

New General Mathematics 1

A Junior Certificate Course

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NEW EDITION

Without Answers

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Preliminary chapter

Addition, subtraction, multiplication, division

Note: Only use this chapter if you cannot add, subtract, multiply and divide whole numbers accurately and quickly.

Revision and practice

You must be able to add, subtract, multiply and divide whole numbers. You learned these skills in Primary School. You will use them throughout Secondary School and in later life.

This Chapter contains tests to give you practice in the basic number skills. There are ten tests in each of the following:

- addition skills (Tests A–J, page 1)
- subtraction skills (Tests A–J, page 2)
- multiplication skills (Tests A–J, page 3)
- division skills (Tests A–J, page 4)

Each test is in the column under the test heading. Do each test as follows:

- (a) Allow 10 minutes to do all the test.
- (b) Try to do every item.
- (c) Use the answers in the back of the book to correct the test.

- (d) If you get an item wrong, find the line that the item was in. Then do all the items in that line right across the page. For example, suppose you get item 6 wrong in Test B. Do all the items 6 in Tests A, B, C, D, and E.
- (e) If you still get these items wrong, see your teacher.

There are many different methods of adding, subtracting, multiplying and dividing. Use the method you learned in Primary School. If you find that you make too many mistakes, see your teacher. Your teacher may show you another method.

Do *not* do all the tests at once. There are enough tests to do one every week of the school year. Your teacher will tell you when to do the tests.

Addition: Revision and practice

	Test A	Test B	Test C	Test D	Test E
1	5 + 3	2 + 7	5 + 4	3 + 4	3 + 2
2	11 + 6	3 + 13	10 + 9	12 + 7	4 + 14
3	7 + 8	9 + 4	6 + 7	2 + 9	5 + 7
4	44 + 4	6 + 31	43 + 2	1 + 76	81 + 8
5	3 + 9 + 1	3 + 5 + 7	2 + 8 + 9	5 + 8 + 5	9 + 6 + 1
6	48 + 3	9 + 28	66 + 5	7 + 83	35 + 5
7	23 + 41	82 + 17	49 + 40	64 + 34	22 + 26
8	28 + 26	53 + 64	47 + 33	36 + 55	81 + 65
9	75 + 58	27 + 93	45 + 59	68 + 68	82 + 49
10	445 + 386	383 + 750	939 + 859	637 + 295	486 + 485

11	423 + 397 <hr/>	544 + 961 <hr/>	246 + 255 <hr/>	680 + 361 <hr/>	874 + 727 <hr/>
12	312 89 + 651 <hr/>	629 502 143 + 817 <hr/>	153 153 75 + 9 <hr/>	374 1280 + 6400 <hr/>	378 344 + 206 <hr/>

	Test F	Test G	Test H	Test I	Test J
1	1 + 8	7 + 3	0 + 2	6 + 3	1 + 9
2	15 + 2	4 + 12	7 + 11	18 + 2	3 + 16
3	8 + 6	8 + 9	8 + 3	4 + 7	4 + 8
4	6 + 90	75 + 4	5 + 62	50 + 9	7 + 22
5	7 + 5 + 5	7 + 8 + 3	6 + 4 + 6	8 + 9 + 2	4 + 5 + 6
6	4 + 59	46 + 6	7 + 74	55 + 8	6 + 39
7	55 + 23	34 + 43	53 + 36	23 + 71	30 + 28
8	34 + 70 <hr/>	92 + 76 <hr/>	39 + 49 <hr/>	94 + 25 <hr/>	36 + 57 <hr/>
9	36 + 64 <hr/>	79 + 39 <hr/>	72 + 68 <hr/>	83 + 38 <hr/>	94 + 97 <hr/>
10	471 + 963 <hr/>	704 + 727 <hr/>	491 + 654 <hr/>	736 + 819 <hr/>	356 + 357 <hr/>
11	385 + 315 <hr/>	608 + 405 <hr/>	546 + 556 <hr/>	435 + 367 <hr/>	809 + 195 <hr/>
12	961 86 + 422 <hr/>	256 165 54 + 32 <hr/>	434 504 614 + 24 <hr/>	34 85 75 + 26 <hr/>	693 631 + 676 <hr/>

Subtraction: Revision and practice

	Test A	Test B	Test C	Test D	Test E
1	6 - 2	7 - 4	9 - 5	8 - 0	3 - 1
2	18 - 5	14 - 11	15 - 3	19 - 15	16 - 6
3	10 - 3	20 - 5	10 - 8	20 - 11	10 - 5
4	14 - 6	11 - 7	18 - 9	13 - 5	12 - 3
5	28 - 4	99 - 8	35 - 1	47 - 3	65 - 2
6	49 - 23	56 - 42	28 - 17	85 - 32	78 - 15
7	62 - 58	73 - 65	36 - 28	95 - 87	28 - 19
8	33 - 17 <hr/>	85 - 36 <hr/>	72 - 47 <hr/>	96 - 29 <hr/>	64 - 37 <hr/>

9	$\begin{array}{r} 271 \\ - 93 \\ \hline \end{array}$	$\begin{array}{r} 314 \\ - 46 \\ \hline \end{array}$	$\begin{array}{r} 850 \\ - 83 \\ \hline \end{array}$	$\begin{array}{r} 426 \\ - 58 \\ \hline \end{array}$	$\begin{array}{r} 537 \\ - 39 \\ \hline \end{array}$
10	$\begin{array}{r} 523 \\ - 265 \\ \hline \end{array}$	$\begin{array}{r} 635 \\ - 577 \\ \hline \end{array}$	$\begin{array}{r} 748 \\ - 258 \\ \hline \end{array}$	$\begin{array}{r} 571 \\ - 374 \\ \hline \end{array}$	$\begin{array}{r} 854 \\ - 548 \\ \hline \end{array}$
11	$\begin{array}{r} 204 \\ - 146 \\ \hline \end{array}$	$\begin{array}{r} 402 \\ - 273 \\ \hline \end{array}$	$\begin{array}{r} 500 \\ - 156 \\ \hline \end{array}$	$\begin{array}{r} 603 \\ - 405 \\ \hline \end{array}$	$\begin{array}{r} 307 \\ - 128 \\ \hline \end{array}$
12	$\begin{array}{r} 2037 \\ - 849 \\ \hline \end{array}$	$\begin{array}{r} 3503 \\ - 1597 \\ \hline \end{array}$	$\begin{array}{r} 5662 \\ - 5074 \\ \hline \end{array}$	$\begin{array}{r} 4080 \\ - 1976 \\ \hline \end{array}$	$\begin{array}{r} 2005 \\ - 1229 \\ \hline \end{array}$

	Test F	Test G	Test H	Test I	Test J
1	5 - 4	8 - 3	7 - 5	3 - 3	9 - 3
2	17 - 10	13 - 1	16 - 13	19 - 4	18 - 11
3	20 - 6	10 - 9	20 - 12	10 - 7	20 - 17
4	15 - 8	17 - 8	16 - 7	14 - 9	13 - 6
5	61 - 0	42 - 2	88 - 6	76 - 3	37 - 2
6	99 - 65	36 - 14	59 - 44	64 - 24	84 - 53
7	51 - 49	84 - 75	47 - 39	41 - 32	35 - 26
8	$\begin{array}{r} 45 \\ - 29 \\ \hline \end{array}$	$\begin{array}{r} 60 \\ - 26 \\ \hline \end{array}$	$\begin{array}{r} 93 \\ - 68 \\ \hline \end{array}$	$\begin{array}{r} 61 \\ - 46 \\ \hline \end{array}$	$\begin{array}{r} 82 \\ - 59 \\ \hline \end{array}$
9	$\begin{array}{r} 763 \\ - 76 \\ \hline \end{array}$	$\begin{array}{r} 625 \\ - 86 \\ \hline \end{array}$	$\begin{array}{r} 211 \\ - 72 \\ \hline \end{array}$	$\begin{array}{r} 142 \\ - 57 \\ \hline \end{array}$	$\begin{array}{r} 374 \\ - 95 \\ \hline \end{array}$
10	$\begin{array}{r} 982 \\ - 298 \\ \hline \end{array}$	$\begin{array}{r} 614 \\ - 219 \\ \hline \end{array}$	$\begin{array}{r} 433 \\ - 224 \\ \hline \end{array}$	$\begin{array}{r} 641 \\ - 368 \\ \hline \end{array}$	$\begin{array}{r} 770 \\ - 482 \\ \hline \end{array}$
11	$\begin{array}{r} 405 \\ - 198 \\ \hline \end{array}$	$\begin{array}{r} 708 \\ - 539 \\ \hline \end{array}$	$\begin{array}{r} 201 \\ - 165 \\ \hline \end{array}$	$\begin{array}{r} 306 \\ - 287 \\ \hline \end{array}$	$\begin{array}{r} 502 \\ - 396 \\ \hline \end{array}$
12	$\begin{array}{r} 5231 \\ - 506 \\ \hline \end{array}$	$\begin{array}{r} 6814 \\ - 5807 \\ \hline \end{array}$	$\begin{array}{r} 7486 \\ - 2979 \\ \hline \end{array}$	$\begin{array}{r} 3113 \\ - 2066 \\ \hline \end{array}$	$\begin{array}{r} 6104 \\ - 825 \\ \hline \end{array}$

Multiplication: Revision and practice

	Test A	Test B	Test C	Test D	Test E
1	6×7	9×8	6×9	7×4	7×8
2	$2 \times 8 \times 2$	$5 \times 2 \times 6$	$2 \times 4 \times 8$	$2 \times 8 \times 5$	$9 \times 2 \times 2$
3	4×21	43×2	5×11	61×1	3×12
4	9×13	6×14	2×17	8×16	5×18
5	39×8	78×4	63×7	69×5	25×9
6	36×100	1000×52	860×10	100×4	7×1000

7	428×7	$8\,431 \times 5$	325×9	$3\,781 \times 3$	472×6
8	$3\,126 \times 4$	335×6	$7\,735 \times 3$	172×7	$8\,549 \times 2$
9	$\begin{array}{r} 43 \\ \times 52 \\ \hline \end{array}$	$\begin{array}{r} 254 \\ \times 57 \\ \hline \end{array}$	$\begin{array}{r} 74 \\ \times 61 \\ \hline \end{array}$	$\begin{array}{r} 820 \\ \times 29 \\ \hline \end{array}$	$\begin{array}{r} 93 \\ \times 17 \\ \hline \end{array}$
10	$\begin{array}{r} 562 \\ \times 308 \\ \hline \end{array}$	$\begin{array}{r} 425 \\ \times 409 \\ \hline \end{array}$	$\begin{array}{r} 257 \\ \times 830 \\ \hline \end{array}$	$\begin{array}{r} 621 \\ \times 341 \\ \hline \end{array}$	$\begin{array}{r} 542 \\ \times 209 \\ \hline \end{array}$

	Test F	Test G	Test H	Test I	Test J
1	9×4	6×8	9×7	7×7	9×9
2	$3 \times 8 \times 2$	$3 \times 7 \times 3$	$4 \times 6 \times 2$	$3 \times 3 \times 5$	$2 \times 3 \times 7$
3	53×0	2×23	30×3	4×22	3×23
4	7×16	4×14	3×15	6×19	7×13
5	48×3	28×7	83×6	24×9	37×4
6	10×60	508×100	$1\,000 \times 90$	$10 \times 1\,000$	100×300
7	$8\,307 \times 2$	904×8	$3\,941 \times 4$	861×8	$6\,275 \times 5$
8	156×9	$2\,369 \times 3$	138×8	$6\,141 \times 5$	346×8
9	$\begin{array}{r} 713 \\ \times 35 \\ \hline \end{array}$	$\begin{array}{r} 87 \\ \times 71 \\ \hline \end{array}$	$\begin{array}{r} 389 \\ \times 17 \\ \hline \end{array}$	$\begin{array}{r} 62 \\ \times 36 \\ \hline \end{array}$	$\begin{array}{r} 105 \\ \times 47 \\ \hline \end{array}$
10	$\begin{array}{r} 485 \\ \times 168 \\ \hline \end{array}$	$\begin{array}{r} 647 \\ \times 392 \\ \hline \end{array}$	$\begin{array}{r} 705 \\ \times 516 \\ \hline \end{array}$	$\begin{array}{r} 470 \\ \times 288 \\ \hline \end{array}$	$\begin{array}{r} 452 \\ \times 219 \\ \hline \end{array}$

Division: Revision and practice

Note: $12 \div 6$, $\frac{1}{6}$ of 12, $\frac{12}{6}$, $6 \overline{)12}$ are different ways of writing 12 divided by 6.

	Test A	Test B	Test C	Test D	Test E
1	$9 \div 3$	$\frac{1}{4}$ of 8	$\frac{8}{2}$	$3 \overline{)6}$	$6 \div 2$
2	$\frac{64}{8}$	$3 \overline{)24}$	$42 \div 6$	$\frac{1}{3}$ of 72	$\frac{34}{9}$
3	$\frac{1}{2}$ of 72	$\frac{40}{2}$	$2 \overline{)90}$	$52 \div 2$	$\frac{1}{2}$ of 86
4	$5 \overline{)65}$	$56 \div 4$	$\frac{1}{6}$ of 72	$\frac{81}{3}$	$3 \overline{)51}$
5	$196 \div 7$	$\frac{1}{4}$ of 150	$\frac{124}{4}$	$8 \overline{)184}$	$162 \div 6$
6	$\frac{62\,000}{100}$	$\frac{1}{10}$ of 440	$\frac{50\,000}{1\,000}$	$600 \div 100$	$10 \overline{)8\,000}$
7	$\frac{1}{6}$ of 258	$\frac{853}{7}$	$5 \overline{)690}$	$3\,828 \div 4$	$\frac{1}{8}$ of 464

8	$8\overline{)1624}$	$3\overline{)927}$	$4\overline{)2032}$	$5\overline{)535}$	$4\overline{)2408}$
9	$43\overline{)1247}$	$59\overline{)18054}$	$32\overline{)2752}$	$92\overline{)10580}$	$57\overline{)3591}$
10	$109\overline{)38586}$	$317\overline{)8242}$	$980\overline{)53900}$	$373\overline{)7087}$	$693\overline{)83853}$

	Test F	Test G	Test H	Test I	Test J
1	$7 \div 7$	$5 \div 1$	$\frac{4}{2}$	$\frac{1}{3}$ of 3	$0 + 2$
2	$9\overline{)45}$	$63 \div 7$	$\frac{1}{2}$ of 49	$\frac{48}{8}$	$7\overline{)70}$
3	$\frac{36}{2}$	$2\overline{)98}$	$36 \div 2$	$\frac{1}{2}$ of 76	$\frac{36}{2}$
4	$84 \div 6$	$\frac{1}{8}$ of 96	$\frac{117}{9}$	$4\overline{)68}$	$128 \div 8$
5	$\frac{1}{8}$ of 192	$\frac{189}{9}$	$5\overline{)135}$	$234 \div 9$	$\frac{1}{4}$ of 116
6	$\frac{405\ 000}{1\ 000}$	$\frac{1}{100}$ of 1 000	$10\overline{)62\ 000}$	$\frac{1\ 000}{1\ 000}$	$\frac{70\ 000}{100}$
7	$\frac{2\ 277}{2}$	$4\overline{)2\ 268}$	$894 \div 6$	$\frac{1}{5}$ of 1 165	$\frac{4\ 566}{6}$
8	$7\overline{)910}$	$6\overline{)6\ 012}$	$3\overline{)618}$	$6\overline{)4\ 224}$	$9\overline{)972}$
9	$54\overline{)11\ 232}$	$35\overline{)7\ 455}$	$48\overline{)41\ 952}$	$37\overline{)3\ 478}$	$23\overline{)11\ 661}$
10	$261\overline{)9\ 657}$	$702\overline{)28\ 080}$	$572\overline{)9\ 152}$	$425\overline{)15\ 725}$	$158\overline{)9\ 796}$

Chapter 1

Number systems

Syllabus Reference 5.1.1

Counting

It is most likely that mathematics began when people started to count and measure. Counting and measuring are part of everyday life. Nearly every language in the world contains words for numbers and measures.

People have always used their fingers to help them when counting. This led them to collect numbers in groups: sometimes fives (fingers of one hand), sometimes tens (both hands) and even in groups of twenty (fingers and toes). For example, a herd boy with 23 cattle might say that he had 'four fives and three' cattle, or 'two tens and three' cattle, or 'one twenty and three' cattle. It would depend on the custom and language of his people. In every case, the number of cattle would be the same.

When people group numbers in fives we say that they are using a **base five** method of counting. Most people use **base ten** when counting. For this reason, base ten is used internationally. The numbers that people use for counting are called **natural numbers**.

Exercise 1a (Oral Discussion)

- 1 Say the numbers from 1 to 30 in your mother tongue.
- 2 What is the base of counting of your mother tongue?
- 3 There are many languages in Zimbabwe. Find the bases of counting in as many languages as you can.

Table 1.1 gives the words for the numbers 1–20 in the Ndebele, Shona and Tonga languages. Use Table 1.1 to answer questions 4–8.

4 Look at the Ndebele numbers.

- (a) What do the words for 2, 8, 12, 18, 20 have in common?

Table 1.1

Ndebele	Shona	Tonga
1 <i>kuuye</i>	<i>mati</i>	<i>chakutangana</i>
2 <i>kubili</i>	<i>piri</i>	<i>chabili</i>
3 <i>kuthathu</i>	<i>tatu</i>	<i>chatatu</i>
4 <i>kane</i>	<i>china</i>	<i>chane</i>
5 <i>kublano</i>	<i>chanu</i>	<i>chisanu</i>
6 <i>kusivhupa</i>	<i>tanhatu</i>	<i>chisana achimwe</i>
7 <i>kuyosikhombisa</i>	<i>chinove</i>	<i>chisana azibili</i>
8 <i>kuyitshya galo mbile</i>	<i>rusere</i>	<i>chisana azitatu</i>
9 <i>kuyitshya galo kuye</i>	<i>pfumbamwe</i>	<i>chisana azine</i>
10 <i>kulitshumi</i>	<i>gumi</i>	<i>ikumi</i>
11 <i>kulitshumi lanye</i>	<i>gumi nesimwe</i>	<i>ikumi achimwe</i>
12 <i>kulitshumi lambili</i>	<i>gumi nembiri</i>	<i>ikumi azibili</i>
13 <i>kulitshumi lantathu</i>	<i>gumi antathu</i>	<i>ikumi azitatu</i>
14 <i>kulitshumi lane</i>	<i>gumi nelina</i>	<i>ikumi azine</i>
15 <i>kulitshumi lanhlano</i>	<i>gumi nesihanu</i>	<i>ikumi azisanu</i>
16 <i>kulitshumi lesihupa</i>	<i>gumi natarhatu</i>	<i>ikumi lilamusanu achimwe</i>
17 <i>kulitshumi lesikhombisa</i>	<i>gumi nenomwe</i>	<i>ikumi lilamusanu azibili</i>
18 <i>kulitshumi letshiyegalo</i>	<i>gumi nesere mbile</i>	<i>ikumi lilamusanu azitatu</i>
19 <i>kulitshumi letshiyegalo kuye</i>	<i>gumi nepfumbamwe</i>	<i>ikumi lilamusanu azine</i>
20 <i>kunganiatshumi anabili</i>	<i>makumi maviri</i>	<i>makumi obili</i>

- (b) Translate the words for 8 and 18 into English.
 - (c) What does the word for 20 mean?
 - (d) What base of counting is the Ndebele language in?
- 5 Look at the Shona numbers:
- (a) What do the words for 2, 12, 20 have in common?

- (b) Each of the words for the numbers 11 to 19 begins *gumi ne-*. What does this mean in English?
 (c) What does the word for 20 mean?
 (d) What base of counting is the Shona language in?

6 Look at the Tonga numbers.

- (a) How is the word for 7 made from those for 2 and 5?
 (b) What do the words for 3, 8, 13, 18 have in common?
 (c) Translate the words for 9, 14, 19 into English.
 (d) What does the word for 20 mean?
 (e) What base of counting is the Tonga language in?

7 Do the three languages have anything in common?

8 In which language is the structure of the words for the numbers closest to that of English?

9 In English the word *score* means *twenty*. If a man reaches the age of three score and ten, how old is he?

10 Why is base ten the most common?

Other bases of counting: seven, sixty, twenty-four

There are seven days in a week. Suppose that a baby is 2 weeks and 5 days old. This means that it is 2 lots of 7 days and 5 days old, 19 days altogether.

Example 1

Find the total of 1 week 5 days, 6 days and 3 weeks 4 days. Give the answer (a) in weeks and days, (b) in days.

(a) working:

wk	d
1	5
	6
3	4
<hr/>	<hr/>
6	1

method: In the days column,

$$\begin{aligned} (5 + 6 + 4) \text{ days} &= 15 \text{ days} \\ &= 2 \times 7 \text{ days} + 1 \text{ day} \\ &= 2 \text{ weeks} + 1 \text{ day} \end{aligned}$$

Write down 1 day and carry 2 weeks.
 answer: 6 weeks and 1 day

$$\begin{aligned} \text{(b) } 6 \text{ wk } 1 \text{ d} &= 6 \times 7 \text{ days} + 1 \text{ day} \\ &= 42 \text{ days} + 1 \text{ day} \\ &= 43 \text{ days} \end{aligned}$$

Example 2

Today is Wednesday. What day of the week will it be in 160 days' time?

Change 160 days to days and weeks.

$$160 \text{ d} = 160 \div 7 \text{ wk} = 22 \text{ wk } 6 \text{ d}$$

After 22 weeks it will be Wednesday again. Count a further 6 days on from Wednesday:

Thursday, Friday, Saturday,
 Sunday, Monday, Tuesday

It will be Tuesday in 160 days time.

There are 60 seconds in a minute and 60 minutes in an hour.

Example 3

Find the number of seconds in 3 min 49 s.

$$\begin{aligned} \text{Number of seconds in 3 min} &= 3 \times 60 \text{ s} \\ &= 180 \text{ s} \end{aligned}$$

$$\begin{aligned} \text{Number of seconds in 3 min 49 s} &= 180 \text{ s} + 49 \text{ s} \\ &= 229 \text{ s} \end{aligned}$$

Example 4

Add the following times together. Give the answer in hours and minutes.

3 h 40 min, 2 h 25 min, 28 min, 1 h 35 min

working:

	h	min
	3	40
	2	25
		28
	1	35
	<hr/>	<hr/>
	8	8

method: In the minutes column:

$$\begin{aligned} (40 + 25 + 28 + 35) \text{ min} \\ &= 128 \text{ min} \\ &= 2 \times 60 \text{ min} + 8 \text{ min} \\ &= 2 \text{ h} + 8 \text{ min} \end{aligned}$$

Write down 8 min and carry 2 h.

answer: 8 h 8 min

Tally system

Tally marks were probably the first numerals. Herdsmen made tally marks to represent the numbers of animals they had. The tally marks were scratched on stones or sometimes cut on sticks (Fig. 1.1).



Fig. 1.1

We still use the tally system. It is very useful when counting a large number of objects. We usually group tally marks in fives: hence $\text{||||} \text{ ||}$ means *three fives and two*, or *seventeen*. Notice that the fifth tally is marked across the other four: $\text{||||} = 4$, $\text{||||} = 5$.

Exercise 1c (Oral)

What numbers do the following tally marks represent?

- | | | | | | |
|----|--------------------------|----|--------------------------|----|--------------------------|
| 1 | $\text{ } \text{ }$ | 2 | | 3 | $\text{ } \text{ }$ |
| 4 | $\text{ } \text{ }$ | 5 | $\text{ } \text{ }$ | 6 | $\text{ } \text{ }$ |
| 7 | $\text{ } \text{ }$ | 8 | $\text{ } \text{ }$ | 9 | $\text{ } \text{ }$ |
| 10 | $\text{ } \text{ }$ | 11 | $\text{ } \text{ }$ | 12 | $\text{ } \text{ }$ |

The place-value system

The tally system is inconvenient when numbers are large or fractional. The most convenient way of representing numbers is to use a **place-value system**. The base ten place-value system uses ten symbols: 1, 2, 3, 4, 5, 6, 7, 8, 9 for one to nine and 0 for zero. These symbols are sometimes called **digits**. *Digit* comes from *Digitus*, which is Latin for finger or toe.

In this system, the numbers of units, tens, hundreds, ..., are each represented by a single digit. However, each digit must be written in a special position or place. For any whole number, the **units place** is at the right-hand end of the numeral, the **tens place** is next to the units

place on the left, and so on. For example, the number 3902 means 3 thousands, 9 hundreds, 0 tens and 2 units (Fig. 1.2).

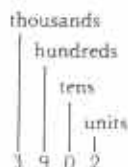


Fig. 1.2

As numbers grow we use more places. The symbol 0, zero, shows that a place is empty. Hence ten thousand is 10 000, using two digits and five places.

An advantage of the place-value system is that it can represent fractions. Just as there are places for whole numbers such as hundreds, tens, units, so we can make places for decimal fractions such as tenths, hundredths, thousandths, and so on. However, it is necessary to show where the whole numbers end and the decimal fractions begin. We use a **decimal comma** to separate the whole numbers from the decimal fractions (Fig. 1.3).

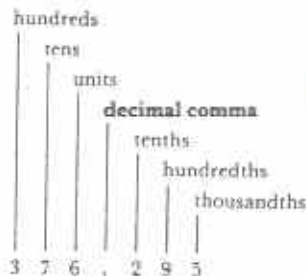


Fig. 1.3

In Fig. 1.3, the 2 in the first position after the decimal comma shows that there are *two tenths*. 9 is in the second position after the comma; this stands for *nine hundredths*.

Exercise 1d

- 1 The value of the 5 in 6508 is five hundred. What is the value of each of the following?
- (a) the 4 in 6402 (b) the 4 in 2984

- (c) the 2 in 6402 (d) the 2 in 2984
 (e) the 2 in 10269 (e) the 9 in 2984
 (g) the 9 in 10269 (h) the 6 in 6402
 (i) the 6 in 10269 (j) the 0 in 6402
 (k) the 0 in 10269 (l) the 1 in 10269

2 What is the value of each of the following?

- (a) the 8 in 1,85 (b) the 8 in 16,08
 (c) the 5 in 1,85 (d) the 5 in 5,691
 (e) the 6 in 5,691 (f) the 0 in 19,08
 (g) the 1 in 1,85 (h) the 1 in 5,691
 (i) the 1 in 19,08 (j) the 7 in 0,072

3 A boy does the sum $352 + 79$. He writes:

$$\begin{array}{r} 352 \\ + 79 \\ \hline \end{array}$$

Why is it not correct to write the sum like this? How should he write the sum?

4 The following additions and subtractions have been set out badly. Set them out correctly. (Do not work out the answers.)

(a) $\begin{array}{r} 3\ 107 \\ 26 \\ + 147 \\ \hline \end{array}$ (b) $\begin{array}{r} 6\ 203 \\ - 97 \\ \hline \end{array}$

(c) $\begin{array}{r} 1\ 429 \\ + 6\ 580 \\ \hline \end{array}$ (d) $\begin{array}{r} 6\ 700 \\ - 34 \\ \hline \end{array}$

5 Rearrange the digits in the following numbers to make (i) the highest possible number, (ii) the lowest possible number.

- (a) 263 (b) 728 (c) 506
 (d) 2815 (e) 2407

6 The following additions and subtractions have been set out carelessly. Set them out correctly. (Do not work out the answers.)

(a) $\begin{array}{r} 60,91 \\ + 3,2 \\ \hline \end{array}$ (b) $\begin{array}{r} 26,3 \\ - 1,7 \\ \hline \end{array}$

(c) $\begin{array}{r} \$4,49 \\ + \$56,20 \\ \hline \end{array}$ (d) $\begin{array}{r} 42,5 \\ - 9,65 \\ \hline \end{array}$

Chapter 2

Sets (1)

Syllabus Reference 5.2.1

Sets

A **set** is a collection of objects. The **members** of a set may have something in common, such as the set of mathematical instruments in Fig. 2.1. Each member of this set can be used for drawing and measuring shapes.

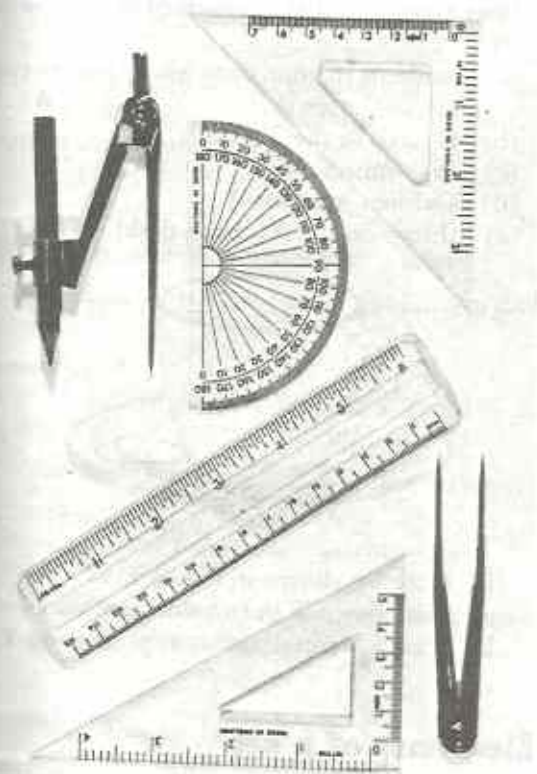


Fig. 2.1

Alternatively, there may be no obvious connection between the members of a set. Fig. 2.2 shows a set whose members do not appear to have anything in common.

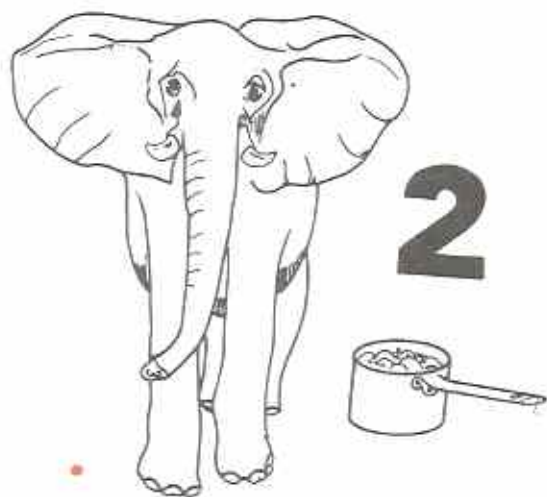


Fig. 2.2

A set can be described, or defined, in two ways: either by making a list of its members, or by describing the property or rule that connects its members. For example, if M is the set shown in Fig. 2.1, then, by listing its members,

$$M = \{60^\circ \text{ set square; } 45^\circ \text{ set square; protractor; ruler; compasses; dividers}\}$$

or, by using the defining property,

$$M = \{\text{mathematical instruments}\}$$

Notice the use of the capital letter to stand for the set and the curly brackets to contain the set. Read $M = \{\text{mathematical instruments}\}$ as 'M is the set of mathematical instruments'. If X is the set shown in Fig. 2.2, then, by listing its members,

$$X = \{\text{elephant; } 2; \text{cooking pot}\}$$

Another way of representing a set is to show its members within a boundary as in Fig. 2.3.

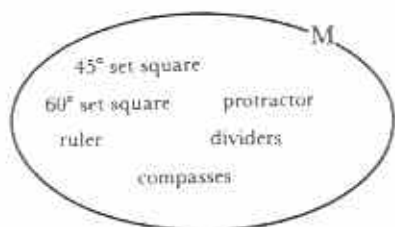


Fig. 2.3 $M = \{\text{mathematical instruments}\}$

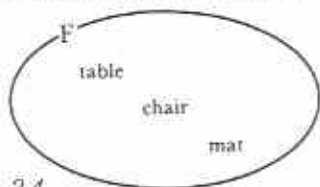
A diagram like that in Fig. 2.3 is called a **Venn diagram**.

Example 1

Given the set $F = \{\text{table; chair; mat}\}$,

- pick out a member that is out of place, giving a possible reason;
- state a common defining property for the members of F ;
- hence give three more members of F ;
- draw a Venn diagram to show F .

- mat*: the only one with no legs
or *table*: people often sit on the other two
or *chair*: the only word that does not contain the letter *t*
- $F = \{\text{household furniture}\}$
or $F = \{\text{things which cost less than \$100}\}$
or $F = \{\text{words containing the letter } a\}$
- Assuming $F = \{\text{household furniture}\}$, then other members of the set could be: bed, chest, cupboard, bookshelf, cabinet, and so on.



(d) Fig. 2.4

Notice in parts (a) and (b) that there is more than one possible correct answer. This shows the importance of defining sets carefully.

Exercise 2a

Questions 1–4 are suitable for class discussion.

- In each of the following sets, pick out the member that is out of place. Give reasons.
 - $\{\text{Harare; Maputo; Lusaka; Malawi}\}$
 - $\{\text{bee; aeroplane; wasp; fly}\}$
 - $\{2; 4; 6; 7; 8\}$
 - $\{\text{table; cow; chair; chicken; dog}\}$

- $\{(2 + 4); (11 - 5); (2 \times 5); (24 \div 4)\}$
- Use the following list when answering questions 2 and 3.

engine, cow, driver, petrol, passenger tractor, soldier, conductor, peace, wheel

- Write down the members of the given list which belong to the following sets.
 - $\{\text{human beings}\}$
 - $\{\text{living things}\}$
 - $\{\text{liquids}\}$
 - $\{\text{things normally found on a bus}\}$
- Give a common defining property for each of the following sets. There may be more than one correct answer in each case.
 - $\{\text{tractor; cow}\}$
 - $\{\text{petrol; passenger; peace}\}$
 - $\{\text{engine; wheel; tractor}\}$
 - $\{\text{peace}\}$
- Find three members of each of the following sets.
 - $\{\text{students in your class whose names contain the letter I}\}$
 - $\{\text{records in the Top 20}\}$
 - $\{\text{even numbers}\}$
 - $\{\text{Cabinet Ministers}\}$
 - $\{\text{things on the teacher's desk}\}$

5



Fig. 2.5

If T is the set shown in Fig. 2.5,

- write down T in two different ways;
- draw a Venn diagram representing T .

Elements of a set

The members of a set are often called **elements**.

Example 2

List the elements of the sets $A = \{\text{even numbers less than } 50\}$ and $B = \{\text{natural counting numbers}\}$.

$$A = \{\text{even numbers less than } 50\}$$

$$= \{2; 4; 6; 8; \dots; 48\}$$

$$B = \{\text{natural counting numbers}\}$$

$$= \{1; 2; 3; 4; \dots\}$$

Notice the following:

- 1 there is a semi-colon between one element and the next;
- 2 when it is impracticable or impossible to write down all the elements we use dots, \dots , to show that we have left some out. The dots stand for 'and so on'.

The following symbols are used to show whether or not elements belong to a set:

\in meaning 'is a member of' or 'belongs to'
 \notin meaning 'is *not* a member of' or 'does *not* belong to'.

For example, in Figs 2.1 and 2.2:

protractor \in {mathematical instruments}
 but cooking pot \notin {mathematical instruments}
 are short for 'a protractor is an element of the set of mathematical instruments' and 'a cooking pot does not belong to the set of mathematical instruments' respectively.

The number of elements in a set

$n(A)$ is short for the number of elements in set A .

Example 3

If $W = \{\text{whole numbers between } 2 \text{ and } 10\}$ find $n(W)$.

If $W = \{\text{whole numbers between } 2 \text{ and } 10\}$
 then $W = \{3; 4; 5; 6; 7; 8; 9\}$ *
 Set W contains 7 elements (by counting),
 hence $n(W) = 7$.

* *Note:* It is assumed that the numbers 2 and 10 are *not* included in the set.

Equal sets

Two sets are equal if they contain the same elements.

Example 4

If $S = \{b; a; d; c\}$ which of the following sets are equal to S ?

(a) $P = \{b; a; d; c; a; d\}$

- (b) $Q = \{d; a; c; e\}$
 (c) $R = \{\text{first four letters of the alphabet}\}$
 (a) P contains the same elements as S with elements s and d repeated.
 Hence $P = S$, since any element which is repeated is counted once only.
 (b) $e \in Q$ but $e \notin S$,
 hence $Q \neq S$ (\neq means 'is not equal to').
 (c) $R = \{\text{first four letters of alphabet}\}$
 $R = \{a; b; c; d\}$
 Hence $R = S$, since the elements of R are simply a rearrangement of those of S .

Exercise 2b

- 1 List the elements of the following sets, using three dots, \dots , if necessary.
 - (a) {letters of the alphabet}
 - (b) {odd numbers less than 30}
 - (c) {all the even numbers}
 - (d) {months in a year}
- 2 Write down the next three members of the following sets.
 - (a) $\{3; 7; 11; 15; \dots\}$
 - (b) $\{2; 4; 8; 16; \dots\}$
 - (c) $\{7; 14; 21; 28; \dots\}$
 - (d) $\{10; 21; 32; 43; \dots\}$
 - (e) $\{1; 4; 9; 16; \dots\}$
- 3 Make each of the following statements true by writing \in or \notin in place of the $*$.
 - (a) $15 * \{3; 6; 9; 12; 15; 18; \dots\}$
 - (b) $17 * \{1; 2; 3; \dots; 7; 8; 9\}$
 - (c) $2\frac{1}{2} * \{\text{whole numbers}\}$
 - (d) If $A = \{\text{African countries}\}$, then *Zambia* $* A$
 - (e) If $T = \{\text{four-sided shapes}\}$, then *square* $* T$
 - (f) $11 * \{1; 3; 5; 7; \dots; 19\}$
- 4 Re-write the following using set notation.
 - (a) A pencil is a member of the set of writing instruments.
 - (b) A pencil is not a member of the set of animals.
 - (c) Bread does not belong to the set of vehicles.
 - (d) 3 does not belong to the set of letters of the alphabet.
 - (e) 3 belongs to the set of digits.
 - (f) 3 is a digit.

- (a) Draw the subsets $A = \{\text{numbers less than } 6\}$, $B = \{\text{odd numbers}\}$.
- (b) Write down those elements which are members of both A and B .

9 If $\mathcal{E} = \{\text{days of the week}\}$ and $T = \{\text{days beginning with T}\}$, draw a Venn diagram showing the elements of \mathcal{E} and T .

10 Given $\mathcal{E} = \{1; 2; 3; \dots; 9; 10\}$, $N = \{\text{even numbers}\}$, $F = \{\text{numbers divisible by } 5\}$, $G = \{\text{numbers greater than } 3\}$.

- (a) Write down the following statement using symbols:
5 and 10 are both greater than 3.
- (b) If a is a number such that $a \in N$, $a \in F$ and $a \in G$, what is a ?
- (c) If b is a number such that $b \in N$ and $b \in G$, what is b ?
- (d) Draw a Venn diagram showing \mathcal{E} , N , F and G .

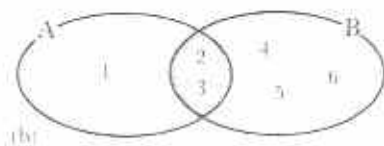
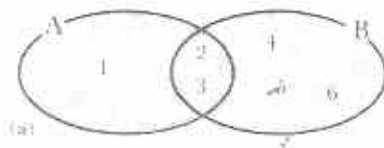


Fig. 2.10 (a) $A \cup B = \{1; 2; 3; 4; 5; 6\}$
(b) $A \cap B = \{2; 3\}$

Example 6

Write down the union and intersection of the following pairs of sets. Represent the results on a Venn diagram.

- (a) $P = \{\text{Friday; Saturday; Sunday}\}$,
 $Q = \{\text{Sunday; Monday}\}$
- (b) $X = \{r; e; k; a\}$, $Y = \{t; i; o; n\}$
- (a) $P \cup Q = \{\text{Friday; Saturday; Sunday; Monday}\}$
 $P \cap Q = \{\text{Sunday}\}$

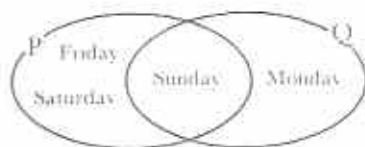


Fig. 2.11

- (b) $X \cup Y = \{r; e; k; a; t; i; o; n\}$
 $X \cap Y = \emptyset$



Fig. 2.12

In example 6(b), $X \cap Y = \emptyset$. Two sets are said to be **disjoint** if their intersection is the empty set. Hence X and Y are disjoint sets.

Exercise 2e

- If $A = \{3; 5; 6; 8; 9\}$ and $B = \{2; 3; 4; 5\}$ write down the sets $A \cup B$ and $A \cap B$. Show A and B on a Venn diagram.
- If $C = \{\text{grapefruit; orange; pear}\}$ and $D = \{\text{grapefruit; pear; apple; pawpaw}\}$ write down the sets $C \cup D$ and $C \cap D$.

Union and intersection

The **union** of two sets is a third set which includes all the elements of the first two. For example, if $A = \{1; 2; 3\}$ and $B = \{2; 3; 4; 5; 6\}$, then the union of A and B is a third set C , where $C = \{1; 2; 3; 4; 5; 6\}$.

In symbols $A \cup B = C$ meaning 'A union B equals C'.

Notice that $A \cup B = B \cup A$.

The **intersection** of two sets is a third set which includes only those elements which are in both of the first two sets. For the sets A and B above, the intersection is D , where $D = \{2; 3\}$.

In symbols $A \cap B = D$ meaning 'A intersects with B to give D'.

Notice that $A \cap B = B \cap A$.

Union and intersection can be represented on Venn diagrams.

In Fig. 2.10(a) the shaded region represents the union of A and B . In Fig. 2.10(b) the shaded region represents the intersection of A and B .

- 3 What is the union of {January; February; March} and {April; May; June}? What is their intersection? Represent the two sets on a Venn diagram.
- 4 Name the intersection of the set of all capital cities in Africa and the set of all towns in Zimbabwe.
- 5 Fig. 2.13 is a Venn diagram representing a universal set \mathcal{E} and the subsets A and B.

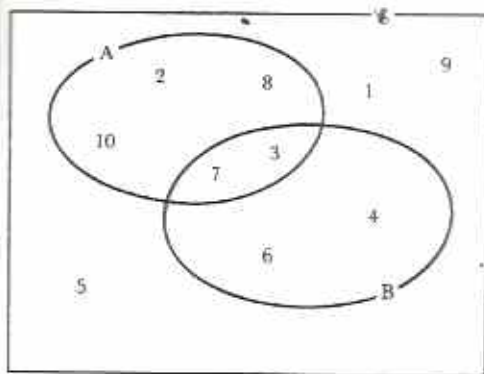


Fig. 2.13

List the elements of the following sets.

- (a) $A \cup B$ (b) $A \cap B$
 (c) $\mathcal{E} \cap B$ (d) $A \cup \mathcal{E}$
- 6 Which of the following pairs of sets are disjoint?
- (a) {even numbers}, {odd numbers}
 (b) {houses in Zimbabwe}, {houses in Kwekwe}
 (c) {rivers of Africa}, {Thames, Mississippi; Zambezi}
 (d) {letters in pupil}, {letters in teacher}
- 7 Let $\mathcal{E} = \{\text{students in your class}\}$
 $B = \{\text{students in the back row}\}$
 $S = \{\text{students whose names begin with S}\}$
 $T = \{\text{students taller than the teacher}\}$

Write down the members of the following.

- (a) $B \cap S$ (b) $B \cup S$
 (c) $S \cup T$ (d) $S \cap T$
 (e) $B \cap T$ (f) $\mathcal{E} \cap S$
- 8 Let $\mathcal{E} = \{1; 2; 3; 4; \dots; 10\}$
 $A = \{\text{odd numbers}\}$
 $B = \{\text{numbers less than 7}\}$
- Write out the members of the following.
- (a) $A \cup B$ (b) $A \cap B$
 (c) $A \cup \mathcal{E}$ (d) $A \cap \mathcal{E}$
 (e) $\mathcal{E} \cap B$ (f) $\mathcal{E} \cup B$
- 9 The members of a football team form a universal set \mathcal{E} , where
 $\mathcal{E} = \{\text{Bob; Farai; Kudzai; Nda; Netsai; Rex; Ron; Sam; Tendai; Thabo; Tom}\}$
 If $A = \{\text{boys whose names end in -ai}\}$
 $B = \{\text{boys with 3-lettered names}\}$
 $T = \{\text{boys whose names begin with T}\}$,
 list the following sets:
- (a) $A \cup B$ (b) $A \cup T$ (c) $B \cup T$
 (d) $A \cup \mathcal{E}$ (e) $B \cup \mathcal{E}$ (f) $T \cup \mathcal{E}$
 (g) $A \cap B$ (h) $B \cap T$ (i) $A \cap T$
 (j) $A \cap \mathcal{E}$ (k) $B \cap \mathcal{E}$ (l) $T \cap \mathcal{E}$

- 10 Given $\mathcal{E} = \{a; b; c; d; e; f\}$ and $X = \{a; b; c; d\}$, $Y = \{c; d; e\}$ and $Z = \{b; d; f\}$, list the following sets. Where brackets are given, do the parts inside the brackets first.

- (a) $X \cup Y$ (b) $X \cup Z$ (c) $Y \cup Z$
 (d) $X \cap Y$ (e) $X \cap Z$ (f) $Y \cap Z$
 (g) $X \cup \mathcal{E}$ (h) $\mathcal{E} \cup Y$ (i) $Z \cup \mathcal{E}$
 (j) $\mathcal{E} \cap X$ (k) $Y \cap \mathcal{E}$ (l) $\mathcal{E} \cap Z$
 (m) $X \cup X$ (n) $Y \cap Y$ (o) $Z \cup \mathcal{E}$
 (p) $X \cup (Y \cup Z)$ (q) $(X \cup Y) \cup Z$
 (r) $X \cap (Y \cap Z)$ (s) $(X \cap Y) \cap Z$
 (t) $X \cap (Y \cup Z)$ (u) $(X \cap Y) \cup Z$
 (v) $X \cup (Y \cap Z)$ (w) $(X \cup Y) \cap Z$
 (x) $(X \cap Y) \cup (X \cap Z)$
 (y) $(X \cup Y) \cap (X \cup Z)$

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Factors and multiples

Syllabus Reference 5.1.1

Factors

$40 \div 8 = 5$ and $40 \div 5 = 8$.

We say that 8 and 5 are **factors** of 40.

If a number can be divided by another number without remainder, the second number is a factor of the first.

The numbers 1, 2, 4, 5, 8, 10, 20, 40 all divide into 40.

They are the set of factors of 40.

Factors of 40 = {1; 2; 4; 5; 8; 10; 20; 40}

We can write 40 as a product of two factors in eight ways:

$$40 = 1 \times 40 = 2 \times 20 = 4 \times 10 = 5 \times 8 \\ = 8 \times 5 = 10 \times 4 = 20 \times 2 = 40 \times 1$$

Exercise 3a

Write down the set of factors of the following numbers.

- 1 12 2 18 3 20 4 24
5 28 6 30 7 32 8 48
9 63 10 72

Which members of {2; 3; 4; 5; 6; 7; 8; 9} are factors of the following numbers?

- 11 27 12 36 13 54 14 56
15 60 16 90 17 120 18 144
19 180 20 210

Prime numbers

A **prime number** has only two factors, itself and 1.

2, 3, 5, 7, 11, 13, ... are prime numbers. 1 has only one factor, itself. 1 is *not* a prime number.

$P = \{2; 3; 5; 7; 11; 13; \dots\}$

where P is the set of prime numbers. P is an infinite set.

Exercise 3b

- Write down the set of prime numbers between 1 and 30.
- Copy the 1–100 number square in Fig. 3.1 into your exercise book.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Fig. 3.1

You are to shade every number which is *not* a prime number.

- 1 is *not* a prime number. Shade the square with 1 in it.
- 2 is a prime number. Leave this square unshaded, then shade every second number following 2. This has been done in Fig. 3.1.
- 3 is a prime number. Leave this square unshaded, then shade every third number following 3.
- Carry on with the next unshaded number, 5, then shade every fifth number after 5.
- Carry on until you can shade no more numbers. This method of finding prime numbers is called the **sieve of Eratosthenes**. Eratosthenes was a Greek who lived over 2 000 years ago.

Prime factors

The **prime factors** of a number are the factors of the number which are prime. It is possible to write every non-prime number as a product of its prime factors. For example:

$$15 = 3 \times 5$$

$$24 = 2 \times 2 \times 2 \times 3$$

$$42 = 2 \times 3 \times 7$$

In set notation:

$$\text{prime factors of } 15 = \{3; 5\}$$

$$\text{prime factors of } 24 = \{2; 3\}$$

$$\text{prime factors of } 42 = \{2; 3; 7\}$$

Notice that although $24 = 2 \times 2 \times 2 \times 3$, the set of prime factors of 24 is simply $\{2; 3\}$. It is not necessary to repeat the factor 2.

To find the prime factors of a number:

- Start with the lowest prime number, 2. Find out if this will divide into the number. If it will not divide, try the next prime number, 3. And so on, trying 5, 7, 11, 13, ... in turn.
- If a prime number will divide, check if it will divide again before moving on to the next prime.

Example 1

- Express 15 288 as a product of its prime factors.
- Hence write down the set of prime factors of 15 288.

$$\begin{aligned} \text{(a) } 15\,288 &= 2 \times 7\,644 \\ &= 2 \times 2 \times 3\,822 \\ &= 2 \times 2 \times 2 \times 1\,911 \\ &= 2 \times 2 \times 2 \times 3 \times 637 \\ &= 2 \times 2 \times 2 \times 3 \times 7 \times 91 \\ &= 2 \times 2 \times 2 \times 3 \times 7 \times 7 \times 13 \end{aligned}$$

The working can be set out as a continued division as follows:

2	15 288
2	7 644
2	3 822
3	1 911
7	637
7	91
13	13
	1

$$\text{Hence } 15\,288 = 2 \times 2 \times 2 \times 3 \times 7 \times 7 \times 13.$$

$$\text{(b) Prime factors of } 15\,288 = \{2; 3; 7; 13\}$$

Exercise 3c

Use the above method (a) to express the following as products of their prime factors, (b) write down their sets of prime factors.

1	12	2	18	3	28	4	30
5	72	6	84	7	108	8	105
9	180	10	216	11	288	12	875
13	900	14	880	15	1 512	16	17 325

In Example 1 we saw that

$$15\,288 = 2 \times 2 \times 2 \times 3 \times 7 \times 7 \times 13.$$

In **index form** $15\,288 = 2^3 \times 3 \times 7^2 \times 13$.

The use of index form saves space and can help to prevent errors in counting and copying.

Exercise 3d

1 Write the following in index form.

$$\text{(a) } 7 \times 7 \times 7 \qquad \text{(b) } 3 \times 3 \times 3 \times 3 \times 3$$

$$\text{(c) } 2 \times 2 \qquad \text{(d) } 10 \times 10 \times 10 \times 10$$

$$\text{(e) } 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$$

$$\text{(f) } 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$$

$$\text{(g) } 4 \times 4 \times 4 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$$

$$\text{(h) } 11 \times 11 \times 17 \times 17 \times 17 \times 17 \times 17 \times 17$$

2 Re-write your answers to Exercise 3c(a) in index form where possible.

3 Express the following as a product of primes in index form.

$$\text{(a) } 24 \qquad \text{(b) } 48 \qquad \text{(c) } 63$$

$$\text{(d) } 92 \qquad \text{(e) } 136 \qquad \text{(f) } 441$$

$$\text{(g) } 360 \qquad \text{(h) } 625 \qquad \text{(i) } 512$$

$$\text{(j) } 720 \qquad \text{(k) } 729 \qquad \text{(l) } 1\,000$$

$$\text{(m) } 1\,232 \qquad \text{(n) } 1\,404 \qquad \text{(o) } 1\,225$$

$$\text{(p) } 7\,290$$

4 Write down the set of prime factors of each number given in question 3.

Common factors

Let F_{12} be the set of factors of 12, then

$$F_{12} = \{1; 2; 3; 4; 6; 12\}$$

and $F_{21} = \{1; 3; 7; 21\}$

$$F_{33} = \{1; 3; 11; 33\}$$

Notice that 1 and 3 are common to all three sets. 1 and 3 are **common factors** of 12, 21 and 33. If C is the set of common factors, then:

$$\begin{aligned} C &= F_{12} \cap F_{21} \cap F_{33} \\ &= \{1; 3\} \end{aligned}$$

There may be more than two common factors of a set of numbers. For example.

$$F_{28} = \{1; 2; 4; 7; 14; 28\}$$

$$F_{42} = \{1; 2; 3; 6; 7; 14; 21; 42\}$$

$$F_{70} = \{1; 2; 5; 7; 10; 14; 35; 70\}$$

$$F_{28} \cap F_{42} \cap F_{70} = \{1; 2; 7; 14\}$$

Notice that since 2 and 7 are common prime factors of 28, 42 and 70, then $14 (= 2 \times 7)$ must also be a common factor of these numbers.

1 is a common factor of all numbers.

Exercise 3e

Find the common factors, other than 1, of the following.

- | | |
|------------------|------------------|
| 1 10 and 14 | 2 15 and 24 |
| 3 21 and 28 | 4 9 and 15 |
| 5 8 and 18 | 6 27 and 30 |
| 7 6 and 12 | 8 10 and 20 |
| 9 30 and 36 | 10 9 and 18 |
| 11 8 and 100 | 12 24 and 40 |
| 13 14, 24 and 38 | 14 21, 35 and 56 |
| 15 27, 36 and 39 | 16 44, 66 and 88 |
| 17 15, 60 and 75 | 18 48, 60 and 72 |

Highest common factor (HCF)

$\{1; 2; 7; 14\}$ is the set of common factors of 28, 42 and 70. 14 is the greatest element of the set. We say that 14 is the **highest common factor** of 28, 42 and 70. **HCF** is short for highest common factor.

Example 2

Find the HCF of 18, 24 and 42.

Compare the sets of factors of the given numbers.

$$F_{18} = \{1; 2; 3; 6; 9; 18\}$$

$$F_{24} = \{1; 2; 3; 4; 6; 8; 12; 24\}$$

$$F_{42} = \{1; 2; 3; 6; 7; 14; 21; 42\}$$

By inspection, $\text{HCF} = 6$.

Alternatively: Express the numbers as products of prime factors and multiply the common prime factors together to give the HCF:

$$18 = 2 \times 3 \times 3$$

$$24 = 2 \times 2 \times 2 \times 3$$

$$42 = 2 \times 3 \times 7$$

The common prime factors are 2 and 3.

$$\text{HCF} = 2 \times 3 = 6.$$

Example 3

Find the HCF of 216, 288 and 360.

The given numbers are large. Express each number as a product of prime factors.

working:

2	216	2	288	2	360
2	108	2	144	2	180
2	54	2	72	2	90
3	27	2	36	3	45
3	9	2	18	3	15
3	3	3	9	5	5
	1	3	3		1
			1		

In index notation,

$$216 = 2^3 \times 3^3$$

$$288 = 2^5 \times 3^2$$

$$360 = 2^3 \times 3^2 \times 5$$

2^3 is the greatest power of 2 contained by all three numbers.

Hence the HCF contains 2^3 .

3^2 is the greatest power of 3 contained by all three numbers.

Hence the HCF also contains 3^2 .

$$216 = (2^3 \times 3^2) \times 3$$

$$288 = (2^3 \times 3^2) \times 2^2$$

$$360 = (2^3 \times 3^2) \times 5$$

$$\text{HCF} = 2^3 \times 3^2 = 8 \times 9 = 72.$$

Exercise 3f

1 Find the HCF of the following. Leave the answers in prime factors.

(a) $2 \times 3 \times 3 \times 5;$

$$2 \times 2 \times 3 \times 3 \times 3;$$

$$3 \times 3 \times 5 \times 7$$

(b) $2 \times 2 \times 3 \times 5;$

$$2 \times 2 \times 3 \times 3 \times 7;$$

$$2 \times 3 \times 3 \times 7$$

(c) $2 \times 3 \times 3 \times 5;$

$$2 \times 2 \times 2 \times 5;$$

$$2 \times 2 \times 3 \times 5 \times 7$$

(d) $2 \times 2 \times 2 \times 3 \times 3 \times 7;$

$$2 \times 2 \times 3 \times 3 \times 3;$$

$$2 \times 3 \times 3 \times 3 \times 7$$

$$(e) \begin{aligned} &2 \times 3 \times 3 \times 5 \times 7; \\ &3 \times 5 \times 5 \times 7 \times 7; \\ &2 \times 2 \times 3 \times 5 \times 5 \times 7 \end{aligned}$$

2 Find the HCF of the following. Leave the answers in prime factors and use index notation.

$$\begin{array}{ll} (a) 2^3 \times 3^2 \times 7; & (b) 2^2 \times 3^3 \times 5; \\ & 2^2 \times 3 \times 5^2; \\ & 2^2 \times 3^3 \times 5; \\ (c) 2^2 \times 3^3 \times 5^2; & (d) 2^3 \times 3^3 \times 5^2; \\ & 3^2 \times 5^3 \times 7; \\ & 2 \times 3^2 \times 5^2 \times 7; \\ (e) 2^3 \times 5^2 \times 7; & \\ & 2^2 \times 3^2 \times 5; \\ & 3^3 \times 5^3 \times 7^2 \end{array}$$

3 Find the HCF of the following.

$$\begin{array}{ll} (a) 9, 15 \text{ and } 24 & (b) 18, 24 \text{ and } 32 \\ (c) 12, 30 \text{ and } 42 & (d) 24, 40 \text{ and } 64 \\ (e) 35, 50 \text{ and } 65 & (f) 30, 45 \text{ and } 75 \\ (g) 42, 70 \text{ and } 56 & (h) 36, 72 \text{ and } 63 \\ (i) 63, 42 \text{ and } 21 & (j) 144, 216 \text{ and } 360 \\ (k) 280, 105 \text{ and } 175 & (l) 126, 234 \text{ and } 90 \\ (m) 160, 96 \text{ and } 224 & (n) 189, 270 \text{ and } 108 \\ (o) 288, 180 \text{ and } 108 & (p) 324, 432 \text{ and } 540 \end{array}$$

Multiples

Common multiples

Each number in the set $\{6; 12; 18; 24; 30; \dots\}$ has 6 as a factor.

We say that the set contains **multiples** of 6.

$$\text{Multiples of } 6 = \{6; 12; 18; 24; 30; \dots\}$$

In the same way,

$$\text{multiples of } 4 = \{4; 8; 12; 16; 20; 24; \dots\}$$

Exercise 3g (Oral)

Give five multiples of each of the following.

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ 6 & 9 & 10 & 11 & 12 \\ 20 & & & & \end{array}$$

Let M_6 be the set of multiples of 6, then

$$M_6 = \{6; 12; 18; 24; 30; \dots\}$$

and $M_4 = \{4; 8; 12; 16; 20; 24; \dots\}$

Notice that 12 and 24 are in both sets. 12 and 24 are **common multiples** of 6 and 4. If K is the set of common multiples, then:

$$\begin{aligned} K &= M_6 \cap M_4 \\ &= \{12; 24; \dots\} \end{aligned}$$

Exercise 3h (Oral)

Give three common multiples of the following numbers.

$$\begin{array}{lll} 1 & 3 \text{ and } 4 & 2 \quad 2 \text{ and } 5 & 3 \quad 3 \text{ and } 7 \\ 4 & 2, 3 \text{ and } 5 & 5 \quad 3, 4 \text{ and } 5 & 6 \quad 2, 3 \text{ and } 7 \end{array}$$

Lowest common multiple (LCM)

30, 60 and 90 are all common multiples of 6, 10 and 15. 30 is the lowest number that 6, 10 and 15 will divide into. We say that 30 is the **lowest common multiple** of 6, 10 and 15. **LCM** is short for lowest common multiple.

Example 4

Find the LCM of 8, 9 and 12.

$$8 = 2 \times 2 \times 2$$

Any multiple of 8 must contain $2 \times 2 \times 2$.

$$9 = 3 \times 3$$

Any multiple of 9 must contain 3×3 .

$$12 = 2 \times 2 \times 3$$

Any multiple of 12 must contain $2 \times 2 \times 3$.

The lowest product containing all three is

$$2 \times 2 \times 2 \times 3 \times 3.$$

The LCM of 8, 9 and 12 is

$$2 \times 2 \times 2 \times 3 \times 3 = 72.$$

Example 5

Find the LCM of 24, 28, 36 and 50.

In indices $24 = 2^3 \times 3$

$$28 = 2^2 \times 7$$

$$36 = 2^2 \times 3^2$$

$$50 = 2 \times 5^2$$

The prime factors are 2, 3, 5, and 7. The LCM contains the highest powers of 2, 3, 5 and 7 that appear in the factorized numbers.

$$\text{LCM} = 2^3 \times 3^2 \times 5^2 \times 7 = 12\,600$$

To find the LCM of a set of numbers:

- express the numbers as a product of prime factors;
- find the lowest product of factors which contains all of the prime factors of the numbers.

Exercise 3i

1 Make two 1–100 number squares like that of Fig. 3.1.

- On the first number square, shade all the

multiples of 2 using shading like this: ///
Then shade all the multiples of 3 using shading like this: \\ .

What can you say about those numbers which have been cross-shaded, XXXX ?

- (b) Repeat the method of part (a) for multiples of 3 and 5. Hence find (i) the LCM of 3 and 5, (ii) the 4th common multiple of 3 and 5.

- 2 Find the LCM of the following. Leave the answers in prime factors.

- (a) $2 \times 2 \times 3$; (b) $2 \times 3 \times 3$;
 $2 \times 3 \times 3 \times 5$; $2 \times 2 \times 2 \times 3$;
 $2 \times 2 \times 5$; $2 \times 2 \times 3 \times 5$
 (c) $2 \times 2 \times 2 \times 3 \times 3$; (d) $3 \times 3 \times 5$;
 $2 \times 3 \times 5 \times 5$; $2 \times 5 \times 5 \times 7$;
 $2 \times 2 \times 3 \times 3 \times 5$; $2 \times 3 \times 7$;
 $3 \times 5 \times 7$

- (e) $2 \times 3 \times 5$;
 $3 \times 5 \times 7$;
 $2 \times 3 \times 3 \times 3$;
 $3 \times 5 \times 5 \times 7$

- 3 Find the LCM of the following. Leave the answers in prime factors, in index form.

- (a) $2 \times 3 \times 5$; $2^2 \times 3 \times 5$; $2 \times 3 \times 5^2$
 (b) $2 \times 3^2 \times 7$; $2^2 \times 3 \times 5$; $2 \times 5^2 \times 7$
 (c) $3 \times 5 \times 7^2$; $2 \times 3^2 \times 5$; $2^2 \times 3 \times 7$
 (d) $2^2 \times 3 \times 5^2$; $2^2 \times 3 \times 7^2$; $3^2 \times 5^2 \times 7$
 (e) $2^3 \times 3^2 \times 5$; $3 \times 5^3 \times 7^2$;
 $2^4 \times 3 \times 7^2$; $3^2 \times 5^2 \times 7^3$

- 4 Find the LCM of the following.

- (a) 4 and 6 (b) 6 and 8
 (c) 6 and 9 (d) 7 and 8
 (e) 8 and 12 (f) 9 and 12
 (g) 2, 3, and 4 (h) 3, 4 and 6
 (i) 4, 6 and 9 (j) 6, 8 and 12
 (k) 8, 10 and 15 (l) 6, 8 and 10
 (m) 10, 12 and 15 (n) 5, 6 and 9
 (o) 7, 8 and 12 (p) 9, 12 and 15
 (q) 10, 16 and 18 (r) 4, 5, 6 and 9
 (s) 6, 8, 10 and 12 (t) 9, 10, 12 and 15

Table 3.1

Any whole number is exactly divisible by	
2	if its last digit is even
3	if the sum of its digits is divisible by 3
4	if its last two digits form a number divisible by 4
5	if its last digit is 5 or 0
6	if it is even and the sum of its digits is divisible by 3
8	if its last three digits form a number divisible by 8
9	if the sum of its digits is divisible by 9
10	if its last digit is 0

There is no easy rule for division by 7.

Notice the following:

- (a) If a number is divisible by another number, it is also divisible by the factors of that number. For example, a number divisible by 8 is also divisible by 2 and 4.
 (b) If a number is divisible by two or more numbers, it is also divisible by the LCM of those numbers. For example, a number divisible by both 6 and 9 is also divisible by 18. 18 is the LCM of 6 and 9.

Example 6

Test the following numbers to see which are exactly divisible by 9.

- (a) 51 066 (b) 9 039 (c) 48 681

method: Find the sum of the digits of each number.

- (a) $5 + 1 + 0 + 6 + 6 = 18$
 18 is divisible by 9.
 Hence 51 066 is also divisible by 9.
 (b) $9 + 0 + 3 + 9 = 21$
 21 is not divisible by 9.
 Hence 9 039 is not divisible by 9.
 (c) $4 + 8 + 6 + 8 + 1 = 27$
 27 is divisible by 9.
 Hence 48 681 is divisible by 9.

Rules for divisibility

Table 3.1 gives some rules for divisors of whole numbers.

Example 7

Which of the numbers 5, 6 and 8 will divide into 2 328 without remainder?

- (a) Test for division by 5:
2 328 does not end in 5 or 0.
2 328 is *not* divisible by 5.
- (b) Test for division by 6:
 $2 + 3 + 2 + 8 = 15$
15 is divisible by 3 and 2 328 is even.
2 328 is divisible by 6.
- (c) Test for division by 8:
The last three digits of 2 328 form the number 328.
 $328 \div 8 = 41$, hence 328 is divisible by 8
2 328 is divisible by 8.
- 2 328 is divisible by 6 and 8; it is *not* divisible by 5. (Notice that since 2 328 is divisible by 6 and 8, it is also divisible by 24. 24 is the LCM of 6 and 8.)

Table 3.2

number	divisors									
	2	3	4	5	6	8	9	10		
(a) 13545	X	✓	X	✓	X	X	✓	X		
(b) 32550										
(c) 83241										
(d) 21015										
(e) 6328										
(f) 3240										
(g) 51294										
(h) 4770										
(i) 435										
(j) 360										
(k) 7138										
(l) 1848										

Exercise 3j

- 1 Find out which of the following numbers are exactly divisible by 4.

92 138 547 1926
2 756 9 428 10 782 56 016

- 2 Find out which of the following numbers are exactly divisible by 3.

184 267 534 917
1 287 3 506 10 523 28 104

- 3 Find out which of the following numbers are exactly divisible by 8.

1 756 3 042 2 872 19 176
48 225 307 138 172 744 563 362

- 4 Find out which of the following numbers are exactly divisible by 9.

