}$ **3** $\frac{1}{1\,000\,000}$
4 x^3 **5** $\frac{1}{a^3}$ **6** 1
7 $\frac{1}{a^5}$ **8** x^9 **9** p^5
10 b^3 **11** 1 **12** $\frac{1}{c^2}$
13 8 **14** 9 **15** 9
16 $6a$ **17** $\frac{3a}{2}$ **18** $18a$

Exercise 4h (p. 22)

1 15 **2** 14 **3** 18 **4** 21
5 22 **6** 20 **7** 30 **8** 40
9 50 **10** 70 **11** 24 **12** 28
13 27 **14** 25 **15** 35 **16** 44
17 42 **18** 45 **19** 48 **20** 54
21 55 **22** 60 **23** 63 **24** 90
25 56 **26** 66 **27** 75 **28** 81
29 84 **30** 88

Exercise 4i (p. 23)

1 6 **2** 6 **3** 15 **4** 11
5 21 **6** 2 **7** 5 **8** 15
9 3 **10** 3 **11** 7 **12** 14

Exercise 4j (p. 23)

1 $\frac{3}{5}$ **2** $\frac{4}{7}$ **3** $\frac{1}{2}$
4 $\frac{6}{9}$ $(= \frac{2}{3})$ **5** $\frac{3}{4}$ **6** $\frac{1}{5}$
7 $\frac{3}{8}$ **8** $\frac{6}{10}$ $(= \frac{3}{5})$ **9** $\frac{6}{7}$
10 $\frac{5}{6}$ **11** $1\frac{1}{2}$ **12** $1\frac{1}{3}$
13 $1\frac{1}{4}$ **14** $1\frac{1}{5}$ **15** $4\frac{1}{2}$

Exercise 5a (p. 24)

1 $x = 6$ **2** $x = 6$ **3** $x = \frac{1}{2}$
4 $x = 2$ **5** $x = 3$ **6** $x = 1$
7 $a = 3$ **8** $z = 5$ **9** $n = 6$
10 $y = 3{,}4$ **11** $r = 3\frac{1}{2}$ **12** $d = 3\frac{1}{3}$

Exercise 5b (p. 24)

1 (a) $2x$ (b) $6n$ (c) $6m + 4$
(d) $2y - 5$ (e) $a - 3$ (f) $5d$

(g) $3h$ cents (h) $3t - 7$ (i) $2k - 9$ cents
(j) $2g + 23$ goals

2 (a) $4x$ cm (b) $3a + 4$ metres
(c) $6c$ cm (d) $8b$ metres
(e) $2h + 20$ metres (f) $5t$ cm

Exercise 5c (p. 25)

1 29 **2** 13 **3** 5
4 40 **5** 17 **6** 6
7 7c **8** 5 **9** 19c, 10c
10 56; 79 goals **11** 8 m **12** 3 m, 6 m
13 15 cm **14** 15 m, 5 m **15** 3 m
16 3 cm

Exercise 5d (p. 26)

1 $a = 2$ **2** $a = -3$ **3** $b = -6$
4 $x = 1\frac{3}{5}$ **5** $a = -1$ **6** $y = \frac{3}{4}$
7 $n = -\frac{3}{4}$ **8** $m = \frac{2}{3}$ **9** $a = 5$
10 $t = 4$ **11** $n = 2$ **12** $c = 1$
13 $q = -1$ **14** $x = 3$ **15** $m = -4$
16 $x = -\frac{1}{2}$ **17** $h = 3$ **18** $a = 4\frac{2}{3}$
19 $f = 2$ **20** $e = 6$ **21** $x = 3\frac{1}{6}$
22 $x = 1$ **23** $x = -12$ **24** $n = \frac{2}{3}$

Exercise 5e (p. 26)

1 $x = 4$ **2** $x = 8$ **3** $a = 6$
4 $y = 2$ **5** $x = -10$ **6** $x = 3$
7 $s = 6$ **8** $b = -9$ **9** $f = -1$
10 $x = -3$ **11** $a = -14$ **12** $b = 2$
13 $e = 3$ **14** $d = \frac{3}{4}$ **15** $x = \frac{1}{4}$
16 $x = -2$ **17** $y = 5$ **18** $y = -6$
19 $x = 24$ **20** $x = -5$ **21** $z = 3$
22 $y = 2$ **23** $v = -3$ **24** $n = 2\frac{1}{2}$

Exercise 5f (p. 27)

1 9 **2** 15 **3** 6
4 7; 8 **5** 13; 15 **6** $16\frac{1}{2}$ m
7 72 kg **8** 17 **9** 8
10 30c **11** 166 **12** 18c

Exercise 5g (p. 28)

1 $x = 15$ **2** $x = 2\frac{1}{2}$ **3** $a = 36$
4 $a = 6$ **5** $z = 10$ **6** $x = 2$
7 $x = 14$ **8** $a = 19$ **9** $a = -3$
10 $y = 19$ **11** $n = 5$ **12** $a = 3$
13 $x = 2$ **14** $x = -3$ **15** $x = 8$
16 $x = 2$ **17** $z = \frac{1}{2}$ **18** $x = -14$
19 $x = 12$ **20** $x = 4$ **21** $m = 6$
22 $x = 1\frac{2}{3}$ **23** $x = 7$ **24** $t = 3$
25 $e = 2$ **26** $d = \frac{3}{4}$ **27** $m = 1$
28 $a = 5\frac{1}{3}$ **29** $a = 5$ **30** $x = 4$

Exercise 5h (p. 29)

1 15 **2** 32 **3** 7
4 5 **5** 40; 42 **6** (a) $x + 24$ (b) 12
7 36c, 45c **8** (a) $\frac{1}{2}x$ (b) $\frac{1}{3}x$ (c) \$96
9 (a) $\frac{4d}{5}$ (b) $\frac{3d}{4}$ (c) $d = 30$
10 (a) $y - 3$ years (b) $y + 4$ years (c) 17 years

Exercise 5i (p. 30)

1 $x = 5$ **2** $r = 9$ **3** $m = 4$
4 $y = 3\frac{1}{2}$ **5** $s = \frac{2}{3}$ **6** $n = -\frac{4}{11}$
7 $t = 3\frac{3}{8}$ **8** $z = 3\frac{1}{3}$ **9** $p = 4\frac{1}{2}$
10 $a = 1\frac{1}{6}$ **11** $x = 2\frac{1}{2}$ **12** $q = -2\frac{1}{3}$
13 $b = 6$ **14** $y = 5$ **15** $r = 8$
16 $c = \frac{5}{8}$ **17** $d = 15$ **18** $s = -\frac{3}{4}$
19 $z = \frac{8}{15}$ **20** $r = 5\frac{1}{4}$ **21** $t = \frac{2}{3}$
22 $x = 3$ **23** $y = 7\frac{1}{2}$ **24** $d = 6$
25 $f = 3$ **26** $x = 1\frac{1}{2}$ **27** $h = 7$
28 $x = 9$ **29** $x = 4$ **30** $x = \frac{3}{4}$

Exercise 5j (p. 32)

1 (a) $\frac{15}{n}$ kg (b) 20 fish

2 (a) $\$\frac{67,2}{x}$ (b) 7 watches

3 (a) $\frac{3}{v}$ hours (b) $5\frac{1}{3}$

4 82 books
5 35 mangoes
6 48 km/h
7 (a) $\frac{180}{x}$ (b) $\frac{180}{4x}\left(\text{or } \frac{45}{x}\right)$ (c) 9 cents
8 \$24

9 (a) $\frac{8}{v}$ (b) $\frac{15}{2v}$ (c) 6 km/h
10 45 km/h

11 (a) $\frac{14,5}{n}$ kg (b) $\frac{21}{2n}$ kg (c) 40 oranges
12 28 fish

Exercise 5k (p. 33)

1 $x = 5$ **2** $x = 3$ **3** $y = 1\frac{1}{2}$
4 $t = 3\frac{1}{2}$ **5** $z = -3$ **6** $r = -2$
7 $x = -9$ **8** $k = 2\frac{1}{2}$ **9** $a = 1$
10 $x = \frac{3}{5}$ **11** $x = 2$ **12** $a = 10$
13 $y = 7$ **14** $b = -9$ **15** $e = -6$
16 $c = 12\frac{1}{2}$ **17** $n = 4\frac{1}{6}$ **18** $d = -1$
19 $a = \frac{1}{2}$ **20** $x = 6$

Exercise 6a (p. 35)

1 (a) 1:3 (b) 1:2 (c) 3:5

2 (a) 9 cm (b) 4 cm (c) 4 cm
(d) 7,3 cm (e) 13 cm (f) 15 cm
(g) 4,5 cm (h) 7,5 cm (i) 15,3 cm
(j) 14,3 cm

3 (a) 60 m (b) 55 m (c) 10 m
(d) 75 m (e) 820 m (f) 18,6 m
(g) 430 km (h) 7,4 km (i) 2,26 m

4 (a) building: 1 cm to 1 m
running track: 1 cm to 20 m
window: 1 cm to 20 cm
roundabout: 1 cm to 10 m
(b) $w = 4$ m, $l = 10$ m, $h = 120$ cm, $d = 36$ m

Exercise 6b (p. 38)

1 15 m 2 68 m 5 210 m

6 AC = 12,2 m, XK = 10,8 m, PH = 8,5 m, QY = 7,8 m

7 64 m 8 $121\frac{1}{2}°$ 9 8,9 m

10 PC = 37 m

11 AD = 124 m

12 approximately 47 m

Exercise 6c (p. 40)

1 (a) 360 km (b) 650 km (c) 210 km
(d) 220 km (e) 190 km (f) 300 km
(g) 490 km (h) 60 km

2 (a) Botswana Road, Mozambique Road
(b) A1 trunk road, Airport Road
(c) Zambia Road, Azania Crescent, A6 trunk road
(d) Zambia Road, Azania Crescent
(e) Mozambique Road, Azania Crescent
(f) Azania Crescent, Mozambique Road
(g) Botswana Road, Namibia Avenue, A1, Airport Road
(h) Azania Crescent then as (g)
(i) Mozambique Road then as (g)
(j) Botswana Road, Azania Crescent

3 (a) 1 500 m (b) 2 400 m (c) 2 750 m
(d) 750 m (e) 1 000 m (f) 1 750 m
(g) 2 600 m (h) 100 m (i) 150 m
(j) 2 100 m

4 (a) (i) 1 900 m (ii) 2 750 m
(b) (i) 3 250 m (ii) 7 500 m
(c) (i) 1 400 m (ii) 2 000 m
(d) (i) 1 650 m (ii) 2 750 m
(e) (i) 3 500 m (ii) 4 250 m

Exercise 6d (p. 41)

1 (a) 3 (b) brown
(c) yellow and green
(d) blue (e) live (L)
(f) 2 (g) 5,1 cm

2 (a) 6 (b) living room
(c) kitchen (d) 3
(e) living room (f) 11
(g) bedroom 3 (h) living room, kitchen
(i) cupboards (j) 8 m; 4 m
(k) $5\frac{1}{2}$ m; 3 m (l) 123 m^2

Exercise 7a (p. 43)

1 (a) \$30, \$17,50, \$45, \$41, \$13, \$35,50
(b) 7 m, 8 m, 4,2 m, 1,8 m, 2,5 m, 9,3 m

2 (b) (i) 37,5 km/h, (ii) 5,3 s

3 (a)

time (min)	0	1	2	3	4	5	6
distance (m)	0	100	200	300	400	500	600

(c) (i) 570 m, (ii) 3,35 min

4 (a)

length (m)	1	2	3	4	5	6
cost (\$)	6	12	18	24	30	36

(c) (i) \$22,80, (ii) 2,3 m

5 (a)

petrol (litres)	0	10	20	30	40	50
distance (km)	0	70	140	210	280	350

(c) (i) approx. 155 km, (ii) approx. 33 litres

6 (a)

petrol (litres)	0	10	20	30	40	50	60
cost (\$)	0	4	8	12	16	20	24

(c) (i) \$8,80, (ii) 37,5 litres

7 (a)

sugar (kg)	1	2	3	4	5	6
cost (\$)	0,80	1,60	2,40	3,20	4,00	4,80

(c) (i) \$2,00, (ii) 3,75 kg

8 (a)

time (h)	0	1	2	3	4	5
distance (m)	0	− 7,5	− 15	− 22,5	− 30	− 37,5

(c) (i) 3,3 h (ii) − 11 km

9 (a)

distance (km)	3	6	9	12	15
time (min)	2	4	6	8	10

(c) (i) 5,2 km, (ii) 6,7 min

10 (a)

time (h)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
distance (km)	9	18	27	36	45	54

(c) (i) 29 km, (iii) 2,2 h

Exercise 7b (p. 45)

1 (a)

tickets	0	1	2	3	4	5
cost ($)	0	2	4	6	8	10

2 (a)

pills	10	20	30	40	50	60
cost (c)	80	160	240	320	400	480

(d) $1,36

3 (a)

sides	3	4	5	6	7
angle sum	180	360	540	720	900

4 (a)

tyres	1	2	3	4	5
cost ($)	96	192	288	384	480

Exercise 7c (p. 47)

1 (b) (i) 1,8°C, (ii) 43 min

2 (b) (i) 182 mm, (ii) 720 g

3 (a)

week	0	1	2	3	4	5	6
mass (kg)	3,4	3,7	4,0	4,3	4,6	4,9	5,2

(c) 37 days (5,3 weeks)

4 (a)

petrol (kℓ)	1	2	3	4	5
delivery ($) basic ($)	60 280	60 560	60 840	60 1 120	60 1 400
total ($)	340	620	900	1 180	1 460

(c) (i) $1 320, (ii) approx. 3,4 kℓ

5 (a)

area (m^2)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
handling ($) basic ($)	5 8	5 16	5 24	5 32	5 40
total ($)	13	21	29	37	45

(c) (i) $1\frac{7}{20}$ m^2, (ii) $26,60

Exercise 7d (p. 50)

1 (a) £50 (b) £75 (c) £95 (d) £12,50
2 (a) $55 (b) $90 (c) $115 (d) $52,50
3 (a) 4 min (b) 500 m
(c) (i) 1 250 m, (ii) 1 750 m
(d) 2 100 m (e) $14\frac{1}{2}$ min
4 (a) 50% (b) 40% (c) 90% (d) 60%
(e) 33% (f) 67% (g) 17% (h) 83%
(i) 13% (j) 43% (k) 53% (l) 97%
(m) 25% (n) 75% (o) 38% (p) 82%
5 (a) 24 (b) 6 (c) 9 (d) 21
(e) 16 (f) 14 (g) 19 (h) 4
6 (a) (i) $2\frac{1}{2}$ h (ii) 3,2 h
(b) 80 km/h, 62,5 km/h
(c) $\frac{185 - 60}{3 - 1} = \frac{125}{2} = 62{,}5$ km/h
7 (a) 45 km (b) A: 120 km; B: 137,5 km
(c) $37\frac{1}{2}$ km (approx.) (d) A: 0,6 h; B: 0,8 h
8 (a) 1100 (b) 1230 (c) $\frac{1}{4}$ hour (d) 2 km
(e) $\frac{1}{2}$ hour (f) 4 km (g) $\frac{3}{4}$ hour (h) 5 km
9 (a) 1200 (b) 1230 (c) 2,7 km (d) 1,7 km
10 (a) 4 km/h (b) $5\frac{1}{3}$ km/h (c) 4 km/h
(d) 12 km/h

Exercise 8a (p. 53)

1 (a) direct (b) direct (c) inverse (d) direct
(e) inverse (f) inverse (g) direct (h) inverse

2 (a) directly (b) $2,16

3 (a) inversely (b) 10 cups

4 (a)

length (m)	2	4	8
cost ($)	27	54	108

(b) directly

5 (a)

cost ($)	2	3	6
number	15	10	5

(b) inversely

6 (a)

mass (g)	200	125	50
number	10	16	40

(b) inversely

7 (a) 10 pieces (b) $\frac{1}{2}$ m (50 cm)

8 (a)

time (h)	2	5	10
speed (km/h)	25	10	5

(b) $16\frac{2}{3}$ km/h

9 (a)

speed (km/h)	10	15	20
time (h)	3	2	$1\frac{1}{2}$

(b) 1 h 40 min ($1\frac{2}{3}$ h)

10 (a)

length (m)	5	8	20
number	8	5	2

(b) $2\frac{1}{2}$ m

11 72 km **12** 30 **13** 21
14 1 680 **15** 250 **16** 50 min ($\frac{5}{6}$ h)

6 (a)

speed (km/h)	30	60	90
time (h)	4	2	$1\frac{1}{3}$
$\frac{1}{\text{time}}$	0,25	0,5	0,75

(c) $2\frac{2}{3}$ h

7 (a)

speed (km/h)	6	12	60
time (h)	10	5	1
$\frac{1}{\text{time}}$	0,1	0,2	1

(c) (i) 4 h
(ii) 8 km/h

8 (a)

time (h)	8	4	2
speed (km/h)	10	20	40
$\frac{1}{\text{speed}}$	0,1	0,05	0,025

(c) (i) 16 km/h
(ii) 3,2 h
(ii) 3,2 h

9 (a)

price/item	30c	60c	100c
n	20	10	6
$\frac{1}{n}$	0,05	0,1	0,17

(c) (i) 25
(ii) 40c

10 (a)

share ($)	2	4	5
n	10	5	4
$\frac{1}{n}$	0,1	0,2	0,25

(c) (i) 8
(ii) $2,22

Exercise 8b (p. 56)

1 (a) 74 yen, 161 yen
(b) 34 cents, 80 cents

2 (b) 17,5 km; 52,5 km; 63 km

3 (b) 2 h; 1,6 h; 1,2 h

4 (a) $10,67; $31,04; $43,65
(b) 19, 27, 41 litres

5 (a) 94 g, 163 g
(b) 39 cm^3, 82 cm^3

Exercise 8a (p. 53)

1 (a) direct (b) direct (c) inverse (d) direct
(e) inverse (f) inverse (g) direct (h) inverse

2 (a) directly (b) \$2,16

3 (a) inversely (b) 10 cups

4 (a)

length (m)	2	4	8
cost (\$)	27	54	108

(b) directly

5 (a)

cost (\$)	2	3	6
number	15	10	5

(b) inversely

6 (a)

mass (g)	200	125	50
number	10	16	40

(b) inversely

7 (a) 10 pieces (b) $\frac{1}{2}$ m (50 cm)

8 (a)

time (h)	2	5	10
speed (km/h)	25	10	5

(b) $16\frac{2}{3}$ km/h

9 (a)

speed (km/h)	10	15	20
time (h)	3	2	$1\frac{1}{2}$

(b) 1 h 40 min ($1\frac{2}{3}$ h)

10 (a)

length (m)	5	8	20
number	8	5	2

(b) $2\frac{1}{2}$ m

11 72 km **12** 30 **13** 21

14 1 680 **15** 250 **16** 50 min ($\frac{5}{6}$ h)

6 (a)

speed (km/h)	30	60	90
time (h)	4	2	$1\frac{1}{3}$
$\frac{1}{\text{time}}$	0,25	0,5	0,75

(c) $2\frac{1}{2}$ h

7 (a)

speed (km/h)	6	12	60
time (h)	10	5	1
$\frac{1}{\text{time}}$	0,1	0,2	1

(c) (i) 4 h
(ii) 8 km/h

8 (a)

time (h)	8	4	2
speed (km/h)	10	20	40
$\frac{1}{\text{speed}}$	0,1	0,05	0,025

(c) (i) 16 km/h
(ii) 3,2 h
(ii) 3,2 h

9 (a)

price/item	30c	60c	100c
n	20	10	6
$\frac{1}{n}$	0,05	0,1	0,17

(c) (i) 25
(ii) 40c

Exercise 8b (p. 56)

1 (a) 74 yen, 161 yen
(b) 34 cents, 80 cents

2 (b) 17,5 km; 52,5 km; 63 km

3 (b) 2 h; 1,6 h; 1,2 h

4 (a) \$10,67; \$31,04; \$43,65
(b) 19, 27, 41 litres

5 (a) 94 g, 163 g
(b) 39 cm^3, 82 cm^3

10 (a)

share (\$)	2	4	5
n	10	5	4
$\frac{1}{n}$	0,1	0,2	0,25

(c) (i) 8
(ii) \$2,22

Revision exercise 1 (p. 58)

1 (a) 41; 48; 55; 62; … (b) 26; 33; 41; 50; …
(c) 48; 96; 192; 384; … (d) 30; 42; 56; 72; …
2 14
3 (a) 1, 2, 3, 4, 6, 8, 12, 24
4 left hand end 133, 91
right hand end 73, 111
5 98, 63, 36, 17, 18, 19, 20, 21, 44, 75
95, 60, 33, 14, 3, 2, 9, 24, 47, 78
88, 55, 30, 13, 14, 15, 16, 17, 38, 67
6 (a) 4 750 000 (b) 860 000 (c) 22 500 000 000
7 (a) 1,75 (b) 6,3
8 HCF: 42, LCM: 420
9 5 and 8 only **10** 2,94

Revision test 1 (p. 58)

1 D **2** B **3** C **4** E **5** C
6 (a) $(n+1)^2 - (n+1)$ (b) 1 001 000
7 $1\ 296 = 2^4 \times 3^4$, $\sqrt{1\ 296} = 2^2 \times 3^2 = 36$
8 (a) 5 250 000 (b) 999 000 000
9 330, 33
10 (a) 10 (b) 150

Revision exercise 2 (p. 59)

1 (a) (i) {2; 3; 4; 5; 6; 8}, (ii) {2; 8}
(b) empty set
(c) (i) 4, (ii) 1, (iii) 5
3 (a) {0} (b) {− 1; − 2; − 3; − 4; …}
(c) any common fraction would do
4 28
5 (a) $x = -4$ (b) $n = -4$ (c) $d = 7$
(d) $m = \frac{3}{8}$ (e) $a = \frac{3}{2}$ (f) $x = 6$
6 (a) $a = 2$ (b) $x = 3$ (c) $x = 9$ (d) $a = 4\frac{1}{2}$
7 \$44; \$61
8 14; 16
9 (a) $r = 25$ (b) $x = 2\frac{2}{3}$ (c) $m = 5$
(d) $x = 2\frac{1}{2}$ (e) $x = 3\frac{1}{2}$ (f) $a = -3$
10 (a) $\frac{6}{v}$ (b) $v = 4\frac{1}{2}$

Revision test 2 (p. 59)

1 E **2** E **3** B **4** D **5** C
6 (a) {5; 6; 7; 8; 9} (b) {9}
(c) {5; 6; 10; 15} (d) {10}
8 (a) $x = -6$ (b) $a = 2\frac{1}{3}$ (c) $n = -10$
(d) $d = 3\frac{1}{3}$ (e) $m = 4$ (f) $y = -2$
(g) $x = \frac{2}{3}$ (h) $a = 28$ (i) $y = 2$
(j) $z = 6$
9 30 yr; 3 yr
10 (a) $x = 4$ (b) $x = 5$

Revision exercise 3 (p. 60)

1

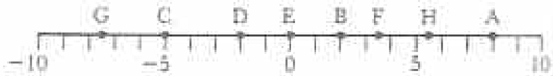

Fig. A5

2 P(3; 4), Q(0; 3), R(1; 2), S(3; 0), T(2; − 1),
U(0; − 2), V(− 2; − 4), W(− 4; − 1),
X(− 3; 1), Y(− 2; 3), Z(− 4; 4)
3 E, F, C, B, G, A, H, D respectively
4 (b) parallelogram (c) (0; ½)
5 (c) CUP
6 (a) 11,9 cm (b) 4,17 h (c) 12,5 h
7 (a) 50 km (b) 40 min
(c) (i) 75 km/h, (ii) 100 km/h

8 (a)

x	0	45	90	135	180
y	180	135	90	45	0

(c) (i) $y = 140$, (ii) $x = 52$
10 (a) \$280 (approx.) (b) \$155

Revision test 3 (p. 62)

1 C **2** E **3** C **4** A **5** C
6 $100 < x \leq 250$
7 (a) (2; 1) (b) 90° (c) rhombus

8 (a)

time (h)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
distance (km)	$1\frac{1}{2}$	3	$4\frac{1}{2}$	6	$7\frac{1}{2}$	9	$10\frac{1}{2}$	12

9 (a) 6,9 km (b) 1,67 h (1 h 40 min)
10 (a) (i) £2,00 (ii) £20
(c) (i) £16,00 (ii) \$70,00

Revision exercise 4 (p. 63)

1 126 km **2** 7,9 cm **3** 2:5
4 225 km **5** 35 m **6** 525 km
7 36 min
8 (a) inversely (b) 66 buckets
9 2,6 hours
10 (a) \$225 000, \$618 750
(b) 6 400 m^2, 8 890 m^2

Revision test 4 (p. 63)

1 E **2** C **3** B **4** B **5** C
6 15 cm **7** (a) 4,8 cm (b) 3,5 m
8 72 cm **9** 6 **10** $3\frac{3}{4}$ hours

General revision test A (p. 64)

1 E **2** D **3** E **4** B **5** C
6 A **7** C **8** C **9** E **10** B
11 (a) 45 (b) 50
12 (a)

$	3	5	6
n	30	18	15
$1/n$	0,033	0,056	0,067

(c) (i) 20 (ii) $3,75
13 a hammer
14 (a) $127 (b) $3 800 (c) $63
15 (a) $x = 2$ (b) $c = 8$ (c) 3
16 (a) 7.45 am (b) 45 min (c) 16 km/h
(d) 6 min (e) $4\frac{1}{2}$ km
17 (a) $k = 6$ (b) $k = -2\frac{1}{2}$ (c) $x = 4$
18 (a) 210 (b) 0,022 (c) 13,5
19 (a) 0,000 003 (b) 0,001 (c) 21 000 000
(d) 180 000