253 579 736 2 331
5 675 17 326 35 406 52 866

- 5 Copy and complete Table 3.2. Make a tick, ✓, in a box if the number at the left of the box is divisible by the number above the box. Make a cross, X, if it is not. The first row of boxes has been done.

- 6 Write down those of the following numbers which are exactly divisible by *all three* of the numbers 5, 8 and 9.

3 825 3 960 5 720
91 848 51 840 87 360

- 7 (a) Find out which of the numbers 5, 8 and 9 will divide into 51 768 exactly.

- (b) Hence state which of the numbers 36, 40, 45, 72 will also divide into 51 768 exactly.

- 8 76 356 36 116 16 869 22 374

- (a) Which of the numbers are divisible by 3?

- (b) Which of the numbers are divisible by 4?

- (c) Which of the numbers are divisible by 6?

- (d) Hence state which of the numbers are divisible by 12.

- 9 (a) A student writes $5 044 \times 9 = 45 295$. Explain how the teacher knows that the student has made a mistake.

- (b) Show that $141 \times 19 = 2 699$ is false without doing the multiplication.

- 10 In the number 8134^* , the * stands for a missing digit.

- (a) Find the digit which will make the number divisible by 9.

- (b) Find the digit which will make the number divisible by 8.

- (c) Find the digit which will make the number divisible by 15.

Problem solving using HCF and LCM

Example 8

Square paving stones are used to cover an area management 16,5 m by 12,75 m. If the stones are all alike, and only whole ones are used, what is the greatest size they can be? How many are there?

method: Find the HCF of 16,5 m and 12,75 m.

working in cm:

2	1650	3	1275
3	825	5	425
5	275	5	85
5	55	17	17
11	11		1
	1		

$$1650 = 2 \times 3 \times 5 \times 5 \times 11$$

$$1275 = 3 \times 5 \times 5 \times 17$$

$$\text{HCF of } 1650 \text{ and } 1275 = 3 \times 5 \times 5 = 75$$

The greatest paving stones measure 75 cm by 75 cm.

$$\begin{aligned} \text{Number of paving stones} &= \frac{1650 \times 1275}{75 \times 75} \\ &= 2 \times 11 \times 17 \\ &= 374 \end{aligned}$$

Example 9

A number of oranges can be divided into equal heaps, each containing 10 or 15 or 24 oranges. Find the smallest number of oranges for which this is possible.

method: Find the LCM of 10, 15 and 24.

working:

$$\begin{aligned} 10 &= 2 \times 5 \\ 15 &= 3 \times 5 \\ 24 &= 2 \times 2 \times 2 \times 3 \\ \text{LCM} &= 2^3 \times 3 \times 5 \\ &= 120 \end{aligned}$$

The smallest number of oranges is 120.

Exercise 3k

- Find the greatest mass that can be taken an exact number of times from 360 g, 504 g and 672 g.
- Find the smallest mass that can be measured out in equal amounts of 6 g or 8 g or 9 g.
- Equal squares, as large as possible, are ruled off on a rectangular board 54 cm by 78 cm. Find the size of each square. How many squares are there?
- A piece of string can be cut into equal lengths, either 20 cm or 24 cm or 30 cm long. Find the length of the shortest piece of string for which this is possible.
- Find the smallest sum of money that is an exact multiple of \$1, 12; 64c and 96c.
- A rectangular field measures 308 m by 228 m. Fence posts are placed along its sides at equal distances apart. If the posts are as far apart as possible, what is the distance between them?
- Find the greatest number which when divided into 179 and 234 will leave a remainder of 3 in each case. (*Hint:* First subtract 3 from both numbers.)
- Four wooden rods are of lengths 140 cm, 238 cm, 168 cm and 210 cm. They are cut into pieces which are all of the same length. Find the greatest length possible for these pieces if no wood is left over.
- A certain mass of sugar can be divided into equal heaps. Each heap contains either 27 g, 45 g, 30 g or 20 g. Find the smallest mass of sugar for which this is possible.
- Find the smallest number of sweets that can be put into bags which all contain either 9, 15, 20 or 24 sweets, with none left over.
- A rectangular piece of ground measures 486 m by 360 m. It is marked off exactly into equal square plots. Find the greatest area of a plot in m^2 .
- Three drums are beaten at intervals of 8 s, 15 s and 9 s respectively. They start together. How long will it be until they next sound together?
- Four cars go round a racing track in 1 min 12 s, 1 min 20 s, 1 min 30 s and 1 min 48 s respectively. They start together. How long will it be until they all pass the starting point together again?
- A rectangular field measures 138 m by 78 m. Poles are placed end to end to fence the field. The poles are all the same length and they do not overlap. Find their greatest possible length.
- Find the greatest number which when divided into 1 250 and 1 000 leaves remainders of 26 and 28 respectively.

Measurement of quantities

Syllabus Reference 5.4.1

The **SI system of units** is an internationally agreed method of measuring quantities such as length, mass, capacity and time. SI is short for *Le Système International d'Unités* (International System of Units). Nearly every country in the world uses the SI system.

Except for the measurement of time, the SI system uses decimal multiples to build up tables connecting the units for each quantity. The basic quantities are length, mass, capacity and time. Other quantities, such as area, volume, speed and density, can be expressed in terms of the basic quantities.

Length

The **metre** is the basic unit of length. The metre was first taken as one ten millionth part of the distance between the North Pole and the Equator. In Table 4.1, Greek prefixes are used for multiples of a metre (distances greater than a metre). Latin prefixes are used for sub-multiples of a metre (distance less than a metre).

Table 4.1

length	abbreviation	relationship to basic unit
1 kilometre	1 km	1 000 m
1 hectometre	1 hm	100 m
1 decametre	1 dam	10 m
1 metre	1 m	1 m
1 decimetre	1 dm	0,1 m
1 centimetre	1 cm	0,01 m
1 millimetre	1 mm	0,001 m

Notice that the value of each unit is ten times that of the unit just below it. $1 \text{ km} = 10 \text{ hm}$, $1 \text{ cm} = 10 \text{ mm}$, etc. Compare this with the decimal place-value system in which the value of a digit is ten times the value of the same digit in the next place to it on the right.

Using a ruler

Example 1

Use a ruler to measure the line l in Fig. 4.1.



Fig. 4.1

Give the length of l in cm and in mm.

Fig. 4.2 shows the line l and part of a ruler beside it.



Fig. 4.2

One end of the line is opposite 6,8 cm on the ruler.

The other end is opposite 2 cm.

$$\begin{aligned} \text{Length of } l &= 6,8 \text{ cm} - 2 \text{ cm} \\ &= 4,8 \text{ cm} \\ &= 48 \text{ mm} \end{aligned}$$

Look at the ruler in Fig. 4.2. The longer marks are numbered 0, 1, 2, The distance between two consecutive long marks is 1 cm. The distance between two consecutive short marks is 1 mm.

Notice the method of subtraction in Example 1. This method has two advantages:

- (1) It is often more convenient to measure near the centre of the ruler.
- (2) It avoids possible confusion with the 0 cm and 1 cm marks.

Exercise 4a

1 Measure the lines in Fig. 4.3. Give each length first in cm and then in mm.

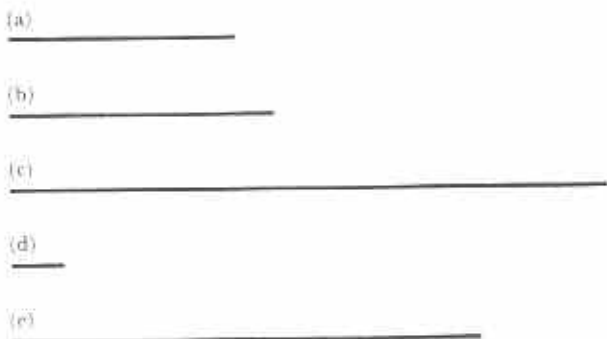


Fig. 4.3

2 Measure the lines in Fig. 4.4. Give each length first in mm and then in cm.

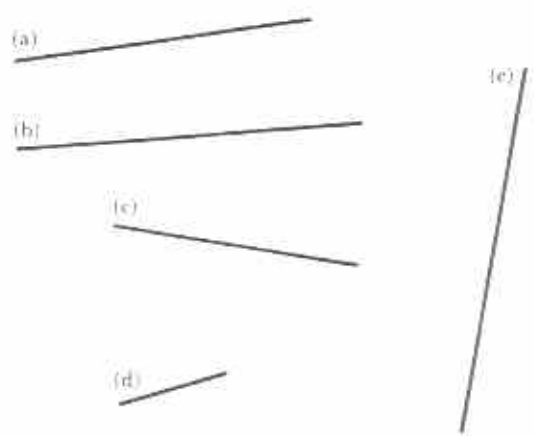


Fig. 4.4

3 Measure the length and breadth of (a) this textbook, (b) your exercise book. Then copy and complete Table 4.2.

Table 4.2

	length		breadth	
	cm	mm	cm	mm
(a) textbook				
(b) exercise book				

- 4 Measure the length and width of the classroom in metres. Convert the result to (a) cm, (b) mm.
- 5 Use a ruler to draw straight lines of the following lengths:
 (a) 6 cm (b) 90 mm (c) 7,5 cm
 (d) 45 mm (e) 5,7 cm (f) 112 mm
 Exchange exercise books with a friend and use a ruler to check the accuracy of each other's work.

In practice, the hectometre, decametre and decimetre are not used very often. The centimetre is useful for measuring short lengths but industry usually gives such lengths in millimetres. This leaves the kilometre, the metre and the millimetre as the most common units:

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ m} = 1000 \text{ mm}$$

The advantage of a decimal system of measurement is that it is easy to write down compound quantities as decimals without doing any calculation.

Example 2

$$1 \text{ metre and } 67 \text{ centimetres} = 1,67 \text{ m}$$

$$= 167 \text{ cm}$$

$$= 1670 \text{ mm}$$

$$= 0,00167 \text{ km}$$

$$4 \text{ km and } 18 \text{ metres} = 4,018 \text{ km}$$

$$= 4018 \text{ m}$$

$$= 4018000 \text{ mm}$$

Exercise 4b

- Questions 1 - 6 are suitable for oral work.
- 1 Express the lengths given in Table 4.3 in (a) km, (b) m, (c) mm.

Table 4.3

km	hm	dam	m	dm	cm	mm
	3	6	4			
		2	5	7		
8	0	9	2			
			6	4	0	8
	2	0	5			

2 Express the following in metres.

- (a) 3 km (b) 5 km (c) 8 km
 (d) 2 km (e) 6 km (f) 10 km
 (g) 9 km (h) 13 km (i) 7 km
 (j) 4 km (k) 3,5 km (l) 4,2 km
 (m) 6,8 km (n) 8,1 km (o) 5,9 km
 (p) 10,4 km (q) 2,7 km (r) 9,3 km
 (s) 12,5 km (t) 1,6 km

3 Express the following in metres.

- (a) 3,85 km (b) 2,44 km
 (c) 8,39 km (d) 6,05 km
 (e) 9,14 km (f) 1,07 km
 (g) 4,124 km (h) 2,993 km
 (i) 7,625 km (j) 5,704 km
 (k) 3,009 km (l) 2,058 km
 (m) 9 km and 400 m (n) 4 km and 620 m
 (o) 3 km and 315 m (p) 1 km and 82 m
 (q) 5 km and 50 km (r) 6 km and 108 m
 (s) 7 km and 8 m (t) 1 km and 4 m
 (u) 9 km and 1 m

4 Express the following in kilometres.

- (a) 5 000 m (b) 8 000 m
 (c) 10 000 m (d) 1 500 m
 (e) 2 400 m (f) 9 500 m
 (g) 6 520 m (h) 4 350 m
 (i) 7 330 m (j) 3 748 m
 (k) 1 375 m (l) 5 822 m
 (m) 2 056 m (n) 9 040 m
 (o) 4007 m (p) 3 km and 416 m
 (q) 7 km and 502 m (r) 6 km and 847 m
 (s) 10 km and 500 m
 (t) 8 km and 420 m (u) 1 km and 440 m
 (v) 9 km and 25 m (w) 5 km and 5 m
 (x) 2 km and 9 m

5 Express the following in metres.

- (a) 173 cm (b) 458 cm (c) 843 cm
 (d) 150 cm (e) 105 cm (f) 280 cm
 (g) 101 cm (h) 100 cm (i) 99 cm
 (j) 53 cm (k) 40 cm (l) 8 cm
 (m) 19 cm (n) 5 cm (o) 50 cm

6 Express the following in metres.

- (a) 1 000 mm (b) 2 000 mm
 (c) 7 000 mm (d) 6 800 mm
 (e) 4 100 mm (f) 1 400 mm
 (g) 3 726 mm (h) 9 504 mm
 (i) 8 119 mm (j) 600 mm
 (k) 300 mm (l) 200 mm
 (m) 51 mm (n) 60 mm
 (o) 3 mm

7 Find the value of the following:

- (a) $93,7 \text{ cm} + 83 \text{ mm} + 2,63 \text{ m}$
 (answer in m)
 (b) $1\,758 \text{ m} + 1\,347 \text{ m} - 2,895 \text{ km}$
 (answer in m)
 (c) $213,6 \text{ m} \times 9$
 (answer in km)
 (d) $69,3 \text{ m} \div 36$
 (answer in cm)

8 A farmer walks round a field with five sides of lengths 376 m, 285 m, 493 m, 329 m and 117 m. How far, in km, does he walk altogether?

9 There are 35 bricks in a pile. Each one is 7,6 cm thick. How high is the pile? (Answer in metres.)

10 There are 40 books in a pile. If each book is 8,5 cm thick, how high is the pile in metres?

11 In Fig. 4.5 a pile of six books, each 6,7 cm thick, are stacked underneath a table.

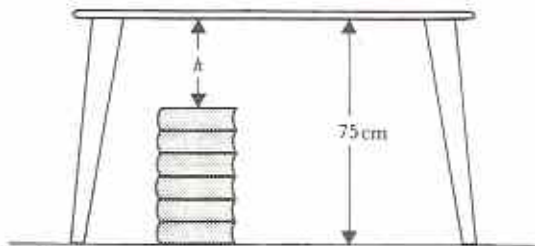


Fig. 4.5

If the table is 75 cm high, what is the distance, h , between the table and the top of the books?

- 12 A piece of writing paper, 29,7 cm long, is ruled into 33 equal spaces. What is the distance between each line in mm?
- 13 An iron rod is 3,028 m long. It is cut into four pieces of equal length. Find the length of each piece in cm.

- 14 A ball of string contains $13\frac{1}{2}$ metres. Lengths of 2,3 m, 1,8 m, 95 cm and 2,37 m are cut off. The remainder is divided into equal pieces each 32 cm long. How many of these equal pieces are there?
- 15 Five lengths of railway line, each 8,95 m long, are placed in line. Find the total length of the line in metres if there are gaps of 5 mm between the ends of the rails.

Mass

The **gramme** is the basic unit of mass. A gramme is the mass of 1 cubic centimetre of water at a temperature of 4°C . This is a very small unit. For this reason, the kilogramme (1 000 grammes) has become the standard unit of mass.

The prefixes are the same as those for lengths. The abbreviations follow those for length but are based on g, the abbreviation of gramme. See Table 4.4.

Table 4.4

mass	abbreviation	relationship to basic unit
1 kilogramme	1 kg	1 000 g
1 hectogramme	1 hg	100 g
1 decagramme	1 dag	10 g
1 gramme	1 g	1 g
1 decigramme	1 dg	0,1 g
1 centigramme	1 cg	0,01 g
1 milligramme	1 mg	0,001 g

The kilogramme, the gramme and the milligramme are the only units used for practical purposes. A further unit is used for large masses, the **tonne** (t).

- 1 tonne = 1 000 kg
 1 kg = 1 000 g
 1 g = 1 000 mg

Capacity

The **litre** is the basic unit of capacity. A litre is the space occupied by 1 kg of water at standard temperature and pressure. Only the kilolitre, litre and millilitre are used for practical and scientific purposes.

Table 4.5

capacity	abbreviation	relationship to basic unit
1 kilolitre	1 kℓ	1 000 ℓ
1 litre	1 ℓ	1 ℓ
1 millilitre	1 ml	0,001 ℓ

Example 3

A bucket holds 18 ℓ of water. It takes 60 cups to fill the bucket. How many ml of water does each cup hold?

60 cups hold 18 litres of water
 60 cups hold $18 \times 1\,000$ ml of water

$$1 \text{ cup holds } \frac{18 \times 1\,000}{60} \text{ ml of water} \\ = 300 \text{ ml}$$

Each cup holds 300 ml of water.

Exercise 4c

- Add the following and give the answers in kilogrammes.
 - 2,3 kg; 5,8 kg; 2,1 kg
 - 785 g; 97 g; 605 g
 - 574 g; 1,706 kg; 605 g
 - 2,8 t; 450 kg; 1,37 t
- Add the following and give the answers in litres.
 - 3,7 litres; 2,4 litres; 1,8 litres
 - 400 ml; 800 ml; 80 ml
 - 1,588 litres; 475 ml; 2,014 litres
 - 2 kℓ; 2,3 kℓ; 850 litres
- Find the value of the following.
 - $2,43 \ell + 1\,875 \text{ ml} - 3,025 \ell$
(answer in litres)
 - $574 \text{ g} + 8,706 \text{ kg} - 6,25 \text{ kg}$
(answer in kg)
 - $7,43 \text{ g} \times 36$ (answer in kg)
 - $6,723 \ell \div 9$ (answer in ml)

- 4 A garage had 582 litres of petrol left in its tanks when it took delivery of 3,865 kℓ of petrol. How much petrol did it then have altogether?
- 5 A lorry is filled with 7,1 tonnes of sand. During the journey 210 kg of sand either falls off or blows away. What mass of sand, in tonnes, is delivered at the end of the journey?
- 6 A sheet of paper has a mass of 4,38 g. What is the mass, in kg, of a packet of 500 sheets of the paper?
- 7 Six identical text-books have a total mass of 1,704 kg. What is the mass of one of the books in grammes?
- 8 Four identical tins of biscuits have a mass of 1,14 kg. What is the mass of three of the tins?
- 9 An unstretched elastic string is 48 cm long. It stretches 15 mm for every 30 g of mass that the string supports. Find the total length of the string when it supports 1,5 kg.
- 10 A wire is bent to form a rectangle 104 mm by 76 mm. 1 cm of the wire has a mass of 1 g. Find the mass of the rectangle.

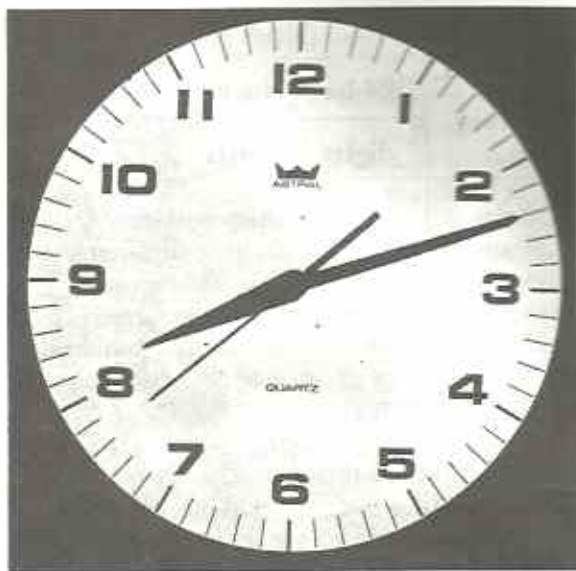


Fig. 4.6

differing time zones. To avoid confusion when reading timetables, railways and airlines use the **24-hour clock**.

Time

The **second** is the basic unit of time. Units of time do not follow the decimal system.

Table 4.6

time	abbreviation	relationship to basic unit
1 second	1 s	1 s
1 minute	1 min	60 s
1 hour	1 h	3 600 s = 60 min

The clock in Fig. 4.6 shows the time as 8.12. A person reading the time will know whether it is morning or evening, i.e. whether the clock shows 8.12 am or 8.12 pm. Ordinary 12-hour clocks are satisfactory for most purposes.

Long train journeys often take more than 12 hours. Journeys by aeroplane often cross into

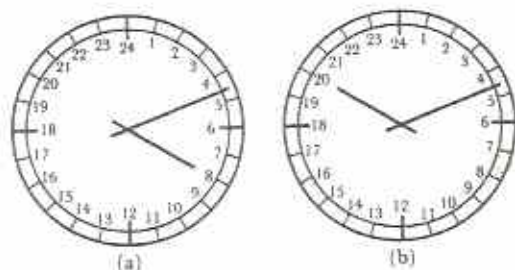


Fig. 4.7
24-hour clock

Fig. 4.7(a) shows the time 8.12 am on a 24-hour clock. Fig. 4.7(b) shows the time 8.12 pm on a 24-hour clock. This appears as 12 minutes past 20 o'clock. This is written as 2012 and said as *twenty-two hours*. 8.12 am is written as 0812 and said as *0-eight-twelve hours*. Table 4.7 gives some ordinary times expressed in terms of the 24-hour clock.

Table 4.7

ordinary time	24-hour clock	
	digits	words
1.30 pm	1330	thirteen-thirty
3.54 am	0354	0-three-fiftyfour
3.54 pm	1554	fifteen-fiftyfour
midday	1200	twelve-hundred
midnight	2400	twentyfour-hundred
12.15 am	0015	0-hundred-fifteen

Notice that all times on the 24-hour clock are given using 4 digits and no points. It is sometimes possible to see 24-hour clocks as in Fig. 4.7. However, in airports it is more common to see digital clocks as in Fig. 4.8.



Fig. 4.8

Exercise 4d

- How many minutes in
 - $2\frac{1}{2}$ h,
 - $1\frac{1}{4}$ h,
 - $1\frac{2}{3}$ h,
 - 180 s
 - 150 s?
- How many seconds in
 - 5 min,
 - $1\frac{1}{3}$ min,
 - $\frac{1}{4}$ h,
 - $\frac{1}{2}$ h,
 - 3 h?
- A motorist took 5 h 11 min for a certain journey. Another motorist took 3 h 54 min for the same journey. What is the difference in their times?
- A machine uses 850 ml of diesel oil in 1 minute. How many litres does it use in $1\frac{1}{2}$ h?
- A borehole pumps 7,2 kl of water in 1 hour. How many litres does it pump in 1 s?
- A car travels 108 km in an hour at a steady speed. How many metres does it go in a minute?
- A man walks at a rate of 96 paces to the minute. Each pace is 0,625 m long. How long does he take to walk 5,4 km?
- Copy and complete Table 4.8.

Table 4.8

ordinary time	24-hour clock	
	digits	words
2.45 pm		
7.30 am		
7.30 pm		
6 am		
10 pm		
12.40 am		

- 9 Express the following in terms of the 24-hour clock.

- 2 pm
- 8.30 pm
- 9 am
- 4 pm
- 5.30 pm
- 5.45 am
- 5.25 am
- 11.55 pm
- 5 to 9 at night
- 10 past 6 in the morning
- noon
- 3 minutes past midnight

- 10 Express the following 24-hour clock times as ordinary times.

- 0500
- 2300
- 1000
- 1930
- 0930
- 0215
- 2145
- 1415
- 1115
- 1819
- 0740
- 2357

- 11 Table 4.9 is the Royal Swazi airline timetable for flights between Harare and Manzini (Swaziland).

Table 4.9

MONDAY		TUESDAY
ZC540	↓	ZC541
FJF		FJF
Y		Y
1330 Lv	MANZINI	Ar 1115
1500 Ar	HARARE	Lv 0945

- What do Lv and Ar stand for?
- How long does it take to fly from Manzini to Harare?
- How long does it take to fly from Harare to Manzini?
- Find out what ZC540, FJF, Y stand for.

12 Table 4.10 is the Air Zimbabwe timetable for flights between Harare and Nairobi (Kenya).

Table 4.10

Effective 3rd April 1982	
Fri	Fri
RH156	RH157
707	707
F/Y	F/Y
1000 × 1345	lv Harare ar ar Nairobi lv 1630 1445

A pilot flew from Harare to Nairobi and back on 10 April.

- What was the flight number from Harare to Nairobi?
- If the actual journey time is 2 h 45 min, what time will the pilot's watch show when he lands at Nairobi?
- What is the difference in local times between Harare and Nairobi?
- How long is the aircraft on the ground at Nairobi?
- What fraction of the time for the total journey Harare-Nairobi-Harare is the 707 in the air?

Use Table 4.11 to answer questions 13 and 14. Table 4.11 is a timetable for flights from Harare to the United Kingdom.

Table 4.11

HARARE-EUROPE/UNITED KINGDOM

Mon	Wed	Thu	Fri	Sat	Sun	
RH	RH	BA	RH	BA	RH	
122	120	052	120	052	120	
707	707	747	707	747	707	
F/Y	F/Y	FCM	F/Y	FCM	F/Y	
2130	2225	2045	2225	2255	2225	Lv HARARE
Tue						Ar FRANKFURT
0610						Lv FRANKFURT
0705						
		Fri		Sun		Ar LONDON/HEATHROW
	Thu	0545	Sat	0755	Mon	Ar LONDON/GATWICK
0815	0745		0745		0745	

- On which day(s) does the flight stop at Frankfurt?
 - What is the flight number of Wednesday's plane?
 - What kind of plane flies on a Sunday?
 - On which day(s) does the flight arrive at London/Heathrow?
- How long is the plane on the ground at Frankfurt?
 - How long does it take to fly from Frankfurt to London/Gatwick?
 - How long does flight BA052 take?
 - How long does flight RH120 take?
- Table 4.12 is a National Railways timetable for train journeys between Harare and Bulawayo.

Table 4.12 Harare - Bulawayo

STATIONS	HARARE		BULAWAYO
		Dep.	2000
	Lochinvar	Arr.	2011
		Dep.	2021
	Norton	Arr.	2057
		Dep.	2107
	Chegutu	Arr.	2216
		Dep.	2227
	Kadoma	Arr.	2258
		Dep.	2308
	Kwekwe	Arr.	0043
		Dep.	0050
	Gweru (M)	Arr.	0210
		Dep.	0305
	Somabhula	Arr.	0340
		Dep.	0342
	Shangani	Arr.	0417
		Dep.	0428
	Heany Junction (O)	Arr.	0557
		Dep.	0605
	Mpopoma	Arr.	0641
		Dep.	0648
		Arr.	0700

- What time does the train depart from Harare?
- What time does the train arrive in Bulawayo?
- How long does the journey from Harare to Bulawayo take?
- How many times does the train stop between Harare and Bulawayo?
- Between which two stations is the train at midnight?

Chapter 5

Fractions and percentages

Syllabus Reference 5.1.1

It is not always possible to use whole numbers to describe quantities (Figs. 5.1, 5.2, 5.3).



Fig. 5.1 I will eat one-third of the loaf now and the rest later

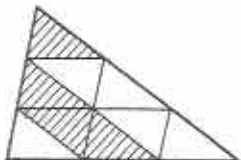


Fig. 5.2 Three-fifths of the men are wearing white shirts

We use **fractions** to describe parts of quantities. We write the fractions like this:

one-third $\frac{1}{3}$ three-fifths $\frac{3}{5}$ four-ninths $\frac{4}{9}$

Fig. 5.3 Four-ninths of the triangle has been shaded








The number below the line is called the **denominator**. The denominator shows the number of equal parts the whole has been divided into. The number above the line is called the **numerator**. The numerator shows the number of parts in the fraction.

Equivalent fractions

Look at the diagrams and the fractions in Table 5.1.

Table 5.1

diagram	number of equal parts	number of parts shaded	fraction shaded
	3	1	$\frac{1}{3}$
	6	2	$\frac{2}{6}$
	9	3	$\frac{3}{9}$
	12	4	$\frac{4}{12}$
	15	5	$\frac{5}{15}$

In each diagram the whole has stayed the same and the same amount has been shaded.



The fractions $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}$ all represent the same amount. Thus $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$. We say that fractions which are equal in this way are **equivalent fractions**. $\frac{1}{3}$ is equivalent to $\frac{5}{15}$ or, simply, $\frac{1}{3} = \frac{5}{15}$.

Exercise 5a (Oral)

Find the missing numbers.

$$1 \quad \frac{1}{9} = \frac{2}{18} = \frac{3}{27} = \frac{4}{36} = \frac{5}{45} = \frac{6}{54} = \frac{7}{63} = \frac{8}{72} = \frac{9}{81} = \frac{10}{90} = \frac{11}{99} = \frac{12}{108} = \frac{13}{117} = \frac{14}{126} = \frac{15}{135} = \frac{16}{144} = \frac{17}{153} = \frac{18}{162} = \frac{19}{171} = \frac{20}{180}$$

$$2 \quad \frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \frac{7}{28} = \frac{8}{32} = \frac{9}{36} = \frac{10}{40} = \frac{11}{44} = \frac{12}{48} = \frac{13}{52} = \frac{14}{56} = \frac{15}{60} = \frac{16}{64} = \frac{17}{68} = \frac{18}{72} = \frac{19}{76} = \frac{20}{80}$$

$$3 \quad \frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24} = \frac{5}{30} = \frac{6}{36} = \frac{7}{42} = \frac{8}{48} = \frac{9}{54} = \frac{10}{60} = \frac{11}{66} = \frac{12}{72} = \frac{13}{78} = \frac{14}{84} = \frac{15}{90} = \frac{16}{96} = \frac{17}{102} = \frac{18}{108} = \frac{19}{114} = \frac{20}{120}$$

$$4 \quad \frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25} = \frac{6}{30} = \frac{7}{35} = \frac{8}{40} = \frac{9}{45} = \frac{10}{50} = \frac{11}{55} = \frac{12}{60} = \frac{13}{65} = \frac{14}{70} = \frac{15}{75} = \frac{16}{80} = \frac{17}{85} = \frac{18}{90} = \frac{19}{95} = \frac{20}{100}$$

$$5 \quad \frac{1}{8} = \frac{2}{16} = \frac{3}{24} = \frac{4}{32} = \frac{5}{40} = \frac{6}{48} = \frac{7}{56} = \frac{8}{64} = \frac{9}{72} = \frac{10}{80} = \frac{11}{88} = \frac{12}{96} = \frac{13}{104} = \frac{14}{112} = \frac{15}{120} = \frac{16}{128} = \frac{17}{136} = \frac{18}{144} = \frac{19}{152} = \frac{20}{160}$$

$$6 \quad \frac{1}{10} = \frac{2}{20} = \frac{3}{30} = \frac{4}{40} = \frac{5}{50} = \frac{6}{60} = \frac{7}{70} = \frac{8}{80} = \frac{9}{90} = \frac{10}{100} = \frac{11}{110} = \frac{12}{120} = \frac{13}{130} = \frac{14}{140} = \frac{15}{150} = \frac{16}{160} = \frac{17}{170} = \frac{18}{180} = \frac{19}{190} = \frac{20}{200}$$

Sets of equivalent fractions can be made by multiplying the numerator and denominator by the same number. For example:

$$\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$

$$\frac{7}{9} = \frac{7 \times 4}{9 \times 4} = \frac{28}{36}$$

Example 1

Express each of the fractions $\frac{3}{4}, \frac{5}{8}, \frac{7}{12}, \frac{2}{3}$ with a denominator of 24. Hence arrange the fractions in ascending order (i.e. from lowest to highest).

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

$$\frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}$$

$$\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$$

The order is $\frac{14}{24}, \frac{15}{24}, \frac{16}{24}, \frac{18}{24}$, i.e. $\frac{7}{12}, \frac{5}{8}, \frac{2}{3}, \frac{3}{4}$.

Note: 24 is the LCM of 4, 8, 12 and 3. We say that 24 is the **lowest common denominator** of the set of fractions.

Exercise 5b

1 Find the missing numbers.

$$(a) \frac{2}{3} = \frac{\quad}{18}$$

$$(b) \frac{5}{6} = \frac{20}{\quad}$$

$$(c) \frac{3}{5} = \frac{\quad}{15}$$

$$(d) \frac{7}{10} = \frac{14}{\quad}$$

$$(e) \frac{3}{7} = \frac{9}{\quad}$$

$$(f) \frac{6}{11} = \frac{18}{\quad}$$

$$(g) \frac{1}{6} = \frac{\quad}{18}$$

$$(h) \frac{3}{5} = \frac{\quad}{20}$$

$$(i) \frac{5}{8} = \frac{25}{\quad}$$

$$(j) \frac{5}{11} = \frac{35}{\quad}$$

$$(k) \frac{3}{8} = \frac{\quad}{48}$$

$$(l) \frac{\quad}{11} = \frac{36}{33}$$

2 Use the given common denominators to arrange the fractions in ascending order (i.e. from lowest to highest).

$$(a) \frac{2}{3}, \frac{5}{6}, \frac{7}{12}, \frac{3}{4} \text{ common denominator } 12$$

$$(b) \frac{3}{4}, \frac{4}{5}, \frac{9}{10}, \frac{17}{20} \text{ common denominator } 20$$

$$(c) \frac{2}{3}, \frac{5}{9}, \frac{7}{12}, \frac{11}{18} \text{ common denominator } 36$$

3 Find the lowest common denominator of the following sets of fractions. Use this to arrange the fractions in ascending order of size.

$$(a) \frac{1}{3}, \frac{2}{9}, \frac{5}{18} \quad (b) \frac{1}{2}, \frac{3}{5}, \frac{8}{15}, \frac{17}{30} \quad (c) \frac{3}{5}, \frac{5}{8}, \frac{7}{10}, \frac{13}{20}$$

Lowest terms

The value of a fraction stays the same if both the numerator and denominator are divided by the same number. For example.

$$\frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3}$$

$$\frac{15}{60} = \frac{15 \div 15}{60 \div 15} = \frac{1}{4}$$

When the numerator and denominator have no common factor, we say that the fraction is in its **lowest terms**, or in its **simplest form**. Thus $\frac{16}{24}$ in its lowest terms is $\frac{2}{3}$; $\frac{1}{4}$ is the simplest form of $\frac{15}{60}$.

To express a fraction in its lowest terms:

- look for common factors of the numerator and denominator;
- divide the numerator and denominator by their common factors;
- repeat until there are no more common factors.

Example 2

Express (a) $\frac{42}{70}$ and (b) $\frac{26}{78}$ in their lowest terms.

(a) by inspection:

$$\frac{42}{70} = \frac{42 \div 7}{70 \div 7} = \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

or by using prime factors:

$$\begin{aligned} \frac{42}{70} &= \frac{2 \times 3 \times 7}{2 \times 5 \times 7} = \frac{2 \times 2 \times 3 \times 7 \div 7}{2 \times 2 \times 5 \times 7 \div 7} \\ &= \frac{1 \times 3 \times 1}{1 \times 5 \times 1} = \frac{3}{5} \end{aligned}$$

(b) by inspection:

$$\frac{26}{78} = \frac{26 \div 2}{78 \div 2} = \frac{13}{39} = \frac{13 \div 13}{39 \div 13} = \frac{1}{3}$$

After practice, you will be able to leave out many of the steps shown in the above example.

Exercise 5c

1 Find the missing numbers.

(a) $\frac{15}{20} \div \frac{1}{3} = \frac{1}{4}$ (b) $\frac{12}{21} = \frac{12}{21} \div \frac{3}{3} = \frac{4}{7}$

(c) $\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$ (d) $\frac{80}{100} = \frac{8}{10} = \frac{4}{5}$

(e) $\frac{25}{100} = \frac{25}{100} \div \frac{25}{25} = \frac{1}{4}$ (f) $\frac{42}{56} = \frac{21}{28} = \frac{3}{4}$

(g) $\frac{18}{27} = \frac{2}{3}$ (h) $\frac{10}{16} = \frac{5}{8}$

(i) $\frac{28}{36} = \frac{7}{9}$ (j) $\frac{48}{100} = \frac{12}{25}$

(k) $\frac{22}{40} = \frac{11}{20}$ (l) $\frac{16}{48} = \frac{1}{3}$

2 Reduce the following fractions to their lowest terms.

(a) $\frac{7}{35}$ (b) $\frac{3}{18}$ (c) $\frac{5}{100}$

(d) $\frac{30}{90}$ (e) $\frac{14}{21}$ (f) $\frac{24}{32}$

(g) $\frac{30}{36}$ (h) $\frac{48}{60}$ (i) $\frac{24}{54}$

(j) $\frac{45}{105}$ (k) $\frac{90}{126}$ (l) $\frac{128}{176}$

Mixed numbers

Some quantities need whole numbers and fractions to describe them (Fig. 5.4).

The number three and a half, written $3\frac{1}{2}$, is a **mixed number**. A mixed number has a whole number part and a fractional part: $3\frac{1}{2} = 3 + \frac{1}{2}$.

It is possible to express a mixed number as a single fraction:

$$3\frac{1}{2} = 3 + \frac{1}{2} = \frac{3}{1} + \frac{1}{2} = \frac{3 \times 2}{1 \times 2} + \frac{1}{2}$$

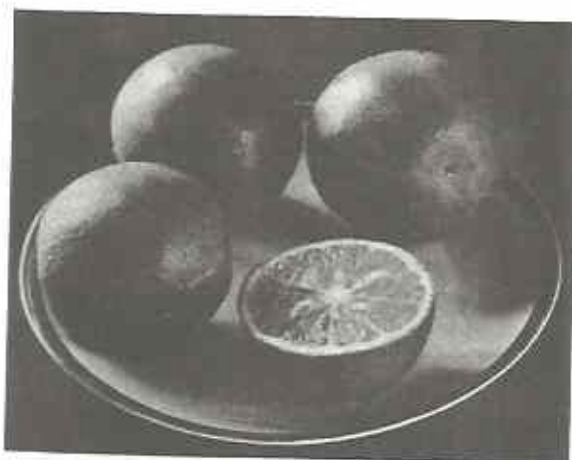


Fig. 5.4 Three and a half oranges

$$= \frac{6}{2} + \frac{1}{2} = \frac{6+1}{2} = \frac{7}{2}$$

$$3\frac{1}{2} = \frac{7}{2}$$

The numerator of the fraction $\frac{7}{2}$ is greater than the denominator. This is an example of an **improper fraction**. If the numerator is less than the denominator, the fraction is a **proper fraction**. For example, $\frac{2}{3}$ is a **proper fraction**.

Numbers such as $9, 3\frac{1}{2}, \frac{2}{3}$ can all be expressed as either improper fractions or proper fractions:

$$9 = \frac{9}{1}$$

$$3\frac{1}{2} = \frac{7}{2}$$

$$\frac{2}{3} = \frac{2}{3}$$

Such numbers are called **rational numbers**.

Example 3

Express $4\frac{5}{6}$ as an improper fraction.

$$4\frac{5}{6} = 4 + \frac{5}{6} = \frac{24}{6} + \frac{5}{6} = \frac{24+5}{6} = \frac{29}{6}$$

or, more quickly,

$$4\frac{5}{6} = \frac{4 \times 6 + 5}{6} = \frac{24+5}{6} = \frac{29}{6}$$

Example 4

Express $\frac{19}{8}$ as a mixed number.

$$\frac{19}{8} = \frac{16+3}{8} = \frac{16}{8} + \frac{3}{8} = 2 + \frac{3}{8} = 2\frac{3}{8}$$

$\frac{19}{8}$, more quickly,

$$\frac{19}{8} = 19 \div 8 = 2 \text{ with remainder } 3$$

But the remainder is also divided by 8:

$$19 \div 8 = 2 + \frac{3}{8} = 2\frac{3}{8}$$

Exercise 5d (Oral)

1 Express as mixed numbers.

- (a) $\frac{4}{3}$ (b) $\frac{5}{2}$ (c) $\frac{11}{4}$ (d) $\frac{7}{5}$
(e) $\frac{11}{5}$ (f) $\frac{25}{7}$ (g) $\frac{25}{6}$ (h) $\frac{19}{8}$
(i) $\frac{26}{9}$ (j) $\frac{41}{5}$ (k) $\frac{80}{7}$ (l) $\frac{65}{6}$
(m) $\frac{89}{9}$ (n) $\frac{40}{11}$ (o) $\frac{113}{3}$ (p) $\frac{119}{9}$
(q) $\frac{123}{7}$ (r) $\frac{57}{10}$ (s) $\frac{33}{20}$ (t) $\frac{129}{100}$

2 Express as improper fractions.

- (a) $1\frac{3}{4}$ (b) $3\frac{1}{3}$ (c) $6\frac{1}{2}$ (d) $4\frac{2}{5}$
(e) $9\frac{1}{4}$ (f) $3\frac{5}{6}$ (g) $4\frac{2}{7}$ (h) $7\frac{2}{3}$
(i) $5\frac{2}{3}$ (j) $6\frac{5}{6}$ (k) $12\frac{2}{3}$ (l) $13\frac{1}{3}$
(m) $5\frac{2}{7}$ (n) $4\frac{1}{12}$ (o) $3\frac{7}{18}$ (p) $8\frac{5}{6}$
(q) $47\frac{1}{2}$ (r) $9\frac{7}{10}$ (s) $6\frac{7}{20}$ (t) $2\frac{13}{100}$

Addition and subtraction of fractions

We can only add or subtract fractions if they have the same denominators.

Example 5

$$\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9}$$

Example 6

$$\frac{13}{15} - \frac{2}{15} = \frac{13-2}{15} = \frac{11}{15}$$

If fractions have different denominators:

- (a) find a common denominator (preferably the lowest);
(b) express each fraction as an equivalent fraction using that denominator;
(c) add or subtract as above.

Example 7

$$\frac{5}{6} + \frac{3}{8}$$

The LCM of 6 and 8 is 24.

$$\begin{aligned}\frac{5}{6} + \frac{3}{8} &= \frac{5 \times 4}{6 \times 4} + \frac{3 \times 3}{8 \times 3} = \frac{20}{24} + \frac{9}{24} \\ &= \frac{20+9}{24} = \frac{29}{24} = 1\frac{5}{24}\end{aligned}$$

Example 8

$$\frac{7}{10} - \frac{4}{15}$$

The LCM of 10 and 15 is 30.

$$\begin{aligned}\frac{7}{10} - \frac{4}{15} &= \frac{7 \times 3}{10 \times 3} - \frac{4 \times 2}{15 \times 2} = \frac{21}{30} - \frac{8}{30} \\ &= \frac{21-8}{30} = \frac{13}{30}\end{aligned}$$

If the numbers are mixed, express them as improper fractions.

Example 9

$$6\frac{5}{12} - 3\frac{3}{4}$$

$$\begin{aligned}6\frac{5}{12} - 3\frac{3}{4} &= \frac{77}{12} - \frac{15}{4} = \frac{77}{12} - \frac{45}{12} = \frac{32}{12} \\ &= \frac{32 \div 4}{12 \div 4} = \frac{8}{3} = 2\frac{2}{3}\end{aligned}$$

Example 10

$$3\frac{2}{5} + 2\frac{2}{3}$$

$$3\frac{2}{5} + 2\frac{2}{3} = \frac{17}{5} + \frac{8}{3} = \frac{51}{15} + \frac{40}{15} = \frac{91}{15} = 6\frac{1}{15}$$

Exercise 5e.

Simplify the following.

- 1 $\frac{3}{4} + \frac{2}{3}$ 2 $\frac{3}{4} - \frac{2}{3}$
3 $\frac{5}{6} + \frac{2}{3}$ 4 $\frac{5}{6} - \frac{2}{3}$
5 $\frac{7}{12} - \frac{3}{8}$ 6 $\frac{7}{12} + \frac{3}{8}$
7 $\frac{11}{15} - \frac{2}{6}$ 8 $\frac{11}{15} + \frac{1}{5}$
9 $1\frac{1}{2} + \frac{3}{4}$ 10 $1\frac{3}{4} - 1\frac{1}{2}$
11 $1\frac{1}{2} + 2\frac{1}{3}$ 12 $1\frac{1}{4} - \frac{2}{3}$
13 $3\frac{7}{8} + 2\frac{3}{4}$ 14 $2\frac{5}{8} + 5\frac{7}{9}$

- 15 $5\frac{3}{4} - 2\frac{4}{5}$ 16 $4\frac{1}{6} - 1\frac{5}{8}$
 17 $1\frac{1}{2} + 2\frac{1}{3} + 3\frac{1}{4}$ 18 $5\frac{3}{4} - 2\frac{7}{8} + 1\frac{1}{2}$
 19 $3\frac{7}{8} + 7\frac{3}{4} - 6\frac{1}{2}$ 20 $5\frac{1}{6} - 3\frac{2}{3} + 6\frac{7}{12}$

Exercise 5f

- What is the sum of $1\frac{7}{12}$ and $3\frac{5}{8}$?
- What is the difference between $4\frac{3}{8}$ and $3\frac{11}{12}$?
- Find the sum of $2\frac{3}{4}$ and $2\frac{4}{5}$. Find the difference between this sum and 6.
- A girl spends $\frac{1}{4}$ of her pocket money on Monday and $\frac{1}{8}$ on Wednesday. What fraction of her money is left?
- A fruit grower uses $\frac{1}{3}$ of his land for bananas, $\frac{3}{8}$ for pineapples, $\frac{1}{6}$ for mangoes and the remainder for oranges. What fraction of his land is used for oranges?
- $\frac{1}{3}$ of a class's timetable is given to English and $\frac{1}{8}$ to Mathematics. What fraction is left for the other subjects?
- During a week, a student spends $\frac{1}{3}$ of his time in bed, $\frac{5}{24}$ of his time in lessons and $\frac{1}{8}$ of his time doing homework. What fraction of his time is left for doing other things?
- A woman goes on a journey. She does $\frac{1}{6}$ of it on a bicycle, $\frac{4}{5}$ on a lorry and walks the rest. What fraction of the journey does she walk?
- A boy plays football for $1\frac{3}{4}$ hours, listens to the radio for $\frac{3}{4}$ hour and then spends $1\frac{1}{4}$ hours doing his homework. How much time does he spend altogether doing these things?
- By how much is the sum of $2\frac{1}{3}$ and $4\frac{1}{2}$ less than $8\frac{1}{10}$?

Multiplication of fractions

Whole number \times fraction

$4 \times \frac{1}{3}$ means 4 lots of $\frac{1}{3}$ of something. We can show this in pictures and numbers (Fig. 5.5):

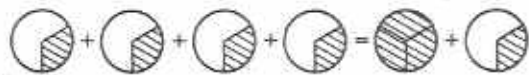


Fig. 5.5

$$4 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3} = 1\frac{1}{3}$$

$$\text{or } 4 \times \frac{1}{3} = \frac{4 \times 1}{3} = \frac{4}{3}$$

Similarly, $3 \times \frac{2}{9}$, (Fig. 5.6):

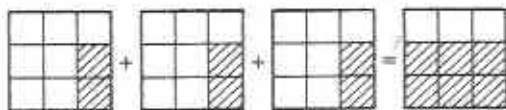


Fig. 5.6

$$3 \times \frac{2}{9} = \frac{2}{9} + \frac{2}{9} + \frac{2}{9} = \frac{6}{9} (= \frac{2}{3})$$

$$\text{or } 3 \times \frac{2}{9} = \frac{3 \times 2}{9} = \frac{6}{9} (= \frac{2}{3})$$

Fraction \times whole number

$\frac{2}{3} \times 3$ means $\frac{2}{3}$ of 3 objects. Let the objects be 3 squares (Fig. 5.7).

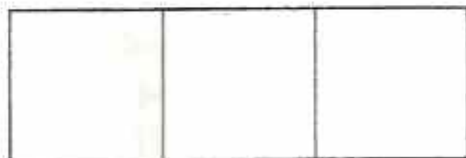


Fig. 5.7

The 3 squares can be divided into fifths. Two of the fifths are shaded in each square (Fig. 5.8).

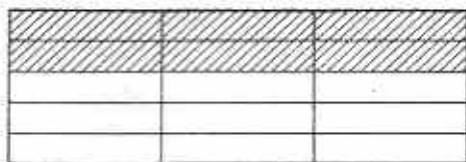


Fig. 5.8

The fifths can be rearranged to make 1 square and $\frac{1}{5}$ of a square altogether (Fig. 5.9).

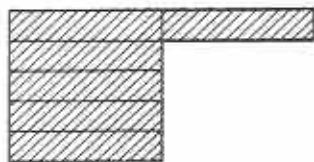


Fig. 5.9

Thus $\frac{2}{5}$ of 3 = $\frac{2}{5} \times 3 = 1\frac{1}{5}$. Also notice that $\frac{2}{5} \times 3 = \frac{2 \times 3}{5} = \frac{6}{5} = 1\frac{1}{5}$ as before.

Exercise 5g

Simplify these.

1 $8 \times \frac{2}{3}$

2 $12 \times \frac{3}{4}$

3 $5 \times \frac{3}{10}$

$$\begin{array}{lll} 4 & 2 \times \frac{2}{5} & 5 & 9 \times \frac{1}{2} & 6 & \frac{2}{3} \text{ of } 15 \\ 7 & \frac{5}{6} \times 8 & 8 & \frac{3}{4} \text{ of } 10 & 9 & \frac{1}{2} \text{ of } 2 \\ 10 & \frac{2}{3} \times 20 & & & & \end{array}$$

Fraction \times fraction

$\frac{2}{3} \times \frac{5}{7}$ means $\frac{2}{3}$ of $\frac{5}{7}$ of something. The pictures in Fig. 5.10 show how to find $\frac{2}{3}$ of $\frac{5}{7}$ of a rectangle.



First, shade $\frac{5}{7}$ of the rectangle.



Then cross-shade $\frac{2}{3}$ of the shaded part.



Fig. 5.10

10 small squares represent $\frac{2}{3}$ of $\frac{5}{7}$ of the rectangle. Since the rectangle contained 21 small squares, then $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$. Notice that

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

To multiply a fraction by a fraction:

- multiply the numerators to make the numerator of the product;
- multiply the denominators to make the denominator of the product; also,
- always change mixed numbers to improper fractions *before* multiplying.

Example 11

Simplify $\frac{3}{8}$ of $2\frac{2}{5} \times 1\frac{3}{5}$.

$$\frac{3}{8} \text{ of } 2\frac{2}{5} \times 1\frac{3}{5} = \frac{3}{8} \times \frac{20}{5} \times \frac{8}{5} = \frac{3 \times 20 \times 8}{8 \times 5 \times 5}^* = \frac{3}{5} = 1\frac{3}{5}$$

* divide the numerator and denominator by the common factors 3, 5 and 8 to simplify.

Exercise 5h

Simplify the following.

- $\frac{1}{2} \times \frac{3}{4}$
- $\frac{2}{3}$ of $\frac{3}{4}$
- $\frac{3}{4}$ of $\frac{1}{3}$
- $\frac{3}{6} \times \frac{2}{3}$
- $\frac{5}{6} \times \frac{3}{8}$
- $\frac{1}{2} \times \frac{8}{9}$
- $1\frac{1}{4} \times \frac{3}{5}$
- $\frac{2}{5}$ of $2\frac{1}{2}$
- $2\frac{1}{2} \times \frac{2}{5}$
- $1\frac{1}{2} \times 1\frac{2}{3}$
- $2\frac{3}{4} \times \frac{1}{5}$
- $\frac{1}{11}$ of $3\frac{2}{3}$
- $\frac{5}{7}$ of $5\frac{1}{4}$
- $5\frac{1}{3} \times 2\frac{1}{4}$
- $2\frac{1}{3} \times \frac{1}{14}$
- $\frac{5 \times 21}{14 \times 15}$
- $\frac{12 \times 25}{10 \times 18}$
- $\frac{16 \times 20 \times 28}{21 \times 24 \times 15}$
- $\frac{4}{7}$ of $8\frac{3}{4}$
- $1\frac{7}{8} \times 3\frac{1}{5}$
- $(2\frac{1}{2})^2$
- $3\frac{3}{4} \times \frac{4}{9} \times 1\frac{1}{5}$
- $1\frac{7}{9} \times 3\frac{3}{4} \times \frac{3}{8}$
- $\frac{12}{25}$ of $(1\frac{1}{4})^2$

Division of fractions

Look at the following working very carefully.

$$2 \div \frac{5}{8} = \frac{2}{\frac{5}{8}} = \frac{2 \times \frac{8}{5}}{\frac{5}{5} \times \frac{8}{5}} = \frac{2 \times \frac{8}{5}}{1} = 2 \times \frac{8}{5}$$

Thus, $2 \div \frac{5}{8} = 2 \times \frac{8}{5}$. $\frac{8}{5}$ is the **reciprocal** of $\frac{5}{8}$; i.e. it is the same fraction turned upside down.

To divide by a fraction, simply multiply by its reciprocal.

Example 12

Find the value of $2\frac{1}{4} \div \frac{3}{7}$

$$2\frac{1}{4} \div \frac{3}{7} = 2\frac{1}{4} \times \frac{7}{3} = \frac{9}{4} \times \frac{7}{3} = \frac{3 \times 7}{4 \times 1} = \frac{21}{4} = 5\frac{1}{4}$$

Notice that this example shows that it is possible to get a result which is greater than either of the given numbers.

We saw before that the value of a fraction is unchanged if we multiply the numerator and

denominator by the same number. This gives another method of dividing by fractions.

$$2\frac{1}{4} \div \frac{3}{7} = \frac{\frac{9}{4}}{\frac{3}{7}} = \frac{\frac{9}{4} \times 28^*}{\frac{3}{7} \times 28} = \frac{9 \times 7}{3 \times 4} = \frac{21}{4} = 5\frac{1}{4}$$

* 28 is the LCM of the denominators of the two fractions.

Example 13

Simplify $\frac{2\frac{2}{3} \times 1\frac{1}{2}}{4\frac{4}{5}}$

Multiplying by the reciprocal:

$$\frac{2\frac{2}{3} \times 1\frac{1}{2}}{4\frac{4}{5}} = \frac{8}{3} \times \frac{3}{5} \div \frac{24}{5} = \frac{8}{3} \times \frac{3}{2} \times \frac{5}{24} = \frac{24 \times 5}{6 \times 24} = \frac{5}{6}$$

or by equal multiplication:

$$\frac{2\frac{2}{3} \times 1\frac{1}{2}}{4\frac{4}{5}} = \frac{\frac{8}{3} \times \frac{3}{2}}{\frac{24}{5}} = \frac{\frac{8}{3} \times \frac{3}{2} \times 30^*}{\frac{24}{5} \times 30} = \frac{8 \times 3 \times 5}{24 \times 6} = \frac{5}{6}$$

* 30 is the LCM of the denominators 2, 3 and 5.

Exercise 5i

Simplify the following.

- | | | | |
|----|---|----|--|
| 1 | $1\frac{2}{3} \div 5$ | 2 | $2\frac{1}{4} \div 3$ |
| 3 | $4\frac{1}{5} \div 3$ | 4 | $6\frac{2}{3} \div 5$ |
| 5 | $1\frac{2}{5} \div 6$ | 6 | $3\frac{3}{5} \div 4$ |
| 7 | $\frac{3}{4} \div \frac{1}{2}$ | 8 | $2\frac{1}{5} \div \frac{1}{5}$ |
| 9 | $6 \div \frac{3}{5}$ | 10 | $10 \div \frac{2}{7}$ |
| 11 | $4\frac{1}{5} \div 7$ | 12 | $13\frac{3}{4} \div 11$ |
| 13 | $\frac{2}{14} \div \frac{3}{7}$ | 14 | $\frac{12}{25} \div \frac{8}{10}$ |
| 15 | $\frac{10}{33} \div \frac{6}{11}$ | 16 | $1\frac{1}{7} \div \frac{4}{21}$ |
| 17 | $2\frac{2}{5} \div 1\frac{1}{3}$ | 18 | $4\frac{2}{5} \div 1\frac{1}{10}$ |
| 19 | $3\frac{1}{3} \div 6\frac{2}{3}$ | 20 | $5\frac{1}{2} \div 2\frac{3}{5}$ |
| 21 | $\frac{3}{8} \div 5\frac{1}{4}$ | 22 | $\frac{5}{6} \div 2\frac{1}{12}$ |
| 23 | $1\frac{4}{5} \div 6\frac{3}{10}$ | 24 | $2\frac{5}{6} \div 8\frac{1}{6}$ |
| 25 | $8\frac{1}{6} \div 3\frac{5}{6}$ | 26 | $7\frac{7}{8} \div 6\frac{5}{12}$ |
| 27 | $9\frac{2}{7} \div 3\frac{9}{10}$ | 28 | $7\frac{1}{8} \div 4\frac{3}{4}$ |
| 29 | $\frac{\frac{8}{10} \times \frac{35}{26}}{\frac{14}{15}}$ | 30 | $\frac{9}{10} \div 3\frac{1}{5}$ |
| 31 | $\frac{7\frac{3}{4}}{9\frac{3}{4}} \div \frac{5}{21}$ | 32 | $\frac{8\frac{1}{6} \times 3\frac{3}{7}}{11\frac{2}{3}}$ |
| 33 | $\frac{3\frac{2}{5} \times 1\frac{5}{8}}{2\frac{1}{10}}$ | 34 | $\frac{5\frac{1}{4} \div 2\frac{4}{5}}{3\frac{3}{4}}$ |
| 35 | $\frac{9\frac{3}{6}}{5\frac{1}{2}} \times 1\frac{1}{4}$ | | |

Exercise 5j

- Find the product of $3\frac{1}{2}$ and $2\frac{2}{5}$.
- Find the value of $6\frac{2}{3}$ divided by $1\frac{7}{9}$.
- What is three-quarters of $3\frac{3}{4}$?
- Does $4\frac{1}{2}$ divided into $1\frac{4}{5}$ mean $4\frac{1}{2} \div 1\frac{4}{5}$ or $1\frac{4}{5} \div 4\frac{1}{2}$? Find the values of $4\frac{1}{2} \div 1\frac{4}{5}$ and $1\frac{4}{5} \div 4\frac{1}{2}$ and say which one is $4\frac{1}{2}$ divided into $1\frac{4}{5}$.
- The mass of each book of an encyclopaedia is $1\frac{3}{4}$ kg. There are 20 books in the encyclopaedia. Find the total mass of the encyclopaedia.
- A tank holds 15 litres of water. The capacity of a cup is $\frac{3}{10}$ of a litre. How many cups of water does the tank hold?
- LP records turn round $33\frac{1}{3}$ times every minute. If one side of an LP plays for 24 minutes, how many times does it turn round?
- One lecture at an evening course lasts $1\frac{1}{2}$ hours. If the course lasts 30 hours altogether, how many lectures are there?
- Some shop-soiled goods are sold for $\frac{2}{5}$ of their original price. What will be the selling price of a pen originally costing 85 cents?
- In a school, $\frac{2}{10}$ of the students play sports. $\frac{3}{5}$ of these play football. What fraction of the students play football?
- It takes $1\frac{3}{4}$ m of cloth to make a skirt. How many skirts can be made from $10\frac{1}{2}$ m of cloth?
- A ball always bounces to $\frac{2}{5}$ of the height from which it falls. If it falls from a height of 1 m, how high will it rise after the second bounce?
- A farmer uses $\frac{9}{16}$ of a field for growing groundnuts. He uses $\frac{2}{7}$ of the remainder for growing maize. What fraction of the field is used for growing maize?
- Three sisters share some money. The oldest gets $\frac{5}{11}$ of the money. The next girl gets $\frac{7}{12}$ of the remainder. What fraction of the money does the youngest girl get?

Percentages

20% means $\frac{20}{100}$. The symbol % is short for per cent. **Per cent** means hundredths. A fraction in the form $\frac{20}{100}$ is called a **percentage**.

100% means $\frac{100}{100} \cdot \frac{100}{100} = 1$. So, when we say we want 100% of something, we mean we want all of it.

To express a percentage as a fraction:

- write the percentage as a fraction of 100;
- reduce the fraction to its lowest terms.

For example, $15\% = \frac{15}{100} = \frac{3 \times 5}{20 \times 5} = \frac{3}{20}$.

Exercise 5k

Express the following percentages as fractions in their lowest terms.

- | | | | |
|--------|--------|--------|--------|
| 1 5% | 2 50% | 3 25% | 4 40% |
| 5 70% | 6 4% | 7 35% | 8 12% |
| 9 80% | 10 75% | 11 24% | 12 65% |
| 13 64% | 14 60% | 15 45% | |

To express a fraction as a percentage make an equivalent fraction with a denominator of 100. For example,

$$\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 40\%$$

$$\frac{176}{300} = \frac{176 \div 3}{300 \div 3} = \frac{58\frac{2}{3}}{100} = 58\frac{2}{3}\%$$

Notice in the last two examples that $\frac{2}{5} \times 100 = 40$ and $\frac{176}{300} \times 100 = 58\frac{2}{3}$. Thus to change a fraction to a percentage it is quicker and easier to multiply the fraction by 100. For example,

$$\frac{3}{8} = \frac{3}{8} \times 100\% = \frac{300}{8}\% = 37\frac{1}{2}\%$$

Exercise 5l

Express the following fractions as percentages.

- | | | | |
|------------------|------------------|-------------------|--------------------|
| 1 $\frac{1}{2}$ | 2 $\frac{1}{4}$ | 3 $\frac{1}{3}$ | 4 $\frac{3}{4}$ |
| 5 $\frac{1}{5}$ | 6 $\frac{1}{10}$ | 7 $\frac{1}{20}$ | 8 $\frac{1}{25}$ |
| 9 $\frac{3}{10}$ | 10 $\frac{2}{5}$ | 11 $\frac{2}{15}$ | 12 $\frac{17}{20}$ |
| 13 $\frac{5}{8}$ | 14 $\frac{1}{6}$ | 15 $\frac{9}{25}$ | |

To express one quantity as a fraction of another

Example 14

Express 7 min 30 s as a fraction of 1 hour.

$$\frac{7 \text{ min } 30 \text{ s}}{1 \text{ h}} = \frac{450 \text{ s}^*}{3600 \text{ s}} = \frac{9 \times 50}{72 \times 50} = \frac{9}{72} = \frac{1}{8}$$

or

$$\frac{7 \text{ min } 30 \text{ s}}{1 \text{ h}} = \frac{7\frac{1}{2} \text{ min}^*}{60 \text{ min}} = \frac{15}{120} = \frac{1 \times 15}{8 \times 15} = \frac{1}{8}$$

*Notice that both quantities must be in the same units before fractions can be reduced.

Exercise 5m

- What fraction of 1 minute is 15 seconds?
- Express 25 cm as a fraction of 3 m.
- What fraction is 500g of 2 kg?
- What fraction of 6 weeks is 6 days?
- What fraction of \$1 is 40c?
- Express 40 min as a fraction of 1 hour.
- Express 650 m as a fraction of 1 km.
- What fraction is 4 min 30 s of 12 min?
- Express 13 weeks as a fraction of 1 year.
- What fraction is 4 mm of 10 cm?
- Express the first quantity as a fraction of the second. Give your answers in their lowest terms.
 - 2 min 15 s, 7 min 30 s
 - 35c, \$2
 - 2,25 kg, 3,75 kg
 - 75 mL, 1 litre
 - 650 m, 1,95 km
 - 2 weeks, 1 year
 - 360 mL, 2 litres
 - 440 m, 1,5 km
 - \$2,25, \$24,75
 - 2 h 20 min, 5 h 15 min

To express one quantity as a percentage of another

Make sure that both quantities are in the same units. Express the first quantity as a fraction of the second. Multiply the fraction by 100 to change it to a percentage.

Example 15

A man hammers a post 2,2 m long into the ground. 66 cm of the post is below ground. What percentage of the post is above the ground?

Fraction of post below the ground

$$= \frac{66 \text{ cm}}{2,2 \text{ m}} = \frac{66 \text{ cm}}{220 \text{ cm}} = \frac{66}{220}$$

Percentage of post below ground

$$= \frac{66}{220} \times 100\% = \frac{6 \times 11 \times 5 \times 20}{11 \times 20} \% \\ = 30\%$$

Percentage of post above ground

$$= 100\% - 30\% = 70\%$$

Exercise 5n

- Express the first quantity as a percentage of the second.
 - 10c, \$1
 - 25c, \$1
 - 400 g, 1 kg
 - 800 mL, 1 litre
 - 3 mm, 1 cm
 - 500 mL, 4 litres
 - 45g, 75g
 - 25m, 1 km
 - 30c, \$1,50
 - \$1, \$2,50
- In an exam a student scored 60 marks out of a possible 80. What percentage is this?
- Three girls are missing from a class of 25 girls. What percentage of the class is missing?
- 26 cm of wood is cut off a board 130 cm long. What percentage has been cut off?
- In a box of 200 oranges, 18 are bad. What percentage is bad? What percentage is good?
- A piece of elastic is 48 cm long. It is stretched to a length of 60 cm. What is the increase in length? What percentage of the original length is the increase?
- A motorist buys a 5-litre can of gear oil. She uses 800 mL. What percentage of oil has she used and what percentage remains?
- The distance between two villages is 8 km. A boy walks from one village to the other. He walks most of the way but runs the last 480 metres. What percentage of the journey did he run?
- A man bought a car for \$7 000. He sold it a year later for \$5 600. What percentage of his money did he lose?
- During a Physics lesson lasting 1 hour, it took 9 minutes to set up and clear away apparatus. What percentage of the lesson time was used in this way?
- A cake has a mass of 2,5 kg. It contains 275 g of fruit. What percentage of fruit is there in the cake?

Symbolic expression

Syllabus Reference 5.6.1

Numbers in boxes

$14 + \square = 17$. What number in the box will make this true? You may have seen problems like this in Primary School. $14 + \square = 17$ will be true if 3 goes in the box: $14 + \boxed{3} = 17$ is true.

Exercise 6a (Oral)

In each sentence, find the number which makes it true.

- | | |
|------------------------|------------------------|
| 1 $3 + 2 = \square$ | 2 $8 - 7 = \square$ |
| 3 $7 + 7 = \square$ | 4 $7 - 7 = \square$ |
| 5 $0 + 8 = \square$ | 6 $4 - 0 = \square$ |
| 7 $9 + 5 = \square$ | 8 $18 - 11 = \square$ |
| 9 $6 + 8 = \square$ | 10 $20 - 14 = \square$ |
| 11 $\square = 6 + 3$ | 12 $\square = 4 - 1$ |
| 13 $\square = 10 + 10$ | 14 $\square = 17 - 7$ |
| 15 $\square = 6 + 10$ | 16 $\square = 19 - 13$ |
| 17 $\square = 4 + 8$ | 18 $\square = 14 - 9$ |
| 19 $\square = 7 + 4$ | 20 $\square = 13 - 5$ |
| 21 $3 + \square = 5$ | 22 $8 - \square = 2$ |
| 23 $6 + \square = 9$ | 24 $10 - \square = 9$ |
| 25 $1 + \square = 8$ | 26 $7 - \square = 3$ |
| 27 $0 + \square = 5$ | 28 $14 - \square = 14$ |
| 29 $7 + \square = 11$ | 30 $12 - \square = 8$ |
| 31 $\square + 2 = 5$ | 32 $\square - 1 = 6$ |
| 33 $\square + 8 = 9$ | 34 $\square - 3 = 1$ |
| 35 $\square + 4 = 8$ | 36 $\square - 2 = 9$ |
| 37 $\square + 7 = 7$ | 38 $\square - 6 = 0$ |
| 39 $\square + 5 = 12$ | 40 $\square - 5 = 7$ |
| 41 $3 = 2 + \square$ | 42 $8 = 10 - \square$ |
| 43 $8 = 3 + \square$ | 44 $10 = 13 - \square$ |
| 45 $10 = 7 + \square$ | 46 $1 = 5 - \square$ |
| 47 $15 = 10 + \square$ | 48 $6 = 16 - \square$ |
| 49 $18 = 6 + \square$ | 50 $7 = 19 - \square$ |
| 51 $6 = \square + 2$ | 52 $5 = \square - 2$ |
| 53 $8 = \square + 5$ | 54 $13 = \square - 3$ |

- | | |
|-----------------------|-----------------------|
| 55 $9 = \square + 7$ | 56 $0 = \square - 8$ |
| 57 $10 = \square + 4$ | 58 $9 = \square - 10$ |
| 59 $16 = \square + 9$ | 60 $11 = \square - 6$ |

Exercise 6b (Oral)

In each sentence there are two or three boxes. The *same* number should go in every box and make the sentence true.

- $\square + \square = 4$
- $\square + \square = 6$
- $\square + \square = 10$
- $12 = \square + \square$
- $8 = \square + \square$
- $2 = \square + \square$
- $\square + \square + \square = 6$
- $\square + \square + \square = 15$
- $\square + \square + \square = 21$
- $9 = \square + \square + \square$
- $18 = \square + \square + \square$
- $30 = \square + \square + \square$
- $10 - \square = \square$
- $16 - \square = \square$
- $\square = 14 - \square$
- $\square = 2 - \square$

What is the value of $5 + \square$? The value depends on what number goes in the box. If 4 goes in the box, the value is 9. If 11 goes in the box the value is 16.

Exercise 6c (Oral)

- Find the value of the following if 5 goes in each box.

(a) $\square + 3$	(b) $\square + 7$	(c) $4 + \square$
(d) $7 + \square$	(e) $6 - \square$	(f) $10 - \square$
(g) $\square - 1$	(h) $\square - 4$	(i) $5 - \square$
- Find the value of the following if 8 goes in each box.

(a) $\square + 9$	(b) $15 + \square$	(c) $16 - \square$
(d) $\square - 6$	(e) $\square + 0$	(f) $\square + 6$

(g) $11 + \square$ (h) $9 - \square$ (i) $\square - 7$

3 Find the value of the following if 9 goes in each box.

(a) $\square + \square$ (b) $\square - \square$
(c) $\square + \square + 2$ (d) $\square + \square - 6$
(e) $\square + 7 + \square$ (f) $\square - 3 + \square$
(g) $5 + \square + \square$ (h) $26 - \square - \square$

Letters for numbers

In mathematics we use letters of the alphabet to stand for numbers instead of boxes. We write $14 + x$ instead of $14 + \square$. Any letter can be used. For example, $14 + a$ would be just as good as $14 + x$. Capital letters are not used; only small letters are used.

When using a letter instead of a number, the letter can stand for any number in general. Thus the value of $14 + x$ depends on the value of x . For example,

- if x stands for 2; $14 + x$ has the value 16;
- if x stands for 12; $14 + x$ has the value 26;
- if x stands for 5; $14 + x$ has the value 19.

When letters and numbers are used together in this way, the mathematics is called **generalised arithmetic** or **algebra**. The word algebra comes to us from an important book written around AD830 by Mohammed Musa al Khwarizmi, a noted Moslem mathematician. The title of the book was *Al-jabr wa'l Muqābalah*.

The statement $14 + x = 17$ is called an **algebraic sentence**. It means fourteen plus a number x makes seventeen, or, fourteen plus x equals seventeen.

In the next exercise, each letter stands for a number. The number that the letter stands for should make the sentence true. For example, $14 + x = 17$ will be true if x stands for 3. We write this:

$$\begin{aligned} \text{If } 14 + x &= 17 \\ \text{then } x &= 3 \end{aligned}$$

Exercise 6d (Oral)

Each sentence is true. Find the number that each letter stands for.

- | | |
|------------------|------------------|
| 1 $x = 2 + 7$ | 2 $x = 3 - 0$ |
| 3 $y = 9 + 5$ | 4 $y = 14 - 4$ |
| 5 $z = 18 + 11$ | 6 $z = 15 - 8$ |
| 7 $a = 5 + 16$ | 8 $a = 23 - 6$ |
| 9 $b = 8 + 18$ | 10 $b = 24 - 19$ |
| 11 $6 + 3 = c$ | 12 $6 - 4 = c$ |
| 13 $12 + 8 = p$ | 14 $18 - 7 = p$ |
| 15 $9 + 7 = q$ | 16 $17 - 8 = q$ |
| 17 $13 + 9 = r$ | 18 $25 - 18 = r$ |
| 19 $5 + 18 = s$ | 20 $21 - 12 = s$ |
| 21 $2 + m = 5$ | 22 $7 - m = 6$ |
| 23 $11 + n = 14$ | 24 $15 - n = 10$ |
| 25 $22 + d = 30$ | 26 $17 - d = 9$ |
| 27 $19 + e = 30$ | 28 $25 - e = 15$ |
| 29 $8 + f = 23$ | 30 $17 - f = 11$ |
| 31 $z + 4 = 8$ | 32 $z - 6 = 2$ |
| 33 $y + 12 = 22$ | 34 $y - 1 = 27$ |
| 35 $x + 6 = 29$ | 36 $x - 14 = 0$ |
| 37 $c + 11 = 30$ | 38 $c - 15 = 15$ |
| 39 $b + 7 = 21$ | 40 $b - 17 = 12$ |
| 41 $5 = 4 + a$ | 42 $6 = 7 - a$ |
| 43 $10 = 5 + r$ | 44 $8 = 13 - r$ |
| 45 $17 = 9 + q$ | 46 $16 = 20 - q$ |
| 47 $26 = 12 + p$ | 48 $4 = 21 - p$ |
| 49 $24 = 17 + s$ | 50 $11 = 29 - s$ |
| 51 $4 = n + 2$ | 52 $9 = n - 5$ |
| 53 $12 = m + 5$ | 54 $16 = m - 9$ |
| 55 $30 = f + 7$ | 56 $10 = f - 17$ |
| 57 $28 = e + 13$ | 58 $6 = e - 18$ |
| 59 $22 = d + 15$ | 60 $21 = d - 6$ |

Exercise 6e (Oral)

Each sentence is true. Find the numbers that the letters stand for.

- | | |
|---------------------|---------------------|
| 1 $x + x = 8$ | 2 $y + y = 12$ |
| 3 $z + z = 14$ | 4 $4 = a + a$ |
| 5 $18 = b + b$ | 6 $16 = c + c$ |
| 7 $e + e + e = 3$ | 8 $f + f + f = 12$ |
| 9 $g + g + g = 30$ | 10 $6 = h + h + h$ |
| 11 $24 = j + j + j$ | 12 $27 = k + k + k$ |
| 13 $12 - m = m$ | 14 $20 - n = n$ |
| 15 $p = 28 - p$ | |

What is the value of $x + 6$? The value of $x + 6$ depends on what x stands for.

If $x = 3$, then $x + 6 = 3 + 6 = 9$

If $x = 8$, then $x + 6 = 8 + 6 = 14$

Exercise 6f (Oral)

- Find the value of the following when $x = 4$.
(a) $x + 5$ (b) $x + 8$ (c) $x - 1$
(d) $x - 4$ (e) $9 + x$ (f) $0 + x$
(g) $10 - x$ (h) $19 - x$
- Find the value of the following when $m = 15$.
(a) $m + 1$ (b) $m + 9$ (c) $m - 3$
(d) $m - 14$ (e) $5 + m$ (f) $13 + m$
(g) $22 - m$ (h) $15 - m$
- Find the value of the following when $a = 7$.
(a) $a + a$ (b) $a - a$
(c) $a + a + 6$ (d) $a + a + 3$
(e) $a + 5 + a$ (f) $a - 2 + a$
(g) $9 + a + a$ (h) $16 + a - a$
(i) $11 - a + a$ (j) $a - 8 + a$
(k) $a - 13 + a$ (l) $2 - a + a$

Exercise 6g (Oral)

- A bicycle costs \$100 and a radio costs \$ x . They cost \$160 altogether. What is the value of x ?
- Esau is a years old. Next year he will be 14 years old. What is the value of a ?
- A bag of sugar has a mass of m kg. The mass of three bags of sugar is 6 kg. What is the value of m ? What would be the total mass of one bag of sugar and a 5 kg bag of flour?
- There are two pencils. One is 14 cm long. The other is x cm long. Someone puts the pencils together as shown in Fig. 6.1. If $x = 10$, what is the total length of the two pencils?

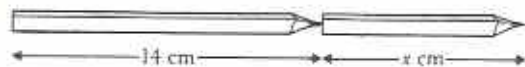


Fig. 6.1

- There are n students in a classroom. 12 more students come into the classroom. If $n = 22$, how many students are there in the classroom altogether?
- A man has 28 goats. He gives n of them to his sons and keeps the rest. If he keeps 10 goats, what is the value of n ?
- A boy has k cents. He spends 15 cents. If he now has 32 cents, what is the value of k ?
- A father is 42 years old and his son is y years old. If the difference of their ages is 28 years, what is the value of y ?

- A girl has 14 sweets. She eats n of them. Find how many sweets she has left when $n = 5$. What is the value of n when she has no sweets left?
- There are 21 matches left in a matchbox. Originally there were m matches in the box and 27 have been used. What is the value of m ?

Algebra from words

Here is a story in pictures (Fig. 6.2)

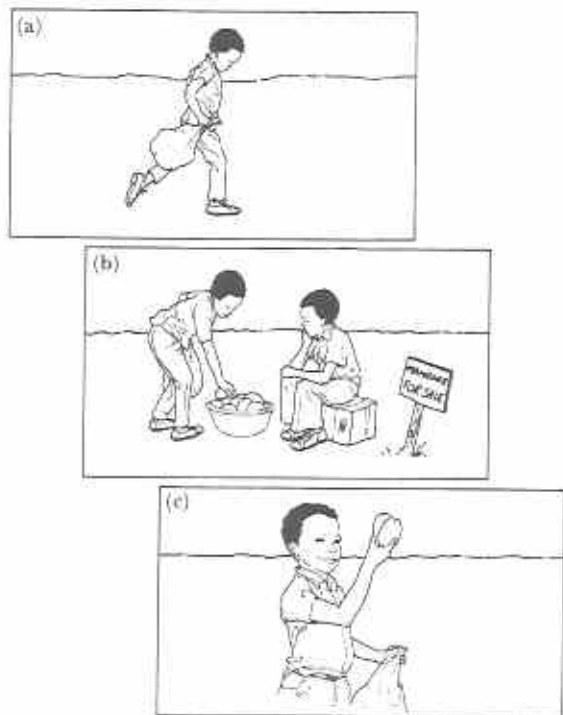


Fig. 6.2

Here is the story in words.

- John has a bag with some mangoes in it.
 - He buys two more mangoes.
 - John now has a bag with some mangoes and two more mangoes.
- Here is the same story with algebraic symbols.

- (a) John has m mangoes.
 (b) John buys 2 mangoes.
 (c) John now has $m + 2$ mangoes.

In this story, m represents the number of mangoes in the bag, $m + 2$ represents the number of mangoes that John has altogether.

Example 1

A boy has m mangoes. He sells two of them. How many mangoes does he now have?

He sells 2 mangoes. He must have 2 less than he had before. He now has $m - 2$ mangoes.

Sometimes a problem may be difficult because letters are used for numbers. If this happens, first change the letter to an easy number. Use that number instead of the letter. This will show what has to be done with the letter.

Example 2

A girl is 14 years old. How old will she be in x years time?

Use 2 and 10 instead of x :

In 2 years time the girl will be $14 + 2$ years old (16 years old).

In 10 years time the girl will be $14 + 10$ years old (24 years old).

So, in x years time the girl will be $14 + x$ years old.

$14 + x$ will not simplify.

The girl will be $14 + x$ years old.

Remember that, in algebra, letters represent numbers only. If a problem contains units, then units must show in the working. For example, the length of rope cannot be 7 or x . The units are missing. The rope might be 7 metres long or x centimetres long. Always use units when necessary.

Example 3

How many cents are there in n Dollars?

Use \$2 and \$5 instead of n Dollars.

Multiply by 100 to change cents to Dollars

$$\$2 = 2 \times 100 \text{ cents} = 200 \text{ cents}$$

$$\$5 = 5 \times 100 \text{ cents} = 500 \text{ cents}$$

So, $\$n = n \times 100 \text{ cents} = 100n \text{ cents}$

Exercise 6h (Oral)

Use the pictures to help you with the algebra.

- 1 In Fig. 6.3 the man is x years old and his daughter is 12 years old. Together their ages come to 50 years. What is the value of x ?



Fig. 6.3

- 2 The height of the blocks in Fig. 6.4 is h cm. If each block is 20 cm thick, find the number that h stands for.

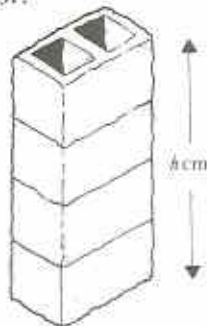


Fig. 6.4

- 3 In Fig. 6.5 each stamp costs n cents. Find n if the 8 stamps cost \$4 altogether.



Fig. 6.5

- 4 When new, the box of matches in Fig. 6.6 contained m matches. Find m if 4 are used and 42 remain.



Fig. 6.6

- 5 The herd of cattle in Fig. 6.7 is worth x Dollars. Find x if each beast is worth \$340.

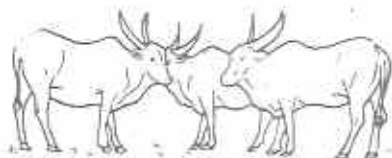


Fig. 6.7

- 6 Use the information in Fig. 6.8 to find the value of d .

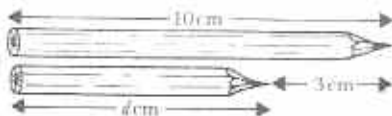


Fig. 6.8

- 7 In Fig. 6.9 there are x oranges in the bag. How many oranges are there altogether?

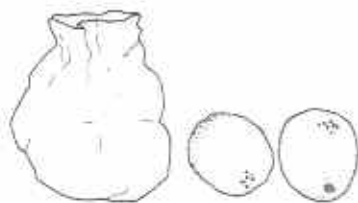


Fig. 6.9

- 8 The telegraph poles in Fig. 6.10 are d metres apart. How far is the student from the right-hand pole?

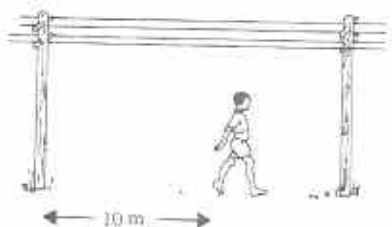


Fig. 6.10

- 9 The mass of one of the dictionaries in Fig. 6.11 is b grammes. What is the total mass of the books in the picture (a) in grammes, (b) in kilogrammes?



Fig. 6.11

- 10 Eggs cost x cents each. Find the total cost of the eggs in Fig. 6.12 (a) in cents, (b) in Dollars.

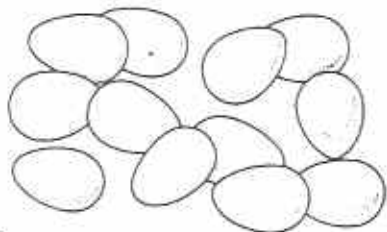


Fig. 6.12

The following exercises will give you practice at making algebraic expressions from English sentences.

Exercise 6i (Oral)

Use addition in this exercise.

- 1 Simba is 13 years old. How old will he be in 8 years from now? How old will he be in y years from now?
- 2 What number is 4 more than 3? What number is 4 more than n ? What number is 7 more than x ?
- 3 A farmer has 6 goats and z cows. How many animals altogether?
- 4 Last year the body-mass of a girl was 48 kg. The increase in her body-mass this year is m kg. What is the girl's body-mass this year?
- 5 A man has k coconuts. A friend gives him one more coconut. How many coconuts does the man have now?
- 6 If today is the 6th of September, what will be the date in n days time? (Assume that the month will still be September.)
- 7 Each side of a square is d centimetres long. What is the perimeter of the square?

- 8 In a test, Rufaro got 7 marks more than Nomsa. Nomsa got x marks. How many marks did Rufaro get? David got y marks more than Rufaro. How many marks did David get?

Exercise 6j (Oral)

Use subtraction in this exercise.

- 1 What number is 2 less than 9? What number is 2 less than n ? What number is 5 less than r ?
- 2 A woman is 30 years old. What was her age 10 years ago? What was her age y years ago?
- 3 A book has x pages. A student reads 30 pages of the book. How many pages has the student not read?
- 4 A boy has m mangoes. If he eats 5 of them, how many does he have left?
- 5 A girl guesses that a line is 15 cm long. She measures the line. She finds that her guess is x cm too big. What is the length of the line?
- 6 A man bought a diamond for $\$x$. He sold it for $\$1\ 600$. What was his profit?
- 7 A man cycles x km towards a village which is 20 km away. How far does he still have to cycle?
- 8 Ruth has s Dollars in her savings account. She uses some of this to buy a dress which costs t Dollars. How much does she have left in her savings?

Exercise 6k (Oral)

Use multiplication in this exercise.

- 1 How many metres in 1 kilometre? How many metres in 4 kilometres? How many metres in x kilometres?
- 2 How many days in one week? How many days in 5 weeks? How many days in w weeks?
- 3 How many calendar months are there in x years?
- 4 What is the number which is 9 times greater than d ?
- 5 What is the cost of 8 books at $\$4$ each? What is the cost of x books at $\$4$ each? What is the cost of 5 books at $\$t$ each?
- 6 A loaf of bread has a mass of x kg. What is its mass in grammes?

- 7 A rubber band is x mm long when unstretched. It is stretched to 3 times its unstretched length. What is the stretched length?
- 8 What is the cost of 10 boxes of matches at m cents each?

Exercise 6l (Oral)

Use division in this exercise.

- 1 How many weeks are there in 14 days? How many weeks are there in d days?
- 2 How many metres are there in 3 000 millimetres? How many metres are there in k mm?
- 3 Six equal blocks have a total mass of b kg. What is the mass of each block?
- 4 Five pencils cost 50 cents. How much does one pencil cost? In another shop 5 pencils cost p cents. How much does one pencil cost in this shop?
- 5 A chicken lays h eggs in 9 days. What is the average number of eggs it lays in one day?
- 6 A stick is 140 cm long. The stick is cut into x pieces, each the same length. What is the length of each piece?
- 7 What is the number which is one-third as big as n ?
- 8 Fig. 6.13 shows 5 equal bricks built in a column. The height of the column is x cm. How thick is each brick?



Fig. 6.13

Exercise 6m

- 1 A radio programme has x minutes of talking and 20 minutes of music. How long is the programme?
- 2 A teacher has 6 pens, n of them are red, the rest are blue. How many blue pens does the teacher have?
- 3 Find the area, in m^2 , of a rectangle which is a m long and 5 m wide.

- 4 How many kilogrammes are there in x grammes?
- 5 A packet of sugar costs 50c. How much would x packets of sugar cost? Give your answer, (a) in cents, (b) in Dollars.
- 6 A school paid \$3 800 to buy desks. It bought d desks altogether. What was the cost of one desk?
- 7 Mary has 20 oranges. Naomi has x oranges less than Mary. How many oranges has Naomi?
- 8 A man walks $3x$ km on the first day, 12 km on the second day and $2x$ km on the third day. How far does he walk in the three days?
- 9 Team A scored three times as many points as Team B. Which team scored most points? If Team A scored n points, how many points did Team B score?
- 10 A square grass mat has a side of length 3 metres. What is the total area of x of these mats?

Exercise 6n

- 1 Find the difference between x Dollars and $7x$ cents. Give your answer in cents.

- 2 A rectangle is such that its length is twice its breadth. If the breadth is d cm, find the perimeter of the rectangle.
- 3 A full matchbox contains x matches. A man has three full matchboxes and a matchbox containing y matches. How many matches does he have altogether?
- 4 A child is x years old. Her mother is four times as old. How old is the mother? What will be the age of (a) the child, (b) the mother in y years time?
- 5 Find the sum of $2x$ kg and $5x$ g. Give your answer in grammes.
- 6 A car cost $\$c$ when new. It was sold for four-fifths of its cost price. How much money was lost on the car?
- 7 A man has n cattle. He sells half of them. He then gives 8 cattle to his daughter's husband. How many cattle does he have left?
- 8 How many packets of pins at 15 cents per packet can I buy for x Dollars?
- 9 During a period of x weeks there were 13 days when it didn't rain. On how many days did it rain?
- 10 A sum of \$5 is shared equally between x girls. One of the girls spends 36c. How many cents does she have left?

Chapter 7

Properties of solids

(Optional Chapter)

Syllabus Reference nil

Nearly everything that we can see and touch takes up space. These things are either **gases**, **liquids** or **solids**. You will study some of the properties of liquids and gases in science.

Many solids, such as stones and trees, have rough and **irregular** shapes. These usually occur in nature. Other solids, such as tin cans and houses, have **regular** shapes. These are usually made by humans.

Fig. 7.1 is a photograph of the conical tower at Great Zimbabwe. The figure also shows that the tower, if continued, would make a cone shape.

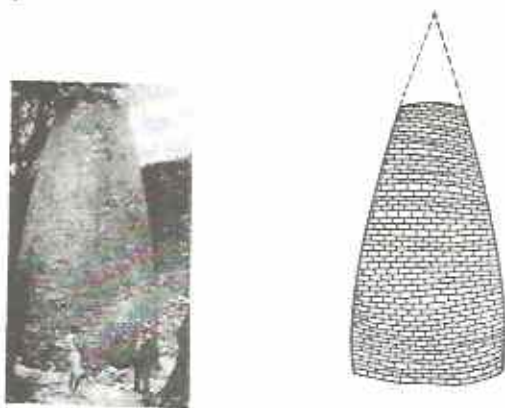
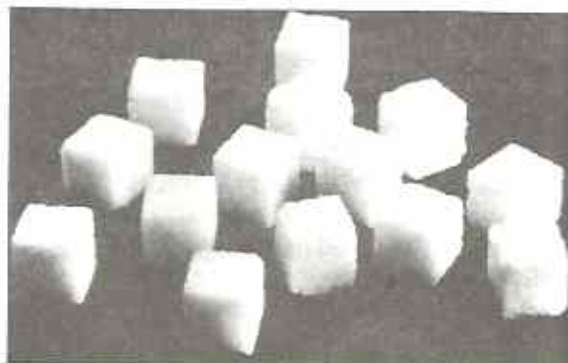


Fig. 7.1 Conical tower at Great Zimbabwe. The drawing shows that the tower, if continued, would be cone shaped.

(b)



(c)



(d)



(a)



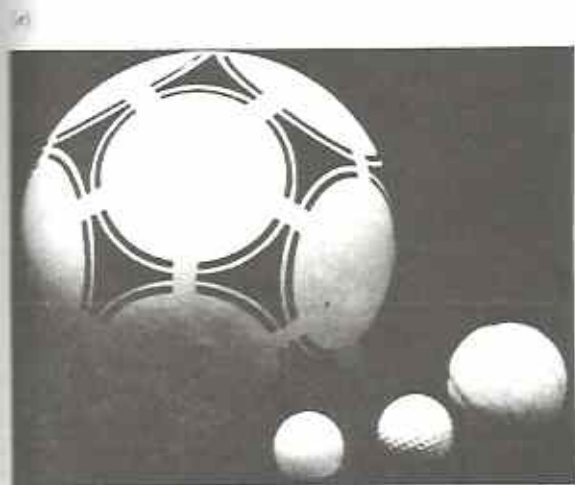


Fig. 7.2 (a) Cuboids (b) Cubes (c) Cone shapes
(d) Cylinders (e) Spheres.

Exercise 7a

- 1 Look at the photographs in Fig. 7.2. Identify as many of the objects as you can.
- 2 Make a table like the one in Fig. 7.3.

No.	Name of object	Freehand drawing	Name of basic shape
1	tin of beans		cylinder
2			
3			

Fig. 7.3

Name at least ten objects in the photographs. Make a freehand sketch of each object and name its basic shape. An example is shown in Fig. 7.3.

Note to teacher:

For the remainder of this chapter, it will be helpful if a supply of solids is made available:

matchbox, soap packet, shoe box, sugar cube, empty tin cans, ball, etc.

Cuboid

Look at the matchbox in Fig. 7.4. The name of its shape is a **cuboid**.

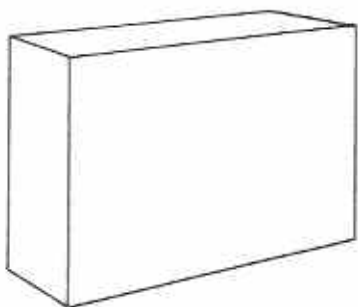


Fig. 7.4 (a) Matchbox
(b) Outline of matchbox: a cuboid.

The cuboid is one of the most common man-made solids. All solids have surfaces, or **faces**. Most solids also have **edges**. Edges meet at a point called a **vertex** (Fig. 7.5).

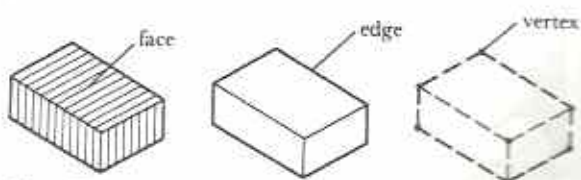


Fig. 7.5

A **face** may be flat (**plane**) or curved. A cuboid

has six plane faces. Each face is in the shape of a rectangle.

An **edge** is a line where two faces meet. Edges may be straight or curved. A cuboid has 12 straight edges.

A **vertex** is a point or corner where three or more edges meet. The plural of vertex is **vertices**. A cuboid has eight vertices.

Planes of symmetry

Any cuboid can be cut through the middle to give two parts, each one the same as the other (Fig. 7.6).

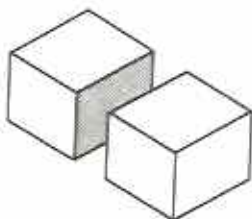


Fig. 7.6

If one part is placed against a mirror as in Fig. 7.7, its reflection looks like the other part.

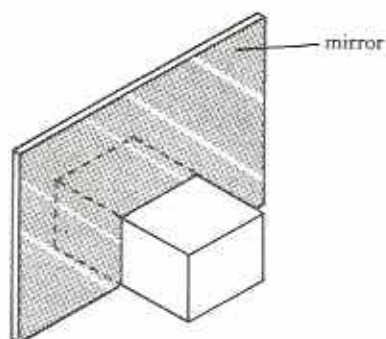


Fig. 7.7

The mirror in Fig. 7.7 is a **plane of symmetry** of the cuboid. Every cuboid has at least three planes of symmetry (Figs 7.7 and 7.8).

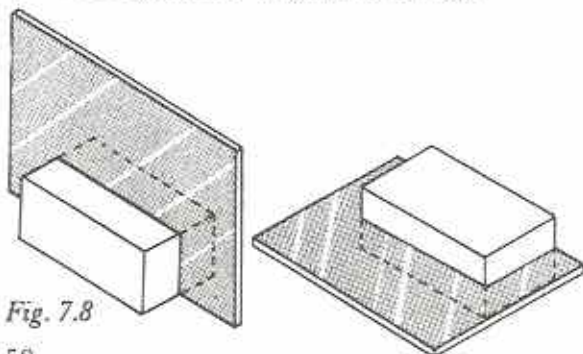


Fig. 7.8

Drawing cuboids

Fig. 7.9 shows four ways of drawing cuboids.

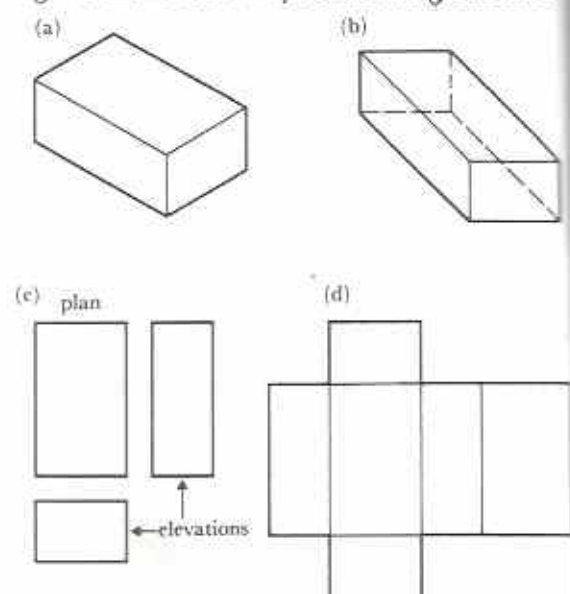


Fig. 7.9 Ways of drawing a cuboid:

- (a) isometric projection (b) skeleton view
(c) engineer's drawing (d) net.

In each case it is impossible to show the solid as it really is. However, a **skeleton view** is very useful since it shows all the edges. Fig. 7.10 shows how to draw a skeleton view. Notice that some edges are hidden from view. We usually show these as broken lines.

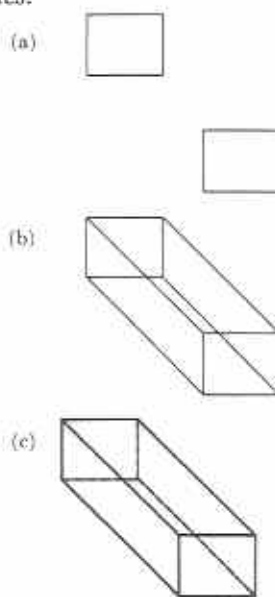


Fig. 7.10 How to draw a skeleton view of a cuboid:

- (a) draw two rectangles
(b) join corner to corner
(c) go over the drawing, make the hidden edges broken.

Net of a cuboid

Get a tray from a matchbox. Cut the edges as shown in Fig. 7.11. Flatten the shape. The flat shape is called the **net** of the open cuboid.

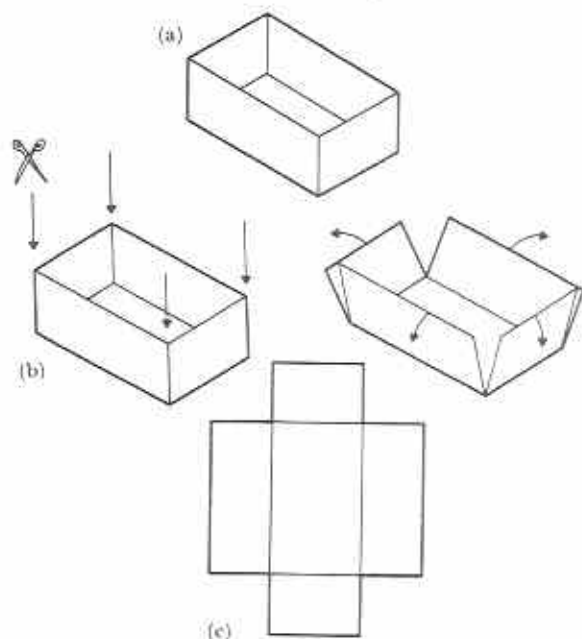


Fig. 7.11 Cutting a matchbox tray to get the net of an open cuboid: (a) this is an open cuboid (b) cut along the edges shown (c) this is the net of the open cuboid.

The shape of the net depends where you make the cuts. You should be able to make the nets shown in Fig. 7.12 from a matchbox tray.

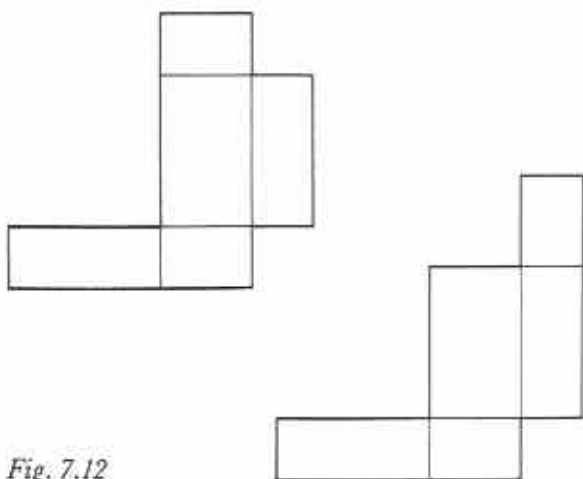


Fig. 7.12

Notice in both cases that the net is made up of five rectangles. The net of a closed cuboid would contain six rectangles.

Cube

A **cube** is a cuboid in which all six faces are squares (Fig. 7.13).

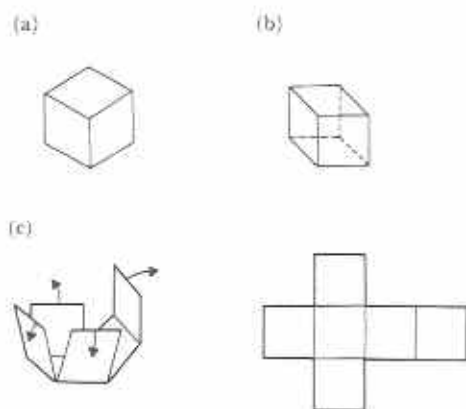


Fig. 7.13 Cubes: (a) isometric projection (b) skeleton view (c) net.

Exercise 7b

- Write down five everyday objects which are cuboids.
- The solids in Fig. 7.14 are made by placing cuboids together. How many cuboids are in each solid?

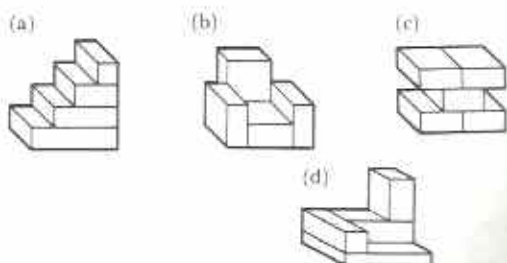


Fig. 7.14

- How many planes of symmetry has each solid in Fig. 7.14?
- Draw a skeleton view of a cuboid and a cube.

- 5 What shape is the face of a cuboid?
- 6 What shape is the face of a cube?
- 7 The cuboid in Fig. 7.15 is made by building up four layers of cubes.

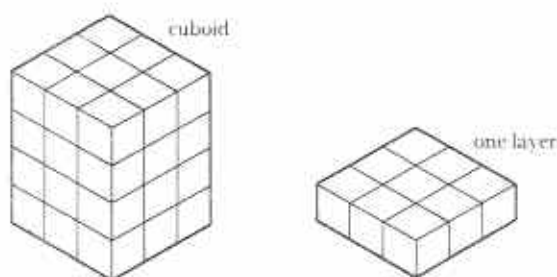


Fig. 7.15

- (a) How many cubes are in one layer?
- (b) How many cubes are in the whole cuboid?

If the outside of the cuboid is painted red:

- (c) How many cubes have three red faces?
- (d) How many cubes have only two red faces?
- (e) How many cubes have only one red face?
- (f) How many cubes have no red faces?

- 8 Which of the diagrams in Fig. 7.16 can be folded to make a cuboid? (It may help if you copy the shapes on to paper and try to fold them.)

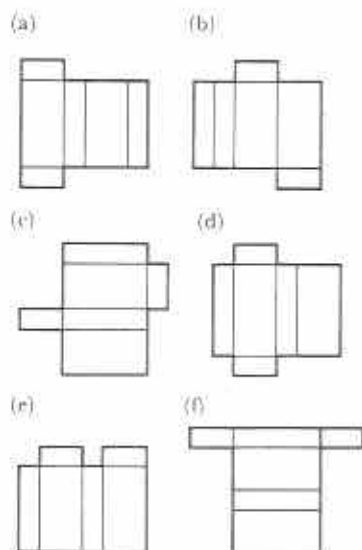


Fig. 7.16

- 9 Use squared paper to draw as many different nets of cubes as you can. Fig. 7.17 gives two examples.

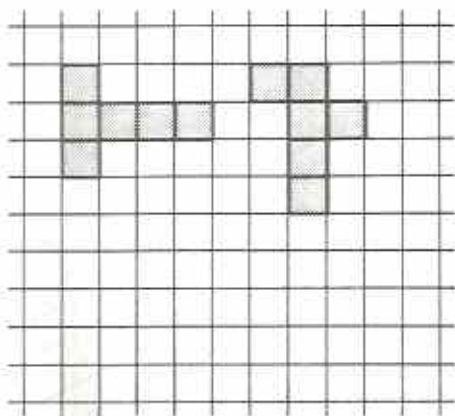


Fig. 7.17

- 10 Draw the net of a cube of edge 5 cm on stiff card. Use clear sticky tape to make the cube from its net.
- 11 How many planes of symmetry has a cube?
- 12 A cuboid is made from wire so that it is 15 cm long, 12 cm wide and 8 cm high. What length of wire is needed? (Make a sketch.)
- 13 A piece of wire of length 66 cm is used to make a skeleton cube. No wire is wasted and the largest possible cube is made. What will be the length of one of the edges of the cube?
- 14 In Fig. 7.18,
 - (a) how many faces can you see?
 - (b) how many edges can you see?
 - (c) how many vertices can you see?

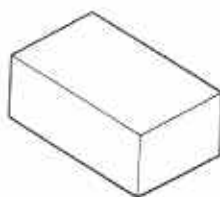


Fig. 7.18

- 15 Copy and complete Table 7.1.

Table 7.1

	number of vertices	number of faces	number of edges
cuboid			
cube			

Cylinder

Look at the tin of milk-powder in Fig. 7.19. The name of its shape is a **cylinder**.

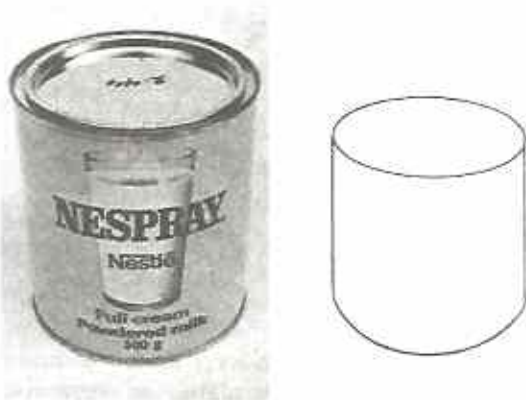


Fig. 7.19 (a) Tin of milk powder
(b) Outline of tin: cylinder

A cylinder has two plane faces and one curved face. It has two curved edges and no vertices. The two plane faces are both circles.

Drawing cylinders

Fig. 7.20 shows two ways of drawing a cylinder.

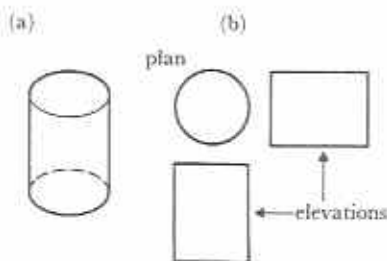


Fig. 7.20 Cylinder: (a) skeleton view
(b) engineer's drawing.

A skeleton view of a cylinder is drawn in much the same way as that of a cuboid. This is shown in Fig. 7.21.

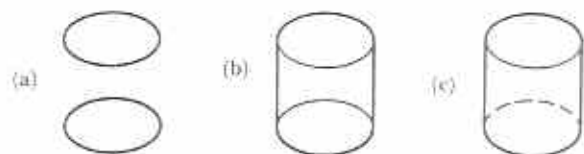


Fig. 7.21 How to draw a skeleton view of a cylinder: (a) draw two flattened circles (ellipses) (b) join the ellipses by two straight lines, as shown (c) make the hidden edge as a broken line.

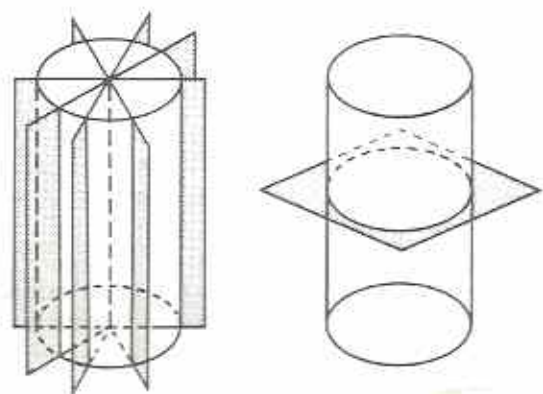


Fig. 7.22 Cylinder: some planes of symmetry

Fig. 7.22 shows that a cylinder can have an uncountable number of planes of symmetry. This is because a circle can have as many lines of symmetry as we care to draw.

Fig. 7.23 shows how a cylinder can be cut to give its net. The net is made up of two circles and one rectangle.

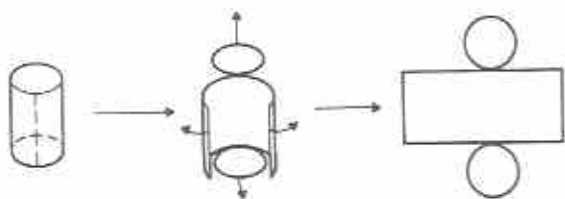


Fig. 7.23 How to cut a cylinder to give its net

Prism

Fig. 7.24 gives some examples of **prisms**.

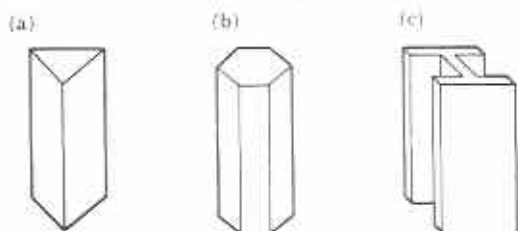


Fig. 7.24 (a) *Triangular prism*
(b) *Hexagonal prism* (c) *I-shaped prism*

Notice that the base and top faces of a prism are always the same shape and size. The names of the prisms come from the shape of their base and top faces. The side faces of right prisms are always rectangles.

The cuboid is a rectangular prism and the cylinder can be thought of as a special prism.

Exercise 7c

- 1 Write down five everyday objects which are cylinders.
- 2 Write down five everyday objects which are prisms.
- 3 The solids in Fig. 7.25 each include cylinders. How many cylinders are in each solid?

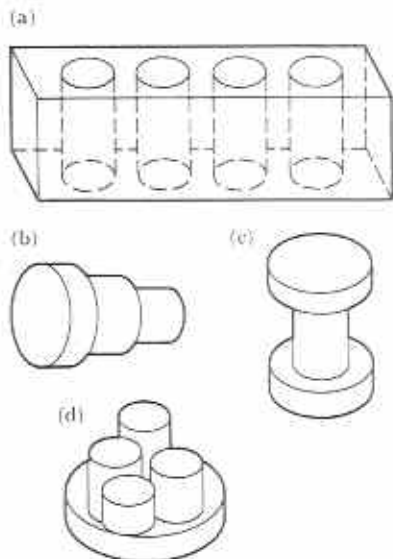


Fig. 7.25

- 4 How many planes of symmetry has each solid in Fig. 7.25?
- 5 How many planes of symmetry has the I-shaped prism in Fig. 7.24?
- 6 Draw a skeleton view of a cylinder and a triangular prism.
- 7 What is the shape of the plane faces of a cylinder?
- 8 What is the shape of the end faces of a triangular prism? What is the shape of the other faces of a triangular prism?
- 9 Name the solids which have the nets shown in Fig. 7.26.

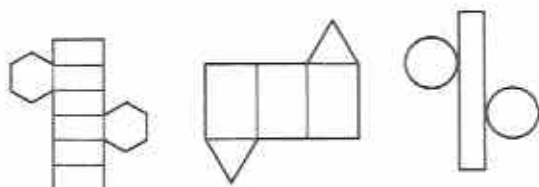


Fig. 7.26

- 10 If the nets in Fig. 7.26 were folded to make the solids, how many planes of symmetry would each solid have?
- 11 How many faces, edges and vertices does a triangular prism have?
- 12 Sketch three different nets of a triangular prism.
- 13 A right prism is such that its top and bottom faces are equilateral triangles of side 8 cm. It takes 90 cm of wire to make a skeleton model of the prism. What is the height of the prism?
- 14 Fig. 7.27 shows two views of a triangular prism. How many vertices, faces and edges can you see in each view? Copy and complete Table 7.2.

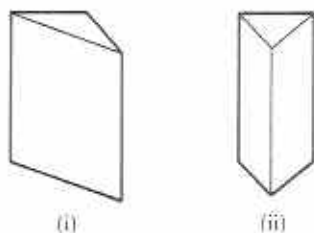


Fig. 7.27

Table 7.2

view	number of vertices seen	number of faces seen	number of edges seen
(i)			
(ii)			

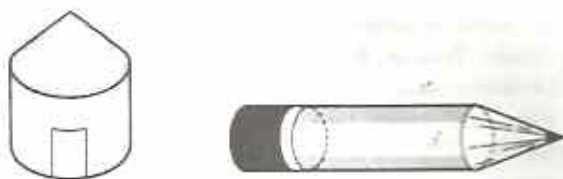


Fig. 7.29 (a) The roof of a round house
(b) The sharp end of a round pencil.

15 Copy and complete Table 7.3 for the prisms given in Fig. 7.24. Include faces, edges and vertices which cannot be seen.

Table 7.3

prism	number of vertices	number of faces	number of edges
triangular			
hexagonal			
I-shaped			

Fig. 7.30 shows how a cone can be cut to give its net. The figure also shows that the base of a cone is circular.

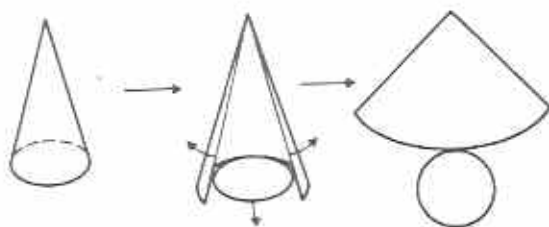


Fig. 7.30 How to cut a cone to give its net

Cone

A mound of rice is often cone-shaped (Fig. 7.28).

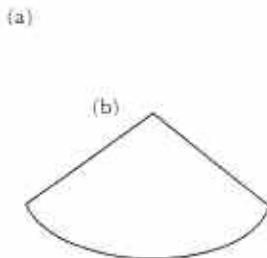
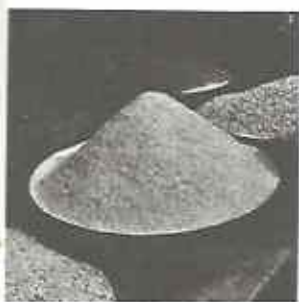


Fig. 7.28 (a) A cone-shaped mound of rice
(b) Outline of mound: cone

Cones are quite common shapes but they are usually part of bigger objects. Two examples are shown in Fig. 7.29.

Pyramid

The names of pyramids come from the shapes of their base faces (Fig. 7.31).

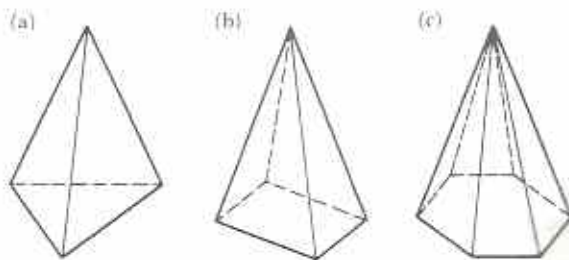


Fig. 7.31 (a) Triangular-based pyramid (tetrahedron)
(b) Square-based pyramid
(c) Hexagonal-based pyramid

Note: Tetrahedron means four faces.

A pyramid with an n -sided base will have n triangular faces meeting at a common point.

This point is usually called **the vertex** of the pyramid. Notice, however, that a pyramid has other vertices around its base.

Fig. 7.32 gives two nets of a square-based pyramid. The nets show that a square-based pyramid has four triangular faces.

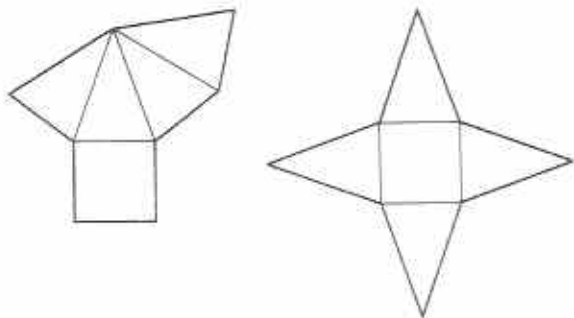


Fig. 7.32

A **right pyramid** is one in which the vertex is above the centre of the base. Fig. 7.33 shows that a right square-based pyramid has four planes of symmetry.

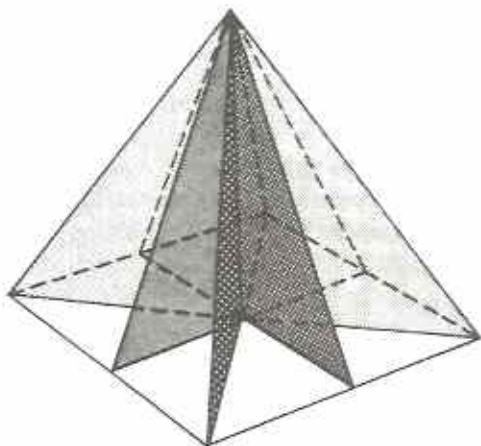


Fig. 7.33 Planes of symmetry in a right square-based pyramid

Exercise 7d

- 1 Write down five everyday objects which are wholly or partly cone-shaped.
- 2 Write down five everyday objects which are wholly or partly pyramid-shaped.
- 3 The solids in Fig. 7.34 include either cones or pyramids. How many cones or pyramids can you see in each solid?

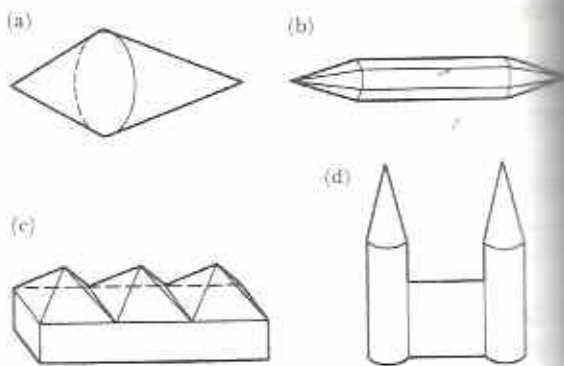


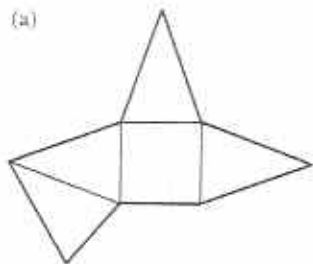
Fig. 7.34

- 4 How many planes of symmetry do the solids in parts (a), (c) and (d) of Fig. 7.34 have?
- 5 Draw a skeleton view of a cone and a square-based pyramid.
- 6 The faces of a pyramid are all triangular. What kind of pyramid is it? What is the special name for such a pyramid?
- 7 Fig. 7.35 shows the bottom part of a cone (compare with Fig. 7.1). How many planes of symmetry has this solid?



Fig. 7.35

- 8 Sketch a pyramid which could have five planes of symmetry.
- 9 A pyramid has a total of six triangular faces. How many faces does it have altogether? What is the shape of its other face?
- 10 Name the solids which have the nets shown in Fig. 7.36.



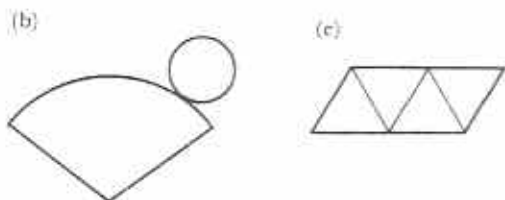


Fig. 7.36

- 11 If a pyramid is made from the net shown in Fig. 7.37, which edge will join to
- (a) DE, (b) EF, (c) AB, (d) HG?
- (e) Which letters will be at the vertex of the pyramid?

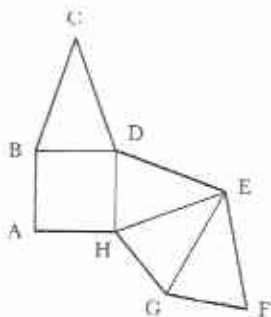


Fig. 7.37

- 12 Fig. 7.38 shows a tower made from a cube and a pyramid. If each edge is 3 cm long, what length of wire will be needed to make a skeleton model of the tower?



Fig. 7.38

- 13 Fig. 7.39 shows a view of a square-based pyramid.



Fig. 7.39

How many vertices, faces and edges can you see in this view of the pyramid?

14 Copy and complete Table 7.4.

Table 7.4

solid	number of vertices	number of faces	number of edges
triangular pyramid			
square pyramid			
hexagonal pyramid			
triangular prism			
solid in Fig 7.38			

- 15 A solid has f plane faces and v vertices. Use your results in Tables 7.1, 7.3, 7.4 to write down an expression in f and v for the number of edges that the solid has.

Sphere

Nearly every ball is sphere-shaped (Fig. 7.40).

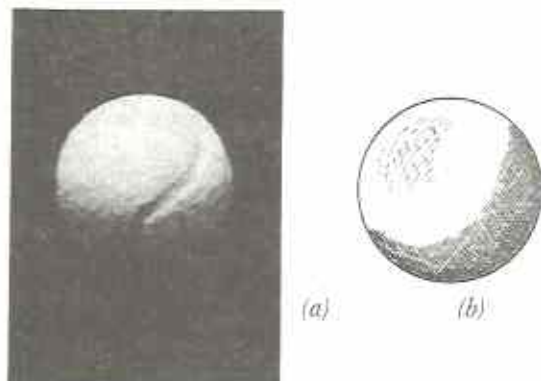


Fig. 7.40 (a) A tennis ball
(b) Outline of a ball: a sphere

Any plane cut through the centre of a sphere will give two parts which are identical to each other. Half a sphere is called a **hemisphere** (Fig. 7.41).

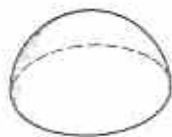


Fig. 7.41

Exercise 7e

- Write down five everyday objects each of which is either a sphere or contains part of a sphere in its shape.
- State which shapes make up the solids in Fig. 7.42.

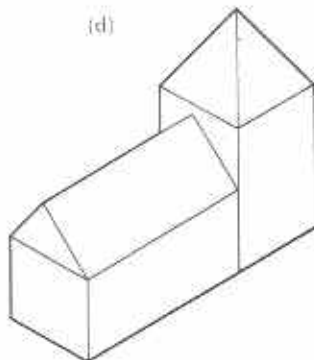


Fig. 7.42

- How many planes of symmetry do the solids in Fig. 7.42 have?
- Which of the following solids will roll smoothly?
 - cube
 - cone
 - sphere
 - square-based pyramid
 - cuboid
 - cylinder
- A skeleton square-based pyramid is made from wire. All its edges are the same length. If 40 cm of wire is used, find the length of the edge of each pyramid. (Make a sketch of the pyramid.)

- Name two solids which have five plane faces.
- If the net shown in Fig. 7.43 is folded to make a cuboid, which edge will join to
 - AB,
 - CD,
 - FG,
 - KL?

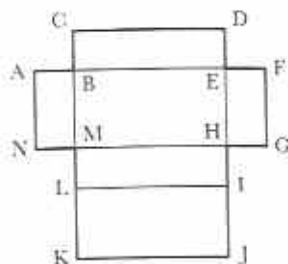


Fig. 7.43

Which points will join to
(e) point A, (f) point D?

- Name two solids which have no vertices.
- Sketch four different nets of a square-based pyramid.
- Name the solids which have the following shapes for *all* of their faces.
 - triangle
 - rectangle
 - square.

Naming vertices, edges and faces

We use capital letters of the alphabet to name the vertices of a solid. The left-hand edge in Fig. 7.44 is edge AD. The top face is face ABFE.

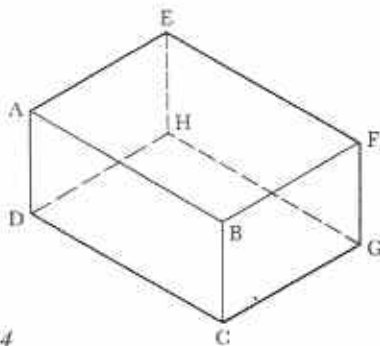


Fig. 7.44

Exercise 7f

- Fig. 7.45 shows the cuboid ABCDEFGH.
 - Faces ABCD and ABGF meet along which edge?
 - Faces BCHG and ABGF meet along which edge?

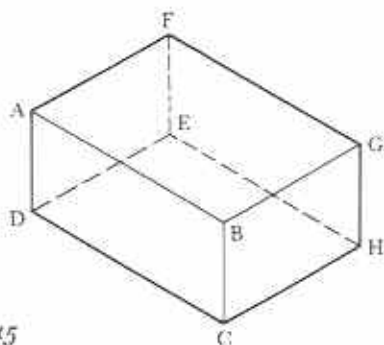


Fig. 7.45

- (c) Which edges meet at vertex G?
- (d) Which edges meet at vertex E?
- (e) Along which edge do faces CDEH and EFGH meet?
- (f) At which vertex do edges AF and FG meet?

2 Fig. 7.46 shows a triangular prism ABCDEF.

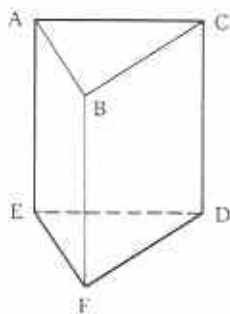


Fig. 7.46

Name the vertices or edges where the following intersect:

- (a) edge AE and edge EF
- (b) edge CD, edge DE and edge DF
- (c) edge BF and face ABC
- (d) face ACDE and edge DF
- (e) face ACDE and face ABC
- (f) face BCDF and face ABFE

3 Fig. 7.47 is a skeleton view of a right square-based pyramid.

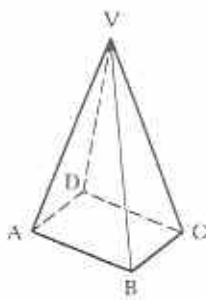


Fig. 7.47

- (a) What is the shape of face VDA?
- (b) What is the true shape of face ABCD?
- (c) How many edges meet at the vertex V? Name the vertices or edges where the following meet.
- (d) edge AB and edge VB
- (e) edge VB and face AVD
- (f) face AVB and face DVC
- (g) face ABCD and face DVC
- VAC is a plane of symmetry of the pyramid.
- (h) Name another plane of symmetry.

Simplifying algebraic expressions (1)

Syllabus Reference 5.6.2

Coefficients

In arithmetic 3×4 is a short way of writing $4 + 4 + 4$. Similarly, in algebra, $3 \times a$ is a short way of writing $a + a + a$. $3 \times a$ is usually shortened to $3a$. Say this as 'three a'.

$$3a = 3 \times a = a + a + a$$

Notice the difference between 34 in arithmetic and $3a$ in algebra. 34 means 3 tens and 4 or $30 + 4$. 34 does *not* mean 3×4 . In algebra, $3a$ *always* means $3 \times a$.

Exercise 8a

Write the following in the shortest possible way.

- 1 $a + a + a$
- 2 $x + x + x$
- 3 $p + p$
- 4 $r + r + r + r + r$
- 5 $t + t + t + t + t + t + t + t$
- 6 $m + m + m + m$
- 7 $z + z + z + z + z + z + z + z + z + z$
- 8 $y + y$
- 9 $c + c + c + c + c + c$
- 10 $k + k + k + k + k$

Exercise 8b (Oral)

1 What do the following mean in arithmetic?

- (a) 3×4 (b) 34 (c) 3×6 (d) 36
 (e) 2×9 (f) 29 (g) 5×1 (h) 51
 (i) 77 (j) 98

2 What do the following mean in algebra?

- (a) $3a$ (b) $5x$ (c) $2y$ (d) $4n$
 (e) $3m$ (f) $6d$ (g) $2f$ (h) $10e$
 (i) $7r$ (j) $5s$

$3a$, $x + 1$, $7 - r$, $7y$ are all examples of **algebraic expressions**. In the expression $3a$, the 3 is called the coefficient of a . The 3 shows that three a 's have been added together. For example, (a) in $7y$, 7 is the coefficient of y ; seven y 's have been added together;

(b) in x , there is only *one* x ; x is the same as $1x$; the coefficient of x is 1.

Note: it is usual to write x not $1x$.

Coefficients are not always whole numbers. Coefficients can also be fractions. In arithmetic $\frac{1}{3} \times 12$ or $\frac{12}{3}$ are short ways of writing $\frac{1}{3}$ of 12 or $12 \div 3$. In algebra $\frac{1}{3}a$ or $\frac{a}{3}$ are short ways of writing $\frac{1}{3}$ of a or $a \div 3$.

In the expression $\frac{1}{3}a$, the coefficient of a is $\frac{1}{3}$. $\frac{1}{3}a = \frac{a}{3}$, so the coefficient of a in $\frac{a}{3}$ is *also* $\frac{1}{3}$. For example,

(c) in $\frac{2}{3}x$, $\frac{2}{3}$ is the coefficient of x ;

(d) in $\frac{2a}{3}$, $\frac{2}{3}$ is the coefficient of a (since $\frac{2a}{3} = \frac{2}{3}a$);

(e) in $\frac{z}{5}$, $\frac{1}{5}$ is the coefficient of z .

Exercise 8c (Oral)

What is the coefficient of x in each of the following?

- | | | |
|-------------------|-------------------|-------------------|
| 1 $3x$ | 2 $7x$ | 3 $4x$ |
| 4 $8x$ | 5 $15x$ | 6 $9x$ |
| 7 $18x$ | 8 $2x$ | 9 x |
| 10 $10x$ | 11 $\frac{1}{5}x$ | 12 $\frac{1}{3}x$ |
| 13 $\frac{1}{4}x$ | 14 $\frac{2}{3}x$ | 15 $\frac{3}{4}x$ |
| 16 $\frac{x}{5}$ | 17 $\frac{x}{10}$ | 18 $\frac{2x}{3}$ |
| 19 $\frac{3x}{4}$ | 20 $\frac{3x}{5}$ | |

Grouping positive and negative terms

Expressions such as $3x$, x , $8x$, $12x$, $\frac{1}{3}x$ are called **terms** in x . We can add terms in x together.

$3x + 2x$ means 3 x 's add 2 x 's. This gives 5 x 's altogether. Thus: $3x + 2x = 5x$; $5x$ uses less space, or is simpler, than $3x + 2x$. Thus two terms in x have been **simplified** to one term in x .

We can also subtract terms. $7y - 4y$ means 7 y 's take away 4 y 's. This leaves 3 y 's. Thus $7y - 4y = 3y$; again, two terms have been simplified to one term.

Exercise 8d (Oral)

Simplify the following.

- | | | |
|--------------|-------------|---------------|
| 1 $2a + 3a$ | 2 $5x - 3x$ | 3 $4b + 4b$ |
| 4 $6y - 2y$ | 5 $4c + c$ | 6 $7z - z$ |
| 7 $p + 2p$ | 8 $8k - 7k$ | 9 $4q + 6q$ |
| 10 $5m - 2m$ | 11 $r + r$ | 12 $3n - 3n$ |
| 13 $5d - 4d$ | 14 $x + 9x$ | 15 $20x - 2x$ |

It is possible to simplify expressions which contain many terms. For example, the expression, $3a - 8a + 5a + 9a - 2a$ means: add $3a$, $5a$ and $9a$ together, take away $8a$ and $2a$. This gives $17a$ take away $10a$. The result is $7a$. We can write this as follows.

$$\begin{aligned} 3a - 8a + 5a + 9a - 2a \\ = 3a + 5a + 9a - 8a - 2a \\ = 17a - 10a \\ = 7a \end{aligned}$$

The first line in this example is important. We have grouped all the terms with a + sign before them together, and all the terms with a - sign before them together. This method is called **grouping positive and negative terms**. It is usually the best way of simplifying big expressions.

Example 1

Simplify $3m - 8m - 2m + 16m - 4m$.

$$\begin{aligned} 3m - 8m - 2m + 16m - 4m \\ = 3m + 16m - 8m - 2m - 4m \\ = 19m - 14m \\ = 5m \end{aligned}$$

Exercise 8e

1 Simplify the following.

- | | |
|----------------|-----------------|
| (a) $4a + 3a$ | (b) $5b - 2b$ |
| (c) $9c + c$ | (d) $12d - 7d$ |
| (e) $11e + 8e$ | (f) $20f - 16f$ |
| (g) $5g + 9g$ | (h) $13h - h$ |

- | | |
|-------------------|---------------------|
| (i) $18j + 7j$ | (j) $21k - 20k$ |
| (k) $6m + 17m$ | (l) $22n - 8n$ |
| (m) $p + 5p$ | (n) $7q - 7q$ |
| (o) $2r - r$ | (p) $12s - 2s$ |
| (q) $22t + 8t$ | (r) $21u - 12u$ |
| (s) $3v + 4v + v$ | (t) $w + 4w + 4w$ |
| (u) $8x + x + 3x$ | (v) $5y + 13y + 2y$ |
| (w) $z + 9z + z$ | (x) $4a + 5a + a$ |

2 Simplify the following. Set out your work as in Example 1 above.

- | | |
|--------------------------|-----------------------|
| (a) $10b + b - 3b$ | (b) $3c + 5c - 8c$ |
| (c) $7d + 4d - 8d$ | (d) $16e + 12e - 13e$ |
| (e) $6f - 2f + 3f$ | (f) $4g - 7g + 10g$ |
| (g) $20h - 15h + 6h$ | (h) $2j - 9j + 9j$ |
| (i) $11k - 16k + 9k$ | (j) $8m - 3m - 2m$ |
| (k) $9n - 5n - 3n$ | (l) $17p - 8p - 2p$ |
| (m) $13q - 4q - 4q$ | (n) $22r - 14r - 8r$ |
| (o) $5s + s + 2s + 4s$ | (p) $3t + 6t + t + t$ |
| (q) $4u + u + 6u - 5u$ | |
| (r) $9v + 2v + 3v - 8v$ | |
| (s) $6w + 4w - 3w - 2w$ | |
| (t) $12x + 3x - 5x - 2x$ | |
| (u) $3y - 5y + 10y - 4y$ | |
| (v) $7z - 2z + 7z - 2z$ | |
| (w) $11a + 5a - 2a + a$ | |
| (x) $4b + 2b - 9b + 8b$ | |
| (y) $20c - 9c - c + 2c$ | |
| (z) $12d - 5d - 3d + 4d$ | |

3 Simplify the following. Set out your work as in Example 1 above.

- | |
|---------------------------------|
| (a) $22e - 5e - 8e - 4e$ |
| (b) $12f - 5f - 3f - 4f$ |
| (c) $6g + 5g - 2g - 4g - 2g$ |
| (d) $7h + h + 3h - 2h - 4h$ |
| (e) $8j - 10j + 6j - 7j + 4j$ |
| (f) $13k - 5k - 9k + 6k - k$ |
| (g) $4m - 22m + 13m + 10m - 3m$ |
| (h) $14n + 2n - 6n - 12n + 7n$ |
| (i) $p - 15p - 5p - 2p + 25p$ |
| (j) $2q - 20q + 11q + 8q + 5q$ |

Grouping like and unlike terms

What is the sum of 5 potatoes and 8 potatoes? 13 potatoes. Similarly, in algebra, $5p + 8p = 13p$. What is the sum of 3 potatoes and 7

torches? All that can be said is that there is a mixture of potatoes and torches. Similarly, in algebra, it is impossible to simplify $3p + 7t$. $5p$ and $8p$ are **like terms**. Their sum is $13p$. $3p$ and $7t$ are **unlike terms**. Their sum is $3p + 7t$.