Exercise 9a (p. 67)

1 (a) $6 < 11$ (b) $-1 > -5$
(c) $0 > -2{,}4$ (d) $-3 < +3$
(e) $x > 12$ (f) $y < -2$
(g) $4 > a$ (h) $a < 4$
(i) $15 < b$ (j) $b > 15$
2 (a) true (b) true
(c) false (d) false
(e) false (f) true
(g) false (h) true
3 (a) > (b) < (c) < (d) >
(e) < (f) > (g) < (h) >

Exercise 9b (p. 68)

1 (a) $h < 5$ (b) $m < 50$
(c) $x > 5$ (d) $t < 5$
(e) $n < 24$ (f) $m < 20$
(g) $s < 100$ (h) $t > 120$
2 (a) $h < 1{,}5$ (b) $c < 800$
(c) $b > 12$ (d) $g > 60$
(e) $p > 28$ (f) $m < 55$
(g) $t > 6$
3 $m > 28$ **4** $y > 7$
5 $x > 15$ **6** $x < 60$
7 (a) length > 6 cm, (b) perimeter > 24 cm
8 (a) length < 7 cm, (c) area < 49 cm^2

Exercise 9c (p. 69)

1 (a) $a \leq 12$ (b) $n \geq 5$
(c) $t \leq 38$ (d) $s \geq 24$
(e) $n < 36$ (f) $v \leq 120$
2 (a) $l \leq 7$ (b) $s \leq 140$
(c) $h \geq 160$ (d) $p \leq 5$
(e) $d \geq 6$ (f) $g \geq 100$
3 $x < 27$, $y \geq 27$
4 $a < 12$, $b \geq 12$, $a < b$
5 (a) circumference $\leq 6\pi$, (b) area $\leq 9\pi$

Exercise 9d (p. 69)

1 (a) $x < 3$ (b) $x > 2$
(c) $x \geq -2$ (d) $x \leq 5$
(e) $x > -4$ (f) $x \leq -4$
2

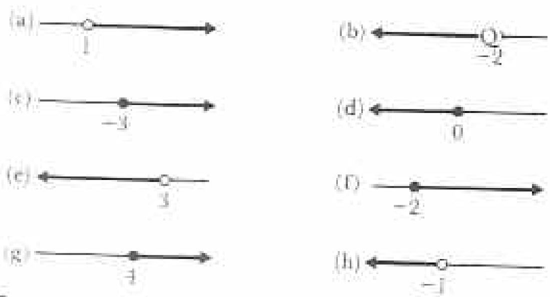

Fig. A6

Exercise 9e (p. 70)

2 (a) $\{(x; y) : x \geq 1\}$ (b) $\{(x; y) : y > -1\}$
(c) $\{(x; y) : x < -2\}$ (d) $\{(x; y) : y \leq 5\}$
4 (a) $\{(x; y) : x < 5\} \cap \{(x; y) : y \leq 7\}$
(b) $\{(x; y) : x \geq -3\} \cap \{(x; y) : y > -4\}$

Exercise 9f (p. 72)

1 (a) $x < 5$ (b) $x \geq 3$
(c) $x < 6$ (d) $x > 5$
(e) $x \leq -6$ (f) $x < -5$
(g) $x < 3$ (h) $x \geq 9$
(i) $x \leq 4$ (j) $x > -2$
(k) $x \geq -2\frac{1}{4}$ (l) $x \geq 2\frac{2}{3}$
(m) $x < 4$ (n) $x \geq 2$
(o) $x < -5$ (p) $x \geq -7$
(q) $x > 5\frac{1}{4}$ (r) $x \geq -\frac{2}{3}$
2 (a) $\{5; 6; 7; \ldots\}$ (b) $\{2; 1; 0; \ldots\}$
(c) $\{-3; -4; -5; \ldots\}$
(d) $\{-3; -2; -1; \ldots\}$
(e) $\{0; -1; -2; \ldots\}$ (f) $\{-2; -1; 0; \ldots\}$
(g) $\{5; 4; 3; \ldots\}$ (h) $\{4; 5; 6; \ldots\}$
(i) $\{-4; -3; -2; \ldots\}$
(j) $\{-4; -5; -6; \ldots\}$
(k) $\{1; 0; -1; \ldots\}$
(l) $\{-5; -4; -3; \ldots\}$
(m) $\{-1; -2; -3; \ldots\}$
(n) $\{4; 5; 6; \ldots\}$
(o) $\{2; 1; 0; \ldots\}$

(p) $\{-4; -3; -2; \ldots\}$
(q) $\{-2; -3; -4; \ldots\}$
(r) $\{3; 2; 1; \ldots\}$

Exercise 9g (p. 73)

1 $x > -4$
2 $a > 2$
3 $m \geq -3$
4 $d \leq -8$
5 $y \geq -4$
6 $z \leq 4$
7 $a < 2$
8 $n \geq -1$
9 $r \leq -2$
10 $t \leq -1\frac{1}{3}$

Exercise 9h (p. 73)

1 $x > 8$
2 $y < 10,7$
3 $n \leq 18$
4 5; 6; 7
5 1; 0; −1; −2
6 $x < 280$
7 19
8 $x \geq 12$
9 $h < 4$
10 x has a value between 3 and 6
11 b has a value from 1 to 17
12 4
13 (a) 5 (b) 7
14 over 2 h
15 over 21 km/h

Exercise 10a (p. 76)

1 the ratios in parts (a) and (b) should be the same
2 the angles in one triangle are equal to the angles in the other
3 in each case the ratio is 5:2
4 (a) all angles = 90°

(b) $\frac{AB}{BC} = \frac{4}{3}, \frac{PQ}{QR} = \frac{8}{1}$; the ratios are different

5 (a) it is unlikely that they will be similar
6 it is unlikely that these cuboids will be similar
8 (a), (d) similar
(b), (c), (e) not similar

Exercise 10b (p. 78)

1 (a) $\triangle$CBA, $104\frac{1}{2}$°, 6 cm, $4\frac{1}{2}$ cm
(b) $\triangle$EFD, 37°, 8 m, 10 m, 53°
(c) $\triangle$HKG, $58\frac{1}{2}$°, 35°, $86\frac{1}{2}$°

2 $\frac{AB}{PQ} = \frac{8,7}{10,7} \simeq 0,8; \frac{AC}{PR} = \frac{4,7}{5,8} \simeq 0,8$

3 (a) $\frac{AB}{BC} = \frac{6,5}{6} \simeq 1,1; \frac{PQ}{QR} = \frac{9,8}{9} \simeq 1,1$

(b) $\frac{AC}{BC} = \frac{3,8}{6} \simeq 0,63; \frac{PR}{QR} = \frac{5,7}{9} \simeq 0,63$

(c) $\frac{AB}{AC} = \frac{6,5}{3,8} \simeq 1,7; \frac{PQ}{PR} = \frac{9,8}{5,7} \simeq 1,7$

4 37°, 53°, 90° in both triangles; the triangles are similar
5 $13\frac{1}{3}$ m, 9 m
6 $12\frac{1}{2}$ cm, 5 cm
7 $\triangle$XOP, $5\frac{1}{4}$ m, 3 m
8 10 cm, $3\frac{3}{5}$ cm
9 $\triangle$OKH, $4\frac{4}{5}$ cm, $4\frac{1}{5}$ cm
10 $10\frac{1}{2}$ m, $1\frac{1}{2}$ m

Exercise 10c (p. 80)

1 the rectangles in set (b) are similar
2 20 cm
3 no: $\frac{AB}{BC} = \frac{1}{3}, \frac{WX}{XY} = \frac{4}{9}$; the ratios are different
4 10 cm long, $17\frac{1}{2}$ cm wide
5 18 cm
6 all cubes are similar
7 6,5 cm
8 1,8 m wide; 2,4 m long
9 18 cm or 2 cm
10 true: (a), (c), (d), (f) false: (b), (e), (g)

Exercise 11a (p. 81)

1 (a) 11 (b) 9 (c) 5 (d) 8
(e) 6 (f) 4 (g) 7 (h) 12
(i) 4 (j) 4,3
2 (a) 4,5 cm (b) $8,20 (c) 4,8 kg
(d) $3\frac{1}{4}$ (e) 0,63
3 38,31 **4** 3 **5** 36 **6** 25°C
7 (a) 15 mm (b) 3 mm
8 27 goals **9** 28 years
10 approximately 13 men
11 (a) 22 marks (b) 55%
12 (a) 10,7(3) hours
(b) advertisement (ii) is accurate; advertisement (i) is not accurate since some batteries do not last 10 hours.

Exercise 11b (p. 83)

1

	(a)	(b)	(c)	(d)	(e)
mode	7	5	12	7	10
median	9	6	11	7	10
mean	10	$6\frac{1}{2}$	10	6,3	10,4

2

	(a)	(b)	(c)	(d)	(e)
mode	4	5	0	7	6
median	$3\frac{1}{2}$	6	3	7	$4\frac{1}{2}$
mean	$3\frac{1}{4}$	6	5	8	$4\frac{1}{2}$

3 (a) 17 students (b) grade B (c) grade C
4 (a) size 7 (b) size $7\frac{1}{2}$
5 mode is 1 km; median is 2 km; mean is 2,3 km
6 (a) 15 girls
(b) modal age is 16 years and median age is 16 years
(c) mean is $15\frac{8}{15}$ years

Exercise 11c (p. 85)
1 mean **2** median **3** mode
4 (a) 8; 7; 7 (b) 4; $3\frac{1}{2}$; 3
(c) $4\frac{1}{3}$; $4\frac{1}{2}$; 1 (d) 5; 4; 3
(e) 0,55; 0,35; 0 (f) 156,4; 155,8; 155,8
5 (a) $71,19 (b) $69,27 **6** 83,3kg
7 (a) 1,15 kg (b) 1 kg **8** 23 yr 3 mo
9 59 kg **10** 82 c/kg **11** 32,5 mm
12 $6 000 **13** 12.5; 12.8; 9.1
14 48 km/h **15** 50 km/h **16** 64 km/h
17 46 km/h **18** 61,4
19 (a) 24 (b) 4 (c) 4 (d) 3,5
20 (a) Rudo 58, Sola 67, Tembo 61, Urban 28, Vera 41
(b) Eng. 52, Hist. 52, Maths 49, Sci. 51
(c) Eng. 52, Hist. 60, Maths 49, Sci. 48

Exercise 11d (p. 87)
1 (a) 150,2 (b) 37,9 (c) $11,85
(d) 1 h 6 min (e) 16.2
2 (a) 226 (b) 121 (c) 33,5
(d) 14.1 (e) $3,81

Exercise 11e (p. 89)
1 (a) bicycles (b) motor bikes
(c) 85 (d) 25
2 (a) 5 900 m (b) 6 200 m
(c) Blanc (d) 1 800 m
3 (a) $\frac{1}{4}$ (b) 25% (c) 25 min
4 (a) 37,5°C (b) 40,5°C (c) 5 (d) 0500