Notice that:

$$3 \text{ potatoes} + 8 \text{ torches} + 6 \text{ potatoes} + 5 \text{ torches} \\ = 9 \text{ potatoes} + 13 \text{ torches.}$$

Just as real things can be grouped, so, in algebra, like terms can be grouped:

$$3p + 8t + 6p + 5t = 3p + 6p + 8t + 5t \\ = 9p + 13t$$

Example 2

Simplify $7h - 4 - 3h + 11$.

Notice that terms in h and number terms are unlike.

$$7h - 4 - 3h + 11 = 7h - 3h + 11 - 4 \\ = 4h + 7$$

Example 3

Simplify $8x - 6y - 9y - 2x$.

$$8x - 6y - 9y - 2x = 8x - 2x - 6y - 9y \\ = 6x - 15y$$

Example 4

Simplify $10r - 3r - 8 - 4r$.

Notice that there are three terms in r .

$$10r - 3r - 8 - 4r = 10r - 3r - 4r - 8 \\ = 10r - 7r - 8 \\ = 3r - 8$$

Exercise 8f

Simplify the following. Set your work out as in the examples above.

- | | |
|--------------------|--------------------|
| 1 $2x + 3x + 7$ | 2 $6x - 3x + 2$ |
| 3 $5x + 6x + 8y$ | 4 $10x - 4x + 1$ |
| 5 $4x + x - 2$ | 6 $7x - 2x - 2y$ |
| 7 $5y + 7x + 2x$ | 8 $3y + 9x - 4x$ |
| 9 $9y + 4x + 8x$ | 10 $15 - 2x + 10x$ |
| 11 $1 + x + x$ | 12 $4y - 8x + 12x$ |
| 13 $3x + 8y + 3x$ | 14 $10x + 5 - 4x$ |
| 15 $10x - 5 + 4x$ | 16 $2x + 11y - 2x$ |
| 17 $x + 2 + 9x$ | 18 $5x - 8 - 4x$ |
| 19 $6y + 7y + 12x$ | 20 $8y - 7y - x$ |
| 21 $14x + 14 + 3$ | 22 $3x + 11 - 2$ |

- | | |
|--------------------------|-----------------------|
| 23 $7y - 3x + 4y$ | 24 $14y - 9x - 6y$ |
| 25 $6a + 2a + 3 + 10$ | 26 $8a - 5a + 14 - 6$ |
| 27 $3x + 8x + 9y - 4y$ | |
| 28 $11x - 2x + 3y + 5y$ | |
| 29 $4a - a + 5b - 4b$ | |
| 30 $7m + 3 + 2m + 1$ | |
| 31 $19n + 19 + n - 10$ | |
| 32 $12p + 3 - 2p - 2$ | |
| 33 $2p + 7t + 5p + 3t$ | |
| 34 $18a - 3b - 6a + 10b$ | |
| 35 $2x - 8 - 3 + 5x$ | |
| 36 $20x - 4y - y - 3x$ | |
| 37 $20 + 2x + x + 3x$ | |
| 38 $12n - 3p - 3p - 5p$ | |
| 39 $6c - 10c + 13c - 7$ | |
| 40 $4y - 2x - 5x - 3y$ | |

Exercise 8g (Further practice)

Simplify the following where possible.

- | | |
|----------------------------------|---------------------|
| 1 $2x + 7x$ | 2 $6d + d$ |
| 3 $a + 8a$ | 4 $9b + 3b$ |
| 5 $3m + 5n$ | 6 $3y + 2y + 5y$ |
| 7 $4h + 9h + 12h$ | 8 $2x + 6y$ |
| 9 $8n - 5n$ | 10 $14k - 6k$ |
| 11 $8e + 5e - 7e$ | 12 $4g - g + 3g$ |
| 13 $15a - 2$ | 14 $19b - 12b$ |
| 15 $5x - 14x + 9x$ | 16 $17d - 8d - 3d$ |
| 17 $6m - 9m + 4m$ | 18 $8a + 14a - 21a$ |
| 19 $35b - 27b$ | 20 $31p - 9q$ |
| 21 $28f - 11$ | |
| 22 $4c - 9c + 15c - 7c$ | |
| 23 $7d + 15d - 9d - 11d$ | |
| 24 $8m - 9m - 5m + 10m$ | |
| 25 $11n - 3n - 6n + 9n - 5n$ | |
| 26 $4x + 9x - 7x - 6x + 8x - 3x$ | |
| 27 $3m + 5n + 4m + 6n$ | |
| 28 $9a + 10b - 5a - 4b$ | |
| 29 $8h - 3 - 5h + 9$ | |
| 30 $7x + 5y - 4$ | |
| 31 $11x + 9y - 7x$ | |
| 32 $13a + 2b - 9a + 7b$ | |
| 33 $5f - 3g - 7f + 9g + 3f - 4g$ | |
| 34 $7m - 2n + 6 - 5m + 7n + 3$ | |
| 35 $3x + 2 - 7x - 4 + 5x + 6$ | |
| 36 $a - 7b + 3c + 8b + 2a$ | |
| 37 $5h + 8k + 2 - 3k - 2h$ | |
| 38 $6m + 3n - 1 - 6m + 4$ | |
| 39 $a - 2b - 4a + 3c + 4a + 5b$ | |
| 40 $6p - 2q + 4r - 2p + 3s + 5q$ | |

Exercise 8h

1. A carpenter has two planks of wood. One plank is x cm long, the other plank is $3x$ cm long. What is the total length of the two planks?
2. A shop sells x books at \$4 each, y books at \$5 each and z books at \$8 each. How many books are sold? How much money is paid for the books?
3. 180 people attend a meeting. There are n men and n women and the rest are children. How many children are at the meeting?
4. A woman earns \$135 per week. Each week she spends \$3x on food and \$2x on rent. How much money is she left with each week?
5. A man weighs k kg. He goes on a diet and loses one-tenth of his weight. What is his weight after going on the diet?
6. A cooperative farmer has $6x$ sheep and $5y$ goats. She sells $3x$ goats and $2y$ sheep. How many animals is she left with?
7. A man has x matches in a matchbox. On Monday he uses one-quarter of them. On Tuesday he uses 5 more. How many matches has he used? How many matches has he left?
8. A girl gets n cents pocket money each week. She saves her money for 5 weeks and buys a present for her mother which costs 79 cents. How much money has she left?
9. A woman has x eggs in a basket. She sells one-third of them to another woman. Two of the eggs get broken. How many eggs are left in the basket?
10. Table 8.1 shows the numbers of goals for and against a team during four weeks of a month.

Table 8.1

	goals for	goals against
Week 1	x	$2y$
Week 2	$2x$	y
Week 3	$3y$	y
Week 4	y	x

- (a) How many 'goals for' in the month?
- (b) How many 'goals against' in the month?

Multiplication with algebraic terms

(a) Just as $5a$ is short for $5 \times a$, so ab is short for $a \times b$.

(b) Just as $3 \times 5 = 5 \times 3 = 15$,
so $a \times b = b \times a = ab$.

It is usual to write the letters in alphabetical order, but ba would be just as correct as ab .

(c) Just as 5^2 is short for 5×5 , so a^2 is short for $a \times a$ and x^3 is short for $x \times x \times x$.

(d) $4x + 4x + 4x = 12x$
 $3 \times 4x = 12x$

and

$$3x + 3x + 3x + 3x = 12x$$

$$4 \times 3x = 12x$$

Thus: $3 \times 4x = 4 \times 3x = 12x$

The terms 3, 4 and x can be multiplied in any order:

$$3 \times 4x = 4 \times 3x = 3x \times 4 = 4x \times 3$$

$$= 4 \times x \times 3 = x \times 3 \times 4 = 12x$$

It is usual to write the numbers before the letters.

Examples 5

Table 8.2

simplify	working	result
$2x \times 3$	$= 2 \times x \times 3 = 2 \times 3 \times x$ $= 6 \times x$	$= 6x$
$5 \times 2y$	$= 5 \times 2 \times y = 10 \times y$	$= 10y$
$7a \times 3b$	$= 7 \times a \times 3 \times b$ $= 7 \times 3 \times a \times b$ $= 21 \times ab$	$= 21ab$
$6x \times 4x$	$= 6 \times x \times 4 \times x$ $= 6 \times 4 \times x \times x$ $= 24 \times x^2$	$= 24x^2$
$5 \times 6ab$	$= 5 \times 6 \times ab$ $= 30 \times ab$	$= 30ab$
$8ab \times 7a$	$= 8 \times a \times b \times 7 \times a$ $= 8 \times 7 \times a \times a \times b$ $= 56 \times a^2 \times b$	$= 56a^2b$
$y \times 11xy$	$= y \times 11 \times x \times y$ $= 11 \times x \times y \times y$ $= 11 \times xy^2$	$= 11xy^2$

Exercise 8i

Simplify the following.

- | | | |
|--------------------|---------------------|---------------------|
| 1 $5 \times a$ | 2 $x \times 4$ | 3 $x \times y$ |
| 4 $y \times a$ | 5 $a \times a$ | 6 $1 \times x^2$ |
| 7 $2a \times 3$ | 8 $3 \times 2a$ | 9 $3a \times 2$ |
| 10 $2 \times 3a$ | 11 $4x \times 7$ | 12 $5 \times 8n$ |
| 13 $2 \times x^2$ | 14 $3y^2 \times 4$ | 15 $16 \times 2x^2$ |
| 16 $4ab \times 5$ | 17 $7 \times 5pq$ | 18 $9ab \times 3$ |
| 19 $6x \times x$ | 20 $y \times 8y$ | 21 $3x \times x$ |
| 22 $p \times 2q$ | 23 $6a \times b$ | 24 $m \times 7n$ |
| 25 $4a \times 3a$ | 26 $5n \times 7n$ | 27 $3x \times 10x$ |
| 28 $9n \times 4n$ | 29 $10q \times 5p$ | 30 $4a \times 5b$ |
| 31 $4ab \times 7a$ | 32 $3b \times 11ab$ | 33 $2xy \times 9y$ |
| 34 $6y \times 5xy$ | 35 $14pq \times p$ | 36 $3a \times 8ab$ |

Division with algebraic terms

In algebra, letters stand for numbers. Just as fractions can be reduced to their lowest terms by equal division of the numerator and denominator, so a letter can be divided by the same letter. For example $x \div x = 1$, just as $3 \div 3 = 1$.

Examples 6**Table 8.3**

simplify	working	result
	$\frac{14a}{7} = \frac{7 \times 2a}{7} = \frac{1 \times 2a}{1}$	$= 2a$
$\frac{1}{3}$ of $36x$	$= \frac{36 \times x}{3} = \frac{3 \times 12x}{3}$	
	$= \frac{1 \times 12x}{1}$	$= 12x$
$\frac{1}{5}$ of y	does not simplify	$= \frac{1}{5}y$ or $\frac{y}{5}$
$5ab \div a$	$= \frac{5 \times a \times b}{a}$	
	$= \frac{5 \times 1 \times b}{1}$	
	$= 5 \times b$	$= 5b$

$$\frac{6xy}{2y} = \frac{6 \times x \times y}{2 \times y} = \frac{3 \times x \times 1}{1 \times 1}$$

$$= 3 \times x = 3x$$

$$x^2 \div x = \frac{x \times x}{x} = \frac{1 \times x}{1} = x$$

$$24x^2y \div 3xy = \frac{24 \times x^2 \times y}{3 \times x \times y}$$

$$= \frac{3 \times 8 \times x \times 1}{3 \times 1 \times 1} = 8 \times x = 8x$$

Exercise 8j

Simplify the following.

- | | | |
|---------------------------|--------------------------|----------------------------|
| 1 $\frac{6a}{3}$ | 2 $12a \div 2$ | 3 $\frac{1}{4}$ of $24x$ |
| 4 $18y \div 6$ | 5 $\frac{1}{5}$ of $15x$ | 6 $\frac{32c}{8}$ |
| 7 $\frac{1}{8}$ of $32x$ | 8 $\frac{21y}{3}$ | 9 $\frac{1}{7} \times 35y$ |
| 10 $\frac{1}{9} \times x$ | 11 $\frac{1}{4}$ of d | 12 $x \times \frac{1}{2}$ |
| 13 $28ab \div 4$ | 14 $16xy \div x$ | 15 $30pq \div 3$ |
| 16 $17mn \div n$ | 17 $22kl \div 11$ | 18 $6ab \div 3$ |
| 19 $d^2 \div d$ | 20 $a \div a^2$ | 21 $\frac{z^2}{z}$ |
| 22 $\frac{3x^2}{x}$ | 23 $6x^2 \div x$ | 24 $\frac{5c^2}{c}$ |
| 25 $\frac{7x^3}{x}$ | 26 $\frac{2x^3}{x}$ | 27 $\frac{5x^3}{x^2}$ |
| 28 $\frac{12x^2}{3x}$ | 29 $\frac{18a^3}{3a}$ | 30 $\frac{26x^2}{2x}$ |
| 31 $\frac{33mn}{3m}$ | 32 $\frac{42xy}{7y}$ | 33 $\frac{54ab}{9b}$ |
| 34 $\frac{72a^2b}{8a}$ | 35 $\frac{48x^2y}{12xy}$ | 36 $\frac{40pq^2}{8pq}$ |

Order of operations

What is the value of $17 - 5 \times 2$? If we do subtraction first, the value is 24 as follows:

$$17 - 5 \times 2 = (17 - 5) \times 2 = 12 \times 2 = 24$$

If we do multiplication first, the value is 7 as follows:

$$17 - 5 \times 2 = 17 - (5 \times 2) = 17 - 10 = 7$$

To avoid confusion, remember the following rules:

(a) If there are no brackets, do multiplication and division before addition and subtraction.

(b) If there are brackets, simplify the terms inside the brackets first.

These rules are sometimes called **rules of precedence**, where 'precedence' means 'order'. The rules of precedence show that the value of $17 - 5 \times 2$ is 7.

Example 7

Find the value of $16 \times 2 - 3 + 14 \div 7$.

$$\begin{aligned} 16 \times 2 - 3 + 14 \div 7 &= 32 - 3 + 2 \\ &= 34 - 3 = 31 \end{aligned}$$

Example 8

Simplify $7 \times 3a - (3a + 5a) \times 2$.

$$\begin{aligned} 7 \times 3a - (3a + 5a) \times 2 &= 21a - 8a \times 2 \\ &= 21a - 16a = 5a \end{aligned}$$

Exercise 8k

Find the value of the following.

- 1 $18 - 6 \times 2$
- 2 $12 + 4 + 2$
- 3 $6 - 18 \div 3$
- 4 $4 \times 5 + 8 + 4$
- 5 $16 \div 2 - 3 \times 2$
- 6 $7 \times 3 + 27 \div 9$
- 7 $(5 + 3) + 3 \times 5$
- 8 $5 + (3 + 3) \times 5$
- 9 $4 \times 6 - (7 - 3)$
- 10 $8 \times 3 - 17 + 15 \div 5$
- 11 $28 \div 4 + 2 - 2 \times 4$
- 12 $6 \times 2 - 2 - 40 \div 4$

Exercise 8l

Simplify the following as far as possible.

- 1 $3x \times 2 + 5x$
- 2 $4 \times 5p - 3p$
- 3 $4n + 3n \times 10$
- 4 $2 \times 7b - 3$
- 5 $6 + 2 \times 5m$
- 6 $4a \times 3 + 5$
- 7 $17y - 5y \times 2$
- 8 $21a - 2 \times 7a$
- 9 $3x + 8x + 2$
- 10 $n + 12n + 4$
- 11 $15 + 5 + 6y$
- 12 $7x + x + 5$
- 13 $5a + 21a + 7$
- 14 $4x - 6 + 3$
- 15 $1 - x \times 0$
- 16 $6a \times 4 - 2 \times 7a$
- 17 $3 \times 5x + 4x + 2$
- 18 $4 \times 8x + 7x \times 3$
- 19 $6u \times 5 - 3 \times 4u - 2u$
- 20 $24x \div 6 + x + 1 \times 5x$
- 21 $7x - 3 \times 4x + 5x \times 2$
- 22 $8v \times 2 + 5v \div 5 - 12v$
- 23 $7a \times 2 + 5 \times 8a - 6a \times 9$
- 24 $5 \times 6x - 4x \times 0 - 7x \times 4$

Chapters 1 – 8

Revision exercise 1 (Chapters 1, 3)

- What is the value of
 - the 5 in 253
 - the 7 in 367
 - the 2 in 2403
 - the 9 in 3937?
- What is the value of
 - the 6 in 4,62
 - the 8 in 0,238
 - the 1 in 3,815
 - the 5 in 1,05?
- If today is Wednesday, what day of the week will it be in (a) 10 days time, (b) 25 days time?
- In a marathon race, the winner finishes in 2 h 14 min 48 s. The last person finishes in 5 h 6 min 19 s. What is the difference between their times?
- Starting with the number 519,
 - rearrange its digits to make (i) the highest possible number, (ii) the lowest possible number;
 - subtract the lowest number from the highest number to give a new three-digit number.
 - Take this new three-digit number and repeat steps (a) and (b).
 - Repeat (a), (b) and (c) until you notice something happening.
 - Will this happen if you start with any three-digit number?
- Express the following as products of their prime factors.
 - 18
 - 26
 - 45
 - 75
- Calculate the value of the following.
 - 7^2
 - 5^3
 - $2^3 \times 3^2$
 - $2^2 \times 3^3$
- Find the HCF of 63 and 90.
- Find the LCM of 12 and 15.
- Find the greatest number which when divided into 593 and 621 will leave a remainder of 5 in each case.

Revision test 1 (Chapters 1, 3)

- The value of the 8 in 18 214 is
 - 8 units
 - 8 tens
 - 8 hundreds
 - 8 thousands
 - 8 ten thousands
- Today is Monday. In 23 days time it will be
 - Saturday
 - Sunday
 - Tuesday
 - Wednesday
 - Thursday
- How many minutes is it between 22 minutes past 1 o'clock and 13 minutes past 3 o'clock?
 - 111
 - 131
 - 171
 - 191
 - 251
- The value of 6^2 is
 - 12
 - 24
 - 26
 - 36
 - 62
- If $M_8 = \{\text{multiples of 8}\}$ and $M_{12} = \{\text{multiples of 12}\}$, what is the smallest element of the set $M_8 \cap M_{12}$?
 - 2
 - 4
 - 24
 - 48
 - 96
- (a) Choose any four digits and arrange them to obtain (i) the largest, (ii) the smallest 4-digit number.
 (b) Find (i) the sum of, (ii) the difference between the largest and smallest numbers.
 (c) Test whether (i) the sum, (ii) the difference is divisible by 9.
- Express the value of the 7 in the number 5 397 225 (a) in words, (b) as a common fraction.
- Express 60 as a product of prime factors.
- Find the HCF of 32, 40 and 56.
- (a) Find out which of the numbers, 2, 5, 8, 9 will divide into 10 170 exactly. (b) Hence state which of the numbers 20, 40, 45, 90 will also divide into 10 170 exactly.

Revision exercise 2 (Chapters 2, 6, 8)

- Which of the following statements are true?

- (a) $12 \in \{2; 4; 6; 8; \dots\}$
 (b) $\{a; b\} \not\subset \{f; a; c; c\}$
 (c) If $Z = \{0\}$, $n(Z) = 0$
 (d) $\{\text{even numbers}\} \cap \{\text{odd numbers}\} = \emptyset$
 (e) $\{1; 2\} \cup \{2; 3\} = \{2\}$

- 2 If $A = \{x; y; z\}$, list all the subsets of A .
 3 Fig. R1 is a Venn diagram showing the elements of sets \mathcal{U} , A , B .

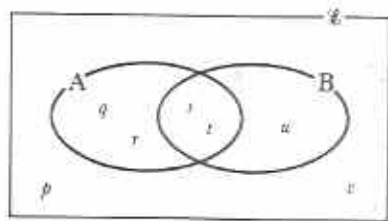


Fig. R1

- (a) Find $n(\mathcal{U})$.
 (b) Find $n(A)$.
 (c) List the elements of $A \cup B$.
 (d) List the elements of $A \cap B$.
- 4 Find the values of the following when $x = 6$.
 (a) $x - 1$ (b) $11 + x$
 (c) $x + 5 - x$ (d) $6x$
- 5 Write down the number which is
 (a) 5 more than m (b) 5 less than m
 (c) 5 times m (d) one-fifth of m .
- 6 If $x = 2$, find the value of
 (a) $3x$ (b) $3x - 1$
 (c) $17 - 5x$ (d) $\frac{14}{x} - 3$

- 7 Simplify the following.
 (a) $6a + 3a$ (b) $10x + 4x - 8x$
 (c) $3n - 8n + 7n$ (d) $4x \times 5$
 (e) $24y \div 3$ (f) $\frac{1}{2}$ of $8c$

- 8 Simplify the following.
 (a) $8n - 5 + 2n$ (b) $5 - x - 5x$
 (c) $3a + 4b - a + 2b$
 (d) $18x + 6 - 4x + 3 \times 2x$
 (e) $3x + 8 \times 3x - 15x \div 5$

- 9 A boy buys 2 packets each containing x biscuits. He eats 6 biscuits. How many biscuits does he have left?

- 10 A trader sells x pencils at 10 cents each, y pens at 20 cents each and 5 rubbers at x

cents each. How much money does the trader get altogether?

Revision test 2 (Chapters 2, 6, 8)

1. If $N = \{\text{odd numbers greater than 11}\}$, which one of the following is an element of N ?

A 9 B 10 C 11 D 12 E 13

2. If $Y = \{5; 1; 4; 1; 2; 6; 1\}$, then $n(Y) =$

A 1 B 5 C 6 D 7 E 20

3. When $x = 8$, the value of $18 - x$ is

A 1 B 8 C 10 D 18 E 26

4. The number which is 6 less than m is

A $m - 6$ B $6 - m$ C $6m$

D $\frac{m}{6}$ E $m + 6$

5. $9 \times 2 - 12 \div 2 + 2 =$

A -43 B -18 C 5

D 14 E 15

6. If $P = \{1; 3; 5; 7; 9\}$ and $Q = \{1; 4; 9; 16\}$, list the elements of the set

(a) $P \cup Q$ (b) $P \cap Q$

7. If $A \subset B$ and $B \subset C$, draw a Venn diagram to show the relationship between A , B and C .

8. Simplify the following.

(a) $a - 5a + 8a - 2a$

(b) $6x - 2y - 5y - 3x$

9. Simplify the following.

(a) $3xy \times 9y$ (b) $2n \times 5an^2$

(c) $36a^2b \div 12ab$ (d) $\frac{5x^2}{x}$

10. A trader buys 30 shirts for $\$x$ altogether. She sells them all for $\$y$ each. What is her profit?

Revision exercise 3 (Chapters 4, 5, 7)

1. A shelf 2,66 m long is used to store copies of a textbook. Each textbook is 2,8 cm thick. How many books can be stored on the shelf?

2. A train is supposed to start from P at 11.58 am and to reach Q at 1.49 pm. If it starts 4 min late and arrives 18 min late, how long does the journey take?

Table R1 is a timetable of flights between Zimbabwe and Mozambique.

Table R1

Sat				Sat
TM402				TM403
737	↓		↓	737
1415	Lv	Maputo	Ar	2115
1545	Ar	Harare	Lv	1945

Use Table R1 to answer questions 3 and 4.

- 3 (a) What is the flight number from Maputo to Harare?
 (b) What time does the Boeing 737 leave Harare?
 (c) How long is the Boeing 737 on the ground at Harare?
- 4 (a) How long does it take to fly from Harare to Maputo?
 (b) If the distance between Harare and Maputo is 900 km, calculate the average flying speed of the 737 in km/h.

5 Reduce the following to their lowest terms.

- (a) $\frac{17}{20}$ (b) $\frac{10}{45}$ (c) $\frac{48}{60}$

6 Simplify the following.

- (a) $\frac{1}{4} + \frac{1}{3}$ (b) $\frac{3}{5} - \frac{1}{4}$
 (c) $\frac{2}{3} \times \frac{3}{10}$ (d) $\frac{5}{6} + \frac{2}{3}$
 (e) $2\frac{3}{4} + 5\frac{1}{8} - 3\frac{1}{2}$ (f) $4\frac{1}{8} \times \frac{4}{15} + 11\frac{2}{3}$

7 Express the following percentages as fractions in their lowest terms.

- (a) 25% (b) 28% (c) 55% (d) $62\frac{1}{2}\%$

8 In a class $\frac{2}{3}$ of the students have mathematical instruments. $\frac{1}{4}$ of these students have lost their protractors. What fraction of students in the class have protractors?

9 How many faces has a triangular prism? How many of these are rectangular?

10 All the edges of a triangular-based pyramid are 10 cm long. A skeleton model of the pyramid is made of wire. What length of wire is needed?

2 Which one of the following is *not* equivalent to $\frac{1}{2}$?

- A $\frac{9}{18}$ B $\frac{11}{22}$ C $\frac{15}{30}$ D $\frac{16}{32}$ E $\frac{21}{42}$

3 If $5\frac{1}{7}$ is expressed as an improper fraction, its numerator will be

- A 8 B 12 C 13 D 35 E 36

4 Which net in Fig. R2 is the net of a triangular prism?

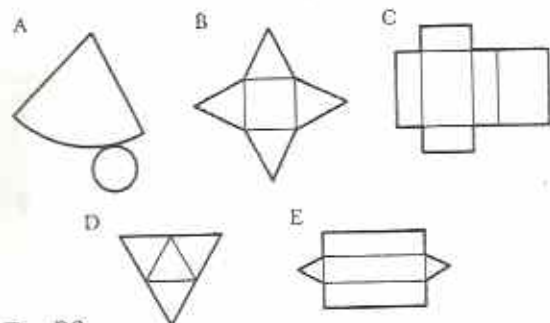


Fig. R2

5 It takes 72 cm of wire to make a skeleton model of a cube. The length of one edge of the cube is

- A 6 cm B 8 cm C 9 cm
 D 12 cm E 18 cm

6 A motorist travels 108 km in 2 h 15 min. If she travels at a steady speed, find how far she goes in 1 hour.

7 Simplify the following.

- (a) $5\frac{1}{4} + 1\frac{1}{6} - 3\frac{2}{3}$ (b) $6\frac{1}{4} \times 1\frac{3}{9}$
 (c) $6\frac{3}{4} + 5\frac{5}{8}$

8 During a 1-hour radio programme there were 18 min of talking; the rest was music. What percentage of the radio programme was music?

9 Fig. R3 shows a cuboid ABCDEFGH.

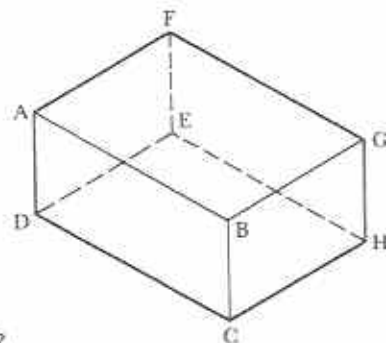


Fig. R3

Revision test 3 (Chapters 4, 5, 7)

- 1 5 300 ml expressed in litres is
 A 0,053 B 0,53 C 5,3 D 53 E 530

- (a) Faces ABGF and BCHG meet along which edge?
 (b) Which edges meet at the vertex H?
 (c) Edges BG and AB meet at which vertex?

- 10 Fig. R4 shows the net of a triangular based pyramid.

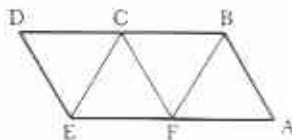


Fig. R4

If the net is folded to make the pyramid,

- (a) which edge will join to edge BC?
 (b) which point will join to point A?

General revision test A (Chapters 1 – 8)

- 1 The value of the 3 in 24,635 is
 A 3 thousandths B 3 hundredths
 C 3 tenths D 3 units E 3 tens
- 2 The HCF of 24 and 60 is
 A 2 B 3 C 4 D 6 E 12
- 3 Fig. R5 is a Venn diagram showing the elements of the sets \mathcal{E} , X, Y.

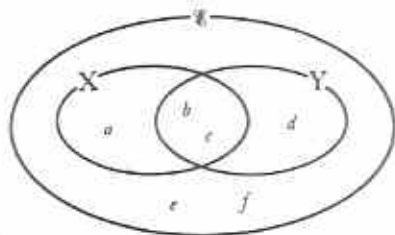


Fig. R5

$$X \cup Y =$$

- A {a; d} B {b; c} C {e; f}
 D {a; b; c; d} E {a; b; c; d; e; f}
- 4 The lowest common denominator of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{3}{10}$ is
 A 15 B 30 C 50 D 60 E 90
- 5 4 pages of a 16-page newspaper are missing. The percentage missing is
 A $\frac{1}{4}$ B 4 C 16 D 25 E 75
- 6 $12x^2y \div 3x =$
 A $4xy$ B $8y$ C $9xy$ D $12xy$ E $16y$

- 7 If $x = 3$, the value of $7x - 2x$ is
 A 6 B 8 C 15 D 21 E 53
- 8 Which one of the following statements about a cylinder is false?
 A A cylinder has two vertices.
 B A cylinder has two plane faces.
 C A cylinder has two curved edges.
 D A cylinder has one curved face.
 E The net of a cylinder contains a rectangle.

Table R2

Mafekeng	D 1400
Gaborone	A 1945 D 1955
Francistown	A 0713 D 0723
Plumtree	A 0955 D 1005
Bulawayo	A 1245

Table R2 is a railway timetable for the journey from Mafekeng to Bulawayo. Use it to answer questions 9, 10, 11.

- 9 Which of the five towns is the train nearest to at midnight?
 A Mafekeng B Gaborone C Francistown
 D Plumtree E Bulawayo
- 10 How long does the train take to travel from Gaborone to Plumtree?
 A 10 h B 10 h 10 min C 10 h 20 min
 D 14 h E 14 h 10 min
- 11 (a) How long does it take to travel from Mafekeng to Bulawayo?
 (b) If it is 880 km from Mafekeng to Bulawayo, what is the average speed of the train correct to 1 s.f.?
- 12 Find the sum of 600 ml, 900 ml and 60 ml. Give your answer in litres.
- 13 If $\mathcal{E} = \{1; 2; 3; \dots; 9\}$, $A = \{2; 4; 6; 8\}$ and $B = \{3; 4; 5; 6\}$,
 (a) list the elements of $A \cup B$,
 (b) list the elements of $A \cap B$,
 (c) draw a Venn diagram showing the elements of \mathcal{E} , A and B.

14 Simplify the following.

(a) $1\frac{1}{20} + \frac{3}{4}$

(b) $5\frac{1}{20} - 4\frac{3}{4}$

(c) $3\frac{3}{4} \times 1\frac{1}{2}$

(d) $3\frac{2}{3} \div 2\frac{2}{5}$

15 A woman gives $\frac{1}{4}$ of a cake to her son, $\frac{1}{4}$ to her daughter and $\frac{1}{3}$ to her husband. What fraction is left for herself?

16 One-sixth of a stick is cut off and then three-tenths of the remaining piece is thrown away. What fraction of the original stick remains?

17 A student walks for $3\frac{1}{2}$ min and runs for $8\frac{1}{2}$ min. What percentage of the journey time is spent running?

18 If $x = 2$, find the value of

(a) $7 - x$

(b) $5x + 3$

(c) $\frac{3}{4}x$

(d) $4 - 2x$

19 I buy 7 metres of cloth at \$ x per metre. How much change will I get from \$30?

20 Express the following times in terms of the 24-hour clock.

(a) 6.25 am

(b) 6.45 pm

(c) 5 to 8 pm

(d) 10 to midnight

Angles (1) Measurement

Syllabus Reference 5.7.1

Angles

We use the word **angle** for amount of turn. For example, Fig. 9.1 shows how the hands of a clock move between 9 o'clock and 10 o'clock.

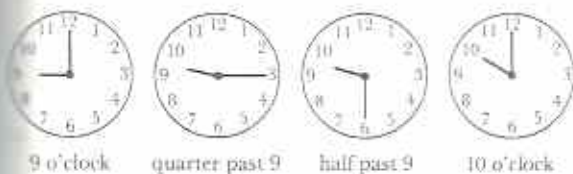


Fig. 9.1

Both hands turn. In one hour the amount that each hand turns is different.

The minute hand makes one complete **turn**, or one **revolution** (Fig. 9.2). The angle turned = 1 revolution.

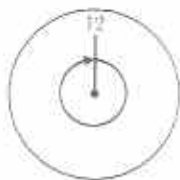


Fig. 9.2

The hour hand turns through $\frac{1}{12}$ of a revolution (Fig. 9.3). The angle turned = $\frac{1}{12}$ revolution.



Fig. 9.3

This example shows that just as we can measure length, so we can measure angle. To avoid fractions, one revolution is divided into 360 equal parts. Each part is called a **degree**. We use the symbol $^\circ$ for degree.

1 revolution = 360 degrees or 360°
 $1^\circ = \frac{1}{360}$ revolution

Angles between lines

Fig. 9.4 shows two lines OA and OB. The lines meet at the point O.

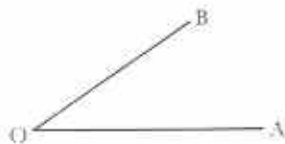


Fig. 9.4

The angle between the two lines is the amount that one line must turn so that it points the same way as the other line (Fig. 9.5).

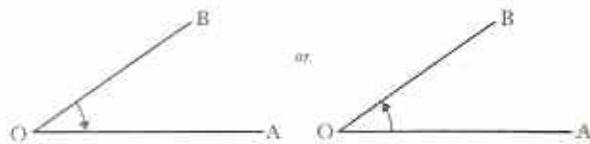


Fig. 9.5

Example 1

What is the angle between the hour hand and the minute hand of a clock at 1 o'clock?

The angle between the hands is the amount that one hand must turn to reach the position of the other (Fig. 9.6). That is $\frac{1}{12}$ of a revolution.

Angle between hands = $\frac{1}{12}$ of a revolution
 $= \frac{1}{12}$ of $360^\circ = 30^\circ$



Fig. 9.6

Notice, in the above example, that the size of the angle does not depend on the size of the hands. At 1 o'clock the angle between the hour hand and the minute hand is 30° whether on a watch or on a clock (Fig. 9.7).

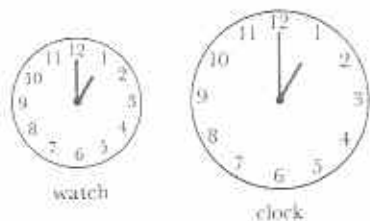


Fig. 9.7

Again, notice from Fig. 9.8 that 330° would also be a correct answer to Example 1. The minute hand would turn through 330° if it went the other way round: $330^\circ + 30^\circ = 360^\circ$.



Fig. 9.8

Exercise 9a

1 Copy and complete Table 9.1

Table 9.1

revs	degrees	revs	degrees
1	360°	$\frac{1}{3}$	
2			90°
3	1080°	$\frac{1}{10}$	
10		$\frac{1}{8}$	
$\frac{1}{2}$	180°	$1\frac{1}{2}$	

2 Find the angles between the hour hand and the minute hand of a clock at the following times. Give your answers both in revolutions and in degrees.

- (a) 2 o'clock (b) 3 o'clock (c) 4 o'clock
 (d) 5 o'clock (e) 6 o'clock (f) 7 o'clock
 (g) 9 o'clock (h) 10 o'clock (i) 11 o'clock

3 Where does the hour hand of a clock point at $\frac{1}{2}$ past 2?

What is $\frac{1}{2}$ of 30° ?

What is the angle between the hour hand and the minute hand of a clock at $\frac{1}{2}$ past 2?

4 Find the angles between the hour hand and the minute hand of a clock at the following times. Give your answers in degrees.

- (a) $\frac{1}{2}$ past 3 (b) $\frac{1}{2}$ past 10
 (c) $\frac{1}{2}$ past 8 (d) $\frac{1}{2}$ past 12
 (e) $\frac{1}{2}$ past 5 (f) $\frac{1}{2}$ past 6

5 Where does the hour hand of a clock point at $\frac{1}{4}$ past 2?

What is $\frac{1}{4}$ of 30° ?

What is $\frac{3}{4}$ of 30° ?

What is the angle between the hour hand and the minute hand of a clock at $\frac{1}{4}$ past 2?

6 Find the angles between the hour hand and the minute hand of a clock at the following times. Give your answers in degrees.

- (a) $\frac{1}{4}$ past 11 (b) $\frac{1}{4}$ past 9 (c) $\frac{1}{4}$ to 9
 (d) $\frac{1}{4}$ past 12 (e) $\frac{1}{4}$ to 5 (f) $\frac{1}{4}$ past 3

Naming angles

When lines OA and OB meet at the point O (Fig. 9.9), we say that angle AOB or angle BOA is the angle between them. AOB is short for angle AOB. Notice that O, the middle letter, is the vertex of the angle. The lines OA and OB are the arms of the angle.

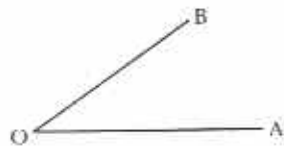


Fig. 9.9

If OA is fixed and OB can turn, we get angles of different sizes. Some of these have special names.

In Fig. 9.10, $\hat{A}OB$ is less than 90° ,
 $\hat{A}OB$ is an **acute angle**.

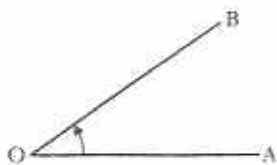


Fig. 9.10

In Fig. 9.11, $\hat{A}OB = 90^\circ$,
 $\hat{A}OB$ is a **right angle** or **quarter turn**.

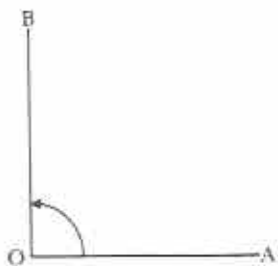


Fig. 9.11

In Fig. 9.12, $\hat{A}OB$ is greater than 90° but less than 180° ,
 $\hat{A}OB$ is an **obtuse angle**.

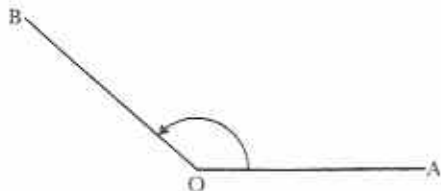


Fig. 9.12

In Fig. 9.13, $\hat{A}OB = 180^\circ = 2$ right angles,
 $\hat{A}OB$ is a **straight angle** or **half turn**.



Fig. 9.13

In both diagrams in Fig. 9.14,
 $\hat{A}OB$ is less than 360° but greater than 180° ,
 $\hat{A}OB$ is a **reflex angle**.

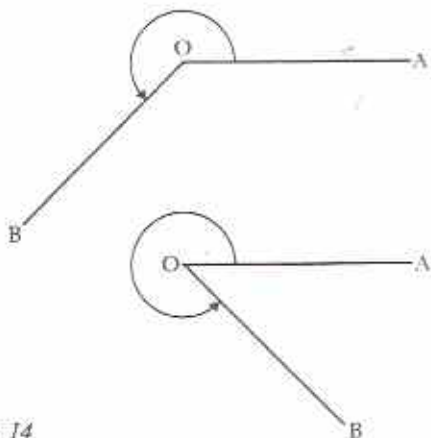


Fig. 9.14

The right angle is especially important. There are many examples where lines meet at right angles. A right angle is often shown on a diagram by drawing a small square at the vertex of the angle (Fig. 9.15).



Fig. 9.15

Measuring angles

We use a **protractor** to measure the number of degrees in an angle. There are many kinds of protractor; two are shown in Fig. 9.16.

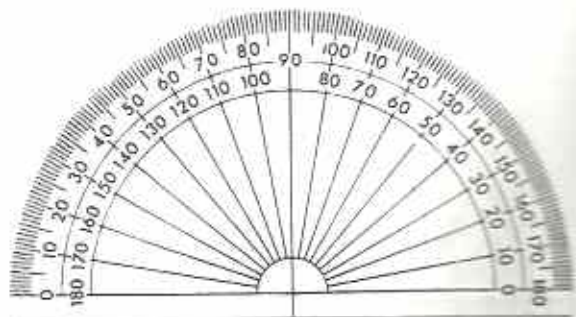


Fig. 9.16 (a) Semi-circular protractor

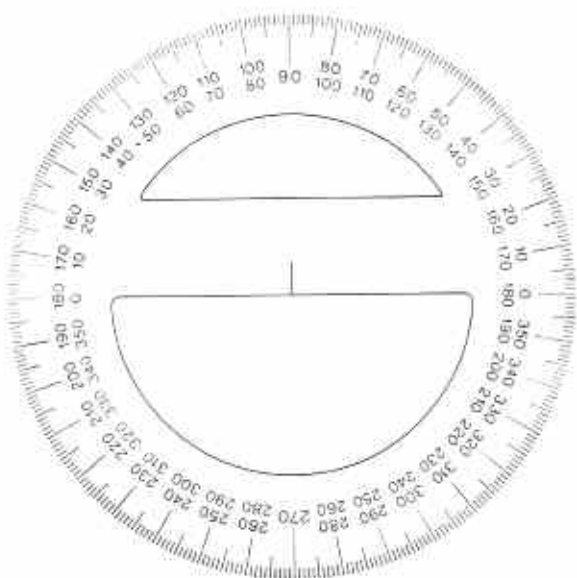
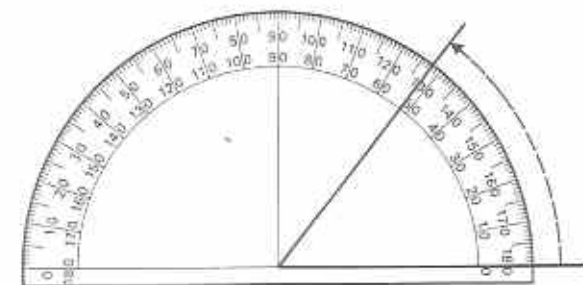


Fig. 9.16 (b) Circular protractor

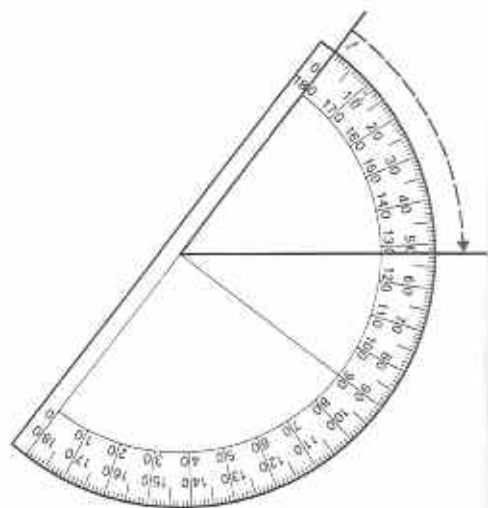
To measure an angle:

(a) Place the protractor over the angle so that its centre is exactly over the vertex of the angle, O , and the base line is exactly along one arm of the angle.

(b) Count the degrees *from* the base line *to* the other arm of the angle. Most protractors have two rows of numbers. This is because you can either measure the angle **clockwise** (as in Fig. 9.17(b)) or **anticlockwise** (as in Fig. 9.17(a)). This can be confusing. In this case the reading is either 52° or 128° . However, since the angle is acute, the correct measurement must be 52° .



(a)



(b)

Fig. 9.17

Example 2

Measure the size of the obtuse angle POQ in Fig. 9.18. Calculate the size of reflex angle POQ .

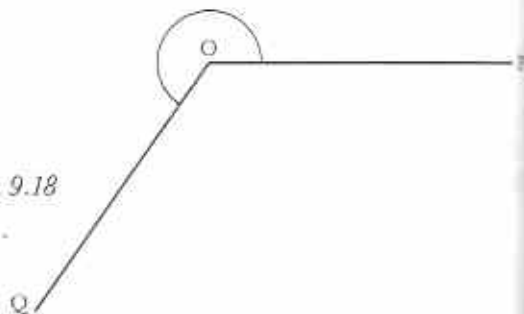
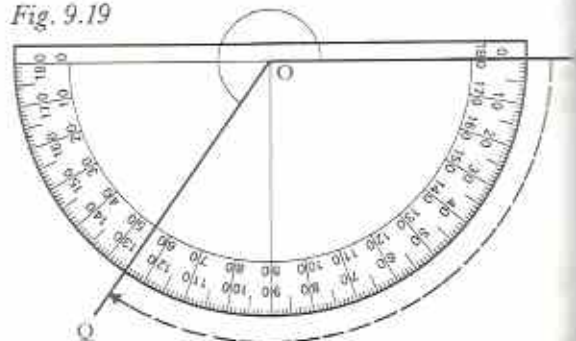


Fig. 9.18

Place the protractor on the angle with its centre on O and its base line on OP as shown in Fig. 9.19.

Fig. 9.19



obtuse $\widehat{POQ} = 125^\circ$ (by measurement)
 reflex $\widehat{POQ} = 360^\circ - 125^\circ = 235^\circ$

Exercise 9b (Oral)

1 In Fig. 9.20,

- name all the acute angles (5 of them),
- name all the obtuse angles (3 of them),
- name all the reflex angles (8 of them).

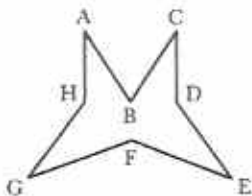


Fig. 9.20

2 In Fig. 9.21, 12 angles have been marked using small letters of the alphabet. State whether each is an acute, right, obtuse, straight or reflex angle.

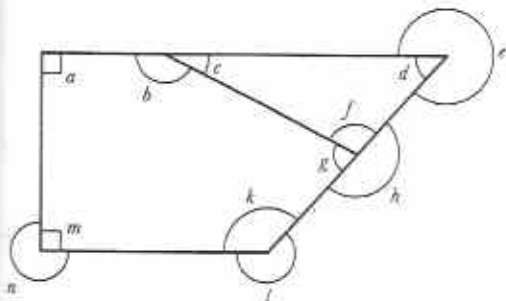
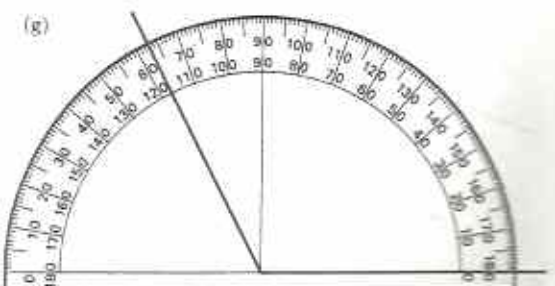
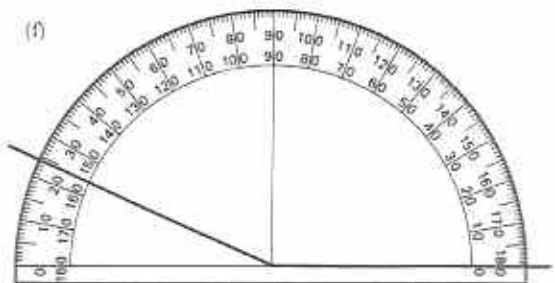
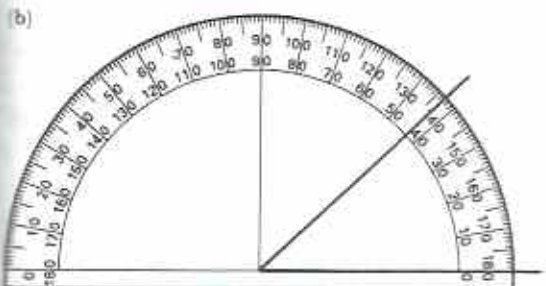
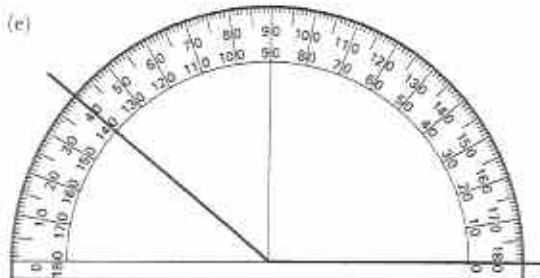
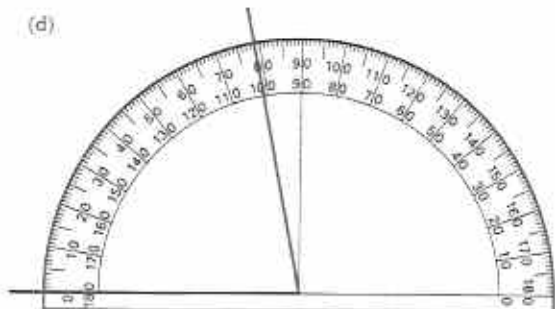
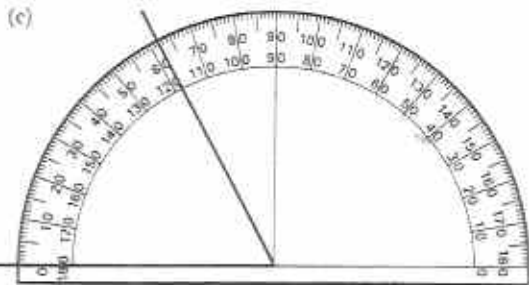
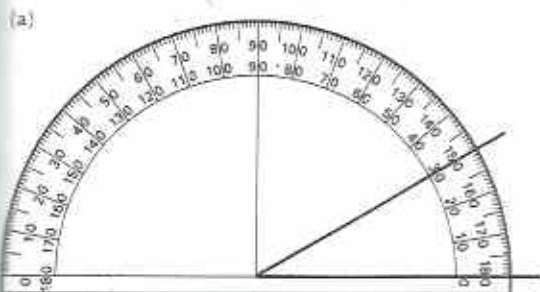


Fig. 9.21

3 Read the sizes of the angles in Fig. 9.22.



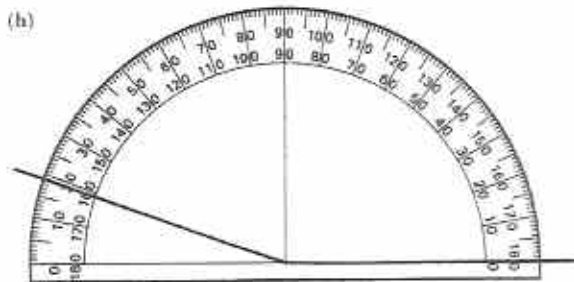


Fig. 9.22

Exercise 9c

You will need a protractor, a ruler and a sheet of thin white paper. Place the sheet of paper over angles 1-10 in Fig. 9.23. Trace the angles on to the paper. Measure each angle (make the arms of the angles longer if necessary).

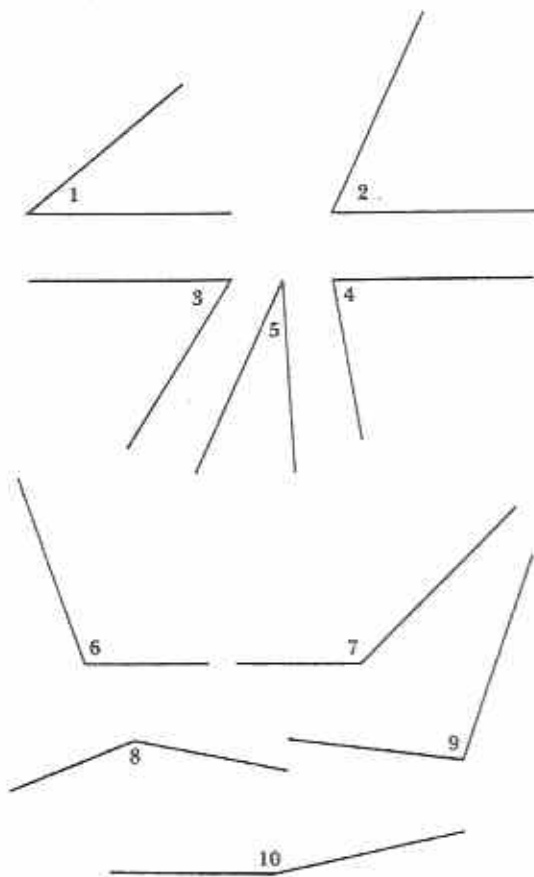


Fig. 9.23

Constructing angles

Example 3

Draw an angle of 47° .

Draw a line and mark a point O on it (Fig. 9.24). Place a protractor so that its centre is over O and its base line is exactly over the line already drawn. Count round until 47° is reached. Make a mark on the paper opposite 47° (Fig. 9.25). Remove the protractor. Join the mark to O with a ruler (Fig. 9.26).



Fig. 9.24

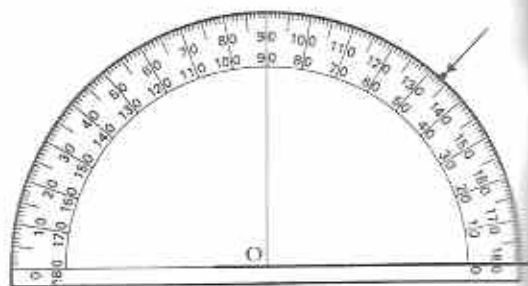


Fig. 9.25

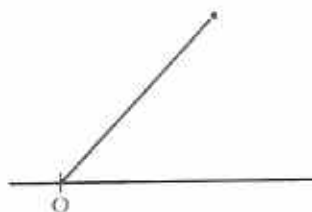


Fig. 9.26

Exercise 9d

- Use a protractor to construct angles of

(a) 40°	(b) 65°	(c) 74°
(d) 130°	(e) 105°	(f) 145°
- Use a protractor to construct reflex angles

(a) 300°	(b) 285°	(c) 215°	(d) 200°
-----------------	-----------------	-----------------	-----------------

 (Hint: Subtract the given angles from 360° to get acute or obtuse angles.)
- Construct a copy of Fig. 9.27 such that $\angle A\hat{C}D = 68^\circ$. Measure $\angle B\hat{C}D$. What do you notice?

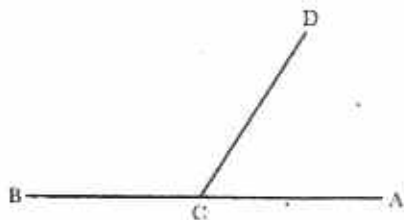


Fig. 9.27

- 4 Make a copy of Fig. 9.28 such that $\widehat{EFG} = 90^\circ$ and $\widehat{EFH} = 25^\circ$. Measure \widehat{HFG} . What do you notice?

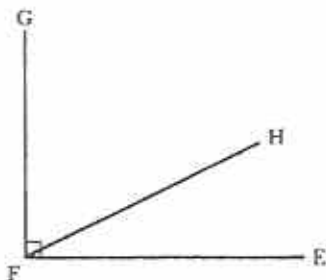


Fig. 9.28

- 5 In Fig. 9.29 WOX and YOZ are straight lines. Make a copy of the diagram such that $\widehat{ZOY} = 53^\circ$. Measure \widehat{ZOW} , \widehat{WOY} and \widehat{YOX} . What do you notice?

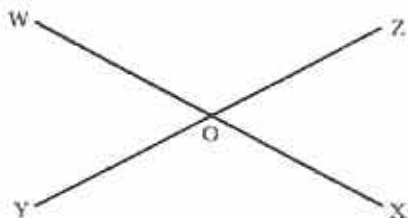


Fig. 9.29

- 6 Draw any large triangle. Measure its three angles. Find the sum of the three angles of the triangle. What do you notice?

Exercise 9e

- 1 Say whether each of the following angles is acute, obtuse or reflex.

(a) 93°	(b) 175°	(c) 86°
(d) 191°	(e) 347°	(f) 28°
(g) 79°	(h) 112°	(i) 63°
(j) 156°	(k) 211°	(l) 167°
(m) 183°	(n) 72°	(o) 98°

- 2 Estimate the sizes of the angles in Fig. 9.30 to the nearest 10° .

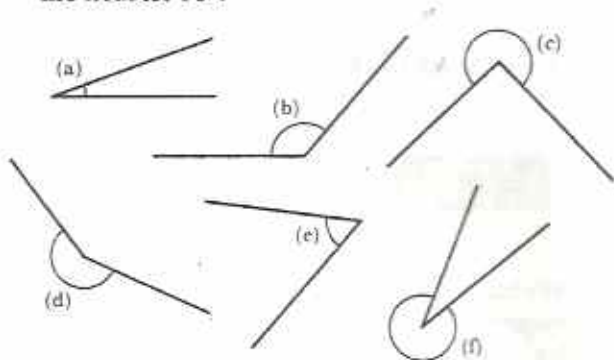


Fig. 9.30

- 3 Use the method of Exercise 9c to trace the angles in question 2 on to a piece of paper. Measure each angle. How good were your estimates?
- 4 Without using a protractor, try to draw angles of the following sizes.
- (a) 90° (b) 45° (c) 80°
 (d) 100° (e) 120°
- 5 Measure the angles you drew in question 4. How accurate were your drawings?
- 6 Draw any four-sided shape. A shape with four sides is called a **quadrilateral**. Measure the four angles of the quadrilateral you have drawn. Find the sum of the four angles. What do you notice?
- 7 Draw a triangle such that two of its sides are 10 cm long. The third side can be any length. Measure the three angles of the triangle. What do you notice? Find the sum of the three angles. What do you notice?
- 8 Draw a triangle such that one of its angles is obtuse. Measure the three angles of the triangle. Find their sum. What do you notice?

Chapter 10

Decimal fractions and percentages

Syllabus Reference 5.1.1

We have seen in Chapter 1 that the place-value system can be extended to include decimal fractions. In Chapter 4 the SI system makes use of this. For example 3,54 kg means $3 \text{ kg} + \frac{5}{10} \text{ kg} + \frac{4}{100} \text{ kg}$ or $3 \text{ kg} + \frac{54}{100} \text{ kg}$. When using measuring instruments, such as a ruler, decimal fractions are used to give intermediate values.

Exercise 10a (Oral)

Each part of Fig. 10.1 shows a scale and an arrow. The reading opposite each arrow is a decimal number. Find the reading in each case, estimating where necessary.

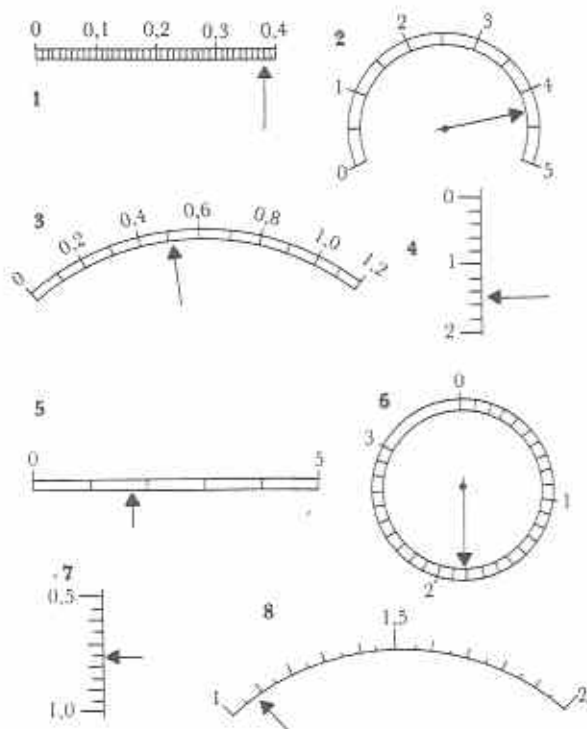


Fig. 10.1

Just as we add, subtract, multiply and divide whole numbers and fractions, we can do the same with decimal numbers and decimal fractions.

Addition and subtraction

Be very careful to set your work out correctly. Units must be under units, decimal commas under decimal commas, ... and so on. For example, $24,8 + 6,5$ is set out as

$$\begin{array}{r} 24,8 \\ + 6,5 \\ \hline \end{array} \quad \text{not} \quad \begin{array}{r} 24,8 \\ 6,5 \\ \hline \end{array}$$

After you have set out your work correctly, add and subtract in the same way as you do with whole numbers, but remember to write down the decimal comma when you come to it.

Exercise 10b

Questions 1-5 each contain ten short questions. Give yourself 10 minutes to do questions 1-5. Correct your work. If you make less than 2 mistakes, go on to question 6. If you make more than 4 mistakes, do questions 3, 4, 5.

1 (5 min)

- | | |
|--|---|
| (a) $0,2 + 0,6$ | (b) $0,6 - 0,3$ |
| (c) $1,3 + 0,8$ | (d) $2 - 0,5$ |
| (e) $2,7 + 3,8$ | (f) $8,8 - 3,4$ |
| (g) $\begin{array}{r} 15,86 \\ + 5,15 \\ \hline \end{array}$ | (h) $\begin{array}{r} 0,56 \\ - 0,18 \\ \hline \end{array}$ |

- (i) $4,23 + 0,78 + 7,52$
 (j) $59,25 - 49,17$

2 (5 min)

- | | |
|-----------------|-----------------|
| (a) $0,3 + 0,5$ | (b) $0,9 - 0,4$ |
| (c) $1,9 + 0,8$ | (d) $3 - 1,6$ |

$$\begin{array}{r} \text{(e)} \ 5,4 + 2,7 \\ \quad 6,33 \\ \quad + 4,07 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(f)} \ 0,47 - 0,25 \\ \quad 7,1 \\ \quad - 3,7 \\ \hline \end{array}$$

$$\text{(i)} \ 8,4 + 7,5 + 31,9 \quad \text{(j)} \ 25,08 - 14,13$$

3 (5 min)

$$\text{(a)} \ 0,1 + 0,8$$

$$\text{(b)} \ 0,4 - 0,1$$

$$\text{(c)} \ 0,6 + 1,7$$

$$\text{(d)} \ 2 - 1,7$$

$$\text{(e)} \ 6,1 + 1,9$$

$$\text{(f)} \ 0,59 - 0,55$$

$$\begin{array}{r} \text{(g)} \quad 12,62 \\ \quad + 13,59 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(h)} \quad 8,2 \\ \quad - 5,7 \\ \hline \end{array}$$

$$\text{(i)} \ 3,67 + 2,74 + 5,4 + 0,23$$

$$\text{(j)} \ 18,33 - 16,55$$

4 (5 min)

$$\text{(a)} \ 0,3 + 0,2$$

$$\text{(b)} \ 0,8 - 0,6$$

$$\text{(c)} \ 0,5 + 0,9$$

$$\text{(d)} \ 3 - 0,3$$

$$\text{(e)} \ 3,5 + 4,6$$

$$\text{(f)} \ 7,5 - 2,5$$

$$\begin{array}{r} \text{(g)} \quad 13,45 \\ \quad + 7,55 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(h)} \quad 0,85 \\ \quad - 0,26 \\ \hline \end{array}$$

$$\text{(i)} \ 4,5 + 0,76 + 6,4 + 1,06$$

$$\text{(j)} \ 24,11 - 14,07$$

5 (5 min)

$$\text{(a)} \ 0,4 + 0,4$$

$$\text{(b)} \ 0,3 - 0,2$$

$$\text{(c)} \ 1,6 + 0,4$$

$$\text{(d)} \ 2 - 1,8$$

$$\text{(e)} \ 7,5 + 1,7$$

$$\text{(f)} \ 9,3 - 6,2$$

$$\begin{array}{r} \text{(g)} \quad 8,92 \\ \quad + 4,09 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(h)} \quad 0,53 \\ \quad - 0,26 \\ \hline \end{array}$$

$$\text{(i)} \ 69,3 + 6,93 + 0,693$$

$$\text{(j)} \ 40,5 - 4,05$$

6 Find the sum of (a) \$1,62, \$2,52 and \$3,42, (b) \$1,12, 64c and 96c.

7 Find the difference between (a) 28,95 m and 17,58 m, (b) 3,062 kg and 855 g.

8 A ball of string is 13,5 m long when new. Lengths of 2,3 m, 2,37 m and 95 cm are cut off. What length of string is cut off altogether? How much string is left? (Give both answers in metres.)

9 A recipe for bread says, 'Mix 30 g of yeast, 15 g of sugar and 15 g of salt. Add the mixture to 1 kg of flour. Add water and stir'. A woman followed the recipe and the total mass of the mixture was 1,65 kg. What mass of water did she add?

10 Find the difference between 9,28 and the sum of 2,31 and 3,92.

Multiplication and division by powers of 10

Table 10.1 shows what happens when 3,07 is multiplied by increasing powers of 10.

Table 10.1

$3,07 \times 1 =$	$3,07$
$3,07 \times 10 = 3,07 \times 10^1 =$	$30,7$
$3,07 \times 100 = 3,07 \times 10^2 =$	$307,$
$3,07 \times 1\,000 = 3,07 \times 10^3 =$	$3\,070,$
$3,07 \times 10\,000 = 3,07 \times 10^4 =$	$30\,700,$

Note that when **multiplying** by powers of 10:

(a) As the power of 10 increases, it is as if the decimal comma stays where it is and the digits in the number move to the left.

(b) The digits move as many places to the left as the power of 10 (or as the number of zeros in the multiplier).

(c) As each place to the right of the digits becomes empty we fill it with a zero to act as a place holder.

(d) If the fraction to the right of the decimal comma becomes zero there is no need to write anything after the comma.