5

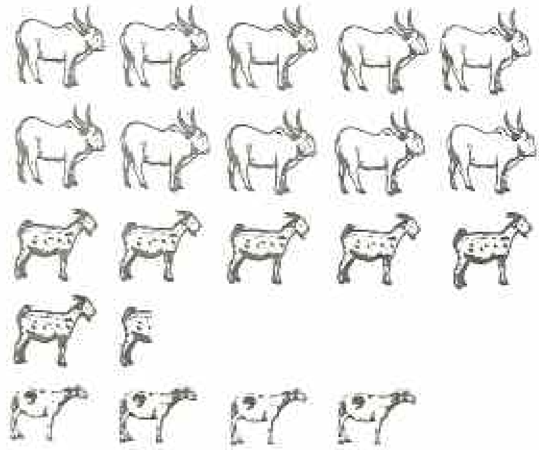

Fig. A7

6

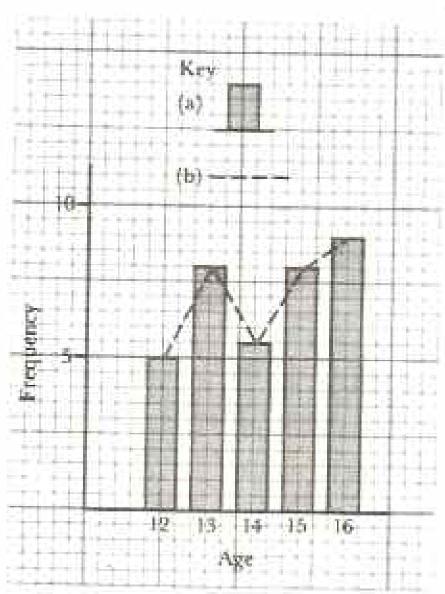

Fig. A8

age (years)	12	13	14	15	16
frequency	5	8	6	8	9

7

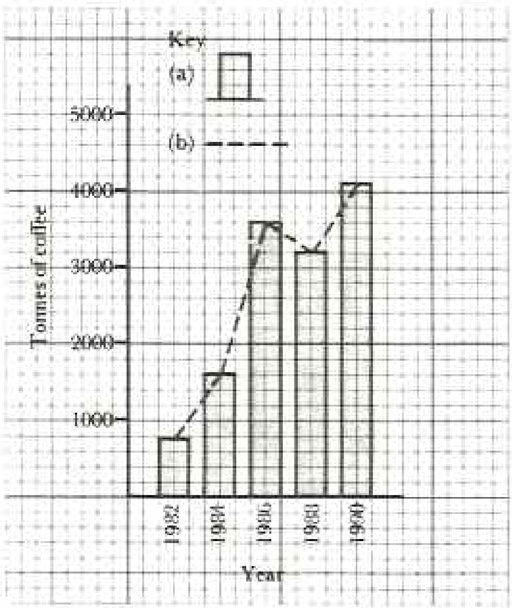

Fig. A9

8

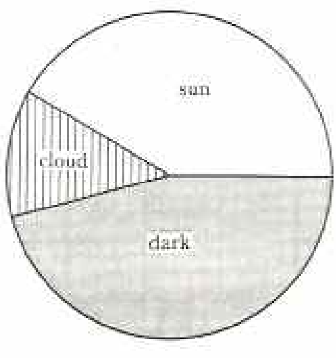

Fig. A10

Exercise 12a (p. 92)

2 (a) square, square, triangle, kite, quadrilateral

Exercise 12b (p. 94)

1 (a) $\triangle$XAZ (*RHS*) (b) $\triangle$BYZ (*SAS*)
(c) $\triangle$YXC (*SSS*) (d) $\triangle$ZDX (*AAS*)
(e) $\triangle$XEZ (*SAS*) (f) $\triangle$GHF (*SAS*)
(g) $\triangle$JZY (*ASA*) (h) $\triangle$XLK (*SAS*)
(i) $\triangle$NMZ (*SAS*) (j) $\triangle$TYS (*AAS*)

2 (a) congruent (*ASA*) (b) congruent (*SAS*)
(c) not congruent (d) congruent (*SSS*)
(e) not necessarily congruent
(f) congruent (*RHS*) (g) not congruent
(h) not necessarily congruent
(i) congruent (*SAS*) (j) congruent (*AAS*)

3 (a) AC = PR, $\hat{A} = \hat{P}$, $\hat{C} = \hat{R}$ (*SAS*)
(b) AC = RP, BC = QP, $\hat{A} = \hat{R}$ (*AAS*)
(c) not congruent
(d) $\hat{A} = \hat{Q}$, $\hat{B} = \hat{P}$, $\hat{C} = \hat{R}$(*SSS*)
(e) AB = PR, $\hat{B} = \hat{R}$, $\hat{C} = \hat{Q}$(*RHS*)
(f) not necessarily congruent
(g) not congruent
(h) AB = RQ, AC = RP, $\hat{A} = \hat{R}$ (*ASA*)
(i) AB = QP, $\hat{A} = \hat{Q}$, $\hat{B} = \hat{P}$ (*SAS*)
(j) AB = PR, AC = PQ, $\hat{B} = \hat{R}$ (*AAS*)

Exercise 12c (p. 96)

1 (a) $a = 40°$ (b) $d = 65°$
(c) $x = 124°$ (d) $m = 51°$
(e) $n = 72°$ (f) $c = 140°$
(g) $u = 28°$, $v = 152°$ (h) $a = 63°$, $b = 117°$
(i) $m = 70°$, $n = 40°$ (j) $x = 76°$, $w = 38°$

2 70° **3** 61° **4** $31\frac{1}{2}°$
5 111° **6** $127\frac{1}{2}°$ **7** 78°
8 62° **9** 106° **10** 90°
13 30° **14** 20°, 80°, 80° **15** 60°
16 32° **17** 30° **18** 115°

Exercise 12e (p. 100)

1 9

2 (a) ABCD, QAPC, AYCX
(b) $\triangle$QAX ≡ $\triangle$PCY, $\triangle$ABY ≡ $\triangle$CDX, $\triangle$QBC ≡ $\triangle$PDA

3 8 cm **4** 4 cm

5 132°, 48°, 132°, 48°

6 32,5 cm

7 rhombus, rectangle **8** $22\frac{1}{2}$

Exercise 13a (p. 101)

1 $2x + 2y$ **2** $35 - 5a$
3 $3n + 27$ **4** $16a - 8b$
5 $-5x - 15y$ **6** $-12p + 4q$
7 $-2m - 2n$ **8** $-3a + 3b$
9 $-4p - 4q$ **10** $-21d + 14$
11 $18k + 27r$ **12** $42s - 6t$
13 $x^2 + 2x$ **14** $y^2 - y$
15 $a^2 + ab$ **16** $3n^2 - 2n$
17 $2ps + 3pt$ **18** $5m - 3mn$
19 $10a^2 - 16ab$ **20** $3x^2 + 27x$
21 $45pr - 40ps$ **22** $-12a^2 + 42ab$
23 $9ab - 12b^2$ **24** $2\pi r^2 + 2\pi rh$

Exercise 13b (p. 101)

1 5 **2** 3 **3** mp **4** $5x$
5 $4a$ **6** $13b$ **7** ab **8** $3de$
9 $8p$ **10** $2ax$ **11** 3 **12** $2a$

Exercise 13c (p. 102)

1 $5(a+z)$
2 $3(2x-5y)$
3 $mp(7n-1)$
4 $5x(y+3)$
5 $4a(3+2a)$
6 $13b(a-2)$
7 $ab(b-a)$
8 $3de(2d-e)$
9 $8p(q+3p)$
10 $2ax(5x+7a)$
11 $3(3xy+8pq)$
12 $2a(15d-14x)$
13 $5m(a-4b)$
14 $a^2(5a-3b)$
15 $\pi r(r+s)$
16 $d(7d-1)$
17 $3d(11b-e)$
18 $3(3pq+4t)$
19 $b(a-2)$
20 $3d(h+5k)$
21 $x(x+9y)$
22 $2a(a+5)$
23 $a(m+1)$
24 $6xy(4x-1)$

Exercise 13d (p. 102)

1 3 400
2 122
3 2 700
4 6 930
5 125
6 44
7 13 400
8 670
9 $3\frac{1}{7}$
10 530
11 30
12 1 400
13 27 000
14 $12\frac{4}{7}$
15 17 400
***16** $\pi(R^2-r^2)$; 176
17 $2\pi r(r+h)$; 660
18 $\pi r^2(h+\frac{1}{3}H)$; 396

Exercise 13e (p. 103)

1 $m(3+u-v)$
2 $a(2-3x-y)$
3 $x(3-a+b)$
4 $p(4m-3n-5)$
5 $(m+1)(a+b)$
6 $(n+2)(a-b)$
7 $x(a-b+4c)$
8 $(a-b)(5x-2y)$
9 $(5u-v)(3h+2k)$
10 $m(u-v+m)$
11 $d(3h+k-4d)$
12 $a(5a+b-c)$
13 $x(4x-3y-2z)$
14 $d^2(3d-e+4f)$
15 $a(4u+v)$
16 $2a(x-3y)$
17 $(3u+2v)(3-a)$
18 $(4a-b)(3x+2y)$
19 $(2a-7b)(h-3k)$
20 $m(5m-2)$
21 $a^2(2a-3b)$
22 $4x(x-1)$
23 $(3m-4n)(2d-3e)$
24 $(x-y)(a+2b-3)$
25 $(2m+n)(p+q-r)$
26 $(h+k)(2r-s)$
27 $(u+v)(4x+y)$
28 $(b-c)(2d+3e)$
29 $(a+2b)(a+2b-3)$
30 $(3m-2n)(3m-2n+5p)$
31 $2(2u-3v)(m-3n)$
32 $(x+2y)(a+x+2y)$
33 $(2x+y)(3u-2x-y)$
34 $(f-g)(4e-f+g)$
35 $3(a-3b)(u+2v)$
36 $5(5m+2n)(a+b)$
37 $(x+3y)(m-n+1)$
38 $(2a-3b)(c+d-1)$
39 $(7u-2v)(1+7u-2v)$
40 $(2u-7v)(2u-7v-1)$

Exercise 13f (p. 104)

1 $(x+y)(a+3b)$
2 $(a+2b)(7+x)$
3 $(x+5)(x+2)$
4 $(p+r)(q+s)$
5 $(a-9)(a+3)$
6 $(4m-1)(2+n)$
7 $(x-2)(5x+3)$
8 $(a-c)(b+d)$
9 $(2b-5)(a+1)$
10 $(3m-1)(1+2m)$

Exercise 13g (p. 104)

1 $(a+c)(b-m)$
2 $(3x+2)(3-x)$
3 $(x-y)(2a-3b)$
4 $(x-7)(x-2)$
5 $(a-b)(5-c)$
6 $(q+4r)(3p-y)$
7 $(a-3)(a-3)$
8 $(2s+5t)(p-r)$
9 $(x-6)(x-1)$
10 $(3k+1)(1-h)$

Exercise 13h (p. 104)

1 $(a+b)(6+m)$
2 $(p+q)(r+s)$
3 $(3+y)(5-x)$
4 $(a-b)(c+d)$
5 $(a+x)(x-y)$
6 $(d-m)(a+c)$
7 $(x-3)(x-5)$
8 $(2a+3y)(4+5b)$
9 $(a-b)(3+c)$
10 $(t+3s)(1+2z)$

Exercise 13i (p. 105)

1 $(m+n)(x+y)$
2 $(x-y)(a+b)$
3 $(u+v)(h-k)$
4 $(a-b)(u-v)$
5 $(a+2b)(m+n)$
6 $(c-d)(x+2y)$
7 no factors
8 $(a-2x)(b-2y)$
9 $(m-n)(a+1)$
10 no factors
11 $(a+1)(a^2+1)$
12 $(h+k)(2m-3n)$
13 $(x-y)(3s+5t)$
14 $(ax+y)(bx+y)$
15 $(h-2m)(k+3n)$
16 no factors
17 $(g+h)(2k-3l)$
18 $(f+2g)(2h-k)$
19 no factors
20 $(h+2k)(l-3m)$
21 $(e-2f)(3c-2d)$
22 $(x-2n)(y+3n)$
23 $(a+2b)(b-2c)$
24 no factors
25 $(4u-v)(2v+3w)$
26 $(m-2n)(n+3p)$
27 $(3x+2y)(y-a)$
28 $3(a-u)(b+v)$
29 no factors
30 $(2c+3d)(4e-f)$
31 $(mu+v)(nu-v)$
32 $5(m-n)(x-y)$
33 $(3a-c)(b-3d)$
34 $(2a-5c)(3b+2d)$
35 $2a(m+n)(u-v)$
36 $(am+2)(bm-3)$
37 $2(2a+b)(x+2y)$
38 $(7m-x)(3n+y)$
39 no factors
40 $(2a-3m)(m+2n)$
41 $(5u-1)(2v+1)$
42 $(a+m)(am-n)$
43 $(xy+a)(2x-y)$
44 $(1-5a)(1+3x)$
45 $(d+2xy)(2dx-3y)$

Exercise 14a (p. 106)

1 \$20
2 \$28
3 \$18
4 \$8
5 \$18
6 \$14
7 \$30
8 \$48
9 \$105
10 \$75
11 \$42
12 \$30
13 \$18
14 \$132
15 \$37,50

Exercise 14b (p. 106)

1 (a) \$7 (b) \$26 (c) \$1 090
(d) \$22 000 (e) \$735 (f) \$1 042,75

Exercise 13c (p. 102)

1 $5(a+z)$ **2** $3(2x-5y)$
3 $mp(7n-1)$ **4** $5x(y+3)$
5 $4a(3+2a)$ **6** $13b(a-2)$
7 $ab(b-a)$ **8** $3de(2d-e)$
9 $8p(q+3p)$ **10** $2ax(5x+7a)$
11 $3(3xy+8pq)$ **12** $2a(15d-14x)$
13 $5m(a-4b)$ **14** $a^2(5a-3b)$
15 $\pi r(r+s)$ **16** $d(7d-1)$
17 $3d(11b-e)$ **18** $3(3pq+4t)$
19 $b(a-2)$ **20** $3d(h+5k)$
21 $x(x+9y)$ **22** $2a(a+5)$
23 $a(m+1)$ **24** $6xy(4x-1)$

Exercise 13d (p. 102)