Similarly, Table 10.2 shows what happens when 3,07 is divided by increasing powers of 10.

Table 10.2

$3,07 \div 1 =$	$3,07$
$3,07 \div 10 = 3,07 \div 10^1 =$	$0,307$
$3,07 \div 100 = 3,07 \div 10^2 =$	$0,0307$
$3,07 \div 1\,000 = 3,07 \div 10^3 =$	$0,00307$
$3,07 \div 10\,000 = 3,07 \div 10^4 =$	$0,000307$

Notice that when **dividing** by powers of 10:

(a) As the power of 10 increases, it is as if the decimal comma stays where it is and the digits in the number move to the right.

(b) The digits move as many places to the right as the power of 10 (or as the number of zeros in the divider).

(c) As each place to the left of the digits becomes empty we fill it with a zero to act as a place holder.

(d) If the number to the left of the decimal comma becomes zero we write zero there.

Example 1

Write the following as decimal numbers.

$$(a) 0,036 \times 10\,000 \quad (b) \frac{23}{1\,000}$$

$$(c) 120 \div 100\,000 \quad (d) 0,00045 \times 100$$

$$(a) 0,036 \times 10\,000 = 0,036 \times 10^4 = 360$$

$$(b) \frac{23}{1\,000} = 23 \div 1\,000 = 23 \div 10^3 = 0,023$$

$$(c) 120 \div 100\,000 = 120 \div 10^5 = 0,00120$$

$$= 0,0012$$

It is not necessary to write zeros to the right of a decimal fraction. For example, 0,200000 is just the same as 0,2.

$$(d) 0,00045 \times 100 = 0,00045 \times 10^2 = 0,045$$

Exercise 10c (Oral)

Give the following as decimal numbers.

1 $2,67 \times 100$	2 $4,55 \times 10,000$
3 $8,03 \times 1\,000$	4 $5,6 \div 1\,000$
5 $8,11 \div 10$	6 $7,04 \div 100$
7 $0,027 \times 10\,000$	8 $0,36 \times 100$
9 $0,0085 \times 1\,000$	10 $\frac{52}{1\,000}$
11 $\frac{63}{10\,000}$	12 $\frac{423}{100}$
13 $0,05 \times 1,000$	14 650×100
15 $90 \times 100,000$	16 $4,37 \times 10^3$
17 $24,5 \times 10^4$	18 $0,207 \times 10^3$
19 $4\,502 \div 10^2$	20 $20,05 \div 10^3$
21 $0,0036 \div 10^4$	22 $5\,140 \div 100\,000$
23 $60 \div 100$	24 $300 \div 10\,000$
25 $0,00026 \times 100$	26 $0,07 \times 10$
27 $0,00008 \times 1\,000$	28 $\frac{2,42}{10\,000}$
29 $\frac{4000,7}{100}$	30 $\frac{70,63}{1\,000}$

- 31 Express \$6,20 in cents.
 32 Express 344c in dollars.
 33 Express 2,41 m in cm.
 34 Express 592 cm in metres.
 35 Express 6,24 tonnes in (a) kg, (b) g.
 36 Express 920 kg in (a) tonnes, (b) g.
 37 Express 420 g in (a) kg, (b) tonnes.
 38 Express 504 ml in (a) l, (b) kl.
 39 Express 20,9 m in (a) mm, (b) km.
 40 Express 7050 litres in (a) kl, (b) ml.

Changing fractions to decimals

To express a fraction as a decimal, make an equivalent fraction with a denominator which is a power of 10.

Example 2

Express $\frac{3}{8}$ as a decimal.

$$\frac{3}{8} = \frac{3 \times 1\,000}{8 \times 1\,000} = \frac{3\,000}{8\,000} = \frac{3\,000 \div 8}{8\,000 \div 8} = \frac{375}{1\,000} = 0,375$$

It is usually quicker to divide the numerator of the fraction by its denominator, taking care to write down the decimal comma as it arises.

$$\frac{3}{8} = 3,000 \div 8 = 0,375$$

$\frac{3}{8}$ gives an exact decimal fraction, 0,375. We say it is a **terminating decimal**.

Exercise 10d

Express the following as terminating decimals.

1 $\frac{1}{2}$	2 $\frac{1}{4}$	3 $\frac{3}{4}$	4 $\frac{1}{5}$
5 $\frac{1}{20}$	6 $\frac{7}{20}$	7 $\frac{1}{25}$	8 $\frac{3}{25}$
9 $\frac{1}{8}$	10 $\frac{13}{50}$	11 $\frac{22}{25}$	12 $\frac{17}{25}$
13 $\frac{9}{25}$	14 $\frac{43}{50}$	15 $\frac{7}{8}$	16 $\frac{11}{12}$
17 $3\frac{1}{8}$	18 $9\frac{3}{8}$	19 $4\frac{13}{30}$	20 $\frac{1}{6}$

Example 3

Express $\frac{7}{9}$ as a decimal.

$$\frac{7}{9} = 7,000 \div 9 = \frac{0,777 \dots}{9 \overline{)7,000 \dots}}$$

$$= 0,777 \dots$$

$\frac{7}{9}$ is a never ending decimal fraction: 0,777 ... The digit 7 is repeated as often as we like. We say it is a **recurring decimal**. We say this is 'zero comma seven recurring' and write it as 0,7̇.

Many fractions give rise to recurring decimals. For example:

$$\frac{4}{11} = 0,363\,636 \dots = 0,3\dot{6}$$

$$\frac{5}{7} = 0,714\,285\,714\,285 \dots = 0,7\dot{1}4\,2\dot{8}5$$

$$\frac{1}{6} = 0,166\,666 \dots = 0,1\dot{6}$$

Exercise 10e

Express the following as recurring decimals.

1 $\frac{1}{3}$	2 $\frac{2}{3}$	3 $\frac{5}{6}$	4 $\frac{5}{9}$
5 $\frac{8}{9}$	6 $\frac{3}{11}$	7 $\frac{6}{11}$	8 $\frac{5}{6}$
9 $\frac{7}{12}$	10 $\frac{1}{7}$		

Changing decimals to fractions

Any decimal fraction can be expressed as a fraction with a denominator which is a power of 10. For example, $0,3 = \frac{3}{10}$, $0,54 = \frac{54}{100}$, $0,207 = \frac{207}{1000}$, and so on.

Example 4

Express 0,376 as a fraction in its lowest terms.

$$0,376 = \frac{376}{1000} = \frac{376 \div 8}{1000 \div 8} = \frac{47}{125}$$

Exercise 10f

Express the following as fractions in their lowest terms.

- | | | |
|----------|----------|----------|
| 1 0,8 | 2 0,75 | 3 0,2 |
| 4 0,45 | 5 0,44 | 6 0,54 |
| 7 0,56 | 8 0,66 | 9 0,84 |
| 10 0,102 | 11 0,105 | 12 0,375 |
| 13 0,725 | 14 0,625 | 15 0,624 |

Multiplication of decimals

Example 5

Find the product of 38,6 and 1,64.

$$38,6 \times 1,64 = \frac{386}{10} \times \frac{164}{100} = \frac{386 \times 164}{1000}$$

Use long multiplication to find the numerator:

$$\begin{array}{r} 386 \\ \times 164 \\ \hline 1544 \\ 23160 \\ 38600 \\ \hline 63304 \end{array}$$

$$38,6 \times 1,64 = \frac{63304}{1000} = 63,304$$

The method in Example 5 can be made shorter as follows:

(a) Multiply the given numbers without decimal commas.

(b) Count the digits after the decimal commas in the numbers being multiplied.

(c) Place the decimal comma so that the product has the same number of digits after the comma.

Example 6

$$0,25 \times 0,009$$

$$25 \times 9 = 225$$

There are five digits after the decimal commas in the given numbers.

$$\text{So, } 0,25 \times 0,009 = 0,00225$$

Exercise 10g (Oral)

Find the following products.

- | | |
|-----------------------|------------------------|
| 1 $0,7 \times 8$ | 2 $0,2 \times 4$ |
| 3 $0,2 \times 6$ | 4 $0,3 \times 9$ |
| 5 $0,8 \times 5$ | 6 $9 \times 0,4$ |
| 7 $7 \times 0,7$ | 8 $7 \times 0,6$ |
| 9 $5 \times 0,6$ | 10 $8 \times 0,5$ |
| 11 $0,6 \times 0,3$ | 12 $0,4 \times 0,7$ |
| 13 $0,1 \times 0,6$ | 14 $0,3 \times 0,4$ |
| 15 $0,3 \times 0,3$ | 16 $0,09 \times 0,2$ |
| 17 $0,7 \times 0,06$ | 18 $0,04 \times 0,08$ |
| 19 $0,5 \times 0,006$ | 20 $0,003 \times 0,05$ |

Exercise 10h

Questions 1-5 each contain five items. Give yourself 10 minutes to do questions 1 and 2. Correct your work. If you make less than 4 mistakes, go on to question 6. If you make more than 3 mistakes, do questions 3, 4, 5.

- (5 min)
(a) $0,6 \times 1,8$ (b) $1,93 \times 0,3$
(c) $(0,05)^2$ (d) $0,64 \times 2,5$
(e) $6,83 \times 0,62$
- (5 min)
(a) $8 \times 0,15$ (b) $27,9 \times 0,5$
(c) $(0,2)^2$ (d) $6,8 \times 3,1$
(e) $7,3 \times 0,144$
- (5 min)
(a) $0,4 \times 1,4$ (b) $510 \times 0,002$
(c) $(0,8)^2$ (d) $5,4 \times 0,052$
(e) $2,71 \times 0,25$
- (5 min)
(a) $5 \times 0,6$ (b) $0,17 \times 3$
(c) $(0,004)^2$ (d) $410 \times 0,0023$
(e) $0,87 \times 0,306$

- 5 (5 min)
 (a) $3 \times 0,9$ (b) $1,3 \times 0,7$
 (c) $(0,3)^2$ (d) $0,77 \times 0,15$
 (e) $3,42 \times 9,9$
- 6 What is 0,25 of 6,36?
- 7 Find the product of 2,03 and 0,055.
- 8 A ream of paper contains 480 sheets. Each sheet is 0,014 cm thick. Find the thickness of the ream of paper.
- 9 A dress needs 3,5 m of cloth. If cloth costs \$3,80 per metre, how much will it cost to buy enough cloth for a dress?
- 10 1 metre of string has a mass of 2,3 g. What is the mass of 112,8 metres of this string? Give your answer, (a) in g, (b) in kg.

Division of decimals

If the divisor is a whole number, divide in the usual way. Be careful to include the decimal comma in the correct place.

Example 7

$$10,71 \div 63$$

$$\begin{array}{r} 0,17 \\ 63 \overline{)10,71} \\ \underline{6,3} \\ 4,41 \\ \underline{4,41} \\ 0 \end{array}$$

$$10,71 \div 63 = 0,17$$

If the divisor contains a decimal fraction, make an equivalent division such that the divisor is a whole number.

Example 8

$$13,05 \div 2,9$$

$$13,05 \div 2,9 = \frac{13,05}{2,9} = \frac{13,05 \times 10}{2,9 \times 10} = \frac{130,5}{29}$$

$$13,05 \div 2,9 = 130,5 \div 29$$

$$\begin{array}{r} 4,5 \\ 29 \overline{)130,5} \\ \underline{116} \\ 14,5 \\ \underline{14,5} \\ 0 \end{array}$$

$$13,05 \div 2,9 = 130,5 \div 29 = 4,5$$

The method in Example 8 can be made shorter as follows.

- (a) Count how many places to the right the digits in the divisor must move to make the divisor a whole number.
 (b) Move the digits of *both* numbers to the right by this number of places.
 (c) Divide as before.

Example 9

$$4,2 \div 0,006$$

$$4,2 \div 0,006 = 4\,200 \div 6 = 700$$

Example 10

$$1,938 \div 0,34$$

$$1,938 \div 0,34 = 193,8 \div 34$$

$$\begin{array}{r} 5,7 \\ 34 \overline{)193,8} \\ \underline{170} \\ 23,8 \\ \underline{23,8} \\ 0 \end{array}$$

$$1,938 \div 0,34 = 193,8 \div 34 = 5,7$$

Exercise 10i (Oral)

Find the value of the following divisions.

- | | | | |
|----|---------------------|----|--------------------|
| 1 | $18 \div 6$ | 2 | $1,8 \div 6$ |
| 3 | $0,18 \div 6$ | 4 | $18 \div 0,6$ |
| 5 | $18 \div 0,06$ | 6 | $1,8 \div 0,6$ |
| 7 | $180 \div 0,6$ | 8 | $0,18 \div 0,06$ |
| 9 | $1\,800 \div 0,06$ | 10 | $0,018 \div 0,6$ |
| 11 | $\frac{30}{5}$ | 12 | $\frac{3}{5}$ |
| 13 | $\frac{30}{0,5}$ | 14 | $\frac{0,3}{5}$ |
| 15 | $\frac{3}{0,005}$ | 16 | $\frac{30}{0,05}$ |
| 17 | $\frac{300}{0,5}$ | 18 | $\frac{3}{0,05}$ |
| 19 | $\frac{0,003}{0,5}$ | 20 | $\frac{0,3}{0,05}$ |

Exercise 10j

Questions 1–5 each contain five items. Give yourself 10 minutes to do questions 1 and 2. Correct your work. If you make less than 4 mistakes go on to question 6. If you make more than 3 mistakes, do questions, 3, 4, 5.

- 1 (5 min)
 (a) $\frac{0,09}{3}$ (b) $\frac{3,6}{0,4}$
 (c) $0,042 \div 0,06$ (d) $\frac{7,3}{0,202}$
 (e) $12,24 \div 3,6$

2 (5 min)

(a) $\frac{1,4}{7}$ (b) $\frac{0,27}{0,9}$

(c) $0,012 \div 0,4$ (d) $\frac{3,68}{0,08}$

(e) $3,172 \div 0,52$

3 (5 min)

(a) $\frac{0,32}{8}$ (b) $\frac{4}{0,5}$

(c) $0,28 \div 0,007$ (d) $\frac{0,615}{0,5}$

(e) $80,5 \div 2,3$

4 (5 min)

(a) $\frac{0,6}{3}$ (b) $\frac{1,8}{0,2}$

(c) $4,8 \div 0,08$ (d) $\frac{4,77}{0,09}$

(e) $6,44 \div 0,028$

5 (5 min)

(a) $\frac{0,7}{7}$ (b) $\frac{0,4}{0,8}$

(c) $36 \div 0,09$ (d) $\frac{0,432}{0,4}$

(e) $351 \div 0,45$

6 If $35 \times 67 = 2\,345$, what is the value of $2,345 \div 6,7$?

7 The mass of 6 equal books is 2,82 kg. Calculate the mass of 1 book (a) in kg, (b) in g.

8 A pile of boards is 36 cm high. Each board is 0,8 cm thick. How many boards are in the pile?

9 6,5 metres of wire costs \$3,51. What is the cost of 1 metre of wire?

10 A test car travels 98,6 km on 7,25 litres of petrol. How many km does it travel on 1 litre of petrol?

Notice also,

$$\frac{1}{3} = 0,333 \dots = 0,3\bar{3}$$
$$\frac{1}{3} = 33,333 \dots \% = 33,3\% = 33\frac{1}{3}\%$$

Exercise 10k (Oral)

1 Express the following percentages as decimals.

(a) 5% (b) 50% (c) 25%

(d) 40% (e) 70% (f) 4%

(g) 15% (h) 12% (i) 80%

(j) 75% (k) 24% (l) 35%

(m) 64% (n) 60% (o) 45%

2 Express the following decimals as percentages.

(a) 0,25 (b) 0,85 (c) 0,36

(d) 0,81 (e) 0,08 (f) 0,125

(g) 0,308 (h) 0,7 (i) 0,07

(j) 0,6 (k) 0,5 (l) 0,022

(m) 0,005 (n) 0,8 (o) 0,13

Exercise 10l

1 Find the value of the following.

(a) $24,2 + 6,78$

(b) $3,55 \text{ kg} + 7,84 \text{ kg}$

(c) $\$5,28 + \$6,74$

(d) $61,7 + 4,05 + 18,86$

(e) $\$34 + \$3,75 + 86c$

(f) $2,2 \text{ kg} - 0,35 \text{ kg}$

(g) $67 \text{ cm} - 281 \text{ mm}$

(h) $\$852,12 - \$78,90$

(i) $50,4 - 5,04$ (j) $\$24 - \$10,14$

(k) $0,5 \times 7$ (l) $(0,4)^2$

(m) $0,08 \times 0,7$ (n) $12,4 \times 0,8$

(o) $21,8 \times 0,73$ (p) $0,1 \div 8$

(q) $0,03 \div 0,4$ (r) $43,4 \div 0,7$

(s) $0,0927 \div 0,03$ (t) $89,6 \div 0,35$

2 Find the value of the following. Give your answers both in decimals and in fractions where possible.

(a) $\frac{1}{4} + 0,34$ (b) $0,6 - \frac{1}{3}$

(c) $5,8 + 2\frac{3}{4}$ (d) $4\frac{1}{2} - 1,06$

(e) $\frac{1}{2}$ of 0,48 (f) $\frac{3}{4}$ of 2,8

(g) $5,4 \times 2\frac{1}{2}$ (h) $3,14 \times (\frac{1}{2})^2$

(i) $1\frac{4}{5} + 0,45$ (j) $2,88 + 3\frac{3}{5}$

(k) $14 \div 5,6$ (l) $0,7 + 5\frac{1}{4}$

Decimals and percentages

Remember that 17% means $\frac{17}{100}$. Thus 17% is the same as 0,17.

$$17\% = \frac{17}{100} = 0,17$$

This shows that to change a percentage to a decimal fraction, divide the percentage by 100. For example,

$$16\% = 16 \div 100 = 0,16$$

To change a decimal fraction to a percentage, multiply by 100. For example,

$$0,732 = (0,732 \times 100)\% = 73,2\%$$

- 3 A student spends 37,5% of her money on food, 28,75% of her money on rent and 15% of her money on books. What percentage of her money is left to spend on other things?
- 4 What percentage of 10 km is 2,5 km?
- 5 What is 30% of \$1,90? A trader bought some caps for \$1,90 each. He sold them at a profit of 30%. What was the selling price of a cap?
- 6 If $1,504 \div 4,7 = 0,32$, what is the value of $320 \times 0,47$?
- 7 A man bought a second-hand car for \$8400. He sold it at a loss of 17,5%. What was the selling price of the car?
- 8 Express 8,705 m in (a) mm, (b) km.
- 9 12,5 g of medicine costs \$10,75. What is the cost of 1 g of medicine?
- 10 A woman works 46,5 hours and is paid \$2,40 per hour. What is her total pay?

Chapter 11

Directed numbers (1) Addition and subtraction

Syllabus Reference 5.1.1

The number line

A **number line** is a picture of the natural numbers. Fig. 11.1 shows a number line. It starts at 0 (zero) and increases in equal steps to the right.

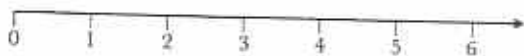


Fig. 11.1

The number line can be used to show addition and subtraction.

Example 1

Use the number line to complete $1 + 4 =$

Fig. 11.2 shows the addition.

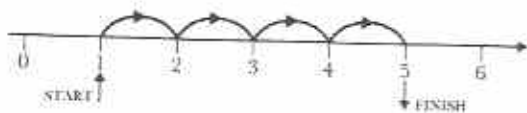


Fig. 11.2

Follow the arrows. Start at 1. Move 4 places to the right. Finish at 5.

$$1 + 4 = 5$$

Example 2

Use the number line to complete $6 - 2 =$

Fig. 11.3 shows the subtraction.



Fig. 11.3

Follow the arrows. Start at 6. Move 2 places to the left. Finish at 4.

$$6 - 2 = 4$$

Exercise 11a

1 Sketch a number line as given in Fig. 11.4.



Fig. 11.4

2 Use your number line to complete the following additions.

- (a) $1 + 5 =$ (b) $2 + 4 =$
(c) $4 + 2 =$ (d) $3 + 1 =$
(e) $4 + 3 =$ (f) $2 + 5 =$

3 Use your number line to complete the following pattern.

$$\begin{aligned} 5 - 2 &= \\ 5 - 3 &= \\ 5 - 4 &= \\ 5 - 5 &= \\ 5 - 6 &= \\ 5 - 7 &= \\ 5 - 8 &= \end{aligned}$$

4 Were you able to complete the last three rows of the pattern in question 3? (If not, read the next section carefully.)

Positive and negative numbers

Fig. 11.5 shows the subtraction $5 - 6$ on the number line.

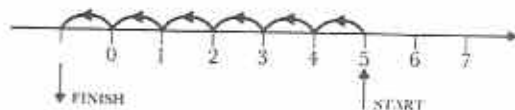


Fig. 11.5

In Fig. 11.5, the finishing point is 1 unit to the left of 0. We could say,

$$5 - 6 = 1L$$

i.e. $5 - 6 = 1$ unit to the left of 0. Similarly,

$$5 - 7 = 2L$$

$$5 - 8 = 3L$$

In practice, we say that numbers to the left of 0 are **negative numbers** and numbers to the right of 0 are **positive numbers**. There are many different symbols for positive and negative numbers. For example, **positive five** can be written $+5$, $(+5)$, $^+5$ or $\overset{+}{5}$ and **negative three** can be written -3 , (-3) , $^-3$ or $\overset{-}{3}$. Fig. 11.6 shows how positive and negative numbers are written on the number line.

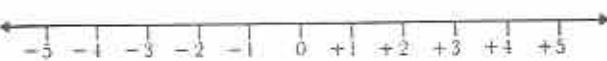


Fig. 11.6

The number line extends in two directions from 0: to the right for positive numbers, to the left for negative numbers.

Exercise 11b (Oral)

1 Fig. 11.7 is a picture of a football team.



Fig. 11.7

The captain, Sam, sits in the middle.

Ben is 2 places to the right of Sam. We can say his position is $(+2)$.

Nda is 5 places to the left of Sam. We can say his position is (-5) .

(i) Use this method to give the position of

- (a) Zenzo, (b) Rex,
(c) Simba, (d) Amos.

(ii) Name the player in position

- (a) $(+4)$, (b) (-4) , (c) (-3) ,
(d) $(+1)$, (e) 0.

2 A line is marked off in metres as shown in Fig. 11.8.

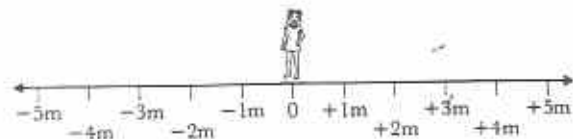


Fig. 11.8

A girl moves to the right and left along the line. Find her final position if she starts at 0 and moves as follows.

- 2 m to the right
- 5 m to the left
- 3 m to the right, then 1 m to the right
- 5 m to the right, then 2 m to the left
- 4 m to the right, then 7 m to the left
- 2 m to the left, then 2 m to the left
- 5 m to the left, then 4 m to the right
- 3 m to the left, then 8 m to the right
- 3 m to the right, then 3 m to the left
- 2 m to the left, then 3 m to the right

Positive and negative numbers in daily life

Consider the following examples.

Example 3

Fig. 11.9 shows a man climbing a tree and a man digging a well.

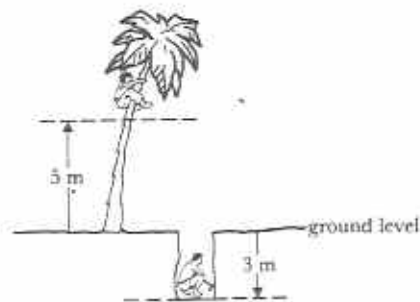


Fig. 11.9

The climber is 5 m above ground level.

The digger is 3 m below ground level.

We can say that their distances from ground level are $+5$ m and -3 m. Positive distances are above ground level. Negative distances are below ground level.

Example 4

Fig. 11.10 shows the readings on two thermometers (a) and (b).

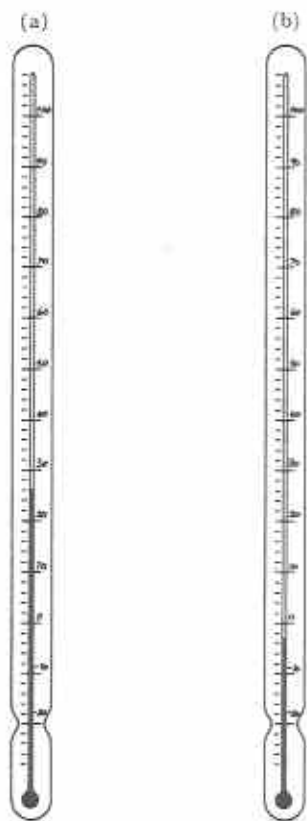


Fig. 11.10

Thermometer (a) shows the shade temperature on an October day. It reads 26°C .

Thermometer (b) shows the temperature inside a refrigerator on the same day. It reads 3°C below 0, i.e. -3°C .

Example 5

A path runs in a straight line from east-west. O is a point on the path. This is shown in Fig. 11.11.

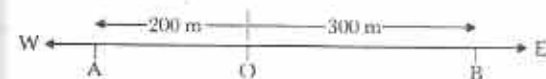


Fig. 11.11

Point A is 200 m west of O. Point B is 300 m east of O. If we take distances east of O to be positive, then the position of A is -200 m and the position of B is $+300\text{ m}$.

Example 6

When an important event is due to take place, the time that it starts is often called 'zero hour'. Times *before* zero hour are *negative*. Times *after* zero hour are *positive*.

For example, a President may decide to broadcast to the nation at 3 pm one day. 3 pm is zero hour. On that day, the President may have lunch with his Ministers at 1 pm. We say that he has lunch at -2 hours. He may finish his speech at 4 pm. He finishes his speech at $+1$ hour.

Examples 3, 4, 5, 6 show that we sometimes use terms like *above*, *below*, *after*, *before* to describe quantities such as distances, temperature and time. In these cases we can also describe the quantities by using positive and negative numbers.

Exercise 11c

- 1 A bird is at the top of a tree 7 m above the surface of a river. A fish swims 2 m below the surface of the river.
(a) Give the positions of the bird and the fish as distances from the surface of the river.
(b) How far is the fish below the bird?
- 2 A map shows that the top of a hill is 200 m above sea-level and the bottom of a lagoon is 15 m below sea-level.
(a) Express these distances as distances from sea-level.
(b) What is the difference between the two heights?
- 3 The temperature inside a freezer is -3°C . The temperature falls by a further 12°C . What is the new temperature?
- 4 What is the difference between temperatures of 12°C and -10°C ?
- 5 A straight road runs east-west through a point O. Distances east of O are positive. Distances west of O are negative.
(i) Express the positions of these points in terms of positive or negative distances.

- (a) P is 8 km west of O
 (b) Q is 9 km east of O
 (c) R is 15 km east of O
 (d) T is 11 km west of O
- (ii) From the above data, find the distance between these points.
 (a) P and Q (b) R and T
 (c) Q and R (d) P and T
- 6 A straight road runs north-south through a point X. Distances north of X are positive. Distances south of X are negative.
 (i) Express the positions of the following points as positive or negative distances.
 (a) A is 330 m north of X
 (b) B is 60 m south of X
 (c) C is 52 m north of X
 (d) D is 128 m south of X
 (ii) From the above data find the distance between these points.
 (a) A and B (b) C and D
 (c) B and C (d) A and D
- 7 Zero hour for a meeting is midday (12 noon). Express the following as positive or negative times.
 (a) 5 h before zero hour
 (b) 7 h after zero hour
 (c) $1\frac{1}{2}$ h after zero hour
 (d) 45 min before zero hour
 (e) 3 pm (f) 10 am (g) 9 pm
 (h) 8.30 am (i) 4.45 pm
- 8 A girl arrives at school 10 minutes early. Another girl is 3 minutes late. How long has the first girl been at school before the second girl arrives?
- 9 A woman was born 19 years before Independence. Her first son was born 5 years after Independence. How old was the woman when her son was born?
- 10 A watch is 9 minutes fast. A clock is 6 minutes slow. A radio announcer says, 'The time is now 11.30 am.' What times do the watch and clock show?
- 11 The top of an oil drill is 10 m above ground level. The drill is 220 m long. What is the distance of the bottom of the drill from ground level?
- 12 Two pieces of wood should be the same length. One is 5 mm too long. The other is 3 mm too short. What is the difference in

the lengths of the two pieces of wood?

- 13 Meat at a temperature of 22°C is put in a refrigerator. The refrigerator cools the meat down by 34°C . What is the temperature of the frozen meat?
- 14 A traveller arrives at an airport $1\frac{1}{2}$ hours before her plane is due to take off. The plane takes off $\frac{3}{4}$ hour late. How long does the traveller wait altogether?
- 15 On Thursday evening Sola has \$19. Gono has no money and owes \$6 to Sola. We can say that Gono has $\$(-6)$. On Friday they each get the same pay, \$64. If Gono repays his debt to Sola, how much money does each have?
- 16 A man owes another man \$18. He only has \$4. What is the most he can pay back? If he pays this back, express his remaining debt as a negative sum of money.
- 17 A woman has \$17. Her younger sister has $\$(-8)$.
 (a) How much does the younger sister owe?
 (b) By how much is the woman better off?
- 18 A man borrows \$12 000 to buy a house. We can say that he has $\$(-12\ 000)$. In the first year he pays back \$1 320. Express his remaining debt as a negative number of Dollars.

Directed numbers

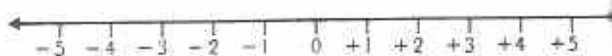


Fig. 11.12

Positive numbers and negative numbers are called **directed numbers**. The sign tells which direction to go from zero to reach the position of the number. Zero is neither positive nor negative. As we move to the right along the number line, the numbers increase. As we move to the left, the numbers decrease (Fig. 11.13).

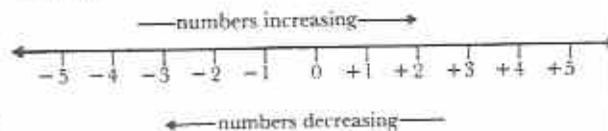


Fig. 11.13

Thus $+2 > -4$, i.e. $+2$ is greater than -4 and $-5 < +1$, i.e. -5 is less than $+1$.

Integers

Directed numbers may be whole or fractional. For example $+2\frac{1}{2}$ and -5.4 are fractional directed numbers.

The positive and negative whole numbers, together with zero, form a set of numbers called **integers**.

If $Z = \{\text{integers}\}$, then $Z = \{\dots, -3, -2, -1, 0, +1, +2, +3, \dots\}$.

If an integer can be divided by 2 to give another integer, it is **even**. If not, it is **odd**.

Even integers

$$= \{\dots, -6, -4, -2, +2, +4, +6, \dots\}$$

Odd integers

$$= \{\dots, -5, -3, -1, +1, +3, +5, \dots\}$$

Notice that zero, 0, is an integer which is neither positive nor negative. Similarly, zero is neither even nor odd.

Exercise 11d (Oral)

Make a number line from -20 to $+20$ by marking a long strip of paper. Use the number line in this exercise.

1 Which is the greatest integer in each of the following?

- (a) $+5, -7, +12, -2, -19, +6$
 (b) $-3, -2, +2, +5, -8, -10$
 (c) $-3, -12, -13, -8, -5, -7$
 (d) $+3, -3, +8, -8, +15, -15$
 (e) $+4, 0, -4, -8, -12, -16$
 (f) $-3, -5, 0, -18, -14, -1$

2 Arrange the following integers in order from least to greatest.

- (a) $-3, 0, -2, +1, -8, +5$
 (b) $+5, -2, +9, +7, -5, -4$
 (c) $+6, -8, -20, +5, -14, 0$
 (d) $-6, +6, -18, +18, -11, +11$
 (e) $-1, 0, -5, -3, -14, -10$
 (f) $-9, -11, +1, -10, +17, +16$

3 Place either $>$ (is greater than) or $<$ (is less than) between each of the two integers to make each a true statement. The first one has been done.

- (a) $-9 < +3$ (b) -15 $+8$
 (c) $+4$ -13 (d) $+18$ $+1$

- (e) -18 -1 (f) 0 -2
 (g) -1 $+1$ (h) -7 -17
 (i) -13 $+13$ (j) 0 -4
 (k) $+5$ -5 (l) -9 0

4 For each of the following write out the complete line. The first line is complete.

	start	move	finish
(a)	$+2$	5 units to the left	-3
(b)	$+5$	7 units to the left	
(c)	-1	7 units to the left	
(d)	$+2$	4 units to the right	
(e)	-3	5 units to the right	
(f)	-10	10 units to the right	
(g)	$+12$	9 units to the left	
(h)	$+14$	30 units to the left	
(i)	-20	28 units to the right	
(j)	$+1$	2 units to the left	
(k)	$+2$		-4
(l)	$+5$		$+3$
(m)	$+6$		-6
(n)	-2		0
(o)	-15		$+15$
(p)	$+15$		-15
(q)	-11		$+13$
(r)	$+12$		-2
(s)	0		$+6$
(t)	0		-17
(u)		2 units to the right	$+5$
(v)		8 units to the left	$+2$
(w)		10 units to the left	-4
(x)		35 units to the right	$+20$
(y)		11 units to the right	$+7$
(z)		13 units to the left	-5

Adding and subtracting positive numbers

Addition

The addition $2 + 3 = 5$ can be written in directed numbers as: $(+2) + (+3) = +5$. We can show this addition on the number line in Fig. 11.14. Follow the arrows from START to FINISH.

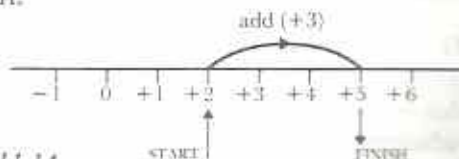


Fig. 11.14

Similarly, we can find the value of $-2 + 6$. In directed numbers this is: $(-2) + (+6)$. Follow the arrows on the number line in Fig. 11.15 from START to FINISH.

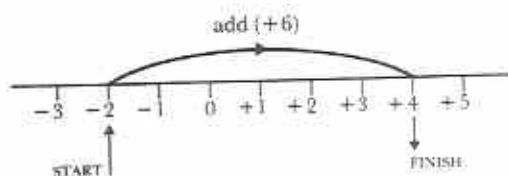


Fig. 11.15

Thus: $-2 + 6 = +4$

To add a positive number: move that number of places to the right on the number line.

Subtraction

The subtraction $4 - 3 = 1$ can be written in directed numbers as: $(+4) - (+3) = +1$. We can show this subtraction on the number line in Fig. 11.16. Follow the arrows from START to FINISH.

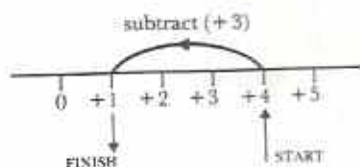


Fig. 11.16

Similarly, we can find the value of $3 - 7$. In directed numbers this is: $(+3) - (+7)$. Follow the arrows on the number line in Fig. 11.17 from START to FINISH.

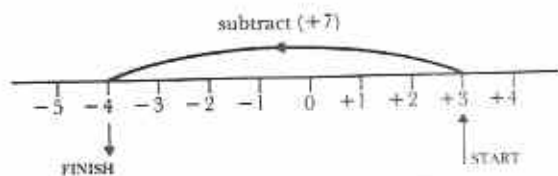


Fig. 11.17

Thus: $3 - 7 = -4$

To subtract a positive number: move that number of places to the left on the number line.

Exercise 11e (Oral)

Complete the following.

1 $(+6) + (+4) = +10$

$(+6) + (+5) =$

$(+6) + (+6) =$

$(+6) + (+7) =$

$(+6) + (+8) =$

$(+6) + (+9) =$

2 $(+6) - (+4) = +2$

$(+6) - (+5) =$

$(+6) - (+6) =$

$(+6) - (+7) =$

$(+6) - (+8) =$

$(+6) - (+9) =$

3 $(-6) + (+4) = -2$

$(-6) + (+5) =$

$(-6) + (+6) =$

$(-6) + (+7) =$

$(-6) + (+8) =$

$(-6) + (+9) =$

4 $(-6) - (+4) = -10$

$(-6) - (+5) =$

$(-6) - (+6) =$

$(-6) - (+7) =$

$(-6) - (+8) =$

5 $(+14) + (+5) =$

6 $(+14) - (+5) =$

7 $(+3) + (+10) =$

8 $(+3) - (+10) =$

9 $(-4) + (+9) =$

10 $(-4) - (+9) =$

11 $(+8) - (+10) =$

12 $(-8) + (+10) =$

13 $(+8) + (+10) =$

14 $(-8) - (+10) =$

15 $(-5) + (+5) =$

16 $(+11) - (+11) =$

17 $(-3) + (+5) - (+8) =$

18 $(+6) - (+9) + (+12) =$

19 $(+12) - (+5) - (+10) =$

20 $(-6) - (+7) + (+14) =$

In many cases, brackets are not given. If a number is not directed, take it to be positive.

Example 7

Simplify $-3 - 9 + 14$.

$$-3 - 9 + 14 = 0 - 3 - 9 + 14$$

$$= 0 - (+3) - (+9) + (+14)$$

The working of example 7 is shown on the number line in Fig. 11.18.

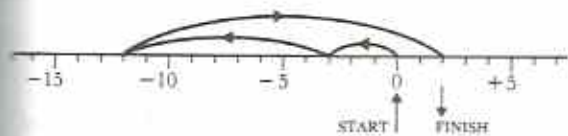


Fig. 11.18

$$3 - (+3) - (+9) + (+14) = +2$$

$$\text{or} \quad -3 - 9 + 14 = 2$$

In practice there is no need to start at 0. The sign in front of the first number shows you where to start.

Exercise 11f (Oral)

Simplify the following.

- | | |
|------------------|-----------------|
| 1 (a) $3 + 1$ | 2 (a) $3 - 1$ |
| (b) $3 + 2$ | (b) $3 - 2$ |
| (c) $3 + 3$ | (c) $3 - 3$ |
| (d) $3 + 4$ | (d) $3 - 4$ |
| (e) $3 + 5$ | (e) $3 - 5$ |
| 3 (a) $-3 + 1$ | 4 (a) $-3 - 1$ |
| (b) $-3 + 2$ | (b) $-3 - 2$ |
| (c) $-3 + 3$ | (c) $-3 - 3$ |
| (d) $-3 + 4$ | (d) $-3 - 4$ |
| (e) $-3 + 5$ | (e) $-3 - 5$ |
| 5 $13 - 9$ | 6 $9 - 13$ |
| 7 $7 - 15$ | 8 $3 - 10$ |
| 9 $-3 - 7$ | 10 $11 - 4$ |
| 11 $-1 - 8$ | 12 $12 - 17$ |
| 13 $19 - 12$ | 14 $-4 - 7$ |
| 15 $-8 + 7$ | 16 $-4 + 12$ |
| 17 $12 - 6 - 4$ | 18 $3 - 11 + 2$ |
| 19 $-3 - 4 + 10$ | 20 $-4 + 9 - 7$ |

Notice that:

- | | |
|--------------------|--------------------|
| (a) $+7 - 4 = +3$ | (b) $+4 - 7 = -3$ |
| (c) $+7 + 4 = +11$ | (d) $-7 - 4 = -11$ |

In (a) and (b), the two signs are different. Disregard signs and take the smaller number from the larger. Then:

- (a) if the larger number has a + sign before it, the result is positive;
 (b) if the larger number has a - sign before it, the result is negative.

In (c) and (d), the two signs are the same. Disregard the signs and add the two numbers.

Then:

- (c) if both signs are +, the result is positive;
 (d) if both signs are -, the result is negative.

This helps to simplify directed numbers without using the number line.

Exercise 11g

Simplify the following. Try not to use the number line.

- | | |
|----------------|----------------|
| 1 $19 - 11$ | 2 $12 - 17$ |
| 3 $-5 - 8$ | 4 $18 - 27$ |
| 5 $31 - 32$ | 6 $-11 - 19$ |
| 7 $-13 - 21$ | 8 $33 - 18$ |
| 9 $43 - 51$ | 10 $29 - 53$ |
| 11 $-17 - 26$ | 12 $27 - 64$ |
| 13 $-33 - 12$ | 14 $-47 - 75$ |
| 15 $81 - 107$ | 16 $121 - 87$ |
| 17 $92 - 155$ | 18 $137 - 173$ |
| 19 $145 - 219$ | 20 $78 - 252$ |

Adding and subtracting negative numbers

Addition

Consider the addition $(+6) + (-2)$. When adding numbers, it does not matter in which order the numbers are added. For example, $3 + 5 = 5 + 3 = 8$.

Thus: $(+6) + (-2) = (-2) + (+6)$

But $(-2) + (+6) = +4$ (see page 91)

Thus: $(+6) + (-2) = +4$

On the number line in Fig. 11.19 we know that we **START** at +6 and **FINISH** at +4. The only way this can happen is if we move 2 places to the *left*.

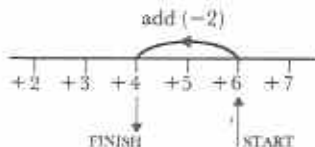


Fig. 11.19

To add a negative number: move that number of places to the left on the number line.

Notice that to add a negative number is equivalent to subtracting a positive number of the same value, for example, see Fig. 11.20.



Fig. 11.20

Example 8

Simplify (a) $2 + (-5)$, (b) $-3 + (-3)$.

$$(a) \quad 2 + (-5) = (+2) + (-5) \\ = (+2) - (+3) = -3$$

$$(b) \quad -3 + (-3) = (-3) + (-3) \\ = (-3) - (+3) = -6$$

Exercise 11h (Oral)

1 Complete the following number patterns.

(a) $(+3) + (+2) = +5$

$(+3) + (+1) =$

$(+3) + 0 =$

$(+3) + (-1) =$

$(+3) + (-2) =$

$(+3) + (-3) =$

(b) $(-1) + (+1) = 0$

$(-1) + 0 =$

$(-1) + (-1) =$

$(-1) + (-2) =$

$(-1) + (-3) =$

$(-1) + (-4) =$

(c) $4 + (+4) = +8$

$4 + (+2) =$

$4 + 0 =$

$4 + (-2) =$

$4 + (-4) =$

$4 + (-6) =$

2 Simplify the following.

(a) $(+4) + (-7)$

(b) $5 + (-3)$

(c) $(-2) + (-6)$

(d) $-8 + (-6)$

(e) $20 + (-20)$

(f) $(+13) + (-10)$

(g) $(-9) + (-9)$

(h) $-18 + (-3)$

(i) $0 + (-5)$

(j) $60 + (-40)$

(k) $(-12) + (-8)$

(l) $43 + (-23)$

(m) $-25 + (-14)$

(n) $100 + (-144)$

(o) $-51 + (-38)$

(p) $16 + (-9) - (+10)$

(q) $-3 + 14 + (-5)$

Subtraction

Consider the subtraction $5 - 3$. One way of doing this is to say, 'What must be added to 3 to get 5?' Since $3 + 2 = 5$, then $5 - 3 = 2$.

Now consider the subtraction $5 - (-3)$. 'What must be added to -3 to get 5?' From earlier work on the number line we know that $-3 + 8 = 5$. Thus: $5 - (-3) = 8$.

This can be shown on the number line as in Fig. 11.21. START at $+5$ and FINISH at $+8$. The only way this can happen is if we move 3 places to the right.

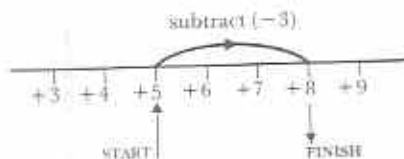


Fig. 11.21

To subtract a negative number, move that number of places to the right on the number line.

Notice that to subtract a negative number is equivalent to adding a positive number of the same value; for example, see Fig. 11.22. This result is so unusual that it is quite easy to remember.



Fig. 11.22

Exercise 11i (Oral)

1 Complete the following number patterns.

(a) $5 - (+2) = +3$

$5 - (+1) =$

$5 - 0 =$

$5 - (-1) =$

$5 - (-2) =$

$$\begin{aligned} \text{(b)} \quad & (-3) - (+3) = -6 \\ & (-3) - (+2) = \\ & (-3) - (+1) = \\ & (-3) - 0 = \\ & (-3) - (-1) = \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & -2 - (+1) = -3 \\ & -2 - 0 = \\ & -2 - (-1) = \\ & -2 - (-2) = \\ & -2 - (-3) = \end{aligned}$$

2 Simplify the following.

$$\begin{array}{ll} \text{(a)} \quad 8 - (-3) & \text{(b)} \quad (-8) - (-3) \\ \text{(c)} \quad (+3) - (-8) & \text{(d)} \quad (-3) - (-8) \\ \text{(e)} \quad (+9) - (-9) & \text{(f)} \quad (-4) - (-4) \\ \text{(g)} \quad (+6) - (-1) & \text{(h)} \quad (-1) - (-2) \\ \text{(i)} \quad -7 - (-6) & \text{(j)} \quad -30 - (-80) \\ \text{(k)} \quad 20 - (-10) & \text{(l)} \quad -40 - (-60) \\ \text{(m)} \quad -41 - (-26) & \text{(n)} \quad -59 - (-37) \\ \text{(o)} \quad 76 - (-74) & \text{(p)} \quad 7 - (-8) + (-5) \\ \text{(q)} \quad -3 - (-21) - 18 \end{array}$$

Chapter summary

- The number line can be extended below zero.
- Numbers above zero are positive. Numbers below zero are negative.
- Positive and negative numbers are called directed numbers.
- Positive and negative whole numbers together with zero form the set of integers.
- Directed numbers can be added or subtracted.
- To add a positive number, move to the right on the number line.
- To subtract a positive number, move to the left on the number line.
- To add a negative number is the same as to subtract a positive number.
- To subtract a negative number is the same as to add a positive number.

Exercise 11j (practice)

1 Which is greater:

- | | |
|--------------------|---------------------|
| (a) -3 or 4 , | (b) -4 or -7 , |
| (c) -5 or 3 , | (d) 0 or -5 , |
| (e) 2 or -15 , | (f) -7 or -10 ? |

2 The temperature during the day in a cold country is 9°C . At night the temperature falls by 13°C . What is the temperature at night?

3 Anna and Simba have no money, but Anna owes \$12 to Simba. When Friday comes they both get the same wages. Anna repays her debt to Simba. Simba now has more money than Anna. How much more?

4 What must be added to:

- | | |
|---------------------------|----------------------|
| (a) 3 to make 8, | (b) -1 to make 2, |
| (c) 16 to make 4, | (d) 3 to make -8 , |
| (e) -35 to make -27 , | |
| (f) 26 to make -4 ? | |

5 What must be subtracted from:

- | | |
|--------------------------|-----------------------|
| (a) 12 to make 8, | (b) 6 to make -10 , |
| (c) -2 to make -7 , | (d) 8 to make 12, |
| (e) -3 to make 4, | |
| (f) -10 to make -3 ? | |

6 A woman has \$23,46 in her bank account. She writes a cheque for \$39,50. How much will she be overdrawn?

7 In the year AD45 a man was 63 years old. In what year was he 5 years old?

8 Simplify the following.

- | |
|-----------------------------|
| (a) $-7 - (16) - 3$ |
| (b) $1 + (-4) - (-3)$ |
| (c) $800 - (+500) - (-150)$ |
| (d) $-50 + (-25) - (+45)$ |
| (e) $6x - 9x - (-5x)$ |
| (f) $-4y + 12y - (-10y)$ |

9 Copy and complete the tables in Fig. 11.23 overleaf. For example, in

- | |
|--|
| (a) $(-1) + (+3) = +2$. |
| Enter the $+2$ across from -1 and under $+3$. |
| (b) $(-1) - (+3) = -4$. |
| Enter -4 across from -1 and under $+3$. |

10 Simplify the following.

- | | |
|--|---------------------------------------|
| (a) $1\frac{3}{4} - 2\frac{1}{4}$ | (b) $-2,8 + 6,3$ |
| (c) $4,8 - (-3,9)$ | (d) $-1\frac{1}{2} - (-3\frac{2}{3})$ |
| (e) $7,2^{\circ}\text{C} - 9,6^{\circ}\text{C}$ | |
| (f) $-5,4^{\circ}\text{C} + 8,6^{\circ}\text{C}$ | |

(a)

		Second						
add		-3	-2	-1	0	+1	+2	+3
First	-3							
	-2							
	-1							+2
	0							
	+1							
	+2							
	+3							

(b)

		Second						
subtract		-3	-2	-1	0	+1	+2	+3
First	-3							
	-2							
	-1							-4
	0							
	+1							
	+2							
	+3							

Fig. 11.23

Plane shapes (1)

Syllabus Reference 5.7.3

Triangles

Tri-angle means three angles. A **triangle** has three sides and three angles. It is the simplest geometrical plane shape. There are many different kinds of triangle (Fig. 12.1).

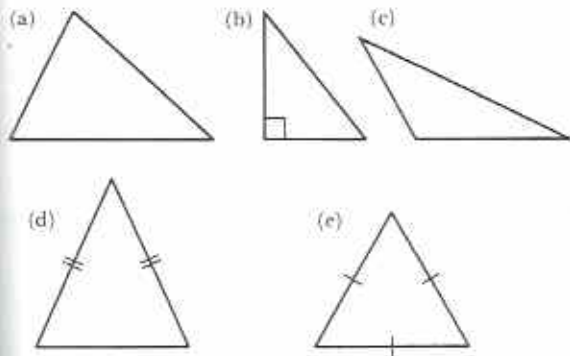


Fig. 12.1 (a) Scalene triangle (b) Right-angled triangle (c) Obtuse-angled triangle (d) Isosceles triangle (e) Equilateral triangle

A **scalene** triangle is a triangle in which all three sides are of different length. A **right-angled** triangle contains one right angle. Similarly, an **obtuse-angled** triangle contains one obtuse angle. *Iso-sceles* means same legs; an **isosceles** triangle has two sides equal in length. *Equi-lateral* means equal sides; an **equilateral** triangle has sides all of the same length.

Exercise 12a (Oral/Class discussion)

For each triangle in Fig. 12.2, (a) measure the sides, (b) measure the angles, (c) find the sum of the angles, (d) name the kind of triangle it is.

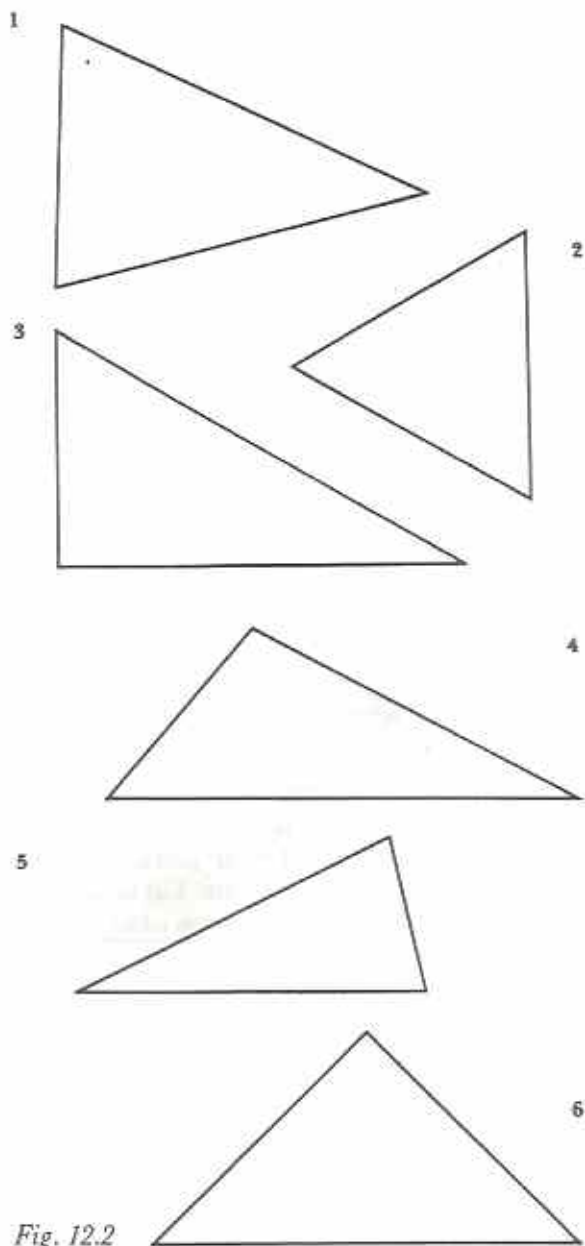


Fig. 12.2

Isosceles triangle

Exercise 12b

- 1 Make an isosceles triangle by folding and cutting a sheet of paper as shown in Fig. 12.3.

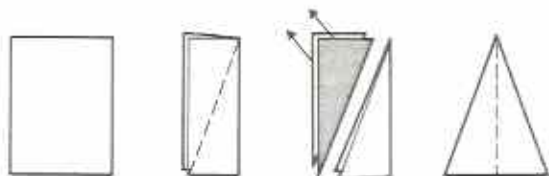


Fig. 12.3

- 2 The line of the fold divides the isosceles triangle into two matching parts. This fold is a **line of symmetry**. Does your isosceles triangle have any other lines of symmetry?
- 3 Measure all three angles of an isosceles triangle. What do you notice?
- 4 In Fig. 12.4, CM is the line of symmetry. On *your* triangle, measure the following: \widehat{ACM} , \widehat{BCM} , \widehat{AMC} , \widehat{BMC} . What do you notice? Also measure the following: AM , BM . What do you notice?

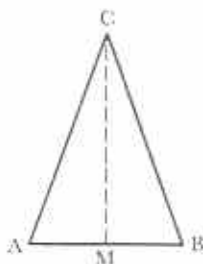


Fig. 12.4

- 5 In Fig. 12.5, $\triangle ACB$ is isosceles, CM is the line of symmetry, $\widehat{ACM} = 20^\circ$ and $BM = 4$ cm. Make a sketch of the diagram. Fill in as many other angles and lengths as you can.

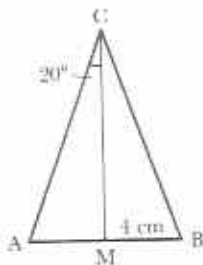


Fig. 12.5

- 6 Why do you think that the triangle in question 5 is named $\triangle ACB$ and not $\triangle ABC$?

Equilateral triangle

Exercise 12c

- 1 The diagrams in Fig. 12.6 show a quick way of drawing an equilateral triangle using ruler and compasses.

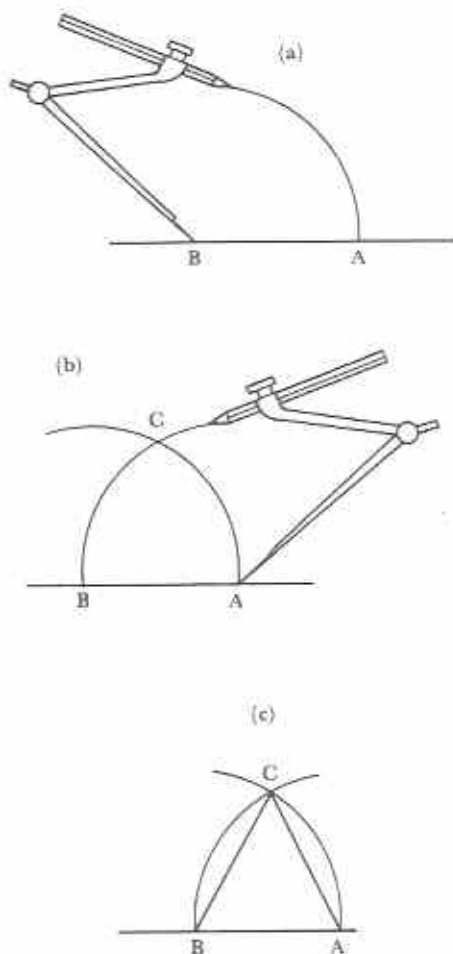


Fig. 12.6

Note that the compasses are kept open at the same position in (a) and (b). Draw some equilateral triangles using this method.

- 2 Measure all three angles of an equilateral triangle you have drawn. What do you notice?

- 3 Write down three differences between an equilateral triangle and an isosceles triangle.
- 4 Fig. 12.7 shows an equilateral triangle and two of its lines of symmetry.



Fig. 12.7

Make a sketch of the diagram. Fill in as many angles as you can.

- 5 Draw a large equilateral triangle on a sheet of paper and carefully cut it out. Fold the triangle to find out if it has any lines of symmetry. How many lines of symmetry has an equilateral triangle? What do you notice about the lines of symmetry?
- 6 How many equilateral triangles can you see in each of the diagrams in Fig. 12.8?

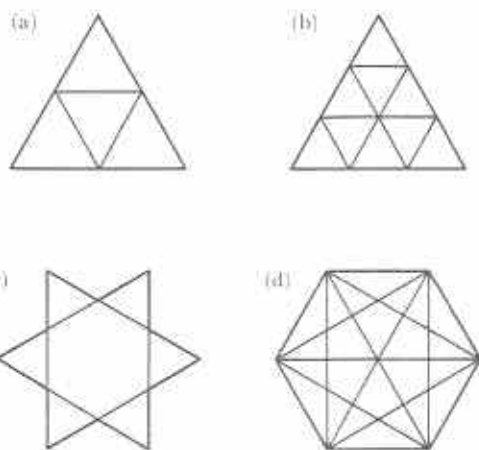


Fig. 12.8

Quadrilaterals

Quadri-lateral means four-sided. A **quadrilateral** is a plane shape with four sides and four angles. There are many different kinds of quadrilateral (Fig. 12.9).

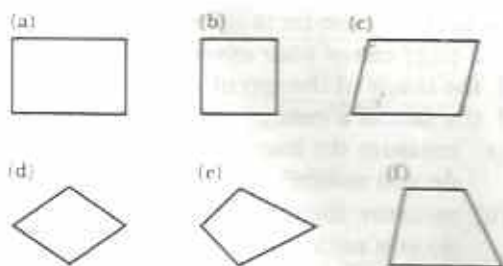


Fig. 12.9

- (a) Rectangle (b) Square (c) Parallelogram
(d) Rhombus (e) Kite (f) Trapezium

Rectangle

Place a matchbox (or other similar small box) on a sheet of paper. Draw round the shape of the face touching the paper. Repeat for two other different faces. Your drawings should look like Fig. 12.10.

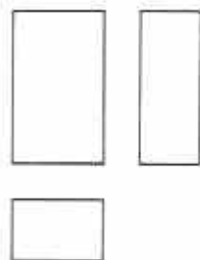


Fig. 12.10

Each shape is a **rectangle**.

A **diagonal** of a rectangle is a straight line from one corner to the opposite corner. Every rectangle has two diagonals as shown in Fig. 12.11.

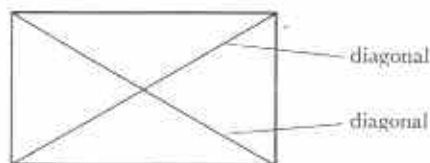


Fig. 12.11

The point where the diagonals cross is the centre of the rectangle.

Exercise 12d

- 1 Name at least ten things in your classroom which have a rectangular shape.
- 2 How many sides has a rectangle?
- 3 How many angles are in a rectangle?

- 4 In each of these rectangles:
- a page out of your exercise book,
 - the shape of the top of your desk,
 - the face of a cuboid (e.g. a chalkbox),
- measure the lengths of all the sides; what do you notice?
 - measure the sizes of all the angles; what do you notice?
- 5 What is the total of the four angles of a rectangle?
- 6 In each rectangle in question 4,
- measure the lengths of both diagonals; what do you notice?
 - measure the distance from the centre to each corner; what do you notice?
- 7 At the centre of each rectangle, where the diagonals cross, there are four angles. How many are acute? What kind of angles are the others?
- 8 One pair of sides of a rectangle point in the same direction (Fig. 12.12). We say they are **parallel** to each other. Are the other two sides parallel to each other?



Fig. 12.12

- 9 Fold a rectangular sheet of paper so that opposite sides meet. Unfold the paper as in Fig. 12.13.

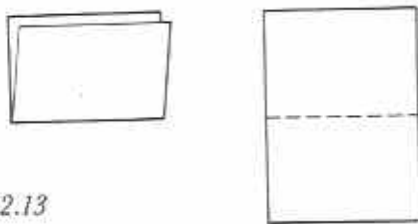


Fig. 12.13

The line of the fold divides the rectangle into two matching parts. This fold is a line of symmetry of the rectangle. Does the rectangle have another line of symmetry? If so, fold the paper another way so that both halves match. How many lines of symmetry has a rectangle?

Square

A **square** is a rectangle in which all the sides are the same length.

Exercise 12e

- 1 Make a square by folding and cutting a rectangular sheet of paper as shown in Fig. 12.14.

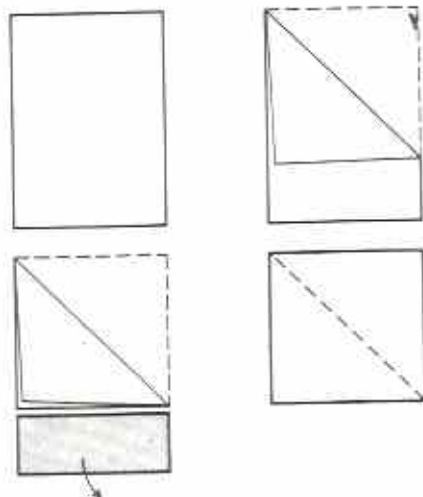


Fig. 12.14

- Name as many things as you can which have square shapes.
- Measure the lengths of the diagonals of a square. What do you notice?
- At the centre, where the diagonals meet, there are four angles. What kind of angles are they?
- Each diagonal meets the corner of a square and makes two angles (Fig. 12.15). Measure these angles. What do you notice?

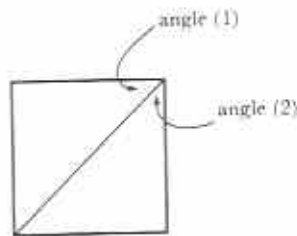


Fig. 12.15

- 6 How many lines of symmetry has a square? Check by folding.

- 7 Write down three properties of both a square and a rectangle.
- 8 Write down three differences between a square and a rectangle.

Parallelogram

A **parallelogram** is a quadrilateral which has opposite sides parallel.

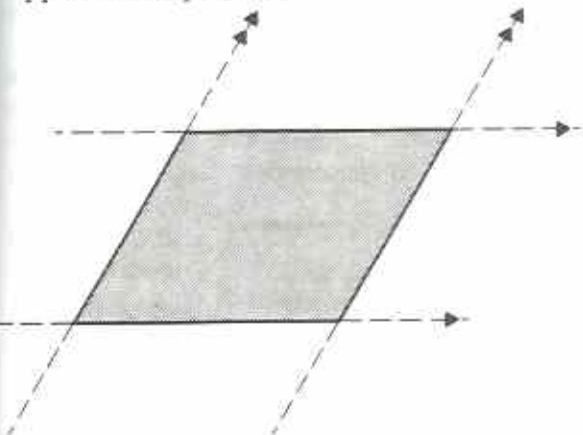


Fig. 12.16

The parallelogram in Fig. 12.16 is formed between two pairs of parallel lines.

Exercise 12f

- 1 Try to name a few things in your home, class or town which have the shape of a parallelogram.
- 2 How many sides has a parallelogram?
- 3 How many angles has a parallelogram?
- 4 (a) Draw a parallelogram with sides at least 5 cm long. The method of using a ruler and set square to draw parallel lines is given in Chapter 20 on page 168.
(b) Draw a large parallelogram on a sheet of newspaper. Cut out the parallelogram.
- 5 Use the parallelograms you made in question 4 to do the following:
 - (a) Measure the lengths of the sides of the parallelograms. What do you notice?
 - (b) Measure the angles of the parallelograms. What do you notice?
 - (c) Find the total of the four angles of each parallelogram.
 - (d) Draw the diagonals of the parallelograms.

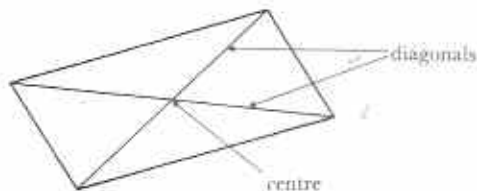


Fig. 12.17

- (e) Measure the lengths of both diagonals of each parallelogram. What do you notice?
- (f) In each parallelogram, measure the distance from the centre to each corner. What do you notice?
- (g) At the centre, where the diagonals cross, there are four angles. How many are acute? What kind of angles are the others?
- (h) Try to fold the newspaper parallelogram so that opposite sides meet. Do they meet completely? Do the folded parts of the parallelogram cover each other completely?
- (i) Try to fold the newspaper parallelogram so that opposite angles meet. Do this for both pairs of opposite angles. Do the folded parts cover each other completely?
- (j) Do your parallelograms have any lines of symmetry?

Rhombus

A rhombus is a quadrilateral which has all four sides equal.



Fig. 12.18

Fig. 12.18 shows a rhombus; it is sometimes said to be diamond shaped.

Exercise 12g

- 1 Try to name a few things in your home, class or town which have the shape of a rhombus.

- How many sides has a rhombus?
- How many angles has a rhombus?
- Use the following method to make a rhombus from a sheet of paper.

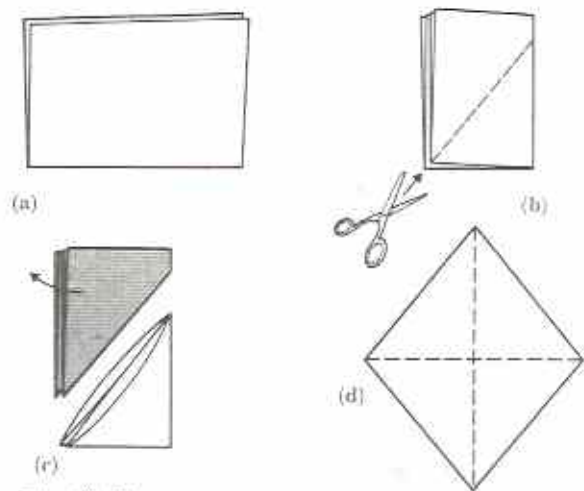


Fig. 12.19

- Fold a rectangular sheet of paper so that opposite sides meet.
 - Fold the paper again. Draw a line as shown in Fig. 12.19(b).
 - Cut along the line through all four thicknesses of paper.
 - Unfold the triangular part. This gives a rhombus.
- Use the rhombus you made in question 4 and the rhombus in Fig. 12.18 to do the following.
 - Measure the lengths of the sides of the rhombuses. What do you notice?
 - Measure the angles of the rhombuses. What do you notice?
 - Find the total of the four angles of each rhombus.
 - Draw the diagonals of each rhombus.

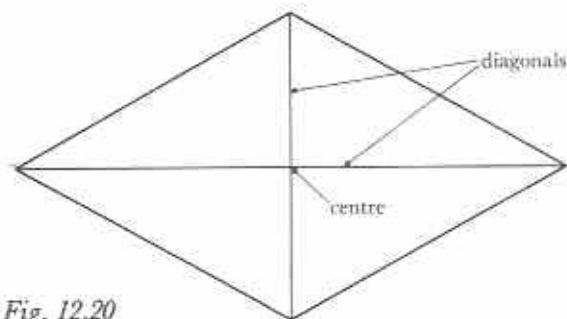


Fig. 12.20

- Measure the lengths of both diagonals of each rhombus. What do you notice?
- In each rhombus, measure the distance from the centre to each corner. What do you notice?
- At the centre, where the diagonals cross, there are four angles. What kind of angles are they?
- How many lines of symmetry has a rhombus? Check by folding your paper rhombus.
- Fig. 12.21 shows angles a and b between the short diagonal and the sides. In each rhombus, measure the angles a and b . What do you notice?



Fig. 12.21

- Fig. 12.22 shows angles x and y between the long diagonal and the sides. In each rhombus, measure the angles x and y . What do you notice?

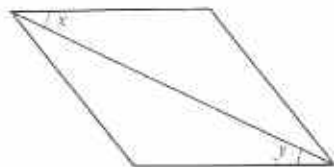


Fig. 12.22

- Write down three properties shown by both a rhombus and a parallelogram.
- Write down three differences between a rhombus and a parallelogram.
- What is the difference between a square and a rhombus? Is a square a rhombus?

Kite

A **kite** is a quadrilateral in which one diagonal is a line of symmetry. Fig. 12.23 shows some kites and their lines of symmetry.

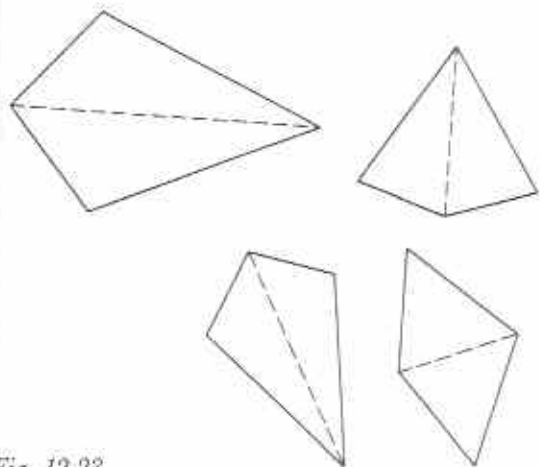


Fig. 12.23

Exercise 12h

- 1 Try to name a few things in your home, class or town which have the shape of a kite.
- 2 How many sides and angles has a kite?
- 3 Use the following method to make a kite from a piece of paper.

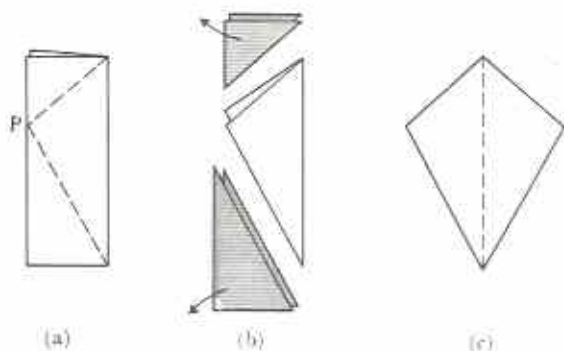


Fig. 12.24

- (a) Fold a rectangular sheet of paper so that opposite sides meet. Mark a point, P, on the long edge such that it is not at the mid-point of the long edge. Draw lines from the point to each end of the fold.
 - (b) Cut off the four corners along the lines you have drawn.
 - (c) Unfold the remaining part. This gives a kite shape.
- 4 Use the kite you made in question 3 and those in Fig. 12.23.
 - (a) Measure the lengths of the sides of the kites. What do you notice?

- (b) Measure the angles of the kites. What do you notice?
 - (c) Find the total of the four angles of each kite.
 - (d) Measure the lengths of both diagonals of each kite. What do you notice?
 - (e) At the centre, where the diagonals cross, there are four angles. What kind of angles are they?
 - (f) How many lines of symmetry has a kite?
- 5 Write down two properties which a kite and a rhombus both have.
 - 6 Write down three things that are different between a kite and a rhombus. Is a rhombus a kite?

Trapezium

A **trapezium** is a quadrilateral which has one pair of parallel sides. Fig. 12.25 shows some trapeziums.

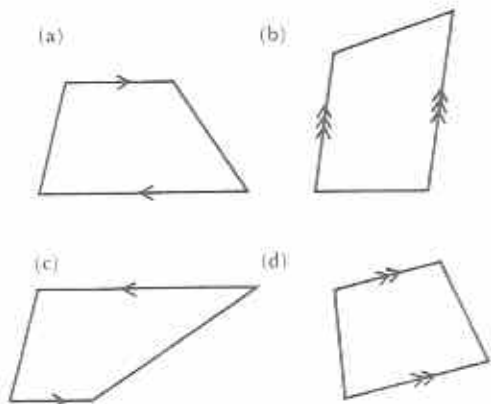


Fig. 12.25

Exercise 12i

- 1 Name four different trapeziums in Fig. 12.26.

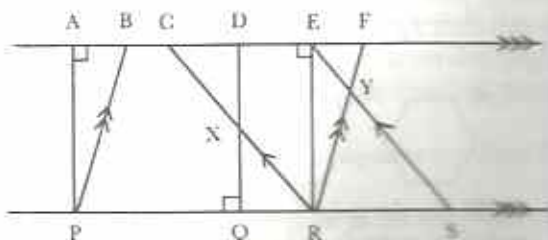


Fig. 12.26

2 In Fig. 12.26, what kind of quadrilaterals are the following?

- (a) ADQP (b) DFRQ (c) CESR
 (d) DERQ (e) AFRP (f) BFRP
 (g) CEYR (h) DERX

3 Do any of the trapeziums in Fig. 12.25 have a line of symmetry?

4 Fig. 12.27 shows two trapeziums. Each has a line of symmetry.

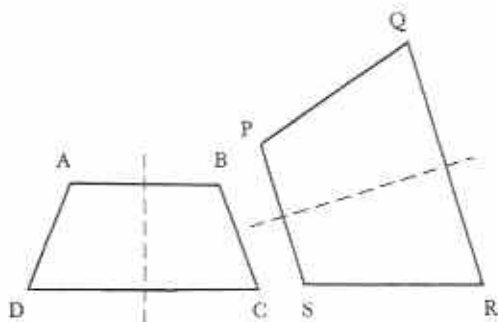


Fig. 12.27

In each trapezium,

- (a) name a pair of equal sides,
 (b) name two pairs of equal angles.

Polygons

A **polygon** is any closed shape which has three or more straight sides. Hence triangles and quadrilaterals are examples of polygons.

A **regular polygon** is one in which the sides are equal in length and the angles are all equal to each other. For example the equilateral triangle and square are regular polygons. Fig. 12.28 shows the first six regular polygons.

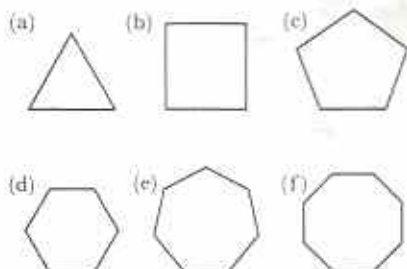


Fig. 12.28 Regular polygons:

- (a) equilateral triangle (b) square (c) pentagon
 (d) hexagon (e) heptagon (f) octagon

Circles

As the number of sides of a regular polygon increases, so their shape approaches that of a **circle**.

The circle is a very common and important shape. It is the shape of the floor of a round house. Most wheels are circular.

It is nearly impossible to draw a good circle without help. Fig. 12.29 shows some ways of drawing a circle.

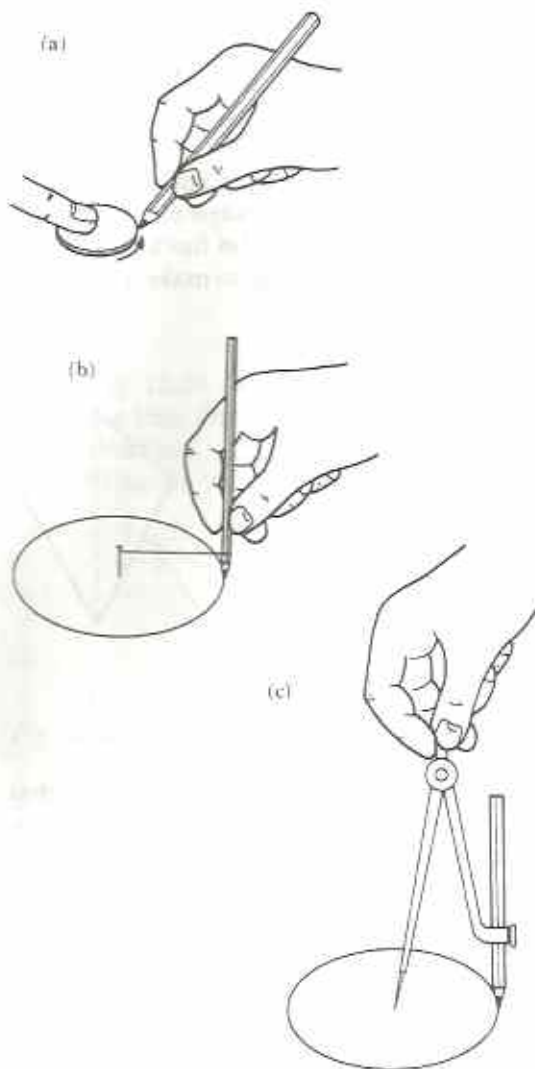


Fig. 12.29 Drawing a circle:

- (a) using a circular object such as a coin
 (b) using string (c) using compasses

The parts of a circle

The **centre** is the point at the middle of a circle.

Lines

In Fig. 12.30, the **circumference** is the curved outer boundary of the circle. An **arc** is a curved part of the circumference. A **radius** is any straight line joining the centre to the circumference. The plural of radius is **radii**. A **chord** is any straight line joining two points on the circumference. A **diameter** is any chord which goes through the centre of the circle.

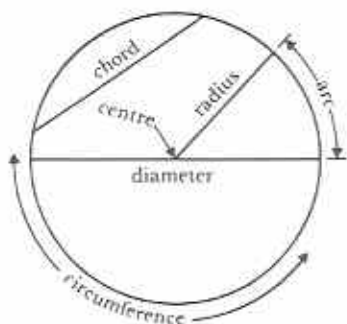


Fig. 12.30

Regions

In Fig. 12.31, a **sector** is the region between two radii and the circumference. A **semi-circle** is a sector between a diameter and the circumference, i.e. half a circle. A **segment** is the region between a chord and the circumference.

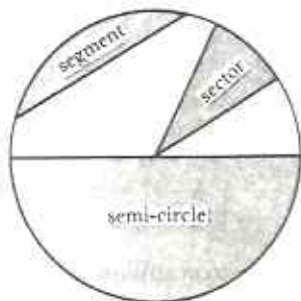


Fig. 12.31

Exercise 12j

Questions 3, 4 and 5 give practice in using compasses.

1 Draw a circle and include the following parts

on it: two radii, a sector, a chord, a segment, a diameter, an arc. Label each part and shade any regions.

- 2 Draw a circle on a piece of paper. Cut out the circle carefully. Check, by folding, to see whether a circle has any lines of symmetry. How many lines of symmetry does a circle have? (Be careful with your answer.)
- 3 Draw three circles, with radii 3 cm, 4 cm and 5 cm, so that each circle has the same centre.
- 4 Draw the pattern shown in Fig. 12.32.

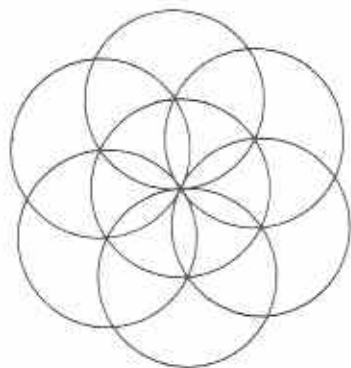


Fig. 12.32

Draw all the circles with a radius of 3,5 cm.

- 5 Draw a circle of radius 3 cm. Mark about 12 points on its circumference. Name the top point P. (See Fig. 12.33.)

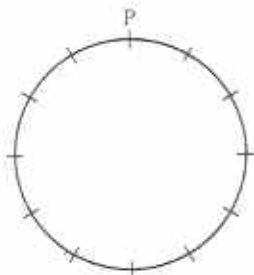


Fig. 12.33

Draw a circle with one of the other points as centre so that the circumference goes through P. Repeat this for all 12 points on the original circle. What shape does your finished pattern look like?

- 6 (a) How many lines of symmetry does Fig. 12.32 have?
(b) How many lines of symmetry does the shape you made in question 5 have?

Equations (1)

Syllabus Reference 5.6.4

Equations

The expression $3x = 18$ is an algebraic sentence. It means: three times an unknown number x is equal to eighteen. A sentence which has an equals sign in this way is called an **equation**. $3x = 18$ is an equation in x . Other examples of equations are: $x + 2 = 5$, $x = 7$, $4 - y = 0$, $3a - 3 = 0$.

The letter in an equation is sometimes called **the unknown**. A sentence like $3x = 18$ may be true or false. It depends on the value of the unknown. For example, $3x = 18$ is false when $x = 2$; it is true when $x = 6$.

Example 1

Is the equation $\frac{x}{4} = 5$ true when $x = 24$?

When $x = 24$, $\frac{x}{4} = \frac{24}{4} = 6$

$6 \neq 5$, thus $\frac{x}{4} = 5$ is *not true* when $x = 24$.

Exercise 13a (Oral)

Say whether the following are true or false.

- 1 $x + 3 = 7$ when $x = 4$
- 2 $5x = 15$ when $x = 3$
- 3 $x - 2 = 9$ when $x = 10$
- 4 $11 + x = 16$ when $x = 7$
- 5 $\frac{12}{x} = 4$ when $x = 3$
- 6 $2x = 22$ when $x = 2$
- 7 $23 - x = 20$ when $x = 3$
- 8 $\frac{x}{8} = 2$ when $x = 16$

- 9 $x - 5 = 7$ when $x = 2$
- 10 $6x = 36$ when $x = 6$
- 11 $4 = 3 + x$ when $x = 1$
- 12 $25 = 2\frac{1}{2}x$ when $x = 10$
- 13 $\frac{24}{x} = 4$ when $x = 4$
- 14 $9 = 14 - x$ when $x = 5$
- 15 $6 = \frac{x}{3}$ when $x = 12$
- 16 $15 = x + 2$ when $x = 17$
- 17 $19 = 19 + x$ when $x = 0$
- 18 $1 = \frac{x}{2}$ when $x = 1$
- 19 $10x = 10$ when $x = 1$
- 20 $12 = 12 - x$ when $x = 12$

Solution of an equation

We can usually find the value of the unknown which makes an equation true. We call this value **the solution** of the equation. $x = 6$ is the solution of $3x = 18$. **To solve** an equation means to find the value of the unknown which makes the equation true.

Example 2

Solve the equation $18 - x = 7$.

The problem is to find a number which when taken from 18 gives 7. The number is 11. $x = 11$ is the solution.

Example 3

Find the solution of $\frac{x}{6} = 5$.

The problem is to find a number which divides by 6 to give 5.

The number is 30.

$x = 30$ is the solution.

Exercise 13b (Oral)

Solve the following equations.

- | | |
|-----------------------|-----------------------|
| 1 $x + 8 = 12$ | 2 $x + 3 = 8$ |
| 3 $20 + x = 28$ | 4 $14 + x = 20$ |
| 5 $14 - x = 11$ | 6 $16 - x = 13$ |
| 7 $x - 2 = 15$ | 8 $x - 3 = 8$ |
| 9 $4x = 20$ | 10 $2x = 50$ |
| 11 $12 = 3x$ | 12 $72 = 9x$ |
| 13 $\frac{x}{2} = 5$ | 14 $4 = \frac{x}{9}$ |
| 15 $\frac{28}{x} = 7$ | 16 $3 = \frac{24}{x}$ |
| 17 $14 = x + 8$ | 18 $8 = 9 - x$ |
| 19 $22 = 11 + x$ | 20 $12 = x - 12$ |
| 21 $7x = 7$ | 22 $\frac{x}{6} = 1$ |
| 23 $x + 5 = 5$ | 24 $x - 3 = 0$ |

The balance method of solving equations

Consider the equation $3x = 18$. The $3x$ on the left-hand side (LHS) equals, or **balances**, the 18 on the right-hand side (RHS). We can show the $3x$ and the 18 balancing on a pair of scales as in Fig. 13.1.

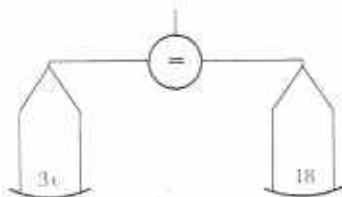


Fig. 13.1

Just as with real scales, the two sides will balance if we add equal amounts to both sides or if we subtract the same from both sides (Fig. 13.2).

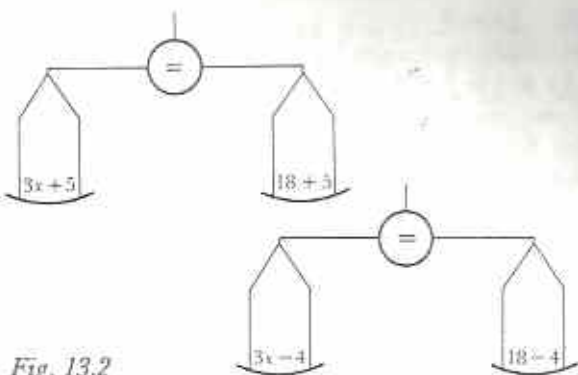


Fig. 13.2

The scales will also balance if we multiply or divide by the same amount on both sides (Fig. 13.3).

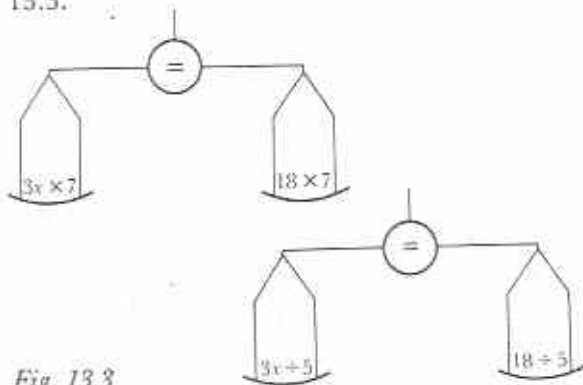


Fig. 13.3

The two sides will stay balanced if we **do the same to both sides**.

To solve $3x = 18$ by the balance method, first find which side the unknown, x , is on. $3x$ is on the LHS. If $3x$ is divided by 3 the result will be x . If we divide the LHS by 3, we must also divide the RHS by 3 to keep the balance.

$$3x = 18$$

Divide both sides by 3

$$\frac{3x}{3} = \frac{18}{3}$$

$$x = 6$$

Example 4

Solve the following equations using the balance method:

(a) $x + 11 = 18$, (b) $\frac{1}{2}y = 7$, (c) $8 = x - 5$.

(a) $x + 11 = 18$

Subtract 11 from both sides

$$x + 11 - 11 = 18 - 11$$

$$\Leftrightarrow x = 7$$

(b) $y = 7$

Multiply both sides by 4

$$y \times 4 = 7 \times 4$$

$$\Leftrightarrow y = 28$$

(c) $8 = x - 5$

Add 5 to both sides

$$8 + 5 = x - 5 + 5$$

$$\Leftrightarrow 13 = x$$

$$\Leftrightarrow x = 13$$

Notice the use of the \Leftrightarrow symbol in Example 4. \Leftrightarrow means 'is equivalent to' and is used to show that two statements are equivalent to each other. For example, $13 = x \Leftrightarrow x = 13$.

Exercise 13c

Use the balance method to solve the following equations. Write down the steps and working as in the above examples.

1 $3x = 21$

2 $5x = 20$

3 $32 = 8x$

4 $27 = 9x$

5 $x + 3 = 8$

6 $x + 5 = 11$

7 $8 + x = 18$

8 $17 + x = 23$

9 $\frac{1}{2}x = 5$

10 $\frac{1}{4}x = 9$

11 $2 = \frac{1}{3}x$

12 $6 = \frac{1}{5}x$

13 $x - 1 = 20$

14 $x - 3 = 1$

15 $13 = x - 7$

16 $0 = x - 6$

17 $9 = x + 2$

18 $20 = x + 14$

19 $15 = 13 + x$

20 $7 = 1 + x$

21 $6x = 6$

22 $\frac{1}{4}x = 2\frac{1}{2}$

23 $x - 10 = 10$

24 $x - 16 = 1$

It is possible to solve the equations in the last exercise from knowledge of numbers. However, the balance method is very useful with more difficult equations. For example, the equation $4x + 5 = 17$ is more difficult to solve directly. Figs 13.4–13.8 show how this equation is solved by the balance method.

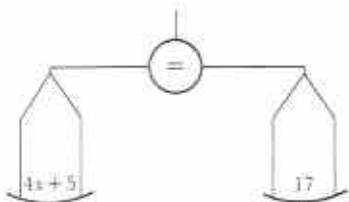


Fig. 13.4

(a) The LHS of $4x + 5 = 17$ contains the unknown. Subtract 5 to leave $4x$ on the LHS. Since 5 is taken from the LHS, 5 must also be taken from the RHS to keep the balance. Subtract 5 from both sides:

$$4x + 5 - 5 = 17 - 5$$

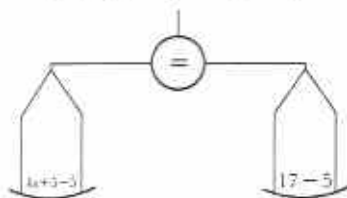


Fig. 13.5

Simplifying:

$$4x = 12$$

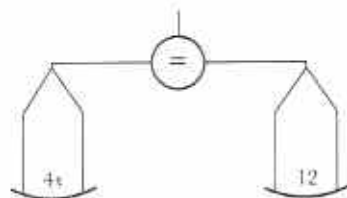


Fig. 13.6

(b) The equation is now easier. Divide the LHS by 4 to leave x . The RHS must also be divided by 4 to keep the balance.

Divide both sides by 4:

$$\frac{4x}{4} = \frac{12}{4}$$

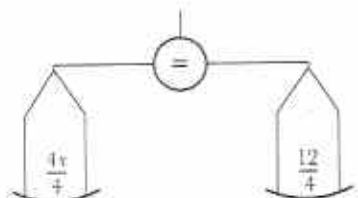


Fig. 13.7

Simplifying:

$$x = 3$$

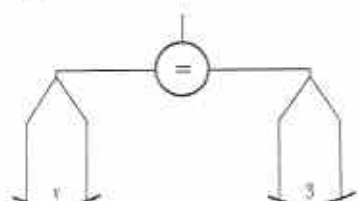


Fig. 13.8

$x = 3$ is the solution of $4x + 5 = 17$

Check: When $x = 3$,

$$\text{LHS} = 4 \times 3 + 5 = 12 + 5 = 17 = \text{RHS}$$

The following examples are also solved by the balance method.

Example 5

Solve $5x - 6 = 29$.

$$5x - 6 = 29$$

Add 6 to both sides

$$5x - 6 + 6 = 29 + 6$$

$$\Leftrightarrow 5x = 35$$

Divide both sides by 5

$$5x \div 5 = 35 \div 5$$

$$\Leftrightarrow x = 7$$

Check: When $x = 7$,

$$\text{LHS} = 5 \times 7 - 6 = 35 - 6 = 29 = \text{RHS}$$

Example 6

Solve $21 = 9 + 2y$.

$$21 = 9 + 2y$$

The unknown is on the RHS

Subtract 9 from both sides

$$21 - 9 = 9 + 2y - 9 = 2y + 9 - 9$$

$$\Leftrightarrow 12 = 2y$$

Divide both sides by 2

$$\frac{12}{2} = \frac{2y}{2}$$

$$\Leftrightarrow 6 = y$$

Check: When $y = 6$,

$$\text{RHS} = 9 + 2 \times 6 = 9 + 12 = 21 = \text{LHS}$$

Exercise 13d

Solve the following equations by the balance method. Write down every step and show all working. Check each solution as in the above examples.

1 $5y + 6 = 21$

3 $3x + 2 = 14$

5 $2n - 3 = 5$

7 $5t - 2 = 18$

9 $6 + 2a = 18$

11 $4 + 3d = 25$

13 $5 = 7b - 9$

2 $4a + 3 = 15$

4 $6p + 2 = 20$

6 $3m - 4 = 26$

8 $8x - 9 = 7$

10 $8 + 5y = 23$

12 $1 + 7q = 22$

14 $16 = 2a - 4$

15 $3 = 4a - 1$

17 $16 = 2a + 4$

19 $19 = 10 + 3x$

21 $5x - 5 = 5$

23 $8x - 24 = 0$

16 $9 = 5x - 1$

18 $17 = 5x + 2$

20 $24 = 10 + 7x$

22 $6x - 11 = 19$

24 $3x + 7 = 7$

The value of the unknown can be fractional.

Example 7

Solve the equation $2x + 7 = 12$.

$$2x + 7 = 12$$

Subtract 7 from both sides

$$2x + 7 - 7 = 12 - 7$$

$$\Leftrightarrow 2x = 5$$

Divide both sides by 2

$$\frac{2x}{2} = \frac{5}{2}$$

$$\Leftrightarrow x = 2\frac{1}{2}$$

Check: When $x = 2\frac{1}{2}$,

$$\text{LHS} = 2 \times 2\frac{1}{2} + 7 = 5 + 7 = 12 = \text{RHS}$$

Exercise 13e

Solve the following equations.

1 $3x + 4 = 17$

3 $5x + 8 = 11$

5 $3 + 2x = 10$

7 $19 = 6 + 9x$

9 $10x - 3 = 5$

11 $9 = 6x - 5$

2 $6x - 5 = 6$

4 $4x - 1 = 2$

6 $2 = 7x - 4$

8 $8x + 3 = 4$

10 $12x + 3 = 21$

12 $6 + 14x = 41$

It is not necessary to write down every step.

Example 8

Solve the equation $3a - 7 = 8$.

$$3a - 7 = 8$$

Add 7 to both sides

$$3a = 15$$

Divide both sides by 3

$$a = 5$$

Check: When $a = 5$,

$$\text{LHS} = 3 \times 5 - 7 = 15 - 7 = 8 = \text{RHS}$$

You can shorten the work if you feel confident enough. However, do not cut out too many steps and always remember to do the same to both sides.

Exercise 13f

Solve the following equations. Write down as many steps as you need. Check each solution.

1 $4n + 3 = 19$

2 $3y + 8 = 41$

3 $9c + 11 = 65$

4 $5b - 12 = 3$

5 $2x - 19 = 5$

6 $7s - 10 = 18$

7 $1 + 2u = 23$

8 $5 + 6m = 29$

9 $3 + 8p = 51$

10 $7 = 3a + 4$

11 $15 = 7t + 1$

12 $29 = 5^f + 4$

13 $47 = 10h - 33$

15 $0 = 8d - 56$

17 $19 = 7 + 4q$

19 $2t + 3 = 14$

21 $6z + 9 = 23$

23 $9y - 4 = 0$

25 $9 = 3h + 7$

27 $5 = 4m + 4$

29 $8 = 11y - 8$

14 $1 = 3y - 14$

16 $32 = 6 + 13k$

18 $21 = 5 + 2n$

20 $5a + 14 = 22$

22 $4x - 5 = 5$

24 $3b - 15 = 16$

26 $11 = 10k + 4$

28 $0 = 6z - 11$

30 $19 = 16y - 21$

Perimeter of plane shapes

Syllabus Reference 5.4.2

The **perimeter** of a shape is the outside boundary or edges of the shape. We often use the word perimeter to mean the length of the boundary of a shape.

Measuring perimeters

The simplest way of finding a perimeter is to measure it with a ruler or tape measure.

Example 1

Measure the perimeter of the quadrilateral ABCD in Fig. 14.1.

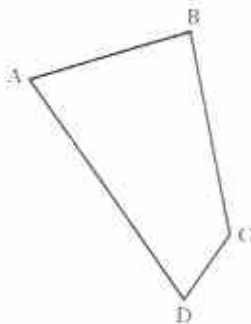


Fig. 14.1

By measurement, $AB = 22$ mm
 $BC = 27$ mm
 $CD = 10$ mm
 $DA = 35$ mm

Perimeter (total) = 94 mm

Example 2

Find, in cm, the perimeter of the regular hexagon ABCDEF in Fig. 14.2.

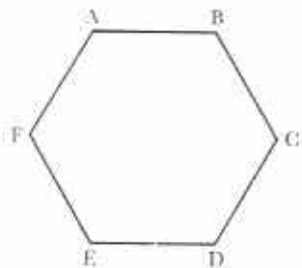


Fig. 14.2

Length of side $AB = 1,6$ cm.
 There are 6 equal sides, so
 perimeter = $6 \times 1,6$ cm
 = 9,6 cm

If a shape has a curved side, use a piece of thread to get the shape of the curve. Make the thread straight and measure its length against a ruler.

Example 3

Measure the perimeter of the shape in Fig. 14.3.

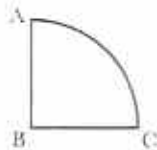


Fig. 14.3

Straight edges: $AB = 14$ mm
 $BC = 14$ mm
 Curved edge: $CA = 22$ mm approximately
 Perimeter (total) = 50 mm approximately

Exercise 14a

1 Use a ruler to measure the perimeters of the shapes in Fig. 14.4. Give your answers in mm.

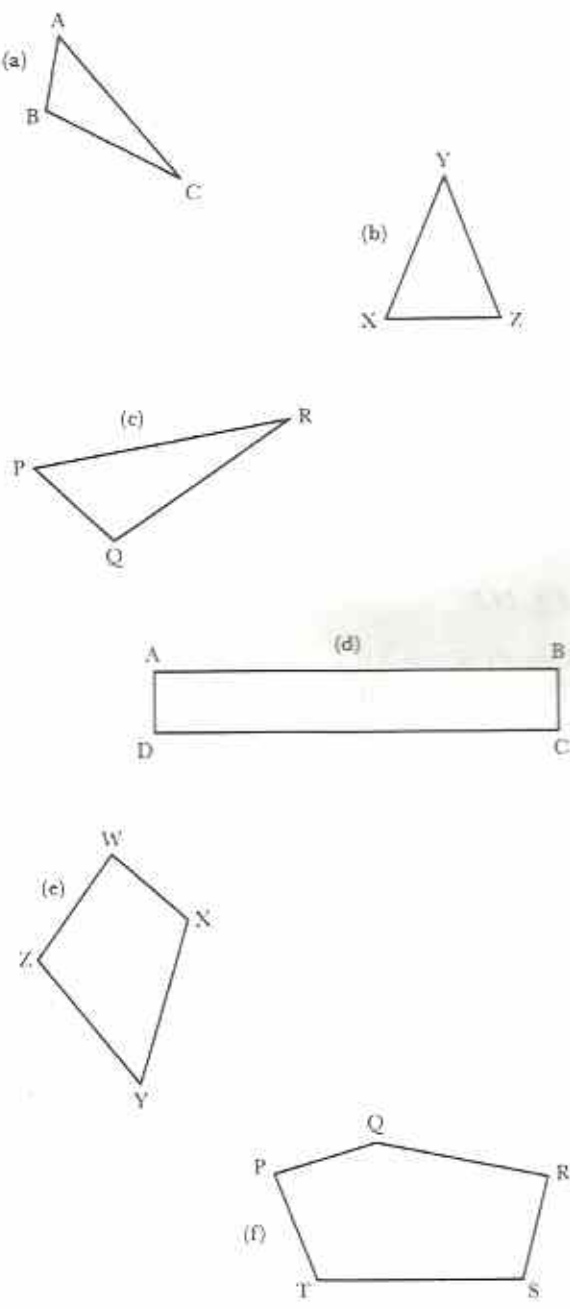


Fig. 14.4

2 Each of the shapes in Fig. 14.5 is regular. Find the perimeter of each shape by measuring one side and multiplying by the number of sides. Give your answers in cm.

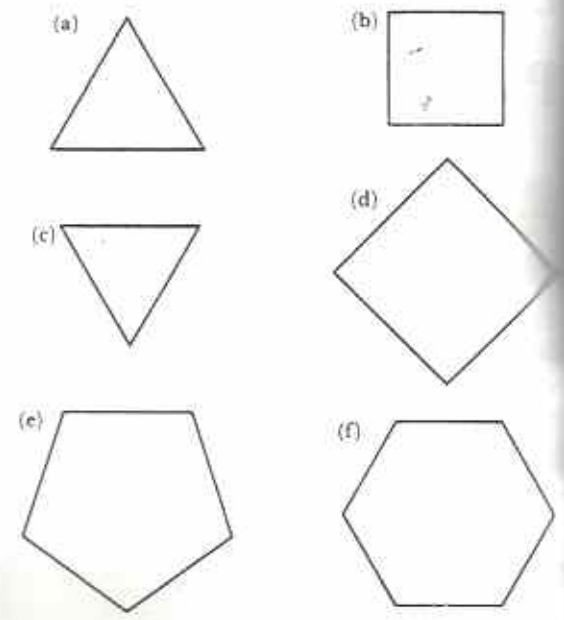


Fig. 14.5

3 Each of the shapes in Fig. 14.6 contains at least one curved edge. Use thread to find the approximate perimeter of each shape by indirect measurement. Give your answers in mm.

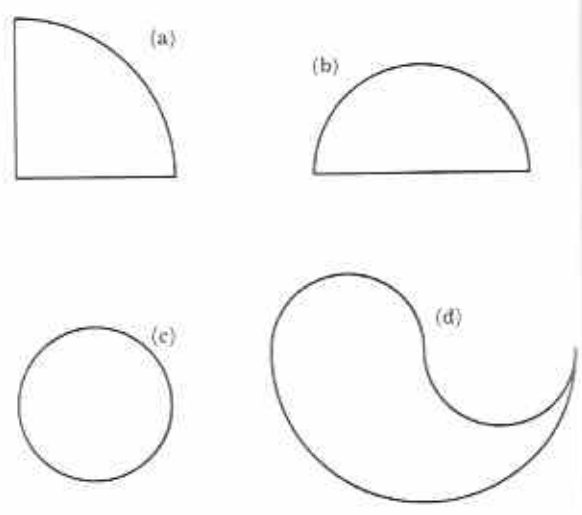


Fig. 14.6

Perimeter of rectangles

We usually call the longer side of a rectangle the **length**, and the shorter side the **breadth**. The letters l and b are used to stand for the length and the breadth in Fig. 14.7. From the diagram:

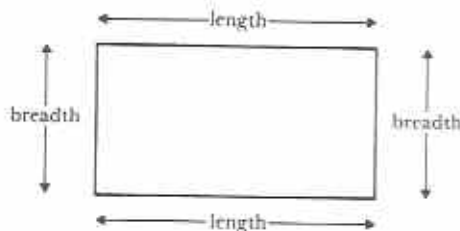


Fig. 14.7

$$\begin{aligned} \text{perimeter of rectangle} &= l + b + l + b \\ &= (l + b) + (l + b) \\ &= 2 \times (l + b) \\ &= 2(l + b) \end{aligned}$$

This **formula** can be used to calculate the perimeter of a rectangle.

Example 4

Calculate the perimeter of a football field which measures 80 m by 50 m.

$$\begin{aligned} \text{perimeter of field} &= 2(l + b) \\ &= 2 \times (80 + 50) \text{ m} \\ &= 2 \times 130 \text{ m} \\ &= 260 \text{ m} \end{aligned}$$

Perimeter of squares

A square is a regular 4-sided shape. If the length of one side of a square is l , then,
perimeter of square = $l \times 4$

$$= 4l$$

The formula for perimeters of rectangles and squares can be useful. However, if you find it difficult to remember formulae, always sketch the given shape and work from that.

Note: formulae is the plural of formula.

Exercise 14b

1 Copy and complete the table of rectangles (Table 14.1).

Table 14.1

	length	breadth	perimeter
(a)	3 cm	2 cm	
(b)	5 cm	4 m	
(c)	16 mm	10 mm	
(d)	$2\frac{1}{2}$ km	1 km	
(e)	6 m	$2\frac{1}{2}$ m	
(f)	$8\frac{1}{4}$ cm	$4\frac{1}{2}$ cm	
(g)	5,1 cm	3,2 cm	
(h)	4,3 cm	0,8 cm	
(i)	7,35 m	7,15 m	
(j)	9 km	0,9 km	

2 Copy and complete the table of squares (Table 14.2).

Table 14.2

	length of side	perimeter
(a)	6 km	
(b)	14 cm	
(c)	$2\frac{1}{4}$ m	
(d)	8,3 cm	
(e)		40 cm
(f)		360 m
(g)		18 mm
(h)		14,4 cm

3 A rectangular field measures 400 m by 550 m. What is the perimeter of the field (a) in metres, (b) in kilometres?

4 A school compound is made up of a rectangle and a square as in Fig. 14.8. Find the perimeter of the compound.

5 A woman fences a 3 m by 4 m rectangular plot to keep her chickens in. The fencing costs \$1,20 per metre. How much does it cost to fence the plot?

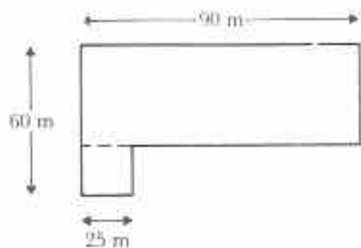


Fig. 14.8

- 6 A length of wire is bent to make a square of side 3.2 cm. What is the length of wire in the square?

If the wire is silver costing 15c per centimetre, what is the cost of the square? 20 of these squares are linked together to make a necklace. What is the value of the necklace?

- 7 A rectangle has a perimeter of 60m. Find the length of the rectangle if its breadth is (a) 10 m, (b) 5 m, (c) 12m.
- 8 A man has 36 square tiles. Each tile measures 1 m by 1 m. He lays the tiles in the shape of a rectangle as in Fig. 14.9.



Fig. 14.9

Find the length, breadth and perimeter of this rectangle. Use squared paper to show different rectangles the man can make using all 36 tiles. Find the perimeters of these rectangles. What do you notice?

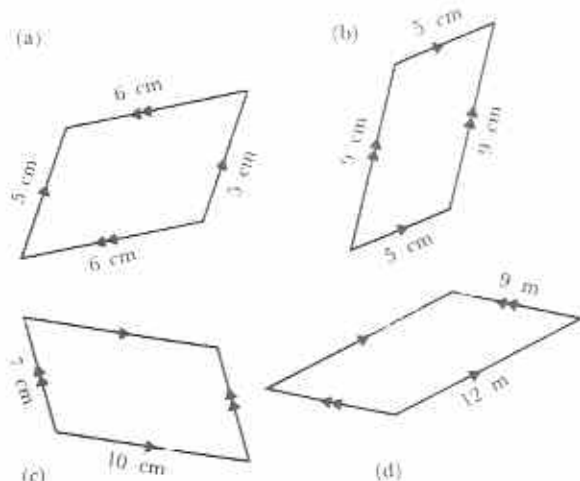


Fig. 14.10

- 9 Find the perimeter of the parallelogram in Fig. 14.10.
- 10 Two sides of a parallelogram are of lengths 5 cm and 8 cm. Calculate the perimeter of the parallelogram.
- 11 The perimeter of a parallelogram is 38 cm and one of its sides is 14 cm long. Find the lengths of the other three sides.
- 12 In Fig. 14.11, a trapezium is made from three equilateral triangles. Find the perimeter of the trapezium if each side of the triangle is 6 cm long.

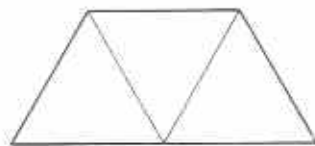


Fig. 14.11

Circumference of circles

The perimeter of a circle is called the **circumference**.

Measuring circumference

Here are two ways of finding the circumference of a cylindrical tin can.

Rolling

- (a) Make a mark on the circumference of the circular end-face (Fig. 14.12). Make a mark A on a long sheet of paper (e.g. newspaper). Start with the two marks opposite each other.
- (b) Roll the tin along the paper.

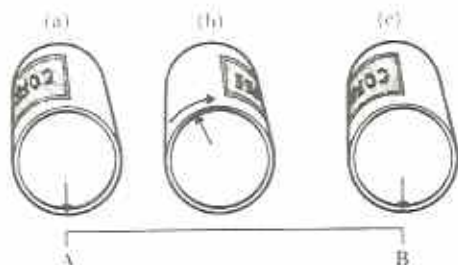


Fig. 14.12

(c) Stop when the mark is against the paper again. Make a mark B on the paper opposite the mark on the tin.

The length AB will be the circumference of the circular end-face.

Winding thread

(a) Wind a piece of thread (or string) once around the tin (Fig. 14.13). Mark the thread at A and B where it crosses.

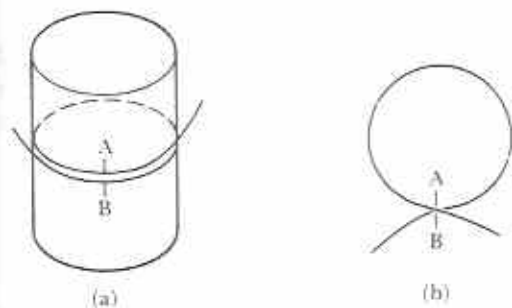


Fig. 14.13

(b) Remove the string from the tin. Pull the string straight and measure AB against a ruler.

The length AB will be the circumference of the circle.

Exercise 14c

Find three cylindrical objects, e.g., a cup, a tin can and a torch battery.

1 Use both of the above methods, rolling and winding thread, to find the circumference of the circular faces of the three objects.

2 Measure the diameters of the three objects. Compare the diameter of each object with its circumference.

- Is the circumference greater than the diameter in each case?
- If so, approximately how many times greater?
- Is this true for all three objects?
- Compare your results with your friends'. Do other people get results like yours?

Your results in Exercise 14c may have been as shown in Table 14.3.

Table 14.3

	circumference	diameter	circumference/diameter
battery	80 mm	25 mm	$\frac{80}{25} = 3\frac{1}{5}$
tin can	31,5 cm	10 cm	$\frac{31,5}{10} = 3,15$

In each case, the circumference is just over 3 times the diameter.

For any circle it will be found that, approximately,

$$\text{circumference} = 3,1 \times \text{diameter}$$

The number 3,1 is not exact. More accurate values are 3,142 or $3\frac{1}{7}$ but even these are not exact. It is impossible to express the number as an exact fraction or decimal. We use the Greek letter π , pi, to represent this number.

$$\text{circumference} = \pi \times \text{diameter}$$

or

$$c = \pi d$$

where c is the length of the circumference and d is the diameter of the circle.

The diameter, d , is twice the radius, r , thus:

$$c = 2\pi r$$

This formula is used to calculate the value of the circumference of a circle of radius r . In any question where this formula is used, the value of π will be given, usually 3,14 or $3\frac{1}{7}$ (i.e. $\frac{22}{7}$).

Example 5

Calculate the circumference of a circle of radius $3\frac{1}{2}$ metres. Use the value $3\frac{1}{7}$ for π .

$$\begin{aligned} \text{Circumference} &= 2\pi r \\ &= 2 \times 3\frac{1}{2} \times 3\frac{1}{7} \text{ m} \\ &= 2 \times \frac{22}{7} \times \frac{7}{2} \text{ m} \\ &= 22 \text{ m} \end{aligned}$$

Example 6

A bicycle wheel has a diameter of 65 cm. During a journey, the wheel makes 1000 complete revolutions.



Fig. 14.14

How many metres does the bicycle travel? (Use the value 3,14 for π .)

$$\begin{aligned} \text{Circumference} &= \pi d \\ \text{Distance travelled in} \\ \text{one revolution} &= 3,14 \times 65 \text{ cm} \\ \text{Distance travelled in} \\ \text{1 000 revolutions} &= 3,14 \times 65 \times 1\,000 \text{ cm} \\ &= \frac{3,14 \times 65 \times 1\,000}{100} \text{ m} \\ &= 31,4 \times 65 \text{ m} \\ &= 2\,041 \text{ m} \end{aligned}$$

Exercise 14d

1 Copy and complete Table 14.4 for circles. Use the value $\frac{22}{7}$ for π .

Table 14.4

	radius	diameter	circumference
(a)	7 m		
(b)		7 cm	
(c)	14 mm		
(d)		42 m	
(e)	350 mm		
(f)		5,6 cm	

- 2 Calculate the perimeter of a circle of radius 70 m. (Use the value $3\frac{1}{7}$ for π .)
- 3 A circular wire ring has a diameter of 20 cm. What is the length of the wire? (Use the value 3,14 for π .)
- 4 The diameter of the circular base of a round hut is 14 m. What is the circumference of the base of the hut? Use the value $\frac{22}{7}$ for π .
- 5 The minute hand of a clock is 10,5 cm long. How far does the tip of the hand travel in 1 hour? (Use the value $3\frac{1}{7}$ for π .)
- 6 A record has a diameter of 30 cm and rotates at $33\frac{1}{3}$ revolutions per minute. How far does a point on the edge of the record travel in a minute? Use the 3,14 for π and give your answer in metres.
- 7 Fig. 14.15 shows a wire circle set inside a wire square of side 7 cm.

- (a) Calculate the perimeter of the square.
- (b) Calculate the circumference of the circle. (Use $\frac{22}{7}$ for π .)
- (c) Calculate the total length of wire used.

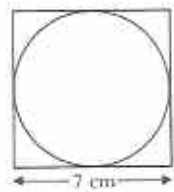


Fig. 14.15

8 A rope is wound 50 times round a cylinder of radius 25 cm (Fig. 14.16). How long is the rope? Use the value 3,14 for π .

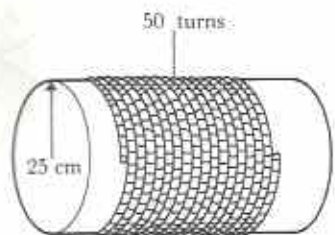


Fig. 14.16

- 9 A bicycle wheel has a diameter of 63 cm. How many metres does the bicycle travel for 100 revolutions of the wheel? (Use the value $\frac{22}{7}$ for π .)
- 10 An arch of a bridge is made by bending a steel beam into the shape of a semi-circle. See Fig. 14.17.

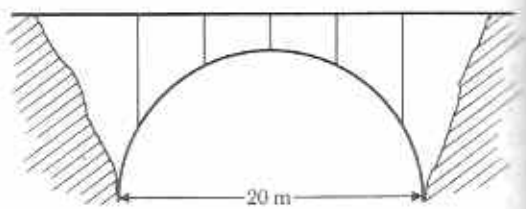


Fig. 14.17

If the span (diameter) of the arch is 20 m, use the value 3,14 for π to find the length of the steel beam to the nearest metre.

11 Fig. 14.18 shows the plan of a corner of a road.

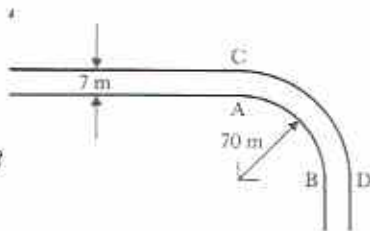


Fig. 14.18

The road is 7 m wide. The corner is in the shape of a quarter of a circle. The inside radius of the corner is 70 m. Chido walks round the corner on the inside, from A to B. Thabo walks round the corner on the outside, from C to D. How much further does Thabo walk than Chido? Use the value $\frac{22}{7}$ for π .

- 12 A cylindrical coil of copper wire has 700 turns each of diameter 4 cm.
 (a) Use the value $\frac{22}{7}$ for π to find the total length of the wire.
 (b) If the mass of the coil is 3,3 kg find the mass of 1 m of the copper wire.
- 13 Water is pulled up from a well in a bucket on a rope. The rope winds on a cylindrical drum 15 cm in diameter. It takes 28 turns of the drum to pull the bucket up from the bottom of the well. How deep is the well? (Use the value $\frac{22}{7}$ for π .)
- 14 A bicycle wheel is 56 cm in diameter. How many complete turns does it make in travelling 1 kilometre? (Use the value $\frac{22}{7}$ for π .)
- 15 A silver chain has 100 links. Each link is made of thin silver wire and is in the shape of a circle of radius 2,5 cm. Find the value of the chain if the silver wire costs 6 cents per centimetre. (Use the value 3,14 for π .)

Example 7

Calculate the perimeter of the shape in Fig. 14.19. All lengths are in centimetres. Use the value $\frac{22}{7}$ for π .

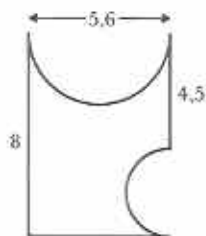


Fig. 14.19

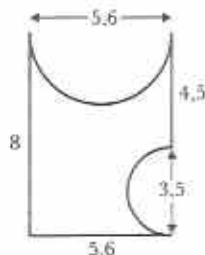


Fig. 14.20

Sketch the shape and enter as many dimensions as possible on the sketch as in Fig. 14.20.

Total length of straight edges
 $= 8 + 5,6 + 4,5$
 $= 18,1$ cm

Length of bigger semi-circle
 $= \frac{1}{2}$ of $\pi \times 5,6$ cm
 $= \frac{1}{2} \times \frac{22}{7} \times \frac{56}{10}$ cm
 $= 11 \times 0,8$ cm = 8,8 cm

Length of smaller semi-circle
 $= \frac{1}{2}$ of $\pi \times 3,5$ cm
 $= \frac{1}{2} \times \frac{22}{7} \times \frac{35}{10}$ cm
 $= 11 \times 0,5$ cm = 5,5 cm

Perimeter of shape
 $= 18,1 + 8,8 + 5,5$
 $= 32,4$ cm

Exercise 14e

Find the perimeters of the shapes in Fig. 14.21. All measurements are in cm. Use the value $\frac{22}{7}$ for π .

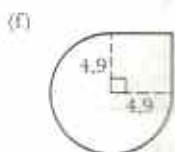
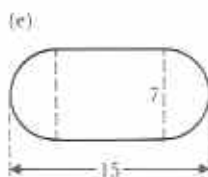
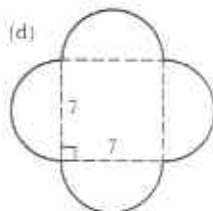
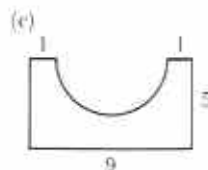
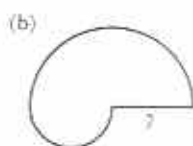
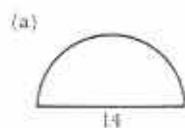


Fig. 14.21

(Hint: make your own sketches and put in as many measurements as you can, especially diameters and radii of circular parts.)

Area of plane shapes (1)

Syllabus Reference 5.4.2

The **area** of a shape is a measure of its surface. The square is used as the shape for the basic unit of area. A square of side 1 m covers an area of **1 square metre** or **1 m²**. A square of side 1 cm (Fig. 15.1) covers an area of **1 square centimetre** or **1 cm²**.

Fig 15.1 1cm²

Exercise 15a

1 Measure the area of the shapes in Fig. 15.2 by counting the squares each contains. If bits are left over, try to estimate by adding them together to make whole squares. Each of the small squares represents 1 cm².

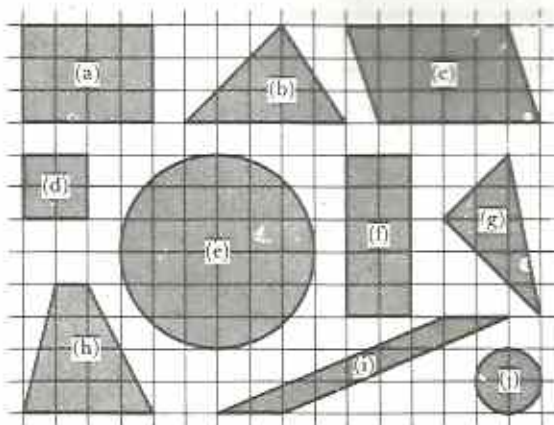


Fig 15.2

2 Find a large leaf. Put the leaf on 1 cm²-squared paper and draw round it. Measure the area of the leaf in cm² by counting squares. Estimate the total number of parts of squares as before.

3 Use the method of question 2 to find the area of your hand. (First draw round your hand on graph paper.)

Area of rectangle and square

A rectangle 5 cm long by 3 cm broad can be divided into cm² as shown in Fig. 15.3.

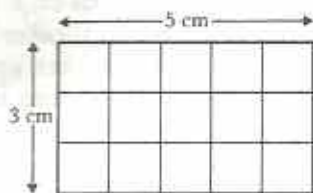


Fig. 15.3

By counting, the area of the rectangle = 15 cm². Notice also that $5 \times 3 = 15$; thus, in general:

$$\text{area of rectangle} = \text{length} \times \text{breadth}$$

Also notice that $5 = 15 \div 3$ and $3 = 15 \div 5$, thus:

$$\text{length of rectangle} = \text{area} \div \text{breadth}$$

$$\text{breadth of rectangle} = \text{area} \div \text{length}$$

A square is a rectangle whose length and breadth are equal, thus:

$$\text{area of square} = (\text{length of side})^2$$

Example 1

Calculate the area of a rectangle 6 cm by 3.5 cm.

$$\text{Area of rectangle} = 6 \text{ cm} \times 3.5 \text{ cm} = 21 \text{ cm}^2$$

Example 2

An assembly area is in the shape of a 30 m by 30 m square. Part of the area is a concrete rectangle 25 m by 5 m; the rest is grass. Calculate the area of the grass.

Make a sketch of the assembly area as shown in Fig. 15.4

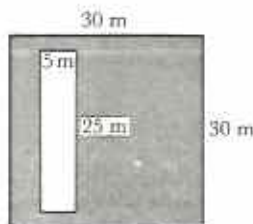


Fig. 15.4

$$\begin{aligned}\text{Area of assembly area} &= (30 \text{ m})^2 = 900 \text{ m}^2 \\ \text{Area of concrete} &= 25 \text{ m} \times 5 \text{ m} \\ &= 125 \text{ m}^2 \\ \text{Area of grass} &= 900 \text{ m}^2 - 125 \text{ m}^2 \\ &= 775 \text{ m}^2\end{aligned}$$

Example 3

The area of a rectangle is 24 cm^2 and one side is 6 cm in length. Find its breadth and perimeter.

$$\begin{aligned}\text{Breadth of rectangle} &= 24 \text{ cm}^2 \div 6 \text{ cm} = 4 \text{ cm} \\ \text{Perimeter} &= 2(6 + 4) \text{ cm} = 2 \times 10 \text{ cm} = 20 \text{ cm}\end{aligned}$$

Example 4

Calculate the area of the shape in Fig. 15.5. All measurements are in metres and all angles are right angles.

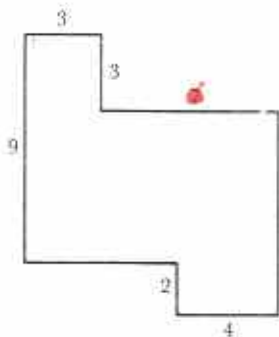


Fig. 15.5

The shape can be split into a 3×3 square and 6×10 and 2×4 rectangles, Fig. 15.6.

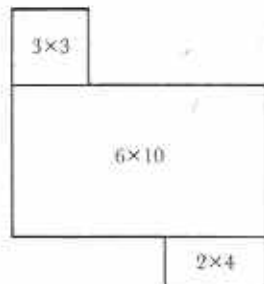


Fig. 15.6

$$\begin{aligned}\text{Area} &= \text{area of square} \\ &\quad + \text{area of the two rectangles.} \\ &= (3 \times 3 + 6 \times 10 + 2 \times 4) \text{ m}^2 \\ &= (9 + 60 + 8) \text{ m}^2 \\ &= 77 \text{ m}^2\end{aligned}$$

Exercise 15b

- 1 Copy and complete the table of rectangles (Table 15.1)

Table 15.1

	length	breadth	area
(a)	3 cm	2 cm	
(b)	5 m	4 m	
(c)	5 m		15 m^2
(d)		3 m	12 m^2
(e)	6 m	$2\frac{1}{2} \text{ m}$	
(f)	5,2 m	3 m	
(g)	2,1 cm	3,4 cm	
(h)	4 m		18 m^2

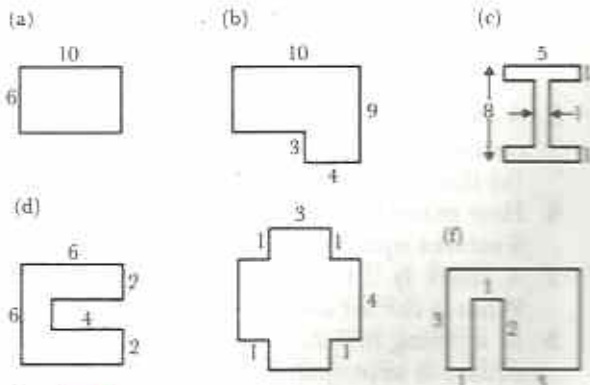


Fig. 15.7

- 2 Copy and complete the table of squares (Table 15.2).

Table 15.2

	length of side	area
(a)	6 cm	
(b)	9 m	
(c)		49 m ²
(d)		16 cm ²
(e)	1,2 m	
(f)	2½ cm	

- 3 Calculate the areas of the shapes in Fig. 15.7. All lengths are in metres and all angles are right angles.
- 4 Calculate the shaded areas in the diagrams in Fig. 15.8. All lengths are in centimetres and all angles are right angles.

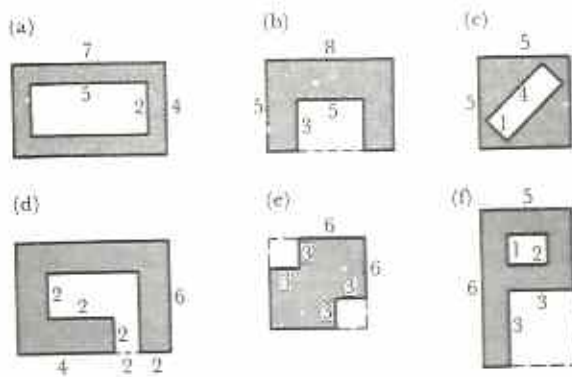


Fig. 15.8

- 5 A floor 4 m long and 3½ m wide is to be concreted. Find (a) the area of the floor, (b) the cost, if concrete costs \$36 per m².
- 6 How many m² of floor are there in a room 6 metres square?
- 7 A sheet is 2,15 m long and 1,6 m wide. What is the area of the sheet?
- 8 In a dining hall 25 m by 12 m, an area 8 m square is kept clear for cooking. What area is left over for dining?

- 9 A 12 m by 12 m square garden has a 1 m wide path through the centre, parallel to one side of the square. This is shown in Fig. 15.9. Calculate the area of garden left over for planting.

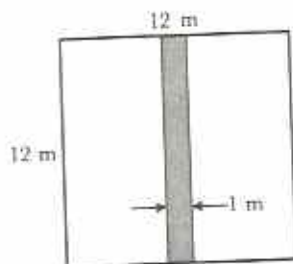


Fig. 15.9

- 10 A cardboard box is 20 cm long, 12 cm wide and 8 cm deep (Fig. 15.10). Calculate the total area of cardboard in the box.

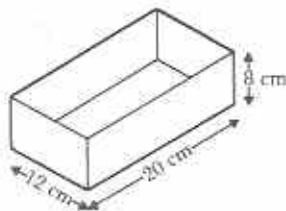


Fig. 15.10

- 11 The box in question 10 is cut down the edges and flattened as shown in Fig. 15.11(a).

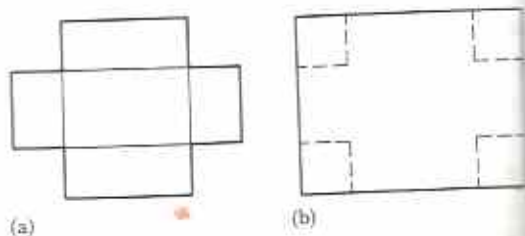


Fig. 15.11

Calculate the area of the smallest square rectangle from which the box could be made, i.e. calculate the area of the rectangle in Fig. 15.11(b). What area would be wasted?

- 12 Calculate the total area of the walls of a room 5 m long, 4 m wide and 2½ m high. (Do not allow for doors and windows.)

- 13 1 litre of paint covers 15 m^2 . How many litres of paint will be needed to paint the walls of the room in question 12?
- 14 Calculate the area of the ceiling of the room in question 12. How many $2\frac{1}{2} \text{ m}$ by 1 m ceiling boards will be needed to cover the ceiling of the room?
- 15 A roll of cloth contains 56 m^2 of material. The cloth is $\frac{1}{2} \text{ m}$ wide. How long is the roll?
- 16 The pages of a booklet measure 20 cm by $12,5 \text{ cm}$.
- (a) Calculate the area of a page of the book.
- The pages of the booklet are numbered 1 to 36.
- (b) How many sheets of paper are needed for a book of 36 pages?
- (c) Calculate the total area of paper used in the booklet.
- 17 A sheet of 150 gsm drawing paper measures $0,8 \text{ m}$ by $0,6 \text{ m}$. 150 gsm means that the mass of the paper is 150 grammes per square metre. Find the mass of 10 sheets of drawing paper.
- 18 A board measures $2\frac{1}{2} \text{ m}$ by $1\frac{1}{4} \text{ m}$ and costs \$25. Calculate the cost of 1 m^2 of the board.
- 19 A boy has 16 matchsticks. He lays them in the shape of a rectangle (Fig. 15.12). Call each matchstick a unit and find the perimeter and area of the rectangle in units and units².



Fig. 15.12

Use matchsticks or squared paper to show the different rectangles which the boy can make using all 16 matchsticks. Find the perimeters and areas of these rectangles. What do you notice?

Area of parallelogram

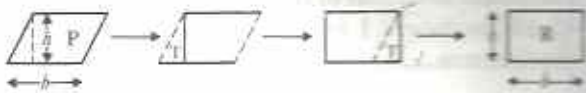


Fig. 15.13

The diagrams in Fig. 15.13 show how a parallelogram, P, can be changed to a rectangle, R, by moving a triangle, T, from one end to the other.

$$\begin{aligned} \text{area of parallelogram, } P &= \text{area of rectangle, } R \\ &= b \times h \end{aligned}$$

In the diagram, the height of the parallelogram is h and its base is b . In general:

area of parallelogram = base \times height
and

$$\text{base of parallelogram} = \text{area} \div \text{height}$$

$$\text{height of parallelogram} = \text{area} \div \text{base}$$

A parallelogram can have two bases and two corresponding heights as shown in Fig. 15.14.

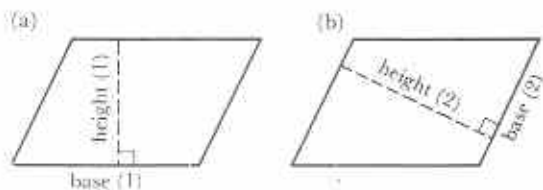


Fig. 15.14 (a) Area = base (1) \times height (1)
(b) Area = base (2) \times height (2)

Example 5

In Fig. 15.15, the base of the parallelogram is 6 cm long and its height is 4 cm . Calculate the area of the parallelogram. If the length of the other side of the parallelogram is 8 cm , calculate its corresponding height, h .



Fig. 15.15

$$\begin{aligned} \text{Area of parallelogram} &= 6 \text{ cm} \times 4 \text{ cm} \\ &= 24 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{height} &= \text{area} \div \text{base} \\ &= 24 \text{ cm}^2 \div 8 \text{ cm} \\ &= 3 \text{ cm} \end{aligned}$$

Exercise 15c

Calculate the areas of the parallelograms in Fig. 15.16 (numbers 1 to 8). All dimensions are in centimetres.

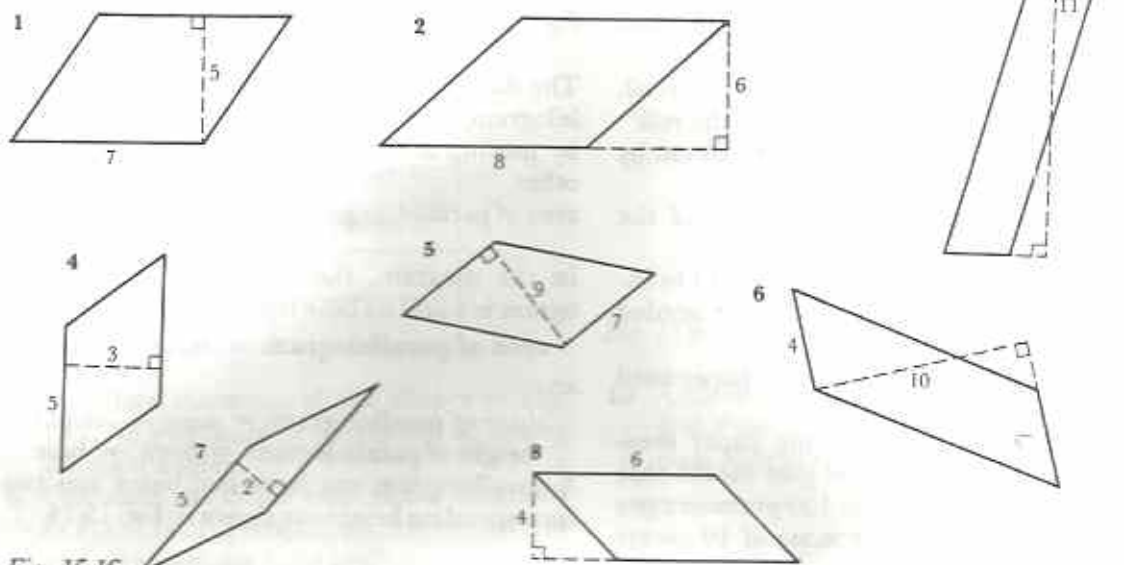


Fig. 15.16

In each of the parallelograms in Fig. 15.17 (numbers 9 to 13), calculate the height, h .