1 3 400 **2** 122
3 2 700 **4** 6 930
5 125 **6** 44
7 13 400 **8** 670
9 $3\frac{1}{4}$ **10** 530
11 30 **12** 1 400
13 27 000 **14** $12\frac{1}{4}$
15 17 400 **16** $\pi(R^2-r^2)$; 176
17 $2\pi r(r+h)$; 660 **18** $\pi r^2(h+\frac{1}{3}H)$; 396

Exercise 13e (p. 103)

1 $m(3+u-v)$ **2** $a(2-3x-y)$
3 $x(3-a+b)$ **4** $p(4m-3n-5)$
5 $(m+1)(a+b)$ **6** $(n+2)(a-b)$
7 $x(a-b+4c)$ **8** $(a-b)(5x-2y)$
9 $(5u-v)(3h+2k)$ **10** $m(u-v+m)$
11 $d(3h+k-4d)$ **12** $a(5a+b-c)$
13 $x(4x-3y-2z)$ **14** $d^2(3d-e+4f)$
15 $a(4u+v)$ **16** $2a(x-3y)$
17 $(3u+2v)(3-a)$ **18** $(4a-b)(3x+2y)$
19 $(2a-7b)(h-3k)$ **20** $m(5m-2)$
21 $a^2(2a-3b)$ **22** $4x(x-1)$
23 $(3m-4n)(2d-3e)$ **24** $(x-y)(a+2b-3)$
25 $(2m+n)(p+q-r)$
26 $(h+k)(2r-s)$ **27** $(u+v)(4x+y)$
28 $(b-c)(2d+3e)$ **29** $(a+2b)(a+2b-3)$
30 $(3m-2n)(3m-2n+5p)$
31 $2(2u-3v)(m-3n)$ **32** $(x+2y)(a+x+2y)$
33 $(2x+y)(3u-2x-y)$
34 $(f-g)(4e-f+g)$
35 $3(a-3b)(u+2v)$ **36** $5(5m+2n)(a+b)$
37 $(x+3y)(m-n+1)$
38 $(2a-3b)(c+d-1)$
39 $(7u-2v)(1+7u-2v)$
40 $(2u-7v)(2u-7v-1)$

Exercise 13f (p. 104)

1 $(x+y)(a+3b)$ **2** $(a+2b)(7+x)$
3 $(x+5)(x+2)$ **4** $(p+r)(q+s)$
5 $(a-9)(a+3)$ **6** $(4m-1)(2+n)$
7 $(x-2)(5x+3)$ **8** $(a-c)(b+d)$
9 $(2b-5)(a+1)$ **10** $(3m-1)(1+2m)$

Exercise 13g (p. 104)

1 $(a+c)(b-m)$ **2** $(3x+2)(3-x)$
3 $(x-y)(2a-3b)$ **4** $(x-7)(x-2)$
5 $(a-b)(5-c)$ **6** $(q+4r)(3p-y)$
7 $(a-3)(a-3)$ **8** $(2s+5t)(p-r)$
9 $(x-6)(x-1)$ **10** $(3k+1)(1-h)$

Exercise 13h (p. 104)

1 $(a+b)(6+m)$ **2** $(p+q)(r+s)$
3 $(3+y)(5-x)$ **4** $(a-b)(c+d)$
5 $(a+x)(x-y)$ **6** $(d-m)(a+c)$
7 $(x-3)(x-5)$ **8** $(2a+3y)(4+5b)$
9 $(a-b)(3+c)$ **10** $(t+3s)(1+2z)$

Exercise 13i (p. 105)

1 $(m+n)(x+y)$ **2** $(x-y)(a+b)$
3 $(u+v)(h-k)$ **4** $(a-b)(u-v)$
5 $(a+2b)(m+n)$ **6** $(c-d)(x+2y)$
7 no factors **8** $(a-2x)(b-2y)$
9 $(m-n)(a+1)$ **10** no factors
11 $(a+1)(a^2+1)$ **12** $(h+k)(2m-3n)$
13 $(x-y)(3s+5t)$ **14** $(ax+y)(bx+y)$
15 $(h-2m)(k+3n)$ **16** no factors
17 $(g+h)(2k-3l)$ **18** $(f+2g)(2h-k)$
19 no factors **20** $(h+2k)(l-3m)$
21 $(e-2f)(3c-2d)$ **22** $(x-2n)(y+3n)$
23 $(a+2b)(b-2c)$ **24** no factors
25 $(4u-v)(2v+3w)$ **26** $(m-2n)(n+3p)$
27 $(3x+2y)(y-a)$ **28** $3(a-u)(b+v)$
29 no factors **30** $(2c+3d)(4e-f)$
31 $(mu+v)(nu-v)$ **32** $5(m-n)(x-y)$
33 $(3a-c)(b-3d)$ **34** $(2a-5c)(3b+2d)$
35 $2a(m+n)(u-v)$ **36** $(am+2)(bm-3)$
37 $2(2a+b)(x+2y)$ **38** $(7m-x)(3n+y)$
39 no factors **40** $(2a-3m)(m+2n)$
41 $(5u-1)(2v+1)$ **42** $(a+m)(am-n)$
43 $(xy+a)(2x-y)$ **44** $(1-5a)(1+3x)$
45 $(d+2xy)(2dx-3y)$

Exercise 14a (p. 106)

1 \$20 **2** \$28 **3** \$18
4 \$8 **5** \$18 **6** \$14
7 \$30 **8** \$48 **9** \$105
10 \$75 **11** \$42 **12** \$30
13 \$18 **14** \$132 **15** \$37,50

Exercise 14b (p. 106)

1 (a) \$7 (b) \$26 (c) \$1 090
(d) \$22 000 (e) \$735 (f) \$1 042,75

2 (a) $44 (b) $52 (c) $80
3 (a) $5 (b) $1,25 (c) $6\frac{1}{4}\%$
4 (a) $3 020 (b) $251,67 (to nearest c)
5 (a) $540 (b) $3 540 (c) $147,50

Exercise 14c (p. 107)
1 (a) $387 (b) $245 (c) $993,60
(d) $69 (e) $23,96
2 $105 3 $127,16
4 60c 5 33c
6 $11,50, $3,50 ($23\frac{1}{3}\%$)

Exercise 14d (p. 108)
1 $103,50
2 (a) $210, (b) $630, (c) $52,50
3 (a) $128,80, (b) $16,30
4 $200,20
5 $717,66
6 $133,60

Exercise 14e (p. 109)
1 (a) (i) $520, (ii) $680
(b) (i) $1 120, (ii) $1 400
(c) (i) $800, (ii) $1 020
(d) (i) $2 520, (ii) $3 000
(e) (i) $488, (ii) $640
(f) (i) $1 469,72, (ii) $1 806,60
2 $1 074,10
3 (a) $6 000 (b) $748 (c) $860,20
(d) $8 239,80
4 (a) $5 800 (b) $437 (c) $588,58
5 $6 291,65

Exercise 14f (p. 110)
1 (a) (i) 80c (ii) $4,80
(b) (i) $1,50 (ii) $11,50
(c) (i) 30c (ii) $3,45
(d) (i) 18c (ii) $1,26
(e) (i) 45c (ii) $1,20
2 (a) (i) 45c profit (ii) 15%
(b) (i) 45c profit (ii) 25%
(c) (i) $36 loss (ii) 10%
(d) (i) $36 profit (ii) $37\frac{1}{2}\%$
(e) (i) 63c loss (ii) 15%
3 $17\frac{1}{2}\%$ 4 $2,47
5 $233\frac{1}{3}\%$ 6 $3
7 $2,40 8 $2 772
9 the son ($7\frac{1}{2}\%$) 10 12%, $2,61
(father's profit = 7%)
11 $18,75, $24 12 20%
13 $45,60 14 20%
15 3%

Exercise 14g (p. 110)
1 $1 198 2 $523,80
3 $57 4 $25,38
5 $568 6 $379,26
7 $316,32 8 $306,60
9 $6 563,76 10 $426,80

Exercise 14h (p. 111)
1 (a) $9,00 (b) $27,00 (c) $32,63
(d) $21,38 (e) $24,75
2 (a) $9,83 (b) $14,75 (c) $17,21
(d) $33,00 (e) $11,50

Exercise 14i (p. 114)
2 $33,48
3 $30,56
5 $9,11
6 $32,95
7 (a) $29,95 (b) $29,10 (c) $34,50
8 $147,97
9 $175,59
10 $312,89 [i.e. $32,35 (water)
+ $74,23 (electricity)
+ $206,31 (rates)]

Exercise 15a (p. 117)
2 (a) M is the mid-point of AC
(b) PM is the perpendicular bisector of AC
3 (e) the 3 folds meet at a point
(f) each fold is a perpendicular bisector of one of the sides of △ABC
4 (b) the 3 perpendicular bisectors meet at a point
5 (b) both perpendicular bisectors meet at the centre of the circle
6 (b) a diameter
(c) a square
7 (d) MN = $\frac{1}{2}$AC
8 (c) 7,1 cm

Exercise 15b (p. 118)
2 (a) $B\hat{A}R = C\hat{A}R$
(b) the bisector of $B\hat{A}C$
3 (e) the 3 folds meet at a point
4 (d) the 3 bisectors meet at a point
5 (d) in isosceles △XYZ the bisector of $\hat{Y}$ is the same line as the perpendicular bisector of XZ
6 (c) $\frac{1}{4}$
7 (c) 2,7 cm, 3,3 cm
8 (d) octagon
(e) 57 mm

Exercise 15c (p. 120)

3 (b) 117 mm
4 (b) 12,7 cm
5 (b) 7,8 cm
6 (c) AC passes through the centre of the circle, i.e. it is a diameter
7 (b) 5,7 cm
8 (b) 6,9 cm

Exercise 15d (p. 121)

4 (d) 5 cm
5 7,9 cm, 13,1 cm
6 (b) 7,3 cm, 9,5 cm
7 (b) 91 mm, 53 mm
8 (b) 7,4 cm

Exercise 15e (p. 121)

1 (b) 71 mm
2 (d) 4,7 cm; yes
3 (c) 69 mm
4 (d) yes (e) 2:1
5 (b) yes
6 (c) 8,8 cm
7 (c) each 59 mm
8 (c) 6,6 cm
9 (d) 57 mm
10 (b) 52 mm
11 (b) 5,8 cm
12 (b) 7,9 cm

Exercise 16a (p. 123)

1 (a) horizontal (b) vertical
(c) vertical (d) horizontal
(e) neither (f) vertical
(g) horizontal (h) vertical
(i) neither (j) horizontal
2 (a) seat of chair, shelves, book on table, top and bottom edges of door, top of door frame, set square on table
(b) light cord, flag-pole, table legs, drawer fronts, left- and right-hand edges of notice board
(c) door handle, light switch, support bracket for shelves, chair legs, broom handle

Exercise 16b (p. 125)

1 (a) 30° (b) 40° (c) 25° (d) 25°
2 (a) 10° (b) 10°
3 (a) 15° (b) 15°
4 (a) 45° (b) 28°
5 52°

Exercise 16d (p. 127)

1 $8\frac{1}{4}$ m **2** 28 m **3** 7,5 m
4 6 m **5** $30\frac{1}{4}$ m **6** 35 m
7 12 m **8** 350 cm **9** 95 m
10 2 400 m

Revision exercise 5 (p. 128)

1 $9x + 13y < 50$
3 (a) $x < 5$ (b) $x < -3$ (c) $x < 2$ (d) $x \leq 3$
4 (a) $9(a - 3)$ (b) $r(3 - 8t)$
(c) $14x(3x - 2y)$ (d) $3ab(14a - 17b)$
5 (a) $(m + n)(a - 3)$ (b) $(a - 7)(a + 3)$
(c) $(y - 2z)(3x - 5a)$
6 (a) $(3a + 2x)(y - b)$ (b) $(s - t)(d - r)$
7 3 700
9 (a) $\{2; 3; 4; \ldots\}$ (b) $\{-8; -7; -6; \ldots\}$
(c) $\{7; 8; 9; \ldots\}$ (d) $\{-12; -13; -14; \ldots\}$
10 $2 \leq x \leq 11$

Revision test 5 (p. 128)

1 B **2** C **3** D **4** B **5** B
6 (a) $\{2; 1; 0; \ldots\}$ (b) $\{-6; -5; -4; \ldots\}$
(c) $\{4; 3; 2; \ldots\}$ (d) $\{1; 0; -1; -2; \ldots\}$
7 (a) $38 - 3n < 20$ (b) 7; 8; 9; 10
8 (a) $7(x - 4)$ (b) $m(5 + 8n)$
(c) $9b(3a + 4b)$ (d) $7pq(5p - 2q)$
9 (a) $a(5a + b)$ (b) $(a - b)(3x - y)$
(c) $(q - r)(p + 8)$ (d) $(x + y)(5 + k)$
10 (a) $\pi r(r + 2h)$ (b) 176

Revision exercise 6 (p. 129)