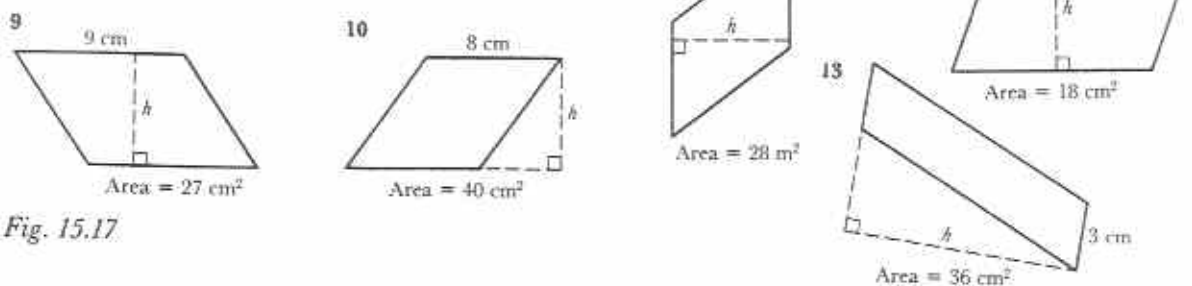


Fig. 15.17

In each of the parallelograms in Fig. 15.18 (numbers 14 to 18), calculate the base, b .

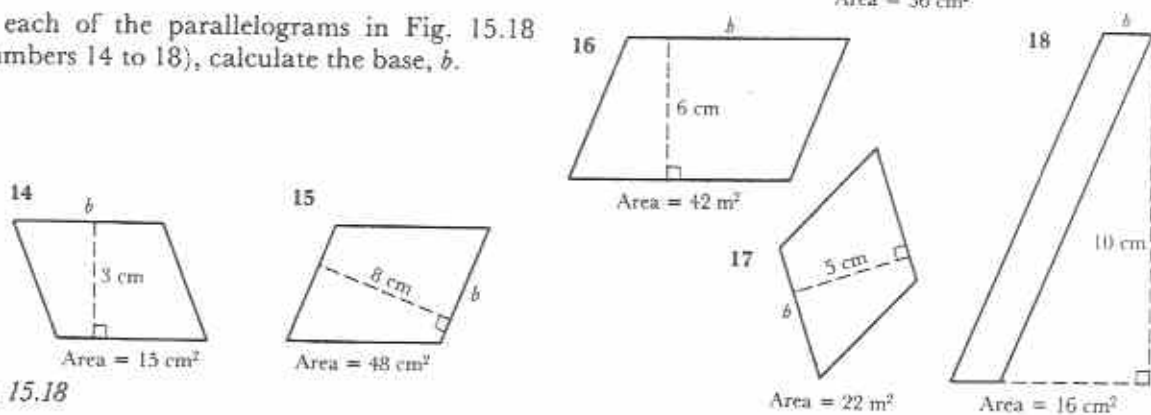


Fig. 15.18

In each of the parallelograms in Fig. 15.19 (numbers 19 to 22), calculate (a) the area, (b) the height h or the base b .

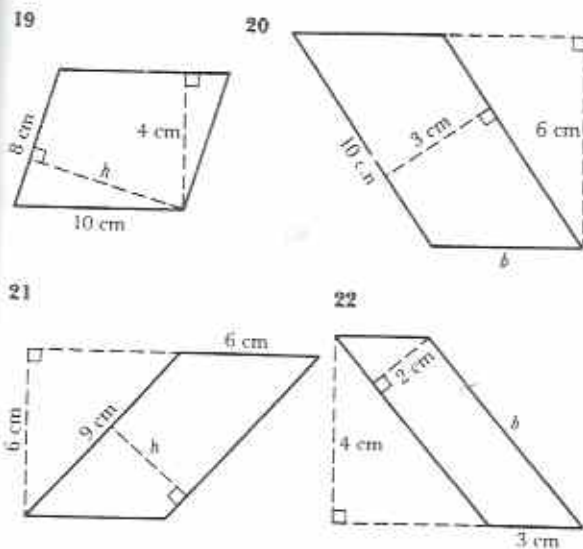


Fig. 15.19



Fig. 15.21

Thus the area of each triangle is half the area of the containing parallelogram (Fig. 15.22).

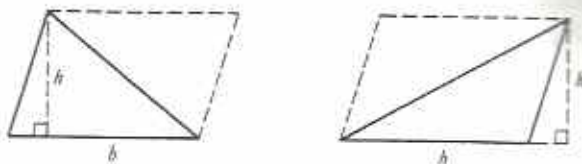


Fig. 15.22

Since, area of parallelogram = base \times height, then:

$$\text{area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

Notice that any side of a triangle can be taken as base. Each base has its corresponding height.

example 6

Calculate the area of the triangle shown in Fig. 15.23.

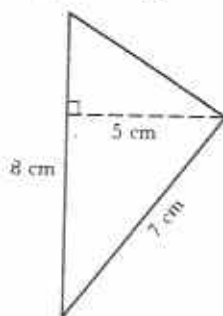


Fig. 15.23

The height is 5 cm. The corresponding base is 8 cm. We do not need the 7 cm side.

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 8 \times 5 \text{ cm}^2 = 20 \text{ cm}^2 \end{aligned}$$

Example 7

Calculate the area of the triangle shown in Fig. 15.24.

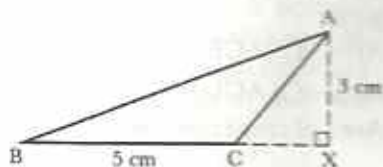


Fig. 15.24

Area of triangle

Right-angled triangle

Any diagonal of a rectangle divides it into two equal right-angled triangles (Fig. 15.20). Thus: area of a right-angled triangle = $\frac{1}{2} \times$ product of the sides containing the right angle.

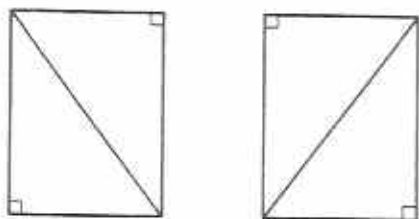


Fig. 15.20

Any triangle

Any diagonal of a parallelogram divides it into two equal triangles (Fig. 15.21).

Notice that the height from A to base BC falls outside the triangle.

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base BC} \times \text{height AX} \\ &= \frac{1}{2} \times 5 \text{ cm} \times 3 \text{ cm} \\ &= 7\frac{1}{2} \text{ cm}^2 \end{aligned}$$

Example 8

Calculate the area of the quadrilateral ABCD in Fig. 15.25.

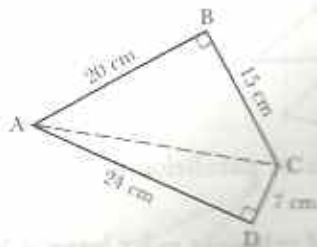


Fig. 15.25

Notice that the diagonal AC divides the quadrilateral into two right-angled triangles.

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times 20 \times 15 \text{ cm}^2 = 150 \text{ cm}^2 \\ \text{Area of } \triangle ADC &= \frac{1}{2} \times 24 \times 7 \text{ cm}^2 = 84 \text{ cm}^2 \\ \text{Area of ABCD} &= 150 \text{ cm}^2 + 84 \text{ cm}^2 = 234 \text{ cm}^2 \end{aligned}$$

Example 9

Calculate the area of the trapezium ABCD in Fig. 15.26.

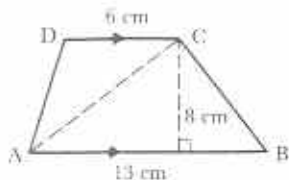


Fig. 15.26

The diagonal AC divides the trapezium into two triangles. The height of each triangle is 8 cm.

$$\begin{aligned} \text{Area of } \triangle ACB &= \frac{1}{2} \times 13 \times 8 \text{ cm}^2 = 52 \text{ cm}^2 \\ \text{Area of } \triangle ACD &= \frac{1}{2} \times 6 \times 8 \text{ cm}^2 = 24 \text{ cm}^2 \\ \text{Area of trapezium} &= 52 \text{ cm}^2 + 24 \text{ cm}^2 \\ &= 76 \text{ cm}^2 \end{aligned}$$

Area of trapezium

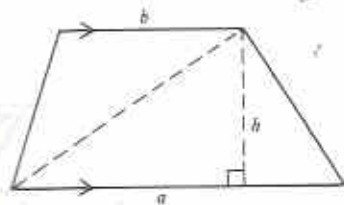


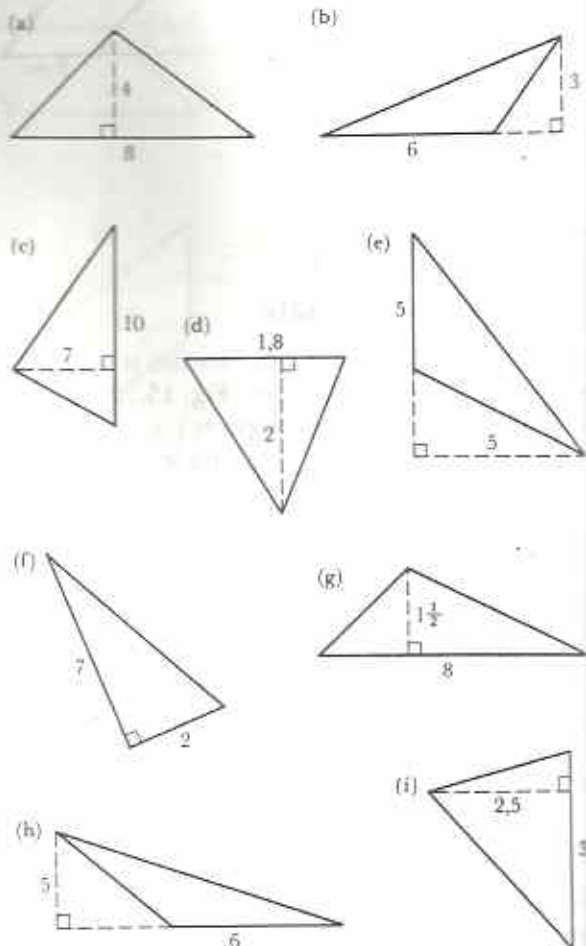
Fig. 15.27

Using the method of Example 9 in Fig. 15.27, area of trapezium = $\frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h$

$$\text{Area of trapezium} = \frac{1}{2} (a + b)h$$

Exercise 15d

1 Calculate the areas of the triangles in Fig. 15.28. All dimensions are in cm.



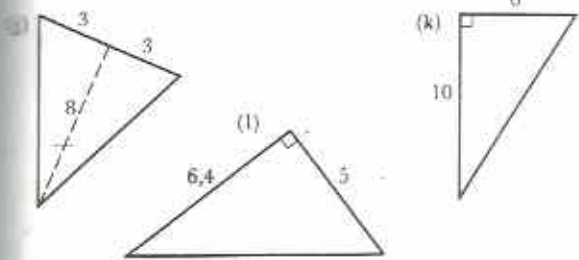


Fig. 15.28

2 Calculate the areas of the quadrilaterals in Fig. 15.29. In each case, draw a diagonal to divide the shapes into two triangles. All dimensions are in cm.

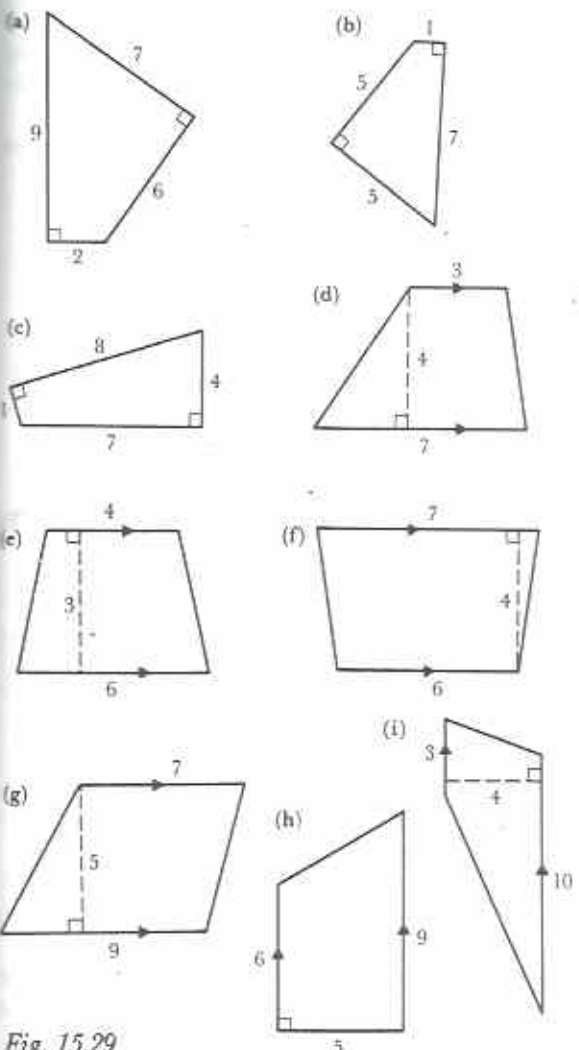


Fig. 15.29

Area of a circle

For the following you will need paper, scissors and a protractor. Draw a circle of any radius, r . Divide it into 12 equal sectors. The angle of each sector is 30° . Cut out the 12 sectors and arrange them to make a 'parallelogram' as shown in Fig. 15.30.

The height of the 'parallelogram' is r and its base is of length πr (i.e. half of the circumference of the original circle).

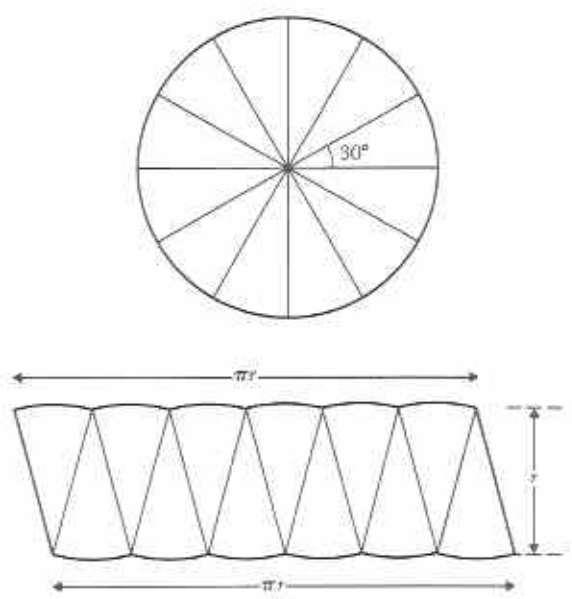


Fig. 15.30

Suppose that the shape is a parallelogram, then,

$$\begin{aligned} \text{area of parallelogram} &= \pi r \times r \\ &= \pi r^2 \text{ and, hence,} \\ \text{area of original circle} &= \pi r^2 \end{aligned}$$

When more sectors of the original circle are taken, the sides of the parallelogram become much straighter. (For example, try this activity with 36 sectors, each of 10° .) Thus, to calculate the area of any circle of radius r , use the formula:

$$\text{area of circle} = \pi r^2$$

Example 10

Find the area of a circle of radius $3\frac{1}{2}$ metres. Use the value $\frac{22}{7}$ for π .

$$\begin{aligned} \text{Area of circle} &= \pi r^2 \\ &= \frac{22}{7} \times (3\frac{1}{2})^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ m}^2 \\ &= \frac{11 \times 7}{2} \text{ m}^2 \\ &= 38\frac{1}{2} \text{ m}^2 \end{aligned}$$

Example 11

Calculate the area of the shape in Fig. 15.31. Use $\frac{22}{7}$ for π .

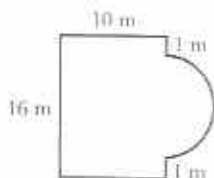


Fig. 15.31

$$\begin{aligned} \text{Area of 16 m by 10 m rectangle} &= 16 \text{ m} \times 10 \text{ m} \\ &= 160 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Diameter of semi-circle} &= 14 \text{ m} \\ \text{Radius of semi-circle} &= 7 \text{ m} \\ \text{Area of semi-circle} &= \frac{1}{2} \pi r^2 \\ &= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \text{ m}^2 \\ &= 77 \text{ m}^2 \\ \text{Area of whole shape} &= 160 \text{ m}^2 + 77 \text{ m}^2 \\ &= 237 \text{ m}^2 \end{aligned}$$

Exercise 15e

- 1 Copy and complete Table 15.3 for circles. Use the value $\frac{22}{7}$ for π .

Table 15.3

	radius	diameter	area
(a)	7 m		
(b)		7 cm	
(c)	140 mm		
(d)		28 m	
(e)		$3\frac{1}{2}$ cm	
(f)	2,1 cm		

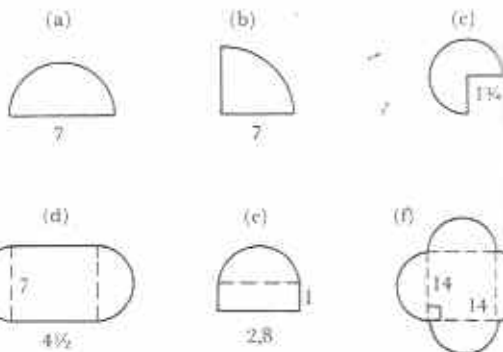


Fig. 15.32

- 2 Calculate the area of each of the shapes in Fig. 15.32. All lengths are in cm. Use the value $\frac{22}{7}$ for π . In each case, make a sketch of the shape and put in as many dimensions as possible, especially the radius of any circular parts.
- 3 Calculate the shaded areas in the shapes in Fig. 15.33. Use the value $\frac{22}{7}$ for π .

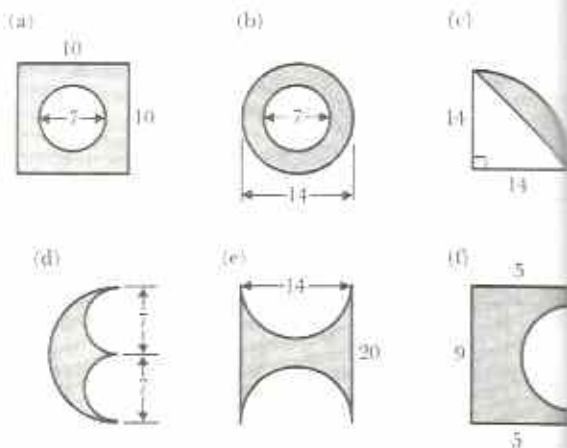


Fig. 15.33

- 4 A circular mat has a diameter of 20 cm. Find its radius and hence calculate the area of the mat. Use the value $3,14$ for π .
- 5 A goat is tied to a peg in the ground. The rope is 3 m long. What area of grass can the goat eat? Use the value $3,1$ for π .
- 6 A protractor is in the shape of a semi-circle of radius 5 cm. Calculate the area of the protractor. Use the value $3,14$ for π .

- 7 There are two circles, one large and one small. The radius of the large circle is three times the radius of the small circle. Find the value of the fraction:

$$\frac{\text{area of small circle}}{\text{area of large circle}}$$

- 8 The floor of a round hut is 10 m in diameter. A weaver charges \$46,50 to make a mat to fit the floor. Use the value 3,1 for π to find the cost of the mat per square metre.
- 9 A design is made by drawing seven small circles inside one large circle as shown in Fig. 15.34.

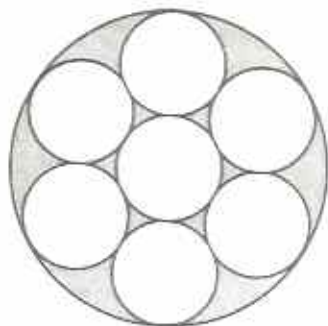


Fig. 15.34

If the diameter of the large circle is 30 cm, calculate

- the radius of the large circle.
- the radius of each small circle.
- the area shaded in the diagram.

Use the value 3,14 for π .

- 10 The sports field shown in Fig. 15.35 has a 90 m by 70 m football field with a semi-circular area at each end. A track runs round the perimeter of the sports field.

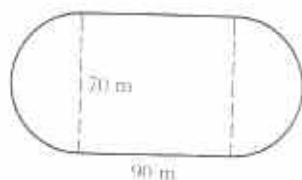


Fig. 15.35

- Use the value $\frac{22}{7}$ for π to calculate,
- the area of the sports field,
 - the length of one lap of the track.

Large areas (hectare, km²)

The cm² and m² are used to measure relatively small areas. Large areas need more appropriate units. The **hectare** is commonly used for measuring areas about the size of a farm or a town. A hectare is the area of a square which measures 100 m by 100 m.

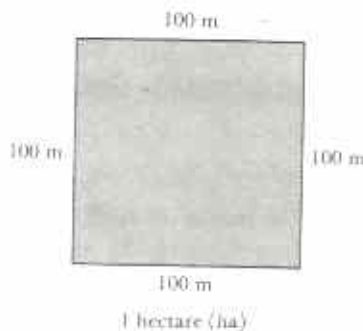


Fig. 15.36

Think of a hectare as being about the size of two full-size football or hockey pitches. The abbreviation of hectare is ha.

$$\begin{aligned} 1 \text{ ha} &= 100 \text{ m} \times 100 \text{ m} \\ &= 10\,000 \text{ m}^2 \end{aligned}$$

The **square kilometre**, km², is used for measuring even larger regions, such as the size of a district or a country. For example, the area of Zimbabwe is 390 580 km².

$$\begin{aligned} 1 \text{ km}^2 &= 1\,000 \text{ m} \times 1\,000 \text{ m} \\ &= 1\,000\,000 \text{ m}^2 \\ &= 100 \text{ ha} \end{aligned}$$

Exercise 15f

- A rectangular park measures 800 m by 300 m. Express its area in hectares.
- A map of greater Harare shows the city to cover an area 25 km by 36 km. Express this area in km².

- 3 Warren Hills golf course is roughly rectangular in shape and measures 0,9 km by 1,6 km. Express its area (a) in km^2 , (b) in ha, (c) in m^2 . Which of these areas do you think is most appropriate?
- 4 A building plot of area 1,5 ha was sold for \$50 000. Find the selling price per m^2 to the nearest 10 c.
- 5 A small game reserve is roughly circular in shape and has a diameter of 8 km. Use the value 3 for π to estimate the area of the reserve in ha.

Everyday arithmetic (1)

Approximation and estimation

Syllabus Reference 5.1.2

Approximation

Rounding off numbers

Suppose that on a certain day the population of a town is exactly 18 279 people. What will be the population one week later? It is impossible to say. However, we will have a good idea of the population. It will be 18 000 **approximately**.

We say that $18\,279 = 18\,000$ **to the nearest thousand**. Notice that 18 279 is between 18 000 and 19 000, but is nearer 18 000.

We can also say that $18\,279 = 18\,300$ **to the nearest hundred**. Notice that 279 is between 200 and 300, but is nearer 300.

In each case we have **rounded off** the original number. We may **round up** or **round down**. 18 279 is rounded down to 18 000, but it is rounded up to 18 300.

Table 16.1 shows how to round off numbers 630, 631, 632, ..., 639 to the nearest ten.

Table 16.1
Rounding off to the nearest ten

630	no need to round off
631 632 633 634	round <i>down</i> to 630 (these numbers are nearer 630 than 640)
635	halfway between 630 and 640: round up to 640*
636 637 638 639	round <i>up</i> to 640 (these numbers are nearer 640 than 630)

* There are other rules for rounding off a 5. However, the above rule will be used in this course.

Example 1

Round off 14 505 to the nearest (a) thousand, (b) hundred, (c) ten.

- (a) $14\,505 = 15\,000$ to the nearest thousand.
Note: 14 505 is a little nearer 15 000 than 14 000.
 (b) $14\,505 = 14\,500$ to the nearest hundred.
 (c) $14\,505 = 14\,510$ to the nearest ten.
Note: the last digit of 14 505 is 5; round up.

Example 2

Approximate 79,65 to the nearest (a) hundred, (b) ten, (c) whole number, (d) tenth.

- (a) $79,65 = 100$ to the nearest hundred.
 (b) $79,65 = 80$ to the nearest 10.
 (c) $79,65 = 80$ to the nearest whole number.
Note: the fraction 0,65 is nearer 1 than 0; round up 79 to the next whole number, 80.
 (d) $79,65 = 79,7$ to the nearest tenth.
Note: the last digit of 79,65 is 5; round up.

Exercise 16a (Oral)

- Round off the following to the nearest (i) thousand, (ii) hundred, (iii) ten.

(a) 18 624	(b) 25 246	(c) 32 781
(d) 7 163	(e) 2 968	(f) 9 476
(g) 14 939	(h) 26 888	(i) 45 072
(j) 5 616	(k) 16 065	(l) 12 505
(m) 9 895	(n) 30 097	(o) 8 350
- Approximate the following to the nearest ten.

(a) 345	(b) 375	(c) 695
(d) 705	(e) 715	(f) 995
- Round off the following to the nearest whole number.

(a) 6,9	(b) 12,3	(c) 78,75
(d) 29,6	(e) 9,5	(f) 99,49

4 Approximate the following to the nearest tenth.

- (a) 0,71 (b) 0,09 (c) 0,15
(d) 0,45 (e) 0,98 (f) 0,02

5 Round off the following to the nearest hundredth.

- (a) 0,164 (b) 0,167 (c) 0,706
(d) 0,702 (e) 0,295 (f) 0,404

6 Approximate the following to the nearest (i) whole number, (ii) tenth.

- (a) 1,38 (b) 4,09 (c) 9,65

7 Round off the following to the nearest (i) tenth, (ii) hundredth.

- (a) 0,372 (b) 0,625 (c) 0,155

8 Approximate the following to the nearest (i) ten, (ii) whole number, (iii) tenth.

- (a) 26,48 (b) 8,35 (c) 5,84

9 Round off 0,798 to the nearest (a) whole number, (b) tenth, (c) hundredth.

10 Round off 69,55 to the nearest (a) hundred, (b) ten, (c) whole number, (d) tenth.

nificant figures. However, they must be written down to keep the correct place values.

$$0,00925 = 0,0093 \text{ to 2 s.f.}$$

Exercise 16b (Oral)

1 Round off the following to (i) 1 s.f., (ii) 2 s.f.,

(iii) 3 s.f.

- (a) 7284 (b) 6035 (c) 14612
(d) 3604 (e) 8009 (f) 5050
(g) 28336 (h) 9852 (i) 9395
(j) 26002

2 Approximate the following to (i) 1 s.f., (ii) 2 s.f., (iii) 3 s.f., (iv) 4 s.f.

- (a) 7,0384 (b) 18,502 (c) 12,675
(d) 3,7998 (e) 234,06

3 Round off the following to (i) 1 s.f., (ii) 2 s.f., (iii) 3 s.f.

- (a) 0,06752 (b) 0,3059 (c) 0,006307
(d) 0,0008666 (e) 0,03355

Significant figures

Significant figures begin from the first non-zero digit at the left of a number. As before, the digits 5, 6, 7, 8, 9 are rounded up and 1, 2, 3, 4 are rounded down. Digits should be written with their correct place value.

Examples 3

Read the following examples and notes carefully.

(a) $546,52 = 500$ to 1 significant figure

$$546,52 = 550 \text{ to 2 s.f.}$$

Note: s.f. is short for significant figures.

$$546,52 = 547 \text{ to 3 s.f.}$$

$$546,52 = 546,5 \text{ to 4 s.f.}$$

(b) $8,0296 = 8$ to 1 s.f.

$$8,0296 = 8,0 \text{ to 2 s.f.}$$

Note: in this case the zero must be given after the decimal comma; it is significant.

$$8,0296 = 8,03 \text{ to 3 s.f.}$$

$$8,0296 = 8,030 \text{ to 4 s.f.}$$

Note: the 4th significant digit is zero; it must be written down.

(c) $0,00925 = 0,009$ to 1 s.f.

Note: 9 is the first non-zero digit. The two zeros after the decimal comma are not sig-

Decimal places

Decimal places are counted from the decimal comma. Zeros after the comma are also counted. Digits are rounded up or down as before. Place values must be kept.

Examples 4

Read the following examples carefully.

(a) $14,9028 = 14,9$ to 1 decimal place

$$14,9028 = 14,90 \text{ to 2 d.p.}$$

$$14,9028 = 14,903 \text{ to 3 d.p.}$$

Note: d.p. is short for decimal places.

(b) $2,3975 = 2,4$ to 1 d.p.

$$2,3975 = 2,40 \text{ to 2 d.p.}$$

$$2,3975 = 2,398 \text{ to 3 d.p.}$$

(c) $0,0072 = 0,0$ to 1 d.p.

$$0,0072 = 0,01 \text{ to 2 d.p.}$$

$$0,0072 = 0,007 \text{ to 3 d.p.}$$

Exercise 16c (Oral)

Round off the following to (a) 1 d.p., (b) 2 d.p.,

(c) 3 d.p.

- 1 12,9348 2 24,1177 3 3,0725
4 1,9375 5 1,9875 6 0,5846
7 0,0625 8 0,0375 9 0,9002
10 0,0089

Example 5

In the 1988–89 budget it was decided to spend \$19 181 160 on Wildlife Management. How might the Finance Minister say this amount in a speech?

He might say:

\$19 million

or, about \$19 million

or, just over \$19 million

He might also say:

\$20 million

or, just under \$20 million

In the first case he has rounded to 2 s.f.

In the second case he has rounded to 1 s.f.

The \$160 at the end of the \$19 181 160 would be a lot of money for a student to spend. However, in terms of the budget for a whole country, \$160 is insignificant. The Finance Minister is not likely to mention it. If a newspaper says, 'Government to spend \$19 million on wildlife', we take this to mean the Government will spend between \$18.5 million and \$19.5 million.

Exercise 16d

- (a) There are two 6's in the amount \$8 614 160. Write down the amount of money that each 6 represents.
(b) There are two 1's in the amount \$8 614 160. Write down the amount of money that each 1 represents.
- The actual age of a man is 39 years 8 months 29 days. A doctor asks the man his age. What will the man say?
- In 1989 it was proposed to spend \$846 744 250 on Primary and Secondary Education. Give at least three different ways in which a Government Minister might say this amount in a speech.
- A newspaper headline says, 'New road to cost \$21.8 million!' Between what amounts will the road cost?
- Table 16.2 gives the provisional amounts spent on two aspects of Social Welfare for the years 1987/88 and 1988/89.

Table 16.2

	Government expenditure (in \$ thousand)	
	1987/88	1988/89
Community and Cooperative and Women's Affairs	18 379	16 682
Youth, Sport and Culture	15 783	17 256

- How might a Finance Minister give the 1987/88 Community and Cooperative and Women's Affairs amount in a speech?
- How might a newspaper print the 1988/89 Youth, Sport and Culture amount?
- Find the total spent in 1987/88 correct to 2 s.f.
- Find the total spent on Community and Cooperative and Women's Affairs during the two years correct to 2 s.f.
- Find the total amount spent on these two aspects of Social Welfare over the 2 years correct to 2 s.f.

False accuracy

Consider the following:

- A student measures a line with a ruler and says it is 162.83 mm long. Since it is impossible to measure 0.83 mm on a ruler, this answer is an example of **false accuracy**. It is more realistic to say that the line is 163 mm long.
- A report estimates the 1990 population of Harare to be 977 332. Since this number is an estimate it would have been more realistic to give the population to 3 s.f.: 977 000.

Most measuring instruments, such as a ruler, protractor, thermometer, balance, measuring cylinder, give results which are correct to only 2 or 3 significant figures. The answers to calculations using measurements by such instruments should likewise be given to 2 or 3 significant figures only.

(b) A student does the following problem:
Calculate the wages of a person who works a 42 hour week for 186c per hour.

The student gets an answer of \$7 812. She looks at her answer and thinks, 'It is nearly impossible for someone to get \$7 812 for a week's work.' She does a rough check:

$$186c \times 42 = \$2 \times 40 = \$80$$

This makes her check her working. She sees that she forgot to change cents to dollars.

$$186c \times 42 = 7\,812 \text{ cents}$$

The correct answer is \$78,12. This agrees with her rough check, \$80.

It is a good habit always to check calculations by making a rough estimate. A quick estimate can stop you making errors. It can also tell you whether your answer is sensible or not.

When making an estimate, it is usually enough to round off numbers to 1 significant figure or to the nearest whole number.

Example 6

(a) Find the rough value of $4\frac{1}{5} \times 1\frac{7}{8}$.

(b) Find the value of $4\frac{1}{5} \times 1\frac{7}{8}$ accurately.

$$(a) \quad 4\frac{1}{5} \times 1\frac{7}{8} = 4 \times 2 = 8$$

$$(b) \quad 4\frac{1}{5} \times 1\frac{7}{8} = \frac{21}{5} \times \frac{15}{8} = \frac{63}{8} = 7\frac{7}{8}$$

Exercise 16f(Oral)

In questions 1–20, round off each number to 1 s.f. Then estimate each answer.

- | | | | |
|----|---------------|----|---------------|
| 1 | 23 + 19 | 2 | 73 - 18 |
| 3 | 24 × 37 | 4 | 572 ÷ 22 |
| 5 | 17 + 58 | 6 | 92 - 26 |
| 7 | 69 × 52 | 8 | 817 ÷ 19 |
| 9 | 37 + 52 | 10 | 653 - 287 |
| 11 | 99 × 95 | 12 | 171 ÷ 18 |
| 13 | \$47 + \$61 | 14 | \$51 - \$17 |
| 15 | \$9,60 × 5,8 | 16 | 43 kg ÷ 8,2 |
| 17 | 133 g + 452 g | 18 | 943 m - 482 m |
| 19 | \$53 × 18 | 20 | \$672 ÷ 24 |

In questions 21–40 round off each number to the nearest whole number. Then estimate each answer.

- | | | | |
|----|------------------------------------|----|-------------------------------------|
| 21 | 6,2 + 3,7 | 22 | 12,3 + 8,7 |
| 23 | 3,4 × 5,8 | 24 | 14,07 ÷ 6,7 |
| 25 | 3,47 + 12,75 | 26 | $5\frac{2}{3} - 2\frac{2}{4}$ |
| 27 | 16,7 × 1,09 | 28 | $15\frac{3}{4} + 4\frac{1}{5}$ |
| 29 | $6\frac{3}{4} + 5\frac{1}{8}$ | 30 | 7,55 - 3,45 |
| 31 | $8\frac{1}{2} \times 7\frac{1}{7}$ | 32 | 9,774 ÷ 3,64 |
| 33 | $\frac{5}{6} + 2\frac{1}{8}$ | 34 | $10\frac{1}{4} \times 3\frac{1}{3}$ |
| 35 | $17\frac{3}{4} - 6\frac{7}{8}$ | 36 | 12,48 ÷ 7,8 |
| 37 | \$4,72 + 89c | 38 | \$13,09 - \$4,81 |
| 39 | \$2,58 × 4,5 | 40 | $19\frac{1}{3} \div 5\frac{1}{3}$ |

In questions 41–50 round off each number to 1 s.f. Then estimate each answer.

- | | | | |
|----|---------------|----|---------------|
| 41 | 0,41 × 0,92 | 42 | 0,075 × 0,025 |
| 43 | 0,333 × 0,667 | 44 | 0,047 × 0,023 |
| 45 | 0,09 × 0,12 | 46 | 0,81 ÷ 0,22 |
| 47 | 0,617 ÷ 0,028 | 48 | 0,49 + 0,12 |
| 49 | 0,067 ÷ 0,25 | 50 | 0,108 ÷ 0,027 |

Exercise 16g

- Calculate the accurate answers to every fifth question in Exercise 16f, i.e. questions 5, 10, 15, ... Use your estimates to check if your answers appear correct.
- In 1990 the value of a house was \$47 600. Its value rises by about 10% each year. Estimate its value in 1991 correct to the nearest \$100.
- A bucket holds 10,5 litres. A cup holds about 320 ml. Estimate the number of cups of water that the bucket holds.
- A cooperative farmer has \$4 000 to spend on cattle. He wants to buy nine calves. Each calf costs \$372 on average. Use estimation to check that the farmer has enough money. Find, accurately, how much change he will get after buying the calves.
- The populations of five towns are 15 600, 17 300, 62 800, 74 000 and 34 400, each to the nearest 100. Find the total population of the five towns to the nearest 1 000.
- 36 football teams meet at the National Sports Stadium. Each team has 12 players. First estimate, then find accurately, the

total number of players at the Sports Stadium.

7 $x = 0,176 \div 0,32$. By doing a rough calculation, decide which one of the following is the value of x :

- (a) 0,18 (b) 0,2 (c) 0,21
(d) 0,3 (e) 0,55

8 A student tries the following problem:

Calculate the cost of 7,8 metres of cloth costing \$8,15 per metre.

Her answer is \$6 357.

- (a) Is this answer sensible?
(b) Estimate a sensible answer by rounding to the nearest whole numbers.
(c) What error do you think the student made?

9 A shop sells about 340 magazines each week. The selling price of a magazine is 48 cents. Estimate the amount of money the shop gets each week from selling magazines.

10 An aeroplane flies 2 783 km in $5\frac{3}{4}$ hours. First estimate, then calculate, the average distance it flies in 1 hour.

11 Fig. 16.1 is a newspaper advertisement showing old prices crossed out and new prices given in place.

SAVE 60%

\$33,99	_____	\$21,47
\$29,99	_____	\$19,29
\$25,99	_____	\$15,84
\$24,99	_____	\$15,23
\$19,99	_____	\$12,57

Fig. 16.1

- (a) Round off all the prices to the nearest \$. Express each new price as a percentage of the old price correct to 1 s.f.
(b) Is it true that 60% is saved?

12 A man's pace is about 70 cm long. He takes 2 858 paces to walk from his house to the post office. Estimate the distance of his house from the post office.

13 Eight oranges have a mass of 965 g. Rufaro bought 5 kg of oranges. Approximately how many oranges did she get?

14 There are 97 exercise books in a pile. The height of the pile is 47 cm. Estimate the thickness of each exercise book in mm.

15 A textbook has a mass of 324 g. A school bought 96 copies of the textbook. Estimate the total mass, in kg, of the parcel the books came in.

16 The perimeter of a school is 1 805 m. In the perimeter fence there are 201 fence posts equally spaced. Estimate the distance between any two posts.

Common measures

The most common units for lengths are millimetres, centimetres, metres and kilometres. We use the lower units (millimetres and centimetres) for short lengths and the higher units (metres and kilometres) for larger distances.

The common units of mass are the gramme, kilogramme and tonne. The common units of capacity are the millilitre, litre and kilolitre. As with length, we use the lower units for smaller quantities and the higher units for greater quantities.

Exercise 16h (Oral)

- 1 State the units of length you would use to measure the following.
- height of a desk
 - height of yourself
 - thickness of a book
 - distance from Harare to Mutare
 - diameter of a coin
 - distance of the classroom from the staffroom
 - length of your fingernail
 - thickness of a coin
 - distance round the earth
 - your waist
- 2 State the units of mass you would use to measure the mass of the following.
- a parcel
 - a lorry's load
 - a mango
 - a book
 - yourself
 - a letter
 - a car
 - packet of sugar

- (i) a table (j) the liquid gas in a gas cylinder

3 State the units of capacity you would use to measure the capacity of the following.

- (a) cup (b) car petrol tank
 (c) medicine bottle (d) test-tube
 (e) bucket (f) reservoir
 (g) teaspoon (h) water storage tank
 (i) sink (j) petrol tanker (lorry)

4 The following are all answers that students gave to questions in a mathematics class. Which of these answers are *not* sensible? If you think an answer is not sensible, say what approximate size you think the answer could be.

- (a) A student walked 60 km in an hour.
 (b) Five tins of milk cost \$23 000.
 (c) The mass of the dictionary is 1,2 kg.
 (d) The height of the man is 181 cm.
 (e) The mass of the woman is 558 kg.
 (f) The pencil was 17 cm long.
 (g) The cup holds 22 litres.
 (h) The water tank has a capacity of 12 kilolitres.
 (i) It took 15 seconds to drive from Gweru to Mutare.
 (j) The diameter of the tennis ball was 25 mm.
 (k) The mass of the record player is 9,8 kg.
 (l) The Principal's pay is \$26 a year.
 (m) The door was 80 cm wide.
 (n) The car is 32 m long.
 (o) The mass of the dog was 12 g.
 (p) The candle was 26 m long.
 (q) The area of the football field is 4 000 cm².
 (r) The diameter of the bicycle wheel is 60 cm.
 (s) The new car cost 45 cents.
 (t) It took 2,6 m of string to tie the parcel.
 (u) The mass of the ruler was 50 kg.
 (v) The sun is 400 m from the earth.
 (w) The boy put 224 cubes of sugar in his tea.
 (x) The plane took 11 hours to fly from Harare to London.
 (y) The height of the flagpole is 3 mm.
 (z) The bottle contained 48 ml of ink.

Body measures

You should know the sizes of parts of your body, such as your hand-span, the length of your foot, your waist measurement and your body mass. You can often use these to estimate other measures.

The following assignments show how to use body measurements to find other distances.

Exercise 16i

You will need a metre rule, a tape measure, an empty litre bottle, a cup and a spring balance from the science laboratory.

- 1 (a) Use a metre rule to measure your hand-span in cm (Fig. 16.2).
 (b) Does anyone in your class have a hand-span less than 15 cm?
 (c) Does anyone in your class have a hand-span greater than 24 cm?
 Use your hand-span to estimate
 (d) the width of your desk to the nearest 10 cm,

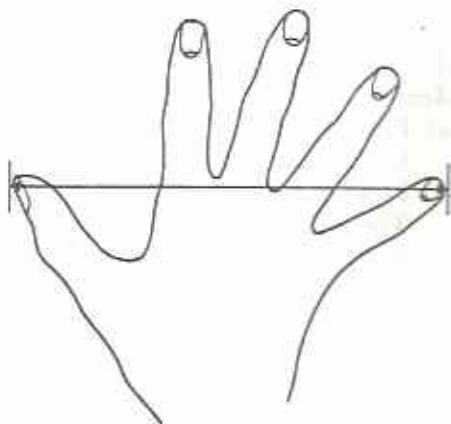


Fig. 16.2 Hand-span: distance between out-stretched tips of small finger and thumb

- (e) the width of the blackboard,
 (f) Use a metre rule to check your estimates in (d) and (e).
 2 (a) Measure the length of a new pencil.
 (b) Use the pencil to estimate the width of your desk.
 (c) Does your answer agree with that of question 1(d)?

- 3 Go outside and walk ten paces (Fig. 16.3)



Fig. 16.3 Pace: length of step in normal walking

- (a) Use a measuring tape, or a metre rule, to measure the distance you have walked.
 (b) Calculate the length of one of your paces.

For example, if you walked 6,3 metres, then

$$\begin{aligned} 1 \text{ pace} &= 6,3 \text{ m} \div 10 \\ &= 0,63 \text{ m} \\ &= 63 \text{ cm} \end{aligned}$$

- 4 Measure the following distances in paces.
 (a) From the front of the classroom to the back of the classroom.
 (b) From the classroom door to the library.
 (c) The length of a football field or netball court.
 (d) From the School gate to the Principal's office.
 (e) Another distance of your own choice.

- 5 Use your result in question 3 to estimate the distances in question 4.

For example, if the football field is 126 paces long, then

$$\begin{aligned} \text{length of football field} &\approx 0,63 \text{ m} \times 126 \\ \text{(rounding off)} &\approx 0,6 \times 130 \\ \text{length of football field} &\approx 78 \text{ m} \end{aligned}$$

- 6 If possible, use a tape measure to check some of your estimates in questions 4 and 5.

- 7 Two students, A and B, walked across the width of a school assembly area. A took 30 paces. B took 36 paces. Which of the following could be reasons why their results are different?

- (a) A is lazy
 (b) B is a girl
 (c) A is quicker than B
 (d) B has longer legs than A
 (e) A has a longer pace than B
 (f) B was carrying a bucket of water

- 8 Find an empty litre bottle and a cup.

- (a) Fill the litre bottle full of water.
 (b) Find how many cups of water the litre bottle holds.
 (c) Calculate the approximate capacity, to the nearest 50 ml, of the cup.
 (d) Find how many cups of water it takes to fill an empty bucket (or large tin).
 (e) Hence calculate the approximate capacity of the bucket (or tin).

- 9 Use a balance from the science laboratory.

- (a) Find a stone which has a mass of about 1 kg.
 (b) Using your hands as a balance, try to find three things which have the same mass as the stone.
 (c) Check your estimate by measuring the three things on the balance.

- 10 Copy and complete Table 16.5 with your personal statistics.

Table 16.5

name	
my height	cm
my mass	kg
my hand-span	cm
my pace	cm
my foot	cm

Chapters 9–16

Revision exercise 4 (Chapters 9, 12)

- How many degrees in
 - a right angle,
 - $\frac{1}{6}$ of a revolution,
 - $2\frac{1}{2}$ revolutions,
 - $1\frac{1}{8}$ revolutions?
- State whether the following angles are acute, obtuse or reflex.
 - 212°
 - 81°
 - 95°
 - 5°
 - 179°
 - 198°
- Use a protractor to measure \widehat{AOB} and \widehat{BOC} in Fig. R6.

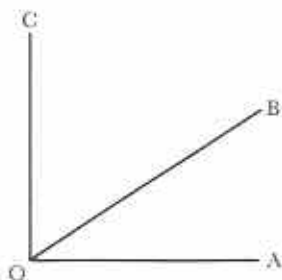


Fig. R6

- Use a protractor to construct angles of
 - 40° ,
 - 90° ,
 - 150° .
- Construct a copy of Fig. R7 such that $\widehat{AOB} = 66^\circ$. Measure \widehat{BOC} .

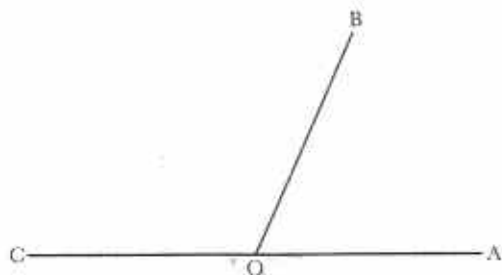


Fig. R7

- Find in degrees the angle between the hour hand and the minute hand of a clock at half past 10.
- How many lines of symmetry do the following have?
 - a rectangle
 - a square
 - an equilateral triangle
 - an isosceles triangle
- Two sides of an isosceles triangle are 3 cm and 10 cm. What must be the length of the third side?

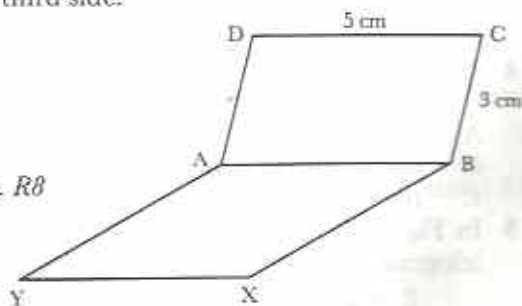


Fig. R8

- In Fig. R8, ABCD is a parallelogram and ABXY is a rhombus. If $BC = 3$ cm and $CD = 5$ cm, find the perimeter of the whole shape.

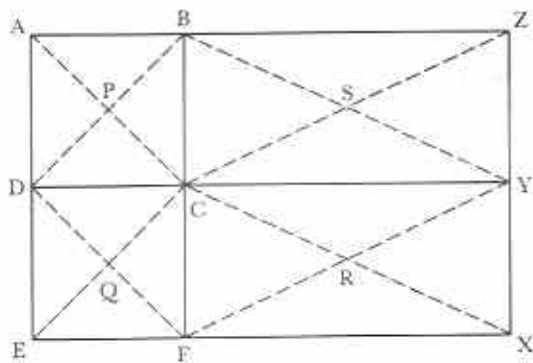


Fig. R9

- 10 In Fig. R9, ABCD and CDEF are squares and CFXY and BCYZ are rectangles. Their diagonals cross at P, Q, R, S respectively. What kind of quadrilateral is
- (a) BPCS, (b) CQFR, (c) PDQC,
 (d) SCRY, (e) SCFY, (f) BDFY,
 (g) PDEC, (h) PQRS?

Revision test 4 (Chapters 9, 12)

- The angle between the hands of a clock at 2 o'clock is
 A 2° B 24° C 30°
 D 60° E 72°
- The number of degrees in $\frac{1}{8}$ of a revolution is
 A 8 B $12\frac{1}{2}$ C $22\frac{1}{2}$
 D 45 E 60
- Which one of the following has *no* lines of symmetry?
 A circle B regular hexagon
 C isosceles triangle D equilateral triangle
 E scalene triangle
- The diagonals of one of the following *always* cross at right angles. Which one?
 A rectangle B square
 C parallelogram D trapezium
 E regular pentagon
- In Fig. R10, PQRT and TQRS are parallelograms.

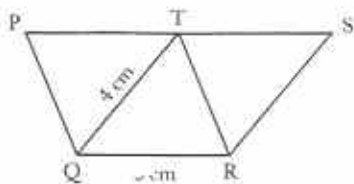


Fig. R10

QR = 3 cm and TQ = 4 cm. What is PS?

- A 3 cm B 4 cm C 5 cm
 D 6 cm E 7 cm
- 6 Use a protractor to measure \widehat{ABC} in Fig. R11.

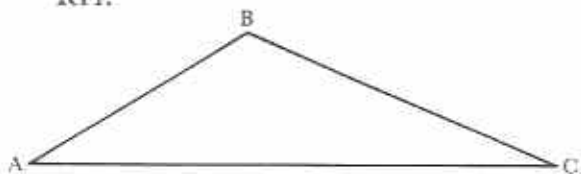


Fig. R11

- Use a protractor to draw an angle of 56° .
- Find, in degrees, the reflex angle between the hour hand and the minute hand of a clock at half past 3.
- Name four quadrilaterals which have at least one pair of parallel sides.
- Sketch a quadrilateral which has only one line of symmetry.

Revision exercise 5 (Chapters 10, 11)

- Simplify the following.
 (a) $3,45 \times 1\,000$ (b) 80×10^2
 (c) $5,15 \div 100$ (d) $247 \div 10^3$
- Simplify the following.
 (a) $9,04 + 6,7$ (b) $9,04 - 6,7$
 (c) $8 \times 0,5$ (d) $0,6 \times 0,04$
 (e) $\frac{0,42}{7}$ (f) $36 \div 0,0004$
- How many cans, each 1,8 litres in capacity, can be filled from a tank containing 54 litres?
- A man bought a house for \$25 350. After 5 years its value had increased by $33\frac{1}{2}\%$. Calculate its value after 5 years.
- Express 0,275
 (a) as a fraction in its lowest terms,
 (b) as a percentage.
- Divide 4,914 by 0,091.
- Simplify the following.
 (a) $(-3) + (+8)$ (b) $(+5) - (+14)$
 (c) $(-7) - (+7)$
- Simplify the following.
 (a) $-4 + (-7)$ (b) $-2 - (-10)$
 (c) $6 + (-9)$
- In the year AD21, a woman was 36 years old. In what year was she 12 years old?
- Simplify the following.
 (a) $2\frac{2}{3} - 4\frac{1}{2}$ (b) $9,8^\circ\text{C} - 18^\circ\text{C}$

Revision test 5 (Chapters 10, 11)

- $1\,200 \div 0,04 =$
 A 30 000 B 3 000 C 300
 D 30 E 3
- If $23 \times 54 = 1\,242$, then $1,242 \div 0,54 =$
 A 0,023 B 0,23 C 2,3
 D 23 E 230
- Which one of the following numbers is the greatest?
 A -2 B -30 C -100 D -50 E -3

- 4 $-20 - (-70) =$
 A -90 B -50 C $+50$
 D $+90$ E None of these
- 5 $3 - (-8) - 5 =$
 A -12 B -2 C $+6$
 D $+12$ E $+16$
- 6 Express the following fractions as recurring decimals.
 (a) $\frac{5}{6}$ (b) $\frac{3}{9}$ (c) $\frac{5}{11}$ (d) $\frac{5}{12}$
- 7 What percentage of the letters in the word *protractor* are vowels?
- 8 A trader bought a pair of shoes for \$12,40. He sold them at a profit of 35%. What was the selling price of the shoes?
- 9 The temperature inside a refrigerator is $2,4^{\circ}\text{C}$. What will be the temperature if it falls by $3,9^{\circ}\text{C}$?
- 10 Simplify the following.
 (a) $3 - 11$ (b) $-9 + 4$
 (c) $8 - (-15)$ (d) $-6 + (-6)$

Revision exercise 6 (Chapters 13, 16)

- 1 Say whether each of the following is true or false.
 (a) $11 - x = 8$ when $x = 19$.
 (b) $36 = 24x$ when $x = 1\frac{1}{2}$
- 2 Solve the following.
 (a) $5x = 35$ (b) $4x = 18$
 (c) $6x - 2c = 12$ (d) $\frac{x}{5} = 20$
 (e) $\frac{28}{x} = 7$ (f) $\frac{3}{4}w = 15$
- 3 Solve the following.
 (a) $x - 4 = 2$ (b) $8 = \frac{1}{2}x$
 (c) $12 - x = 0$ (d) $3a + 5 = 23$
 (e) $12 = 5b - 8$ (f) $7 + 2c = 19$
- 4 Solve the following.
 (a) $2x - 8 = 1$ (b) $2 = 3x - 2$
 (c) $5x + 3 = 10$
- 5 Round off 29 835 to the nearest
 (a) thousand, (b) hundred, (c) ten.
- 6 Round off 0,845 to the nearest (a) tenth, (b) hundredth, (c) whole number.
- 7 Estimate the following.
 (a) $8,6 \times 5,4$ (b) $10\frac{1}{5} \times 5\frac{3}{4}$
 (c) $0,82 \div 0,39$
- 8 A farmer has 385 cattle. The average value

of each beast is \$518. Estimate the total value of the farmer's cattle.

- 9 A box contains eight identical record players. If the mass of the box is 101,6 kg, estimate the mass of one record player.
- 10 Divide 2,647 by 0,9 and give the answer correct to 2 d.p.

Revision test 6 (Chapters 13, 16)

- 1 If $6x + 7 = 55$, then $x =$
 A 7 B 8 C $10\frac{1}{3}$
 D 42 E 48
- 2 67,053 to the nearest tenth is
 A 70 B 67 C 67,0
 D 67,1 E 67,05
- 3 Which one of the following is most likely to be the correct value of $\$3,90 \times 7,8$?
 A 50c B \$20 C \$21,52
 D \$30,42 E \$33,12
- 4 Which one of the following is *not* sensible?
 A The woman's hand-span was 20 cm.
 B The boy ran 100 km in an hour.
 C The height of the tree was 5,8 m.
 D The cup held 280 ml of tea.
 E He cycled from Gweru to Zvishavane in a day.
- 5 What is 0,003 867 to 3 s.f.?
 A 0,004 B 0,003 86 C 0,003 87
 D 386 E 387
- 6 Solve the following.
 (a) $13 - x = 10$ (b) $\frac{a}{3} = 3$
 (c) $y + 8 = 20$ (d) $4n - 3 = 17$
 (e) $50 = 7d + 1$ (f) $12x + 8 = 20$
- 7 Estimate the cost of 20,5 hectares of land at \$1 965 per hectare.
- 8 A hotel bill for nine days was \$397,60. Estimate the daily cost.
- 9 A man's foot is about 28 cm long. He finds that the width of a room is about 11 of his feet. Find the approximate width of the room in metres.
- 10 A student writes 78 words in eight lines of writing.
 (a) Find, to the nearest whole number, the average number of words per line.
 (b) Hence estimate how many lines of writing it will take to write a 1 500-word essay.

Revision exercise 7 (Chapters 14, 15)

- The perimeter of a rectangle is 36 cm. Find the breadth of the rectangle if its length is
(a) 17 cm, (b) 12 cm, (c) 9 cm.
- A wire ring has a diameter of 1 m. Use 3,14 for π to calculate the length of the wire.
- Find the perimeter and area of a rectangle which measures 8 cm by 10 cm.
- Find the circumference and area of a circle of diameter 56 cm. (Use $\frac{22}{7}$ for π .)
- Calculate the area of a parallelogram of height 6 cm and base 9 cm.
- Two sides of a triangle are 7 cm and 4 cm and the angle between them is a right angle. Calculate the area of the triangle.
- A window is in the shape of a semi-circle of diameter 70 cm. Use the value $\frac{22}{7}$ for π to calculate the area of glass in the window.
- A rectangle measures 12 cm by 10 cm. How many 2 cm by 2 cm squares are needed to cover the rectangle completely?
- A lorry has a wheel of diameter 66 cm. Use the value 3 for π to estimate the number of times the wheel turns when the lorry travels 1 km.
- Calculate the perimeter of a rectangle which is 18 cm long and is of area 144 cm².

- Calculate the area of a rectangle which measures 11 cm by 3 cm. Calculate the area of a square with the same perimeter.
- Use 3,14 for π to calculate the area of a circle of radius 3 m.
- Calculate the area of the triangle in Fig. R12. Calculate the height h shown in the diagram.

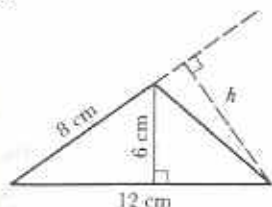


Fig. R12

- A floor 4 m long by $2\frac{1}{2}$ m wide is to be covered with square tiles, each 25 cm by 25 cm. How many tiles are needed?
- Calculate the area of the shape shown in Fig. R13. Use $\frac{22}{7}$ for π .



Fig. R13

Revision test 7 (Chapters 14, 15)

- The perimeter of a rectangle is 26 cm. Its breadth is 4 cm. Its length is
A 9 cm B 11 cm C 13 cm
D 17 cm E 22 cm
- The exact value of π is
A 3,142 B $3\frac{1}{2}$ C 3,14
D 3,1 E impossible to find
- The area of a floor 3 metres square is
A 3 m² B 6 m² C 9 m²
D 300 m² E 90 000 m²
- A triangle and a parallelogram have the same base and same area. If the height of the triangle is 5 cm, the height of the parallelogram is
A 1,25 cm B 2,5 cm C 5 cm
D 10 cm E 25 cm
- The number of cm² in 1 m² is
A 10 B 100 C 1 000
D 10 000 E 100 000

General revision test B (Chapters 9-16)

- Which one of the following has two (and only two) lines of symmetry?
A square
B rectangle
C isosceles triangle
D regular hexagon
E equilateral triangle
- The difference between temperatures of 17 °C above zero and 12 °C below zero is
A 5 °C B 12 °C C 17 °C
D 22 °C E 29 °C
- A square has the same perimeter as a 5 cm by 7 cm rectangle. The area of the square is
A 9 cm² B 25 cm² C 35 cm²
D 36 cm² E 49 cm²
- $16 - x = x$ is true when $x =$
A 0 B 8 C 14
D 16 E 32

- 5 If $3,4 \times 1,8 = 6,12$, then $61,2 \div 0,18 =$
 A 0,34 B 3,4 C 34
 D 340 E 3 400
- 6 In 1990 the estimated population of Zimbabwe was 9 380 000. The area of Zimbabwe is 390 750 km². Estimate the 1990 population density (i.e. the number of people per km²).
 A 0,24 B 0,51 C 2,4
 D 5,1 E 24
- 7 $x = 23\frac{1}{5} \div 8\frac{1}{2}$. Use estimation to decide which one of the following is the accurate value of x .
 A $1\frac{2}{5}$ B $2\frac{4}{5}$ C $3\frac{9}{10}$
 D $4\frac{1}{2}$ E $5\frac{3}{5}$
- 8 A length of wire is given as 6,8 cm correct to 2 s.f. What is the least possible length of the wire?
 A 6,7 cm B 6,74 cm C 6,75 cm
 D 6,8 cm E 6,85 cm
- 9 5 added to a certain number gives a result of -6. The number is
 A -11 B -4 C -1
 D +1 E +11
- 10 $166\frac{2}{3}\%$ of 0,6 is
 A 0,2 B 0,3 C 0,36
 D 1 E 2,7
- 11 Simplify the following.
 (a) $-4 - 9$ (b) $5 - (-12)$
 (c) $-8 - (-3)$ (d) $10 + (-9)$
- 12 A thread is wound 100 times round a reel of diameter 3 cm. Use 3,14 for π to calculate the length of the thread.
- 13 Four discs, each of radius 1 cm, are cut from a 5 cm by 5 cm cardboard square. Use the value 3,14 for π to find the area of cardboard left over.
- 14 What fraction of \$1,75 is 77c? Express this fraction as (a) a decimal, (b) a percentage.
- 15 Solve the following.
 (a) $5 + 8a = 37$ (b) $40 = 14a - 30$
 (c) $2a - 1 = 31$ (d) $3x - 4 = 1$
 (e) $7 = 5 + 5x$ (f) $7 + 8x = 9$
- 16 A man walks at the rate of 88 paces to the minute. If each pace is 0,85 m long, how far does he walk in 10 min?
- 17 Ten tomatoes have a mass of 628 g. A woman buys $2\frac{1}{2}$ kg of tomatoes. Approximately how many tomatoes will she get?
- 18 Express 0,504 6 correct to (a) 3 decimal places, (b) 2 significant figures, (c) the nearest tenth.
- 19 A shopkeeper bought a radio for \$82,50. She sold it at a profit of 42%. What was the selling price?
- 20 Make a drawing like that in Fig. R14 such that $\widehat{ACD} = 114^\circ$. \widehat{A} and \widehat{B} can be any size.

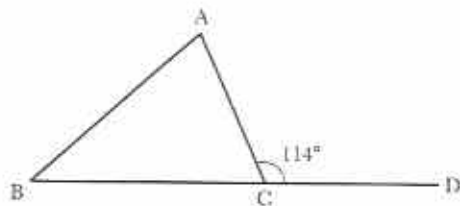


Fig. R14

Measure \widehat{A} and \widehat{B} . Find the sum of \widehat{A} and \widehat{B} .

Angles (2) Angles between lines, in triangles and in quadrilaterals

Syllabus Reference 5.7.1

Exercise 17a revises some of the work you did in Chapter 9. You will need a protractor, ruler and pencil.

Exercise 17a

- Make a drawing like that in Fig. 17.1. BCA is a straight line. \widehat{ACD} can be any size.
 - Measure \widehat{ACD} and \widehat{BCD} .
 - Find the sum of \widehat{ACD} and \widehat{BCD} .
 - Compare your results with other students in your class. What do you notice?

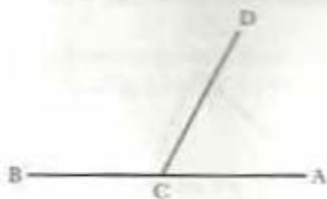


Fig. 17.1

- Draw any two straight lines AB and CD to intersect at a point O (Fig. 17.2).

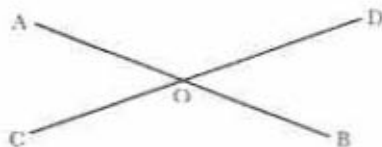


Fig. 17.2

- Measure \widehat{AOC} and \widehat{DOB} . What do you notice?
- Measure \widehat{AOC} and \widehat{AOD} and add them together. What do you notice?
- Guess the size of \widehat{COB} .

(d) Measure \widehat{COB} to see if your guess was correct.

(e) Find the sum of the four angles at O .

- Mark a point O on your paper. Draw any five lines each starting at O . This will give five angles at O (Fig. 17.3). Mark them a, b, c, d, e .

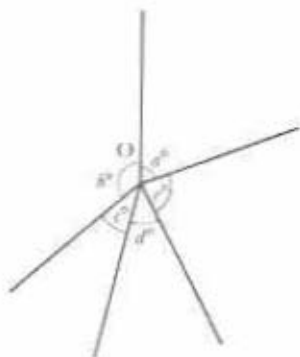


Fig. 17.3

- Measure the five angles, a, b, c, d, e .
- Find the sum of the five angles.
- Compare your results with other students in your class. What do you notice?

While working through the previous exercise you may have discovered some facts about the angles formed when lines meet or cross.

Angles on a straight line

When two angles lie beside each other and have a common vertex, we say they are **adjacent** to each other. In Fig. 17.4 \widehat{AOB} is adjacent to \widehat{BOC} . \widehat{BOC} is adjacent to \widehat{AOB} .

When a straight line stands on another straight line, two adjacent angles are formed.

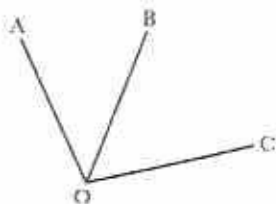


Fig. 17.4

The sum of the two adjacent angles on a straight line is 180° . In Fig. 17.5 $\hat{AOC} + \hat{BOC} = 180^\circ$.

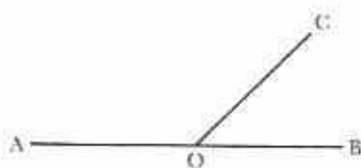


Fig. 17.5

Vertically opposite angles

When two straight lines intersect, they will form four angles. The two angles opposite each other are said to be **vertically opposite**. In Fig. 17.6 AOC is vertically opposite BOD. AOD is vertically opposite BOC. Vertically opposite angles are equal. $\hat{AOC} = \hat{BOD}$. $\hat{AOD} = \hat{BOC}$.

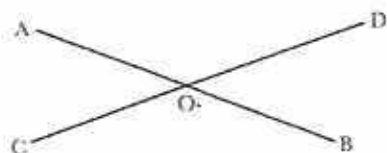


Fig. 17.6

Angles meeting at a point

When a number of lines meet at a point they will form the same number of angles. The sum of the angles at a point is 360° . In Fig. 17.7,

$$\hat{AOB} + \hat{BOC} + \hat{COD} + \hat{DOA} = 360^\circ$$

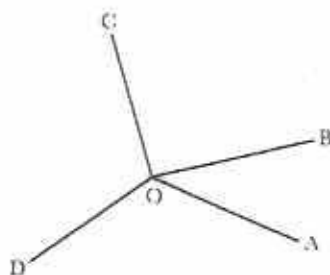


Fig. 17.7

Calculating the sizes of angles

The above facts make it possible to **calculate** the sizes of angles in given figures.

Example 1

In Fig. 17.8 AOB and COD are straight lines. $\hat{BOD} = 62^\circ$ and $\hat{BOE} = 77^\circ$. Calculate the other angles in the figure.