1 21,6 cm
2 (a) 20 cm (b) $11\frac{2}{3}$ cm
3 4 cm **4** 4 cm, 25 cm
5 (a) $1\frac{1}{3}$ (b) $5\frac{1}{3}$ cm
7 (a) △ABG (b) SAS
(c) △CMG ≡ △BMG, △AMC ≡ △AMB
8 (a) $A\hat{B}C = 71°$ (b) $M\hat{G}B = 52°$
(c) $A\hat{G}C = 128°$
9 $A\hat{C}G = 24°$

Revision test 6 (p. 130)

1 E **2** E **3** B **4** B
5 B
6 △OXY, XY = 10 cm
7 11°
9 $B\hat{D}E = B\hat{E}D = 83°$
10 $2x° - 180°$

Revision exercise 7 (p. 131)

1 6%
2 (a) $6 916 (b) $8 736 (c) $1 820

3 22 years **4** (a) 30 (b) $22\frac{1}{2}$ years
5 22,63 ($22\frac{24}{38}$) years
6

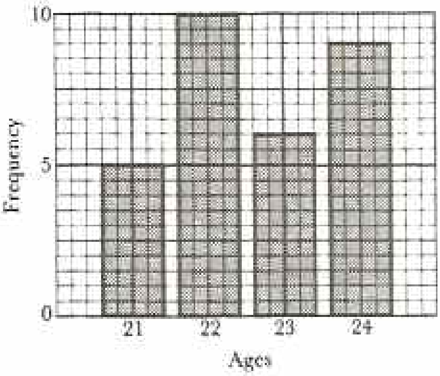

Fig. A11

7 2
8 (a) 23 (b) 22 (c) $22\frac{1}{4}$
9 $192,48
10 (a) 29°C (b) 28°C (c) 28°C
(d) (i) + 5°C (ii) − 4°C

Revision test 7 (p. 131)
1 D
2 B **3** A **4** C **5** D
6 (a) $85 (b) $595 (c) $49,58
7 $120; $234
8 28°C
9 (a) assumed mean = 15.0; true mean = 15.1
(b) (i) + 12 mo (ii) − 14 mo
10 38 points

Exercise 17a (p. 136)
1 (a) $675,04 (b) $290 (c) $385,04
2

Revision exercise 8 (p. 132)
2 parallelogram
3 (b) and (c) 9,4 cm
4 6,9 cm
5 AC is a diameter (= 14 cm)
6 5,8 m
7 $22\frac{1}{2}$ m
8 13,9 m
9 11°
10 150 cm

Revision test 8 (p. 132)
1 E **2** A **3** C **4** C
5 B
6 44° **7** 5,4 cm **8** 27 m
9 27 m **10** 11,5 m

General revision test B (p. 133)
1 B **2** B
3 D **4** C
5 C **6** D
7 E **8** E
9 D **10** C
11 $43,33
12 27 m
14 (a) $4x(13 - 2x)$ (b) $2a(a + b)$
(c) $(x + 5)(x - 9)$ (d) $(3x + 2y)(2a - b)$
15 (a) $42 (b) the mode, $35, is more representative since 77% of the workforce (i.e. 10 out of 13) receive this wage
16 width $11\frac{1}{4}$ cm, height $6\frac{1}{4}$ cm
17 (b) AM = 5,4 m
18 (a) {1; 2; 3; . . .} (b) {4; 5; 6; . . .}
(c) {2; 1; 0; . . .} (d) {6; 5; 4; . . .}
19 $\hat{P} = 30°$, PR = 7,3 cm **20** 9 m

CASH RECEIVED			CASH SPENT		
Date	**Details**	**$**	**Date**	**Details**	**$**
01/06/91	Brought f'ward	775,97	30/06/91	Fixed costs	290,00
30/06/91	Sales	704,30		Balance c.f.	1 190,27
30/06/91		1 480,27	30/06/91		1 480,27
01/07/91	Brought f'ward	1 190,27			

(a) Balance in hand = $1 190,27
(b) Profit = $(1 190,27 − 775,97)
= $414,30

3

CASH RECEIVED			CASH SPENT		
Date	Details	$	Date	Details	$
01 Oct	Brought f'ward	4 159,60	03 Oct	Stationery	701,80
05 Oct	From parents	2 025,00	03 Oct	Textbooks	954,00
15 Oct	From students	431,80	14 Oct	Materials	3 640,00
18 Oct	Concert	330,20	14 Oct	Coop advance	2 200,00
18 Oct	Will legacy	3 000,00	30 Oct	Elec. bill	651,60
				Balance c.f.	1 799,20
31 Oct		9 946,60	31 Oct		9 946,60
01 Nov	Brought f'ward	1 799,20			

Balance in hand = $1 799,20

4 $177 million **5** 1987/88 **6** Taxes

7 (a) Government grants
(b) Goods and services

8 Yes

9 $1 385 million

10 Yes: for the four years it contributed 3,8%; 3,3%; 3,3% and 3,8% respectively

11 85%

12 (a) 1988/89 ($803 million)
(b) 1985/86 ($506 million)

Exercise 17b (p. 138)

1 $2,50 **2** 8 September **3** $100

4 $77,29

5 The account was overdrawn by $4,71
(Note that 41,09 − 45,80 = −4,71)

6 (a) $417,19 (b) $212,88
(c) $204,31 = balance of the account at the end of the month

7 $365,16 on 7 September

8 $150,00 on 6 December

9 31 December and 30 June each year

10 $112,87 **11** $151,48

12 $452,79 (i.e. $1 285,08 − $832,29)

Exercise 17c (p. 140)

1 P20 (b) R26 (c) Me7 000
(d) 1 320 yen (e) £6 (f) US$10

2 (a) Z$20 (b) Z$840 (c) Z$25
(d) Z$40 (e) Z$666,67 (f) Z$400

3 Z$3,33 **4** 10c **5** DM243,24

6 P75,76 **7** Sh14 117,65

8 (a) Z$534 (b) £160,20

9 It rises by Z$1,20

10 (a) Z$317,46 (b) Z$322,58 (c) Z$5,12

Exercise 18a (p. 141)

1 (a) 1,26 m^2 (b) 32 m^2 (c) 84 cm^2
(d) 314 cm^2

2 (a) 10 m^2 (b) 54 cm^2

3 (a) 27 m^2 (b) 28 m^2 (c) 28 m^2
(d) 28 m^2

4 (a) 36 cm^2 (b) 199,8 cm^2 (c) 51,6 cm^2
(d) 232,5 cm^2

5 (a) 1,68 m^2 (b) $25,20

6 384 cm^2 **7** 308 mm^2 (3,08 cm^2)

8 550 min (9 h 10 min) **9** $4 200

10 (a) 231 (b) 2 566 cm^2

Exercise 18b (p. 143)

1 (a) 15 cm^2 (b) 26 cm^2 (c) 40 cm^2
(d) $37\frac{1}{2}$ cm^2 (e) 5,31 cm^2

2 (a) 5 (b) 4 (c) $6\frac{1}{4}$ (d) 7 (e) $5\frac{1}{3}$

Exercise 18c (p. 143)

1 (a) 280 (b) 120 (c) 238 (d) 630
(e) 952 (f) 252 (g) 120 (h) 240

2 (a) 135 (b) $87,75

3 1 320 tiles **4** $2,16

5 187 **6** 14

7 1 132 tiles **8** 1 395

9 11 sheets **10** $12,03

Exercise 18d (p. 145)

1 (a) 22 m² (b) 66 cm² (c) 148,5 m²
(d) 5,28 cm²

2

	length of arc	area of sector
(a)	11 cm	38,5 cm²
(b)	44 m	770 m²
(c)	8,8 cm	18,48 cm²
(d)	13,2 cm	36,98 cm²
(e)	$73\frac{1}{3}$ m	$513\frac{1}{3}$ m²

3 (a) 10,5 cm² (b) 14 cm² (c) 10,5 cm²
4 (a) 7 cm (b) 21 cm (c) 5,25 m (d) 140 m
5 5 cm
6 352 cm²
7 71,5 cm²
8 16,5 m²
9 10,4 cm²
10 907,5 cm²

Exercise 19a (p. 147)

1 (a) 309°K (b) 127°C
2 (a) 12,8 cm (b) 3,5 cm
3 (a) $y = 7$ (b) $x = \frac{7}{9}$
4 (a) 15 m² (b) 8 cm (c) 18 cm
(d) 2,5 m
5 (a) \$300 (b) \$460
6 (a) 3 amps (b) 3 volts (c) 90 ohms
7 (a) 11 m (b) 7 m
8 (a) 60 000 kg (b) 2 m (c) $\frac{8}{5}$ m
9 (a) 44 cm (b) 3,5 m (c) $6\frac{2}{7}$ m
(d) 0,437 5 m
10 (a) 60 km/h (b) $6\frac{1}{4}$ s

Exercise 19b (p. 148)

1 $y = 4; 3; 2; 1; 0$
2 $d = 1; 2; 3; 4; 5$
3 $y = 1; 3; 5; 7; 9$

x	−1	0	1	2	3
$3x$ $+2$	−3 +2	0 +2	3 +2	6 +2	9 +2
y	−1	+2	+5	+8	+11

5

x	0	1	2	3	4	5
17 $-6x$	17 0	17 −6	17 −12	17 −18	17 −24	17 −30
y	17	11	5	−1	−7	−13

6 (a) \$29,50 (b) 146 km
7 (a) 1 h 13 min (b) 2,6 kg
8 (a) \$150 (b) \$1 195 (c) \$2 500
9 (a) 38 litres (b) $4\frac{1}{2}$ h (c) 6,3 h
10 (a) 440 cm² (b) 16 cm

Exercise 19c (p. 149)

1 (a) $y = 0; 40; 160; 360; 640; 1\,000$
(b) $x = \pm\frac{1}{2}; \pm 3; \pm 5; \pm 10$
2 (a) $y = 0; 12; 16; 12; 0$
(b) $x = \pm 4; \pm 3; \pm 2; \pm 1$
3 (a) $m = 100; 4; 1; \frac{1}{4}$
(b) $n = \pm 10; \pm 5; \pm 3\frac{1}{3}; \pm 2$
4 (a) 154 m² (b) 14 cm (c) 3,5 m
5 (a) 308 cm³ (b) 14 cm (c) $2\frac{1}{3}$ cm
6 (a) 44,1 m (b) 6,4 m (c) 10 s
(d) 5 s (e) 44,1 m
7 (a) 95 min (b) 5
8 (a) 12,5 m (b) 5,5 m (c) 50 km
9 (a) $6\frac{1}{2}$ cm (b) 14
10 (a) 1 275 (b) 13 (c) 10

Exercise 19d (p. 151)

1 $x = y - 8$
2 $x = y + 3$
3 $a = b - c, \quad c = b - a$
4 $x = \frac{y}{3}$
5 $x = 4y$
6 $a = \frac{b}{c}, \quad c = \frac{b}{a}$
7 $a = \frac{n}{5x}, \quad x = \frac{n}{5a}$
8 $x = 9y$
9 $x = \frac{y}{2}$
10 $m = np, \quad n = \frac{m}{p}$
11 $x = \frac{y - 11}{6}$
12 $x = \frac{y + 2}{7}$
13 $a = \frac{b + c}{5}$
14 $x = 13 - y, y = 13 -$
15 $q = 2p, p = \frac{1}{2}q$
16 $x = \frac{d + y}{2}, y = 2x -$
17 $d = \frac{p}{4}$ or $\frac{1}{4}p$
18 $r = \frac{c}{2\pi}$
19 $\theta = T - 273$
20 $V = \frac{W}{I}, I = \frac{W}{V}$
21 $r = \frac{A}{\pi l}, \quad l = \frac{A}{\pi r}$
22 $l = \frac{V}{bh}, \quad b = \frac{V}{lh}, \quad h = \frac{V}{lb}$

23 $r = \dfrac{A}{2\pi h}$, $h = \dfrac{A}{2\pi r}$

24 $v = \dfrac{2s}{t}$, $t = \dfrac{2s}{v}$

25 $b = \dfrac{2A}{h}$, $h = \dfrac{2A}{b}$

26 $l = \dfrac{2V}{bh}$, $b = \dfrac{2V}{lh}$, $h = \dfrac{2V}{lb}$

27 $R = \dfrac{100I}{PT}$ 28 $V = IR$, $R = \dfrac{V}{I}$

29 $l = \dfrac{s - 2b}{2}$, $b = \dfrac{s - 2l}{2}$

30 $u = v - at$, $a = \dfrac{v - u}{t}$, $t = \dfrac{v - u}{a}$

Exercise 19e (p. 152)

1 (a) $x = \dfrac{y + 9}{2}$ (b) $x = 7$

2 (a) $x = \dfrac{d - y}{3}$ (b) $x = -4$

3 (a) $h = \dfrac{A}{2\pi r}$ (b) $h = 6$

4 (a) $b = \dfrac{2A}{h}$ (b) $b = 15$

5 (a) $r = \dfrac{w - 59}{2}$ (b) $11\frac{1}{2}$ h

6 (a) $P = \dfrac{100I}{RT}$ (b) $850

7 (a) $T = \dfrac{100I}{PR}$ (b) $3\frac{1}{2}$ years

8 (a) $R = \dfrac{100I}{PT}$ (b) $3\frac{3}{4}$%

9 (a) $d = \dfrac{c}{p}$ (b) 1,9

10 (a) $V = IR$, 12 volts

(b) $R = \dfrac{V}{I}$, 2 400 ohms

Exercise 20a (p. 153)

1 (a) N (b) S 2 (a) E (b) W
3 (a) SW (b) NE 4 (a) SE (b) NW
5 (a) NE (b) SW 6 (a) W (b) E

Exercise 20b (p. 155)

1 baobab: 040°, bridge: 066°, car: 112°, hut: 200°, borehole: 268°, palm tree: 308°

Exercise 20c (p. 157)

1 (a) 056° (b) 240° (c) 120° (d) 270°
(e) 327° (f) 090° (g) 133° (h) 180°
2 040°, 120° 230°, 290°
3 045°, 128°, 200°, 249°, 303°
4

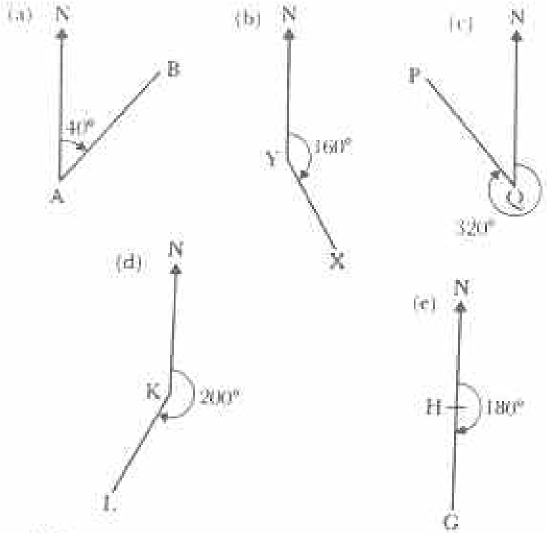

Fig. A12

5 (a) (i) 250°, (ii) 070°
(b) (i) 150°, (ii) 330°
(c) (i) 057°, (ii) 237°
(d) (i) 000°, (ii) 180°
(e) (i) 270°, (ii) 090°
(f) (i) 319°, (ii) 139°
(g) (i) 015°, (ii) 195°

Exercise 20d (p. 158)

1 (a) 20 m (b) 20,5 m
2 $075\frac{1}{2}$°, 60 m
3 36 m
4 358°, 052°
5 061°; 40 m (30 m is an acceptable answer from a good drawing)

Exercise 20e (p. 160)

1 5 km; 037°
2 3,6 km; 236°
3 410 km; 284°
4 22 km; 109°
5 3 760 m; 020°
6 (a) 145 km (b) 270 km
7 (a) 840 km (b) 150 km
8 346°, 195 km
9 (a) 2,4 km (b) 336°
10 (a) 310°; 310 km
(b) 200 km north, 240 km west

Exercise 21a (p. 162)

1 (b) $y = 5$ (c) $x = -1$ (d) (0; 2) and (− 2; 0)
3 (c) the lines are parallel
4 (b) (0; 0,5) and (− 0,3; 0)
5 (b) ($1\frac{1}{2}$; $2\frac{1}{2}$)
6 (b) (− 1; 3)
7 (b) (− 1; 2) (c) 90°
8 (a) (2; − 1) (b) (3; 1) (c) (2; $1\frac{1}{2}$)
(d) (− 2,2; 3,4)

Exercise 21b (p. 164)

1 $x = 1, y = 2$
2 $x = 3, y = 2$
3 $x = 1, y = 0$
4 $x = 0, y = 2$
5 $x = -1, y = 2$
6 $x = 3, y = -1$
7 $x = -1, y = -1$
8 $x = 2\frac{1}{2}, y = 1\frac{1}{2}$
9 $x = 1{,}6, y = 1{,}4$
10 $x = 1{,}3, y = -1{,}2$
11 $x = -1{,}7, y = 2{,}3$
12 $x = -0{,}8, y = -2{,}5$

Exercise 21c (p. 164)

1 $x = 1, y = 2$
2 $x = 3, y = 2$
3 $a = -1, b = 3$
4 $m = -1, n = 2$
5 $x = 3, y = 1$
6 $x = -1, y = -1$
7 $a = 5, b = -2$
8 $x = 2, y = 2$
9 $x = 1, y = 3$
10 $a = 7, b = 5$
11 $x = -2, y = -3$
12 $x = 1, y = -3$

Exercise 21d (p. 165)

1 $a = 4, b = 3$
2 $p = 3, q = -1$
3 $x = 7, y = -2$
4 $x = 2, y = -4$
5 $x = 2, y = -3$
6 $x = 0, y = 3$
7 $a = 2\frac{1}{2}, b = 1$
8 $x = 5, y = -2$
9 $x = 0, y = -2$
10 $h = 2, k = 1\frac{1}{2}$
11 $p = -1, q = -2$
12 $r = -2, s = 11$
13 $x = -3, y = 0$
14 $x = 12, y = -5$
15 $a = 3, b = 4$
16 $u = -2, v = 1$
17 $d = -2, e = 2$
18 $x = 1\frac{1}{2}, y = 3$
19 $f = 2\frac{1}{3}, g = 1\frac{1}{2}$
20 $y = 2\frac{1}{2}, z = -3\frac{3}{4}$

Exercise 21e (p. 166)

1 12; 7
2 39 yr, 14 yr
3 11; 6
4 42c, 18c
5 10c, 45c
6 15c, 12c
7 5 × 5-cents
3 × 10-cents
8 14 yr, 11 yr
9 $x = 3, y = 2$;
150 cm^2
10 $x = 3, y = 2$

Exercise 22a (p. 167)

1 $a^2 + 5a + 6$
2 $c^2 + 5c - 6$
3 $e^2 - e - 6$
4 $d^2 - 3d - 18$
5 $x^2 - 3x + 2$
6 $a^2 + 6a + 9$
7 $b^2 - 10b + 25$
8 $m^2 - 16$
9 $n^2 + n - 20$
10 $d^2 - 4d - 21$
11 $b^2 + b - 30$
12 $p^2 - 8p + 15$
13 $q^2 - 9$
14 $u^2 - 4u - 45$
15 $v^2 - 13v + 36$
16 $2a^2 + 7a + 3$
17 $3b^2 + 14b + 8$
18 $2c^2 - 11c + 15$
19 $2d^2 - 15d - 27$
20 $4x^2 + 4x + 1$
21 $10x^2 - 11x - 6$
22 $6y^2 - 7y - 5$
23 $m^2 + 8mn + 16n^2$
24 $u^2 + 5uv + 6v^2$
25 $9d^2 - 4e^2$
26 $6b^2 + bc - 2c^2$
27 $6s^2 - 13st - 5t^2$
28 $4c^2 - 12cd + 9d^2$
29 $12m^2 - 15mn + 3n^2$
30 $8c^2 - 26ce - 45e^2$

Exercise 22b (p. 168)

1 $a^2 + 3a + 2$
2 $a^2 + 5a + 6$
3 $a^2 + 7a + 12$
4 $b^2 - b - 2$
5 $b^2 - b - 6$
6 $b^2 - b - 12$
7 $c^2 - 7c + 12$
8 $d^2 + 8d + 7$
9 $e^2 + 11e + 18$
10 $f^2 - 9f + 20$
11 $x^2 - 8x + 7$
12 $y^2 - 11y + 18$
13 $h^2 + 12h + 36$
14 $k^2 - 10k + 25$
15 $z^2 - 7z - 18$
16 $a^2 + 10a + 24$
17 $a^2 - 10a + 24$
18 $a^2 + 2a - 24$
19 $a^2 - 2a - 24$
20 $b^2 + 3b - 18$
21 $c^2 - 3c + 2$
22 $m^2 - 2m + 1$
23 $n^2 + 2n + 1$
24 $f^2 + 20f + 99$
25 $e^2 - 8e + 15$
26 $d^2 + 8d - 20$
27 $h^2 - 5h - 24$
28 $a^2 + 6a + 9$
29 $a^2 - 6a + 9$
30 $a^2 - 9$
31 $b^2 - 25$
32 $c^2 - 49$

Exercise 22c (p. 168)

1 (a) + 9 (b) + 2 (c) − 4 (d) − 5 (e) + 14
2 (a) − 1 (b) − 3 (c) + 2 (d) 0 (e) − 10
3 (a) + 7 (b) − 7 (c) − 5 (d) − 38 (e) − 24
4 (a) + 7 (b) − 5 (c) + 3 (d) 0 (e) − 6

Exercise 22d (p. 170)

1 $(x + 5)(x + 1)$
2 $(x + 11)(x + 1)$
3 $(a + 13)(a + 1)$
4 $(b + 7)(b + 1)$
5 $(y + 8)(y + 1)$
6 $(z + 4)(z + 2)$
7 $(c + 5)(c + 3)$
8 $(d + 11)(d + 2)$
9 $(n + 6)(n + 2)$
10 $(r + 5)(r + 4)$
11 $(s + 8)(s + 2)$
12 $(t + 4)(t + 4)$

Exercise 22e (p. 170)

1 $(x - 3)(x - 1)$
2 $(y - 2)(y - 1)$
3 $(z - 17)(z - 1)$
4 $(a - 7)(a - 1)$
5 $(b - 3)(b - 2)$
6 $(c - 6)(c - 1)$
7 $(d - 7)(d - 2)$
8 $(n - 5)(n - 2)$
9 $(p - 8)(p - 3)$
10 $(q - 7)(q - 3)$
11 $(f - 14)(f - 2)$
12 $(x - 5)(x - 5)$

Exercise 22f (p. 170)

1 $(x+5)(x-1)$ 2 $(a-5)(a+1)$
3 $(x+7)(x-1)$ 4 $(b-7)(b+1)$
5 $(n+2)(n-1)$ 6 $(r-3)(r+1)$
7 $(x-11)(x+1)$ 8 $(y+13)(y-1)$
9 $(x-5)(x+3)$ 10 $(x+15)(x-1)$
11 $(s+6)(s-1)$ 12 $(t-6)(t+1)$
13 $(u-3)(u+2)$ 14 $(v+3)(v-2)$
15 $(z+5)(z-4)$ 16 $(c-10)(c+2)$
17 $(x+7)(x-7)$ 18 $(x+2)(x-2)$

Exercise 22g (p. 171)

1 $a^2+8a+16$ 2 b^2-6b+9
3 $25+10c+c^2$ 4 $4-4d+d^2$
5 $1+2m+m^2$ 6 $4n^2+4n+1$
7 $9x^2+6xy+y^2$ 8 $u^2-4uv+4v^2$
9 $25h^2-10hk+k^2$ 10 $p^2+8pq+16q^2$
11 $4a^2+12ad+9d^2$ 12 $9b^2-30bc+25c^2$
13 $49e^2-28ef+4f^2$ 14 $100x^2-20x+1$
15 $1+24y+144y^2$ 16 $9a^2+42ab+49b^2$
17 $c^2-16cd+64d^2$ 18 $81u^2+18uv+v^2$

Exercise 22h (p. 171)

1 10 201 2 9 801 3 10 609
4 9 604 5 1 002 001 6 998 001
7 1 010 025 8 992 016 9 990 025
10 5 184 11 6 889 12 6 241

Exercise 22i (p. 171)

1 $(a+5)^2$ 2 $(b+4)^2$
3 $(c+3)^2$ 4 $(d+10)^2$
5 $(m-3)^2$ 6 $(n-6)^2$
7 $(x-2)^2$ 8 $(y-1)^2$
9 $(z+8)^2$ 10 $(k-7)^2$
11 $(2-b)^2$ 12 $(9+d)^2$
13 $(x+3y)^2$ 14 $(2u-3)^2$
15 $(1-a)^2$ 16 $(5n-3v)^2$
17 $(3a-4b)^2$ 18 $(11-y)^2$

Exercise 22j (p. 172)

1 $(x+1)(x-1)$ 2 $(1+y)(1-y)$
3 $(2m-n)(2m+n)$ 4 $(u+4v)(u-4v)$
5 $(1-ab)(1+ab)$ 6 $(3-2c)(3+2c)$
7 $(2d+3e)(2d-3e)$ 8 $3(1-f)(1+f)$
9 $4(g+1)(g-1)$ 10 $(2h+5)(2h-5)$
11 $(5k-4)(5k+4)$ 12 $(7m-n)(7m+n)$
13 $(pq-3)(pq+3)$ 14 $(5+uv)(5-uv)$
15 $(9-w)(9+w)$ 16 $(10x+1)(10x-1)$
17 $4(2y+z)(2y-z)$ 18 $(4h-k)(4h+k)$
19 $(2c+7d)(2c-7d)$ 20 $(e+2f)(e-2f)$
21 $(6a+7b)(6a-7b)$ 22 $5(c-3d)(c+3d)$
23 $(xy+z)(xy-z)$ 24 $(10-w)(10+w)$

Exercise 22k (p. 173)

1 9 200 2 13 600
3 288 4 9 600
5 10 600 6 400
7 2 600 8 224
9 1 008 000 10 994 000
11 125,6 mm^2 12 26,4 cm^2

Exercise 22*l* (p. 173)

1 3, −5 2 2,1 3 −2, −6 4 5, 0
5 −3, 4 6 5, −3 7 0, −1 8 −5, −3
9 0, −3 10 0, 4 11 −2, 4 12 0, −6
13 0, 7 14 0, −3 15 6, −4 16 −5, 3
17 2, −2 18 −5, 5 19 9 twice 20 −1 twice

Exercise 22m (p. 174)

1 5, $\frac{2}{3}$ 2 $\frac{1}{2}$, −4 3 −3, $-\frac{2}{5}$ 4 $-\frac{3}{4}$, $-\frac{1}{4}$
5 $3\frac{1}{2}$, −2 6 3, 5 7 $\frac{1}{4}$, $-1\frac{1}{2}$ 8 5, $2\frac{1}{2}$
9 $-1\frac{2}{3}$, $\frac{2}{3}$ 10 $-2\frac{1}{3}$, $\frac{1}{4}$ 11 $-3\frac{1}{3}$, 4
12 $\frac{3}{4}$ twice 13 $-\frac{1}{2}$ twice 14 $\frac{1}{2}$, $-\frac{2}{3}$
15 $-1\frac{2}{3}$ twice 16 0, $1\frac{1}{3}$ 17 −3, $\frac{5}{7}$
18 $2\frac{3}{4}$ twice 19 0, $-4\frac{1}{2}$ 20 0, $-3\frac{3}{4}$

Exercise 22n (p. 174)

1 1, 2 2 −2, −3 3 2, −1
4 1, −3 5 2, 5 6 0, 4
7 0, −5 8 −3, −4 9 2, −4
10 1 twice 11 1,4 12 0,9
13 ±3 14 ±5 15 −1, 9
16 5, −7 17 3 twice 18 −4 twice
19 0.4 20 ±2 21 6, 9
22 −3, 18 23 0, −4 24 −1, −3
25 4, −8 26 $\pm 3\frac{1}{2}$ 27 9, −10
28 9, −8 29 13, −3 30 3, −22

Exercise 23a (p. 175)

1 (a) $6,48 (b) $17,28
(c) $22,32 (d) $36,00
(e) $45,36 (f) $51,84
(g) $61,92 (h) $69,84
(i) $90,00 (j) $99,36
2 $30,24 3 $8,20
4 $35 5 $137,28

Exercise 23b (p. 176)

1 (a) $43,20 (b) $144,00
(c) $5 760 (d) $230,40
(e) $540 (f) $8,06
(g) $6,98 (h) $4 104
(i) $2,38 (j) $504

2 (a) $144 (b) $216 (c) $360
(d) $576 (e) $720 (f) $108
(g) $180 (h) $244,80 (i) $129,60
(j) $103,68 (k) $118,80 (l) $155,52
3 $662,40
4 $35,20
5 $1 513,60
6 $29,88
7 $41,59
8 $123 660
10 (a) $174 (b) $132 (c) $54 (d) $288 (e) $405 (f) $165 (g) $480 (h) $1 170
11 (a) $81,60 (b) $228,80 (c) $12,04 (d) $424,80 (e) $119,04 (f) $43 (g) $302,08 (h) $308,76 (i) $800,80 (j) $1 675,52
12 (a) $3,72 (b) $215,76

Exercise 23c (p. 178)

1 (a) 1,17 (b) 6,57 (c) 3,15
(d) $2,25 (e) $3,60 (f) $5,49
(g) $2,79 (h) $8,55 (i) $5,13
(j) $6,21 (k) $7,65 (l) $4,32
2 (a) $9,81 (b) $19,62 (c) $23,98
(d) $40,33 (e) $55,59 (f) $81,75
(g) $30,52 (h) $50,14 (i) $41,42
(j) $98,10 (k) $17,44 (l) $64,31
3 (a) $39,13 (b) $32,76 (c) $45,50
(d) $19,11 (e) $48,23 (f) $26,39
(g) $64,61 (h) $50,05 (i) $7,28
(j) $29,12 (k) $86,45 (l) $43,68
4 (a) $5,40 (b) $7,20 (c) $9,00
(d) $6,93 (e) $4,86 (f) $8,37
(g) 3,51c (h) 4,59c (i) 6,03c
(j) 2,97c (k) 5,94c (l) 7,92c
5 (a) $56,68 (b) $76,30 (c) $85,02
(d) $80,66 (e) $107,91 (f) $89,38
6 (a) $83,72 (b) $80,08 (c) $58,24
(d) $52,78 (e) $61,88 (f) $69,16
7 $107,02
8 $8 087,80
9 $359,70, $327,33
10 $855

Exercise 23d (p. 179)

1 (a) $1,23 (b) 53c (c) $8,17
(d) $1,87 (e) $5,13 (f) 3c
(g) $2,63 (h) $8,17 (i) 47c
(j) 33c (k) $52,50 (l) $56
2 (a) $31,73 (b) $2,16 (c) $1,16
(d) $57,75 (e) $14,28 (f) $33,24
(g) $4,99 (h) $12,30 (i) $85,05
(j) $8,77 (k) $138,60 (l) $2,25
3 $875
4 $868
5 (a) $963 (b) $991,09

Exercise 23e (p. 180)

1 (a) 1,96 (b) 5,29 (c) 46,2
(d) 51,8 (e) 24,0 (f) 74,0
(g) 31,7 (h) 82,4 (i) 9,92
(j) 3,53 (k) 32,6 (l) 20,6
2 (a) 324 (b) 961 (c) 1 020
(d) 225 (e) 841 (f) 1 940
(g) 4 970 (h) 424 (i) 3 930
(j) 3 580 (k) 6 610 (l) 8 260
3 (a) 2,99 (b) 7,90 (c) 6 190
(d) 2 710 (e) 9 310 (f) 2 460
(g) 401 000 (h) 648 000 (i) 92 400
4 (a) 16 900 (b) 168 000 (c) 757 000
(d) 254 000 (e) 7 290 000 (f) 69 700 000
5 yes, in general $(N{,}5)^2 = N \times (N + 1) + 0{,}25$

Exercise 23f (p. 181)

1 (a) 3 (b) 9,49 (c) 1,67 (d) 5,29
(e) 2,17 (f) 6,86 (g) 2,24 (h) 7,10
(i) 6,02 (j) 1,90 (k) 5,07 (l) 1,60
2 (a) 2,65 (b) 8,37 (c) 26,5 (d) 83,7
(e) 1,70 (f) 5,39 (g) 17,0 (h) 53,9
(i) 6,18 (j) 19,5 (k) 61,8 (l) 195
(m) 3,16 (n) 10 (o) 31,6 (p) 100
(q) 1,41 (r) 21,0 (s) 91,9 (t) 269
3 (a) 3,05 (b) 8,84 (c) 21,5 (d) 2,91
(e) 7,83 (f) 24,8 (g) 7,68 (h) 2,41
(i) 70,7 (j) 22,4 (k) 253 (l) 44,5
4 $\sqrt{10} = 3{,}16$ and $\pi = 3{,}14$: a difference of 0,02
$\sqrt{10}$ is a good approximation of π
5 (a) $m = 6{,}32$ (b) $m^2 = 39{,}9$ (c) $m^2 < 40$; this is because the tables contain rounded numbers which are not completely accurate

Exercise 24a (p. 183)

1 Different calculators may give different results for many of these key sequences. The important thing is to get to know how *your* calculator operates.
2 For non-scientific calculators with an eight-digit display:
(a) 99 999 999 (b) 0,000 000 1
3 Looks like SHELL.
4 Looks like SOIL.
5 (a)

7^1	7
7^2	49
7^3	343
7^4	2401
7^5	16807
7^6	117649
7^7	823543
7^8	5764801

(b) Final digits comprise the repeating sequence

7, 9, 3, 1

• (c) Final two digits comprise the repeating sequence

07, 49, 43, 01

Exercise 24b (p. 185)

1 Final answers:
(a) 9 (b) 4 (c) 28 (d) 86
(e) 30 (f) 97 (g) 36 (h) 27
(i) 24,3 (j) 15,5

2 (a) (i), (ii), (iv), (v), (vii) and (viii) are incorrect
(b) Corrections:
(i) 13 (ii) 67 (iv) 908
(v) 915 (vii) 28 (viii) 84

3 The outcomes are all negative:
(a) −5 (b) −10 (c) −33 (d) −37
(e) −67 (f) −216 (g) −655 (h) −56

4 The bill is correct

5 $49,79

Exercise 24c (p. 187)

1 (a) 5 896 (b) 73 (c) 27 (d) 2 116
(e) 42 679 (f) 7 (g) 136 (h) 615

2 (a) (i) 61,199 2 (ii) 61,20
(b) (i) 1 282,386 7 (ii) 1 282,39
(c) (i) 333,423 52 (ii) 333,42
(d) (i) 1,850 898 2 (ii) 1,85
(e) (i) 8* (ii) 8
(f) (i) 28* (ii) 28
(g) (i) 983,952 99 (ii) 983,95
(h) (i) 63,625 5 (ii) 63,63
*Some calculators may give rounding errors in these cases.

3 (a) (i), (iii), (iv), (v), (vi) and (viii) are incorrect
(b) Corrections:
(i) 45 (iii) 5 628
(iv) 46 (v) 33,333 333
(vi) 3,315 (viii) 5,065 110 6

4 1,509 or 1.5000000 = depending on calculator

5 (a) 11111111 (calculator display)
(b) 12345679 (calculator display)

6 31 536 000

7 (b) Age × 365 (c) Age × 365 × 24
(d) Age × 365 × 24 × 60
(e) Most calculators will not cope with this if your age is above 3!

8 (a) $1 443,67 per month
(b) $47,46 per day (365 day year)

9 $8,33

10 (a) 9,17 km (b) 9 166,67 m (c) 152,78 m

Exercise 24d (p. 188)

1 (b) 68 ÷ 17 + 45 (c) 42 ÷ 3 + 22
(d) 18 × 5 + 63 (e) (171 − 15) × 18
(g) (6 + 3) × 487 (h) (31 − 14) × 100

2 (a) 229 (b) 560 (c) 70 (d) 21
(e) 784 (f) 7 (g) 22,361 077 (h) 23,32

3 (a) 26 (b) 12 (c) 12
(d) 435 (e) 3,994 867 4 (f) 282,76

Junior Certificate Practice Examination in Mathematics

Paper I (p. 189)

1	E	**2**	B	**3**	C	**4**	B	**5**	D
6	B	**7**	D	**8**	C	**9**	C	**10**	A
11	C	**12**	E	**13**	C	**14**	A	**15**	D
16	B	**17**	D	**18**	D	**19**	C	**20**	A
21	A	**22**	B	**23**	D	**24**	C	**25**	B
26	C	**27**	C	**28**	B	**29**	A	**30**	E
31	E	**32**	C	**33**	B	**34**	D	**35**	D
36	A	**37**	B	**38**	E	**39**	C	**40**	E
41	D	**42**	E	**43**	B	**44**	D	**45**	C
46	D	**47**	E	**48**	C	**49**	C	**50**	D

Paper II (p. 193)

Section A

1 (a) 28 (b) $16\frac{2}{3}\%$

2 (a) $\frac{100n}{t}$ (b) $y = -3$

3 (b) (i) {o}, (ii) {t; w; o; e; a; m}, (iii) {m; t; w; o}

4 (b) 890 m, 243°

5 1 960 m^2

6 818 g

7 (a) $(4 - h)(4 + h)$ (b) $a - b$ (c) $6x^2 + 25x - 44$

8 (a) $x \geq -4$ (b) $\{(x; y): y > 5\}$

9 18°, 81°, 81°

10 (a) \$205,89 (b) 24 (c) 1,9%

Section B

11 (a) (i) \$10 740, (ii) \$11 784
(b) (i) 2 100, (ii) 4,2

12 (b) (i) 9,5 m, (ii) 6,7 m, (iii) 40°

13 (b) square (c) 4 (d) (0; 3)
(e) X(− 5; 8) or X(3; − 8)

14 (a) $a = 5, b = 4$ (b) $x = -3$ or 5
(c) (i) $(m - 20)$ g, (ii) $4(m - 20) + m = 330$, $m = 82$

15 (a) (i) 15, (ii) 16 yr, (iii) 16 yr, (iv) 15,6 yr
(b) (ii) 1,5 km (iii) 7,5 km/h