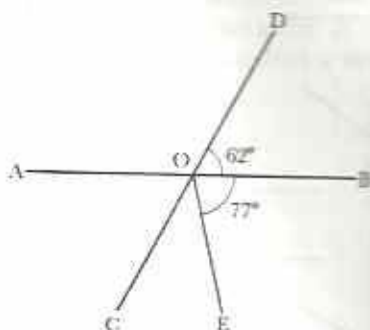


Fig. 17.8

$$\begin{aligned} \hat{AOC} &= 62^\circ \text{ (vertically opposite to } \hat{BOD}\text{)} \\ \hat{AOD} &= 180^\circ - 62^\circ = 118^\circ \text{ (adjacent angles on straight line AOB)} \\ \hat{COE} &= 360^\circ - (62^\circ + 62^\circ + 118^\circ + 77^\circ) \\ &\quad \text{(sum of angles at O)} \\ &= 360^\circ - 319^\circ = 41^\circ \end{aligned}$$

Example 2

In Fig. 17.9 $\hat{APB} = x^\circ$, $\hat{BPC} = \hat{APB}$, \hat{CPD} is twice as big as \hat{APB} and reflex \hat{APD} is five times as big as \hat{APB} . Make an equation in x . Solve the equation and find the four angles.

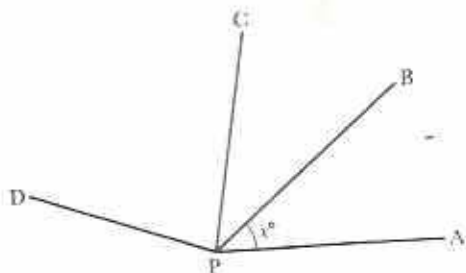


Fig. 17.9

$$\begin{aligned} \hat{APB} &= x^\circ \\ \hat{BPC} &= x^\circ \quad (= \hat{APB}) \\ \hat{CPD} &= 2x^\circ \quad (= 2 \times \hat{APB}) \\ \hat{APD} &= 5x^\circ \quad (= 5 \times \hat{APB}) \end{aligned}$$

$\widehat{APB} + \widehat{BPC} + \widehat{CPD} + \widehat{APD} = 360^\circ$ (sum of angles at P)

$$x^\circ + x^\circ + 2x^\circ + 5x^\circ = 360^\circ$$

$$9x = 360$$

$$x = \frac{360}{9} = 40$$

The four angles are 40° , 40° , 80° and 200° .

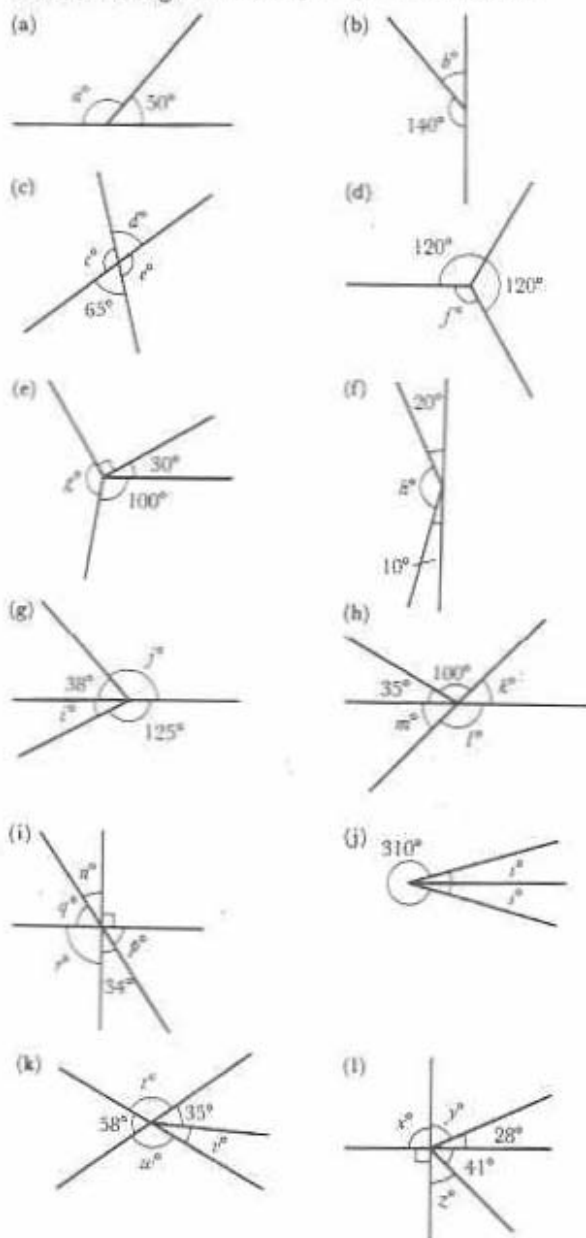


Fig. 17.10

Exercise 17b

1 (Oral) Find the size of the lettered angles in Fig. 17.10. Give reasons.

2 In Fig. 17.11, $\widehat{POR} = 37^\circ$. Calculate the other three angles. Give reasons.

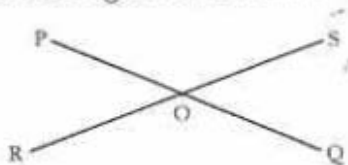


Fig. 17.11

3 In Fig. 17.12, if $\widehat{PXQ} = 61^\circ$ and $\widehat{RXS} = 84^\circ$, calculate \widehat{QXR} .

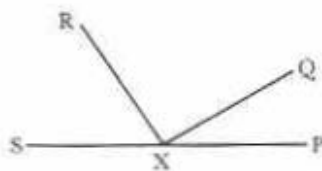


Fig. 17.12

4 In Fig. 17.12, calculate \widehat{RXS} , given that $\widehat{PXQ} = 35^\circ$ and $\widehat{RXQ} = 98^\circ$.

5 In Fig. 17.12, calculate \widehat{PXQ} , if \widehat{QXR} is a right angle and $\widehat{RXS} = 68^\circ$.

6 In Fig. 17.13, $\widehat{BXC} = 36^\circ$ and $\widehat{AXD} = 126^\circ$. If \widehat{BXD} is a right angle, calculate \widehat{CXD} and \widehat{AXB} .

Find the size of \widehat{AXC} . In what way could the drawing be made better?

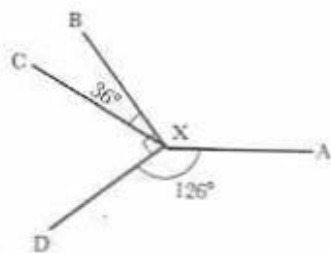


Fig. 17.13

7 In Fig. 17.14, the angles marked x° are equal to each other and the angles marked y° are equal to each other. Make an equation using

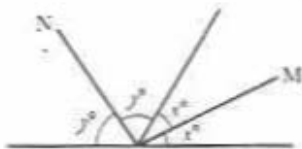


Fig. 17.14

the letters x and y . Hence calculate $\hat{M}\hat{O}\hat{N}$.

- 8 In Fig. 17.15, $\hat{E}\hat{K}\hat{F} = x^\circ$, $\hat{F}\hat{K}\hat{G}$ is twice as big as $\hat{E}\hat{K}\hat{F}$, $\hat{G}\hat{K}\hat{H}$ is three times as big as $\hat{E}\hat{K}\hat{F}$ and $\hat{H}\hat{K}\hat{E}$ is four times as big as $\hat{E}\hat{K}\hat{F}$. Make an equation in x . Solve the equation to find the four angles. Check your answer by finding the sum of the four angles.

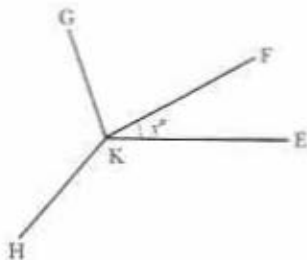


Fig. 17.15

- 9 In Fig. 17.16 $\hat{V}\hat{X}\hat{W} = 2 \times \hat{U}\hat{X}\hat{V}$ and $\hat{W}\hat{X}\hat{Y} = 3 \times \hat{V}\hat{X}\hat{W}$. Calculate $\hat{U}\hat{X}\hat{V}$. (Hint: let $\hat{U}\hat{X}\hat{V} = x^\circ$. Form an equation in x and then solve it.)

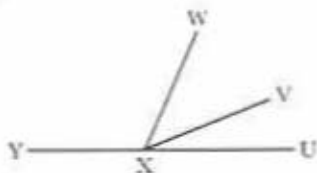


Fig. 17.16

Parallel lines

Usually, if two straight lines are drawn on a plane, they will intersect if the lines are **produced** (i.e. extended) far enough (Fig. 17.17).

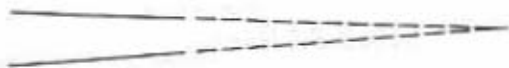


Fig. 17.17

If the lines never meet, however far they are produced, we say that they are **parallel**. For example, the lines in your exercise book are parallel to each other. We sometimes show that lines are parallel by drawing arrow heads on them as in Fig. 17.18.



Fig. 17.18

Notice that the distance between a pair of parallel lines is always the same. A line cutting a pair of parallel lines is called a **transversal** (Fig. 17.19).

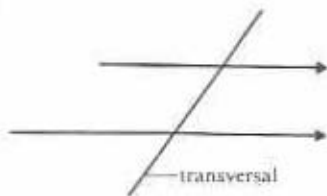


Fig. 17.19

Exercise 17c

- 1 Fig. 17.20 represents a view of a hut.

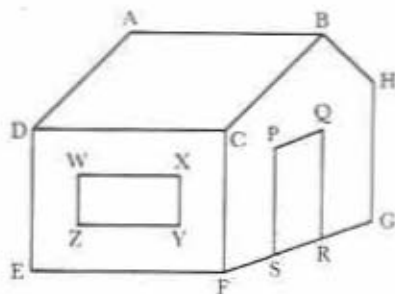


Fig. 17.20

- Name as many lines as you can which are parallel to AB.
 - Name as many lines as you can which are parallel to XY.
 - Name as many lines as you can which are parallel to PQ.
 - Is any line parallel to BC?
 - Is any line parallel to BH?
- 2 Use the ruled lines in your exercise book to draw a pair of parallel lines as in Fig. 17.21. Draw a transversal in any position.

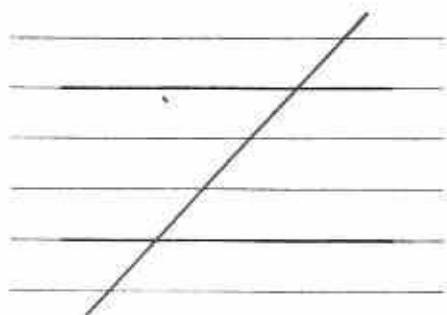


Fig. 17.21

- (a) How many angles have you drawn?
 (b) In Fig. 17.22 the marked angles are called **corresponding angles**. They are in the same, or corresponding, positions, at the two intersections.



Fig. 17.22

Measure the sizes of the two corresponding angles on your own diagram. What do you notice?



Fig. 17.23

- (c) Fig. 17.23 shows two other pairs of corresponding angles. Draw a sketch to show one more pair of corresponding angles.

Measure each pair on your diagram. What do you notice? Corresponding angles are sometimes called **F angles**. Can you think why?

- (d) In Fig. 17.24, the marked angles are called **alternate angles**. Measure the size of this pair of alternate angles on your diagram. What do you notice?

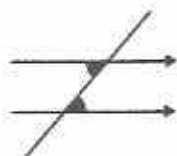


Fig. 17.24

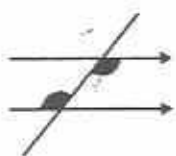


Fig. 17.25

- (e) There is one other pair of alternate angles (Fig. 17.25). Measure these angles on your diagram. What do you notice? Alternate angles are sometimes called **Z angles**. Can you think why?
 (f) In Fig. 17.26, the marked angles are called **allied** or **co-interior angles**. Measure the size of any two allied angles on your diagram. Add your results. What do you notice?

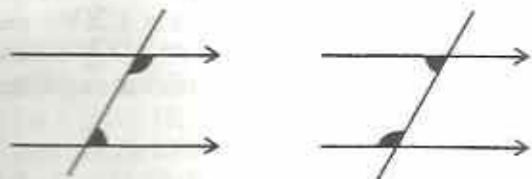


Fig. 17.26

- 3 In Fig. 17.27 name the angle which
 (a) corresponds to $\hat{A}\hat{X}P$, $\hat{B}\hat{X}Y$, $\hat{Q}\hat{Y}D$, $\hat{C}\hat{Y}Q$;
 (b) is alternate to $\hat{B}\hat{X}Y$, $\hat{X}\hat{Y}D$;
 (c) is allied with $\hat{B}\hat{X}Y$;
 (d) is co-interior with $\hat{C}\hat{Y}X$.

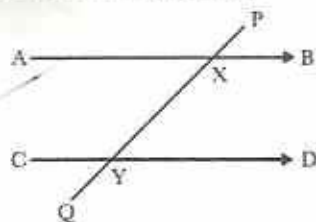


Fig. 17.27

- 4 In Fig. 17.28, which angle corresponds to
 (a) \hat{r} , (b) \hat{p} , (c) \hat{z} , (d) \hat{j} ?
 Which angle is alternate to
 (e) \hat{i} , (f) \hat{m} , (g) \hat{u} , (h) \hat{l} ?
 Which angle is allied with
 (i) \hat{l} (j) \hat{w} ?
 Which angle is co-interior with
 (k) \hat{n} (l) \hat{t} ?

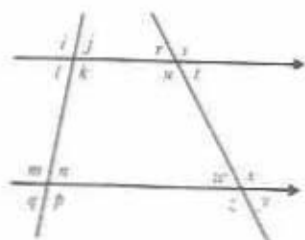


Fig. 17.28

- 5 Fig. 17.29 shows a pair of parallel lines and a transversal intersecting at X and Y. One angle is given as 80° . Sketch a copy of the diagram. Fill in the sizes of all the angles at X. Try to fill in the sizes of the angles at Y.

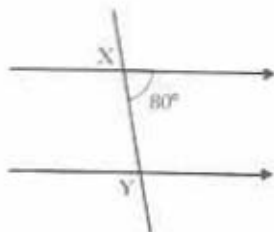


Fig. 17.29

While working through Exercise 17c you may have noticed the following facts about the angles formed when a transversal crosses parallel lines.

Corresponding angles

Corresponding angles are equal (Fig. 17.30).

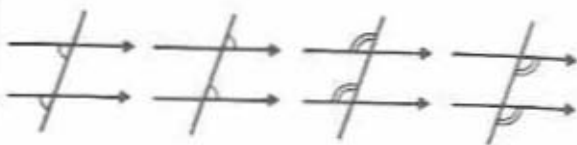


Fig. 17.30

Alternate angles

Alternate angles are equal (Fig. 17.31).

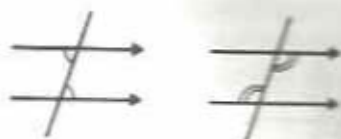


Fig. 17.31

Note: it is possible to have corresponding and alternate angles with lines which are not parallel. In Fig. 17.32, angle a corresponds to angle b ; angle x is alternate to angle y . However, since the lines are not parallel, the angles are not equal, i.e. $a \neq b$ and $x \neq y$.

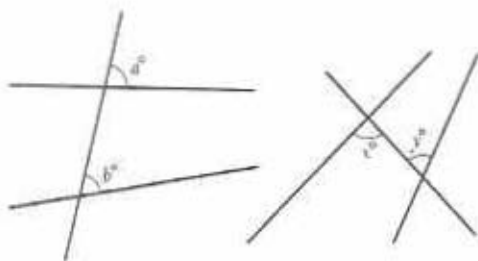


Fig. 17.32

Allied (co-interior) angles

The sum of two allied (or co-interior) angles is 180° (Fig. 17.33).

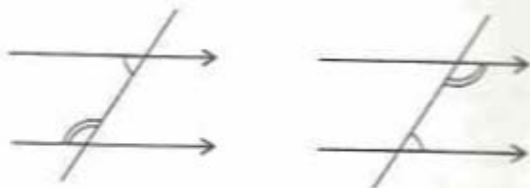


Fig. 17.33

Exercise 17d

1 (Oral) Find the sizes of the lettered angles in Fig. 17.34. Give reasons.

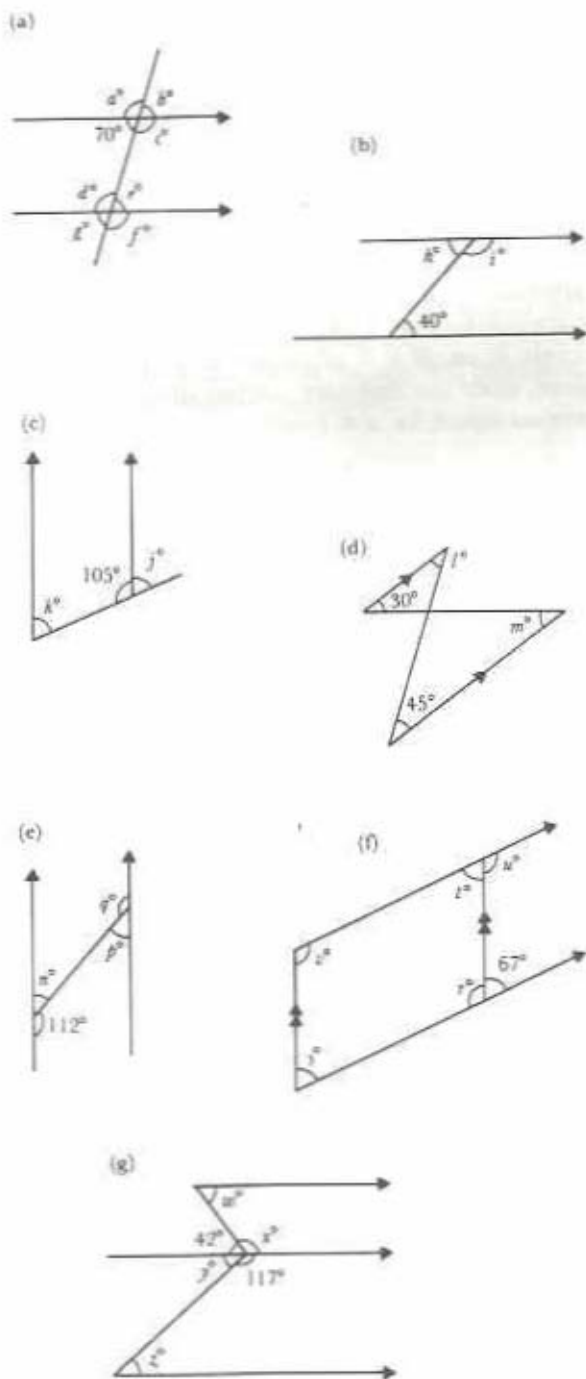


Fig. 17.34

2 Sketch a copy of each diagram in Fig. 17.35. Do not make an accurate drawing. Fill in the sizes of the missing angles.

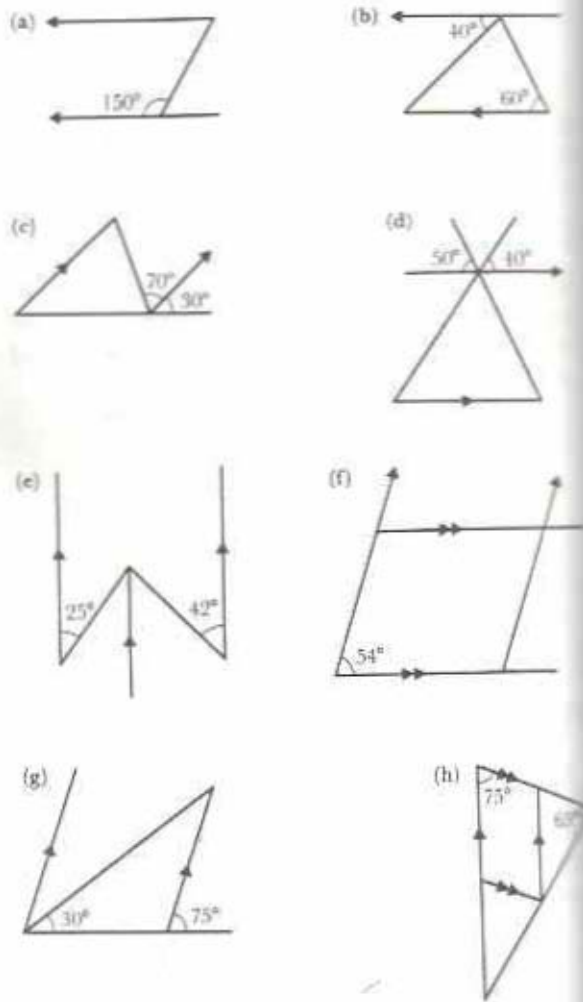


Fig. 17.35

Angles in a triangle

Exercise 17e

- Use a protractor to measure the angles in triangles ABC and PQR in Fig. 17.36.
- Find the sum of the angles of ΔABC (i.e. find $\hat{A} + \hat{B} + \hat{C}$).
- Find the sum of the angles of ΔPQR (i.e. find $\hat{P} + \hat{Q} + \hat{R}$).
- What do you notice about your results in (b) and (c)?

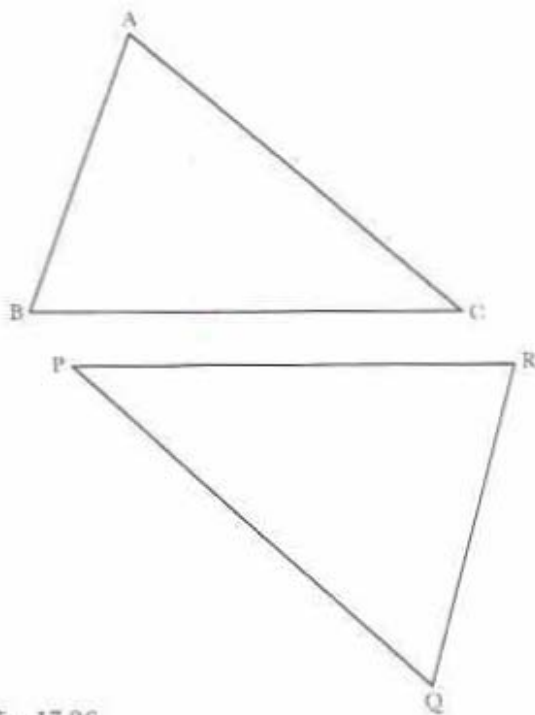


Fig. 17.36

- 2 Draw two large triangles. One of them should include an obtuse angle.
- Measure the angles in each triangle.
 - Find the sum of the angles in each triangle.
 - What do you notice about your results in (b)? Do your friends get the same kinds of results?
- 3
- Draw any triangle. Cut it out carefully along its sides.
 - Tear off the three angles of the triangle as in Fig. 17.37.

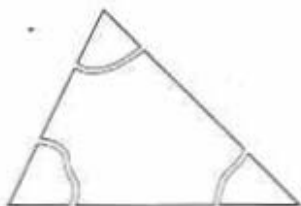


Fig 17.37



Fig. 17.38

- Take the three angles and arrange them so that they are adjacent to each other as in Fig. 17.38.

- What do you notice? What is the sum of the angles on a straight line?

When working through Exercise 17e you may have noticed that the **sum of the angles of a triangle is 180°** . This is true for any triangle.

We can show this in the following way. In Fig. 17.39, ABC is any triangle. Its angles are x° , y° and z° .

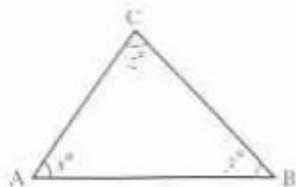


Fig. 17.39

In Fig. 17.40 PCQ is a line through C parallel to AB.

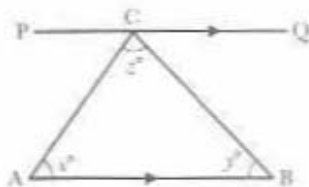


Fig 17.40

We can use the alternate angles fact as in Fig. 17.41 to fill in the missing angles at C.

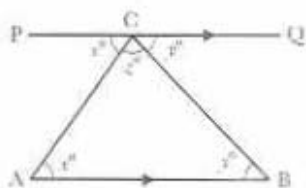


Fig. 17.41

At C, the three angles are adjacent on a straight line. Thus $x^\circ + y^\circ + z^\circ = 180^\circ$. But x° , y° and z° are also the sizes of the angles of the ΔABC . Thus the sum of the angles of any triangle is 180° .

Use this fact in Exercise 17f.

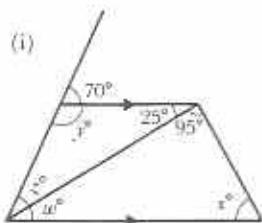
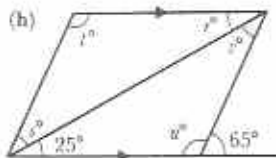
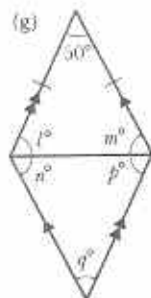
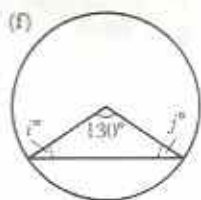
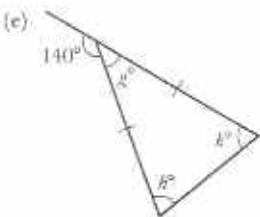
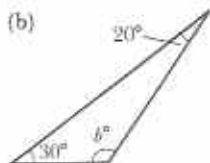
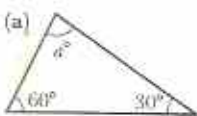
Exercise 17f

1 (Oral) State the sizes of the lettered angles in Fig. 17.42. Give reasons.

Note: in any diagram, lines marked with a small line are equal in length.

2 Calculate the third angle of a triangle in which two of the angles are as follows:

- (a) 47° and 65° (b) 24° and 77°
 (c) 56° and 18° (d) 39° and 21°
 (e) each 58° (f) 103° and 42°
 (g) 69° and 46° (h) 38° and 71°
 (i) 43° and 94° (j) 60° and 60°
 (k) 36° and 55° (l) 58° and 25°



3 The angles in Table 17.1 refer to the diagram in Fig. 17.43. Calculate the missing angles in each row.

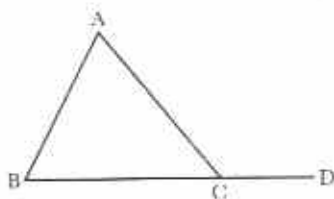


Fig. 17.43

Table 17.1

	\widehat{ABC}	\widehat{BAC}	\widehat{ACB}	\widehat{ACD}
(a)	58°	47°		
(b)	65°			118°
(c)		19°		86°
(d)	46°		35°	

4 In Fig. 17.44, $\widehat{ABC} = x^\circ$, \widehat{BAC} is twice as big as \widehat{ABC} and \widehat{ACB} is three times as big as \widehat{ABC} . Make an equation in x . Solve the equation. Find the three angles of the triangle.

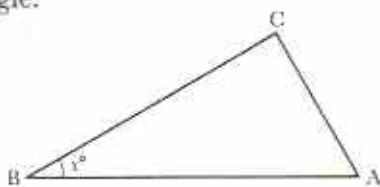


Fig. 17.44

5 In Fig. 17.45, ABCDE is a regular pentagon. Its centre, O, is joined to each vertex.

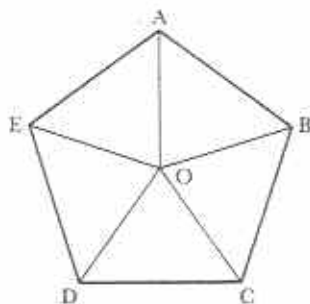


Fig. 17.45

Fig. 17.42

- (a) Calculate the size of each angle at O.
 (b) What kind of triangle is $\triangle AOB$?
 (c) Calculate the angles of $\triangle AOB$.

6 Sketch a copy of each diagram in Fig. 17.46. Fill in the missing angles.

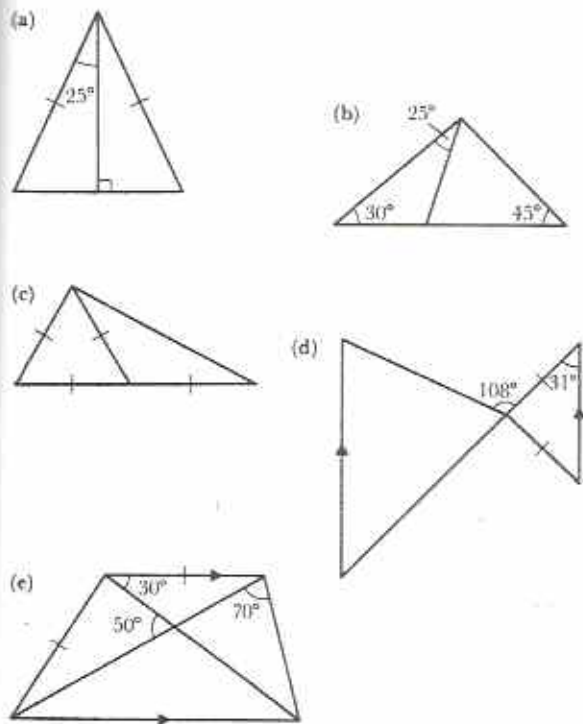


Fig. 17.46

Exterior angles of a triangle

In $\triangle ABC$ (Fig. 17.47) side BC is **produced** to X . To produce a line means to make it longer. \widehat{ACX} is called the **exterior angle** of the triangle. \widehat{ABC} and \widehat{BAC} are called the **opposite interior angles**.

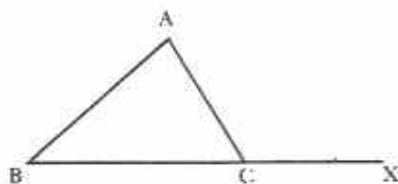


Fig. 17.47

Each side of the triangle can be produced in two directions. In Fig. 17.48 CB is produced to Y .

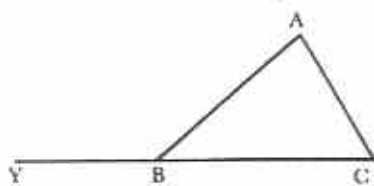


Fig 17.48

Every triangle has three exterior angles. These are labelled x, y and z in Fig. 17.49.

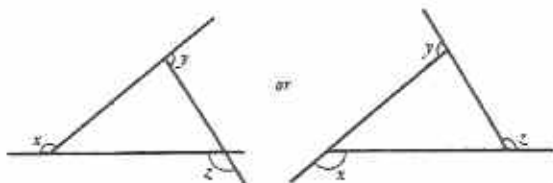


Fig. 17.49

Exercise 17g (Oral/Discussion)

1 Calculate the sizes of the lettered angles in Fig. 17.50.

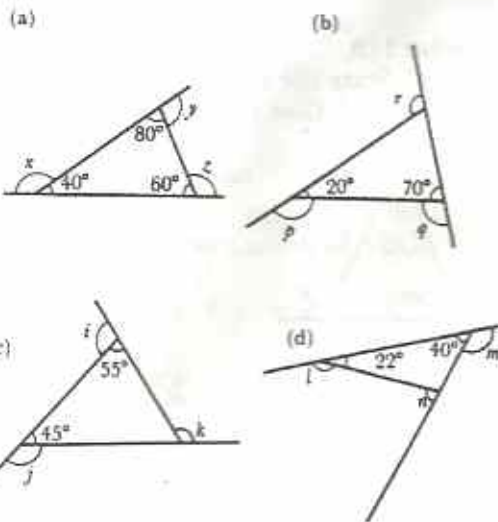


Fig. 17.50

2 Calculate the interior angles in Fig. 17.51.

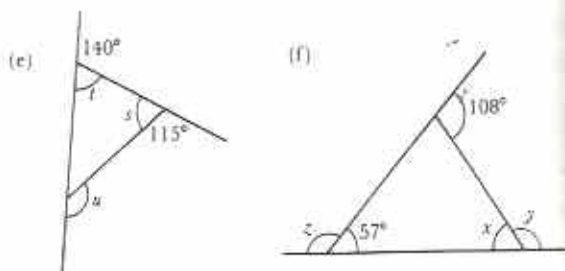
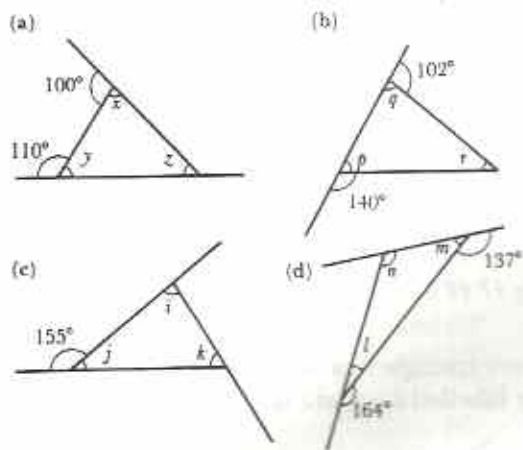


Fig. 17.52

2 In Fig. 17.47, if $\hat{BAC} = 60^\circ$ and $\hat{ACX} = 100^\circ$, calculate \hat{ABC} .

3 Calculate the missing angles in Fig. 17.53.

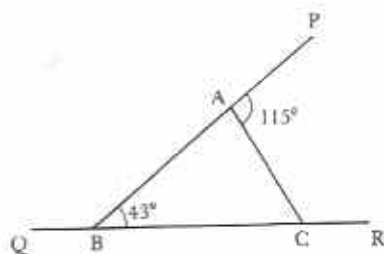


Fig. 17.53

4 Calculate the lettered angles in Fig. 17.54.

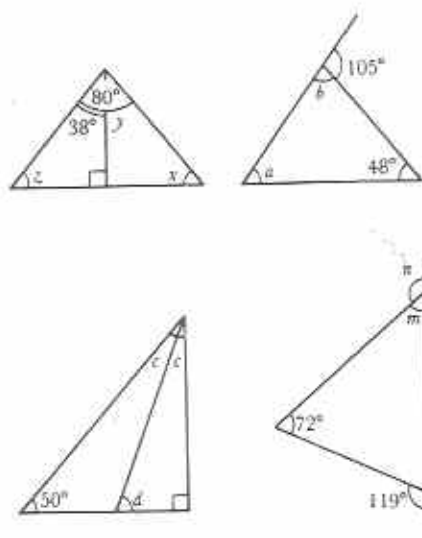


Fig. 17.54

Fig. 17.51

3 For each exterior angle in Figs. 17.50 and 17.51 find the sum of the two opposite interior angles.

4 Find the sum of the exterior angles of each triangle in Figs. 17.50 and 17.51.

The results of Exercise 17g demonstrate the following facts:

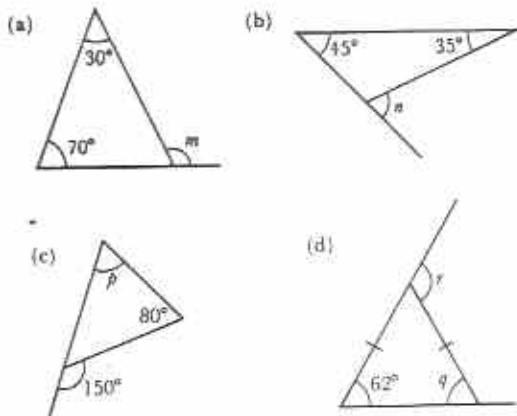
1 The exterior angle of a triangle is equal to the sum of the opposite interior angles.

2 The sum of the exterior angles of a triangle is 360° .

Use these facts in Exercise 17h.

Exercise 17h

1 (Oral) State the sizes of the lettered angles in Fig. 17.52. Give reasons.



- 5 Calculate the values of w , x , y and z in Fig. 17.55.

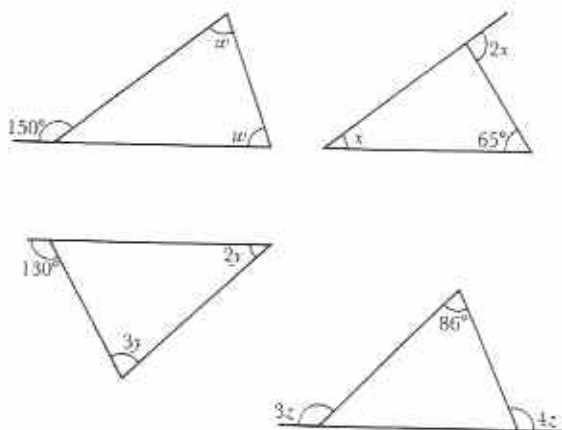


Fig. 17.55

- (b) Take the four angles and arrange them so that they are adjacent to each other as in Fig. 17.57.
 (c) What do you notice? What is the sum of the angles at a point?

When working through Exercise 17i you may have noticed that the **sum of the angles of a quadrilateral is 360°** . We can show that this is true as follows.

Any quadrilateral can be divided into two triangles by drawing its diagonals (Fig. 17.58).

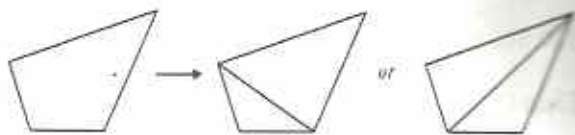


Fig. 17.58

Angles in a quadrilateral

Exercise 17i

- 1 Draw two large quadrilaterals.
 (a) Measure the angles in each quadrilateral.
 (b) Find the sum of the angles in each quadrilateral.
 (c) What do you notice about your results? Do your friends get the same kinds of results?
- 2 Draw any large quadrilateral on a sheet of scrap paper. Cut it out carefully along its sides.
 (a) Tear off the four angles of the quadrilateral as in Fig. 17.56.

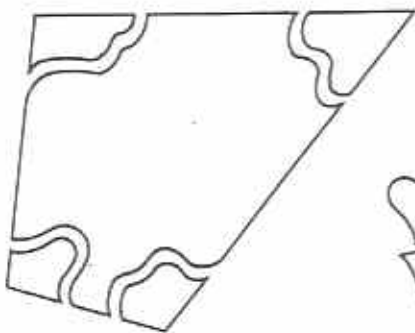


Fig. 17.56



Fig. 17.57

The sum of the angles in each triangle is 180° . Thus the sum of the angles in the quadrilateral is 360° . For example, with the lettering of Fig. 17.59:

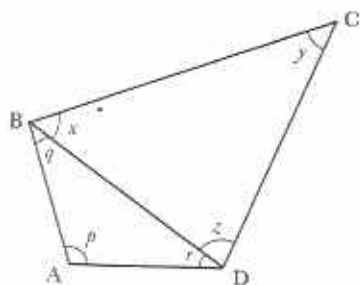


Fig. 17.59

$$p + q + r = 180^\circ \text{ (angles of } \triangle ABD)$$

$$x + y + z = 180^\circ \text{ (angles of } \triangle BCD)$$

thus

$$p + q + r + x + y + z = 180^\circ + 180^\circ = 360^\circ$$

In the quadrilateral,

$$\begin{aligned} \hat{A} &= p, \hat{B} = q + x, \hat{C} = y, \hat{D} = r + z \\ \hat{A} + \hat{B} + \hat{C} + \hat{D} &= p + (q + x) + y + (r + z) \\ &= p + q + r + x + y + z \\ &= 360^\circ \end{aligned}$$

Example 3

Find the value of x in Fig. 17.60. Hence find the other two angles of the quadrilateral.

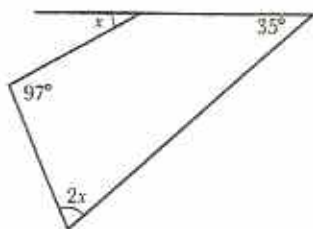


Fig. 17.60

The 4th angle of the quadrilateral = $180^\circ - x$
(angles on straight line)

Thus $(180^\circ - x) + 2x + 97^\circ + 35^\circ = 360^\circ$
(sum of angles of quadrilateral)

$$2x - x + 180^\circ + 97^\circ + 35^\circ = 360^\circ$$

$$x + 312^\circ = 360^\circ$$

$$x = 360^\circ - 312^\circ$$

$$x = 48^\circ$$

$$2x = 2 \times 48^\circ = 96^\circ$$

$$180^\circ - x = 180^\circ - 48^\circ = 132^\circ$$

The other two angles of the quadrilateral are 96° and 132° .

Check: $96^\circ + 132^\circ + 97^\circ + 35^\circ = 360^\circ$.

Exercise 17j

1 Find the sizes of all the angles of the quadrilaterals in Fig. 17.61.

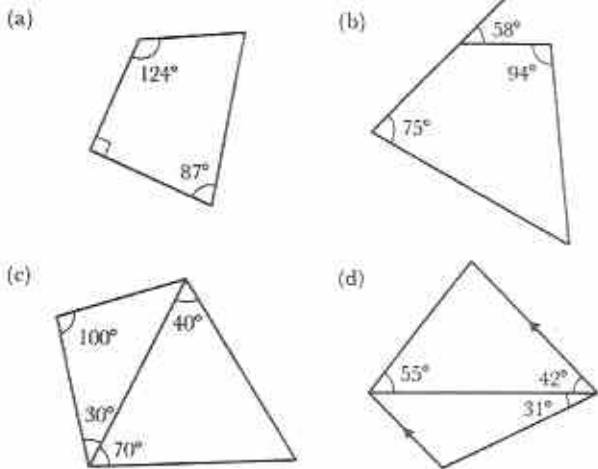


Fig. 17.61

2 Calculate the 4th angle of the quadrilaterals whose other three angles, in order, are:

(a) $100^\circ, 60^\circ, 80^\circ$ (b) $58^\circ, 117^\circ, 122^\circ$

(c) $95^\circ, 85^\circ, 90^\circ$ (d) $109^\circ, 71^\circ, 109^\circ$

(e) $114^\circ, 95^\circ, 114^\circ$

3 Make rough sketches of quadrilaterals (c) and (d) of question 2. What types of quadrilaterals are they?

4 In Fig. 17.62, first find the value of x , then find the unknown angles of the quadrilaterals.

5 The angles of a quadrilateral are $x, 2x, 3x$ and $4x$ in that order.

(a) Make an equation in x .

(b) Find x .

(c) Find the angles of the quadrilateral.

(d) Make a rough drawing of the quadrilateral.

(e) What kind of quadrilateral is it?

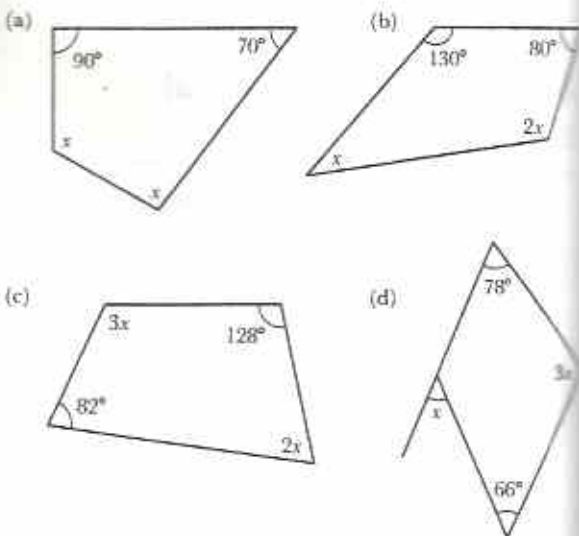


Fig. 17.62

Volume and capacity

Syllabus Reference 5.4.2

Volume

The **volume** of a solid is a measure of the space it takes up. The cube is used as the shape for the basic unit of volume. A cube of edge 1 metre has a volume of **1 cubic metre** or **1 m³**. A cube of edge 1 centimetre has a volume of **1 cubic centimetre** or **1 cm³** (Fig. 18.1).

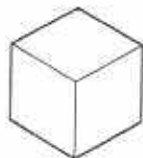


Fig. 18.1 1 cm³

It is difficult to measure volume directly. One way is to build a copy of the solid using basic units. For example, to measure the volume of the 4 cm by 3 cm by 2 cm cuboid in Fig. 18.2, a copy can be built from 1 cm³ cubes as in Figs. 18.3, 18.4 and 18.5.

The volume of the cuboid is 24 cm³.

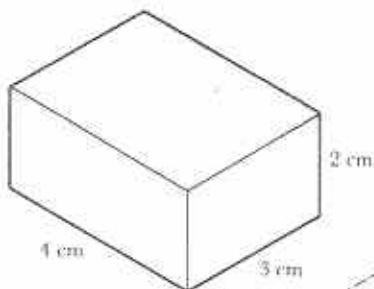


Fig. 18.2

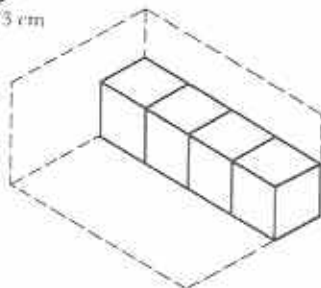


Fig. 18.3
1 row of 4 cubes

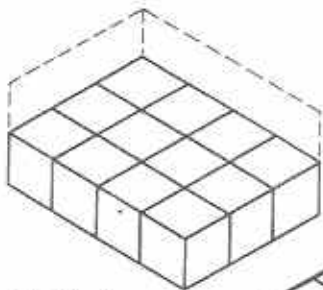


Fig. 18.4
3 rows of 4 cubes,
i.e. 1 layer of 12 cubes

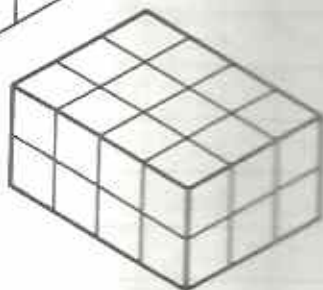


Fig. 18.5
2 layers of 12 cubes, i.e. 24 cubes altogether

Volume of a cuboid

Notice that

- the 4 cm by 3 cm by 2 cm cube in Fig. 18.2 has a volume of 24 cm³; and
- $4 \times 3 \times 2 = 24$.

We can find the volume of any cuboid by finding the product of its length, breadth and height:

volume of cuboid

$$\begin{aligned} &= \text{length} \times \text{breadth} \times \text{height} \\ &= \text{area of base} \times \text{height} \\ &= \text{area of end face} \times \text{length} \\ &= \text{area of side face} \times \text{breadth} \end{aligned}$$

Example 1

Calculate the volume of a rectangular box which measures 30 cm × 15 cm × 10 cm.

$$\begin{aligned} \text{Volume of box} &= (30 \times 15 \times 10) \text{ cm}^3 \\ &= 4\,500 \text{ cm}^3 \end{aligned}$$

Example 2

A room 4 m long by 3 m wide contains 30 m^3 of air. Calculate the height of the room.

$$\text{Volume of room} = 30 \text{ m}^3$$

$$\text{Area of floor (base)} = 4 \text{ m} \times 3 \text{ m} = 12 \text{ m}^2$$

$$\text{Height of room} = \frac{30}{12} \text{ m} = 2\frac{1}{2} \text{ m}$$

Exercise 18a

- 1 Copy and complete the table of cuboids (Table 18.1).

Table 18.1

	length	breadth	height	volume
(a)	5 m	2 m	3 m	
(b)	4 m	5 m	3 m	
(c)	3 cm	2 cm	8 cm	
(d)	5 m	2 m	$2\frac{1}{2}$ m	
(e)	3 cm	3 cm	3 cm	
(f)	6 cm	2 cm		24 cm^3
(g)	10 m	5 m		100 m^3
(h)		4 m	3 m	36 m^3
(i)	5 cm		4 cm	120 cm^3
(j)	$5\frac{1}{2}$ m	3 m	$2\frac{1}{2}$ m	

- 2 How many cm^3 are in a cube of edge 2 cm?
 3 A box has a square base of side 9 cm. Calculate the volume of the box if it is 10 cm deep.
 4 A rectangular room 8 m long by 5 m wide contains 120 m^3 of air. Calculate the height of the room.
 5 A rectangular tank 6 m long by 2 m wide holds 36 m^3 of water. How deep is the water in the tank?
 6 Which has the greater volume, a $4 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm}$ cube or a $3 \text{ cm} \times 7 \text{ cm} \times 3 \text{ cm}$ cuboid?
 7 Calculate the volume of air in a dormitory 10 m long, 5 m wide and 3 m high. If each person should have 15 m^3 of air space, how many people can sleep in the dormitory?

- 8 A room is 3 m high and has a volume of 60 m^3 . Calculate the area of the floor of the room.
 9 During a storm, rain falls to a depth of 1.5 cm. What volume of water will collect in a rectangular tank 30 cm by 10 cm?
 10 A concrete block is made by pouring 1000 cm^3 of concrete into a 10 cm by 25 cm rectangular tray. How thick is the block?

Units of volume and capacity

The cubic metre, m^3 , is the basic unit of volume.

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ m}^3 = (100 \times 100 \times 100) \text{ cm}^3 = 1\,000\,000 \text{ cm}^3$$

Similarly

$$1 \text{ cm} = 10 \text{ mm}$$

$$1 \text{ cm}^3 = (10 \times 10 \times 10) \text{ mm}^3 = 1\,000 \text{ mm}^3$$

When calculating problems about volume, make sure that all dimensions are in the same units.

Example 3

A concrete beam is 20 m long. Its end face is a rectangle 60 cm by 40 cm. Calculate the volume of the beam. Find the mass of the beam if 1 m^3 of concrete has a mass of 2.5 tonnes.

Working in metres:

$$\begin{aligned} \text{volume of beam} &= 20 \text{ m} \times 0.6 \text{ m} \times 0.4 \text{ m} \\ &= 4.8 \text{ m}^3 \end{aligned}$$

$$\text{mass of beam} = 4.8 \times 2.5 \text{ t} = 12 \text{ t}$$

The **capacity** of a container is a measure of the space inside it. The basic unit of capacity is the

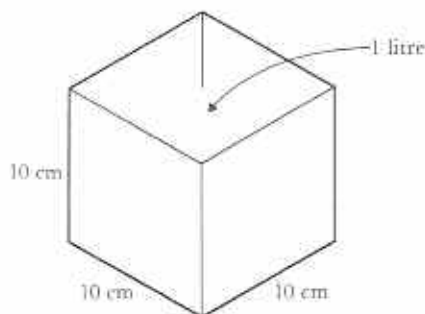


Fig. 18.6

litre; 1 litre of water will just fill a 10 cm by 10 cm by 10 cm cubic container (Fig. 18.6).

Thus, for all practical uses,

$$1 \text{ litre} = (10 \times 10 \times 10) \text{ cm}^3 = 1\,000 \text{ cm}^3$$

Table 18.2 shows the relation between units of capacity and units of volume.

Table 18.2

	capacity	volume
kilolitre	$1 \text{ k}\ell = 1\,000\ell$	$= 1\,000\,000 \text{ cm}^3$ $= 1 \text{ m}^3$
litre	1ℓ	$= 1\,000 \text{ cm}^3$
millilitre	$1 \text{ m}\ell = 0,001\ell$	$= 1 \text{ cm}^3$

Example 4

How many litres of water does a 5 m \times 4 m \times 3 m tank hold?

$$\text{Volume of tank} = (5 \times 4 \times 3) \text{ m}^3 = 60 \text{ m}^3$$

$$\text{but, } 1 \text{ m}^3 = 1\,000 \text{ litres}$$

$$\begin{aligned} \text{Capacity of tank} &= 60 \times 1\,000 \text{ litres} \\ &= 60\,000 \text{ litres} \end{aligned}$$

Exercise 18b

- 1 A rectangular tin measures 10 cm by 10 cm by 20 cm. What is its capacity in litres?
- 2 Calculate the capacity in litres of a tin 20 cm by 20 cm by 10 cm.
- 3 A rectangular tank, $1\frac{1}{2}$ m long and 1 m wide, contains water to a depth of 50 cm. How many litres does it contain?
- 4 How many kilolitres of water are there in a full tank, $5\frac{1}{2}$ m long, 4 m wide and $2\frac{1}{4}$ m deep?
- 5 A school's water tank measures 4 m by 3 m by 2 m. How many litres does it contain when full?

If the school uses about 5 000 litres of water a day, approximately how many days will a full tank last?

- 6 A wooden beam has a rectangular cross face, 24 cm by 15 cm, and is 8 m long. Calculate the volume of the beam.

If the wood has a mass of 700 kg per m^3 , find the mass of the beam.

- 7 A flat rectangular roof measures 7,5 m by 4 m; 12 mm of rain falls on the roof. Find

the mass of water that falls on the roof. (1 cm^3 of water has a mass of 1 gramme.)

- 8 A block of concrete is 1 m long, 50 cm wide and 4 cm thick. Calculate the volume of the block in cm^3 . If 1 cm^3 has a mass of 2,7 g, find the mass of the block in kg.
- 9 A block measures 22 cm by 11 cm by 7 cm. How many of these blocks will be needed to build a wall 5 m long, 22 cm thick and $3\frac{1}{2}$ m high?
- 10 How many 2 cm \times 2 cm \times 2 cm cubes can be packed in a box 1 m long, 20 cm wide and 4 cm deep?

Volume of prisms (optional)

A cuboid can be cut into two equal right-angled triangular prisms as in Fig. 18.7.

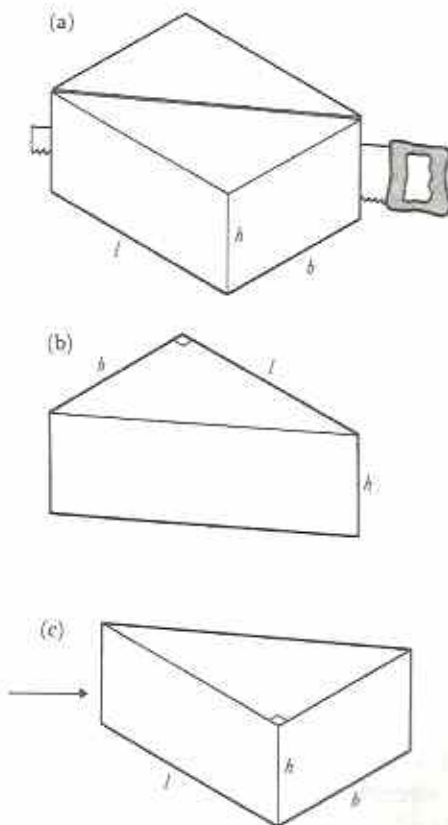


Fig. 18.7 (a) $\text{Volume} = lbh$
(b) $\text{Volume} = \frac{1}{2}lbh$ (c) $\text{Volume} = \frac{1}{2}lbh$

For each prism:

$$\text{volume of prism} = \frac{1}{2}lbh = \left(\frac{1}{2}lb\right) \times h.$$

But, $\frac{1}{2}lb$ = area of end-face because the end-face is a right-angled triangle.

Volume of prism = area of end-face \times height.

Example 5

Calculate the volume of the prism shown in Fig. 18.8.

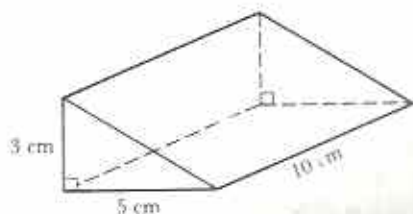


Fig. 18.8

$$\begin{aligned} \text{Area of triangular face} &= \frac{1}{2} \times 5 \times 3 \text{ cm}^2 \\ \text{Volume of prism} &= \frac{1}{2} \times 5 \times 3 \times 10 \text{ cm}^3 \\ &= 75 \text{ cm}^3 \end{aligned}$$

Example 6

A beam has an end-face as shown in Fig. 18.9. Calculate the volume of an 8 m length of the beam.

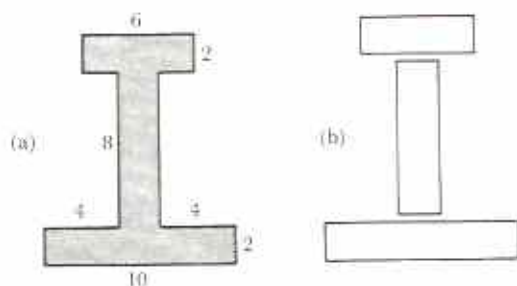


Fig. 18.9

The beam is an I-shaped prism. Its end-face is made up of three rectangles, as shown in Fig. 18.9(b). This gives three cuboids.

Length of each cuboid = 8 m = 800 cm

Volume of beam

$$\begin{aligned} &= (6 \times 2 \times 800 + 8 \times 2 \times 800 \\ &\quad + 10 \times 2 \times 800) \text{ cm}^3 \\ &= 800(12 + 16 + 20) \text{ cm}^3 \\ &= 800 \times 48 \text{ cm}^3 \\ &= 38\,400 \text{ cm}^3 \end{aligned}$$

In Example 6, notice that the area of the end-face of the prism is 48 cm^2 ; its volume is the

product of this area and the length of the prism. In general, for any prism:

volume of prism

= **area of end-face**

\times **distance between end-faces**

Example 7

Fig. 18.10 is a cross-section of an irrigation channel which is made of steel 1 cm thick.

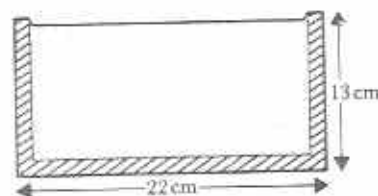


Fig. 18.10

(a) Calculate the mass of a 1 m length of the channel if 1 cm^3 of steel has a mass of 7.5 g. (b) Calculate the capacity of 1 m of the channel in litres.

(a) Area of steel cross-section

$$\begin{aligned} &= (22 \times 13 - 20 \times 12) \text{ cm}^2 \\ &= 46 \text{ cm}^2 \end{aligned}$$

Volume of 1 m of the channel

$$= 46 \times 100 \text{ cm}^3$$

Mass of 1 m of the channel

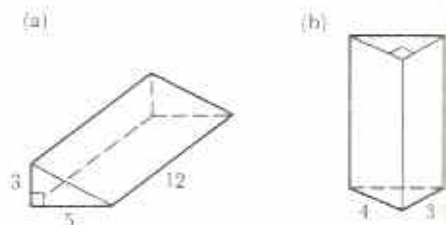
$$\begin{aligned} &= 46 \times 100 \times 7.5 \text{ g} \\ &= 34\,500 \text{ g} \\ &= 34.5 \text{ kg} \end{aligned}$$

(b) Volume contained by 1 m of the channel

$$\begin{aligned} &= 20 \times 12 \times 100 \text{ cm}^3 \\ &= \frac{20 \times 12 \times 100}{1000} \text{ litres} \\ &= 24 \text{ litres} \end{aligned}$$

Exercise 18c

1 Calculate the volumes of the right-angled triangular prisms in Fig. 18.11. All measurements are in cm.



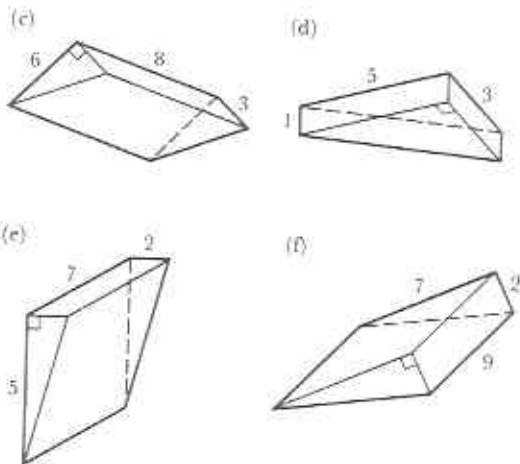


Fig. 18.11

- 2 The diagrams in Fig. 18.12 show the cross-sections of steel beams. All dimensions are in cm. Calculate the volumes of 5 m lengths of the beams. Give your answers in cm^3 .

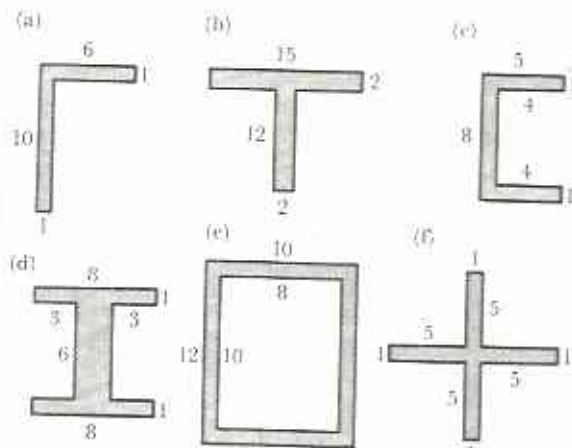


Fig. 18.12

- 3 Fig. 18.13 shows a new pencil. If the area of its end-face is 40 mm^2 , calculate the volume of the pencil in cm^3 .

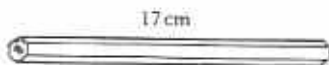


Fig. 18.13

- 4 The end face of the I-girder shown in Fig. 18.14 has an area of 42 cm^2 . If the I-girder is made of steel of density 7.5 g/cm^3 , calculate its mass in kg.

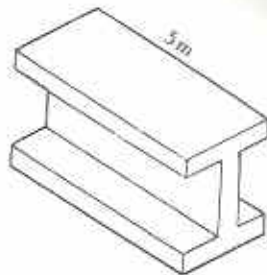


Fig. 18.14

- 5 How many kilolitres of water are contained in an irrigation ditch which is 500 m long and which has an average cross-sectional area of 700 cm^2 ?
- 6 Fig. 18.15 shows a plastic prism. If 1 cm^3 of the plastic has a mass of 2.4 g, calculate the mass of the prism.

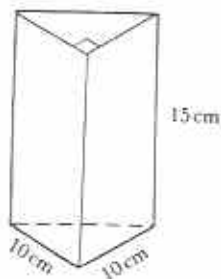


Fig. 18.15

- 7 The area of the base of the cylindrical container in Fig. 18.16 is 0.35 m^2 . It contains water to a depth of 1.26 m. How many times can a bucket of capacity 9 litres be filled from the container?



Fig. 18.16

- 8 Fig. 18.17 shows a special block used in building. If each hole is 15 cm by 15 cm, calculate the volume of concrete in the block in cm^3 .

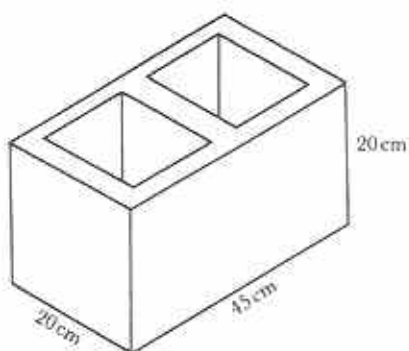


Fig. 18.17

- 9 Fig. 18.18 shows an open wooden box.

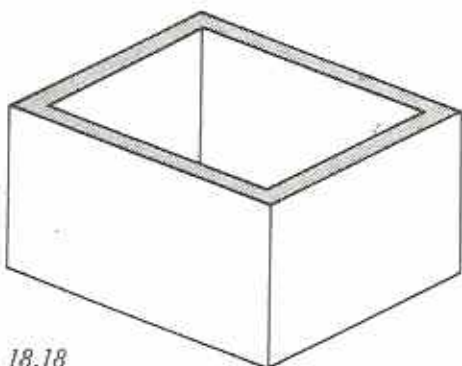


Fig. 18.18

The external dimensions of the box are 36 cm long, 24 cm wide and 12 cm deep and the wood is 1 cm thick. Calculate (a) the volume of wood used in cm^3 , (b) the capacity of the box in litres.

- 10 If the internal dimensions of the box in Fig. 18.18 are 20 cm long, 15 cm wide and 8 cm deep, calculate the capacity of the box in litres. Given that the wood is 2 cm thick, what volume of wood is used in making the box?

Chapter 19

Directed numbers (2) Multiplication and division

Syllabus Reference 5.1.1

Multiplication with directed numbers

Positive multipliers

Multiplication is a short way of writing repeated additions. For example,

$$\begin{aligned} 3 \times 4 &= 3 \text{ lots of } 4 \\ &= 4 + 4 + 4 \\ &= 12 \end{aligned}$$

With directed numbers,

$$\begin{aligned} (+4) + (+4) + (+4) &= 3 \text{ lots of } (+4) \\ &= 3 \times (+4) \end{aligned}$$

The multiplier is 3. It is positive. Thus,

$$\begin{aligned} (+3) \times (+4) &= (+4) + (+4) + (+4) \\ &= +12 \end{aligned}$$

$$(+3) \times (+4) = +12$$

Fig. 19.1 shows $1 \times (+4)$ and $(+3) \times (+4)$ as movements on the number line. The movements are in the same direction from 0.

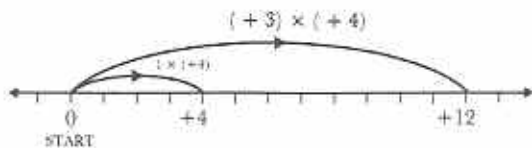


Fig. 19.1

Similarly,

$$\begin{aligned} (-2) + (-2) + (-2) + (-2) + (-2) \\ &= 5 \text{ lots of } (-2) \\ &= 5 \times (-2) \end{aligned}$$

The multiplier is 5. It is positive. Thus,

$$\begin{aligned} (+5) \times (-2) &= (-2) + (-2) + (-2) \\ &\quad + (-2) + (-2) = -10 \end{aligned}$$

$$(+5) \times (-2) = -10$$

Fig. 19.2 shows $1 \times (-2)$ and $(+5) \times (-2)$ as movements on the number line. The movements are in the same direction from 0.



Fig. 19.2

$$\begin{aligned} \text{In general: } (+a) \times (+b) &= + (a \times b) \\ (+a) \times (-b) &= - (a \times b) \end{aligned}$$

Example 1

Simplify the following.

(a) $(+9) \times (+4)$

(b) $(+17) \times (-3)$

(c) $(+\frac{1}{2}) \times (+\frac{1}{4})$

(d) $3 \times (-1,2)$

(a) $(+9) \times (+4) = + (9 \times 4) = + 36$

(b) $(+17) \times (-3) = - (17 \times 3) = - 51$

(c) $(+\frac{1}{2}) \times (+\frac{1}{4}) = + (\frac{1}{2} \times \frac{1}{4}) = +\frac{1}{8}$

(d) $3 \times (-1,2) = (+3) \times (-1,2)$
 $= - (3 \times 1,2) = -3,6$

Exercise 19a

1 Continue the following number patterns as far as the 7th term.

(a) $+15, +10, +5, 0, -5, \dots$

(b) $+6, +4, +2, 0, \dots$

(c) $+30, +20, +10, \dots$

2 Complete the following.

(a) $(+5) \times (+3) = \dots$

$(+5) \times (+2) = \dots$

$(+5) \times (+1) = \dots$

$(+5) \times 0 = \dots$

$(+5) \times (-1) = \dots$

$(+5) \times (-2) = \dots$

$(+5) \times (-3) = \dots$

(b) $(+2) \times (+3) = \dots$

$(+2) \times (+2) = \dots$

$(+2) \times (+1) = \dots$

$$\begin{aligned}
 (+2) \times 0 &= \dots \\
 (+2) \times (-1) &= \dots \\
 (+2) \times (-2) &= \dots \\
 (+2) \times (-3) &= \dots \\
 \text{(c) } (+10) \times (+3) &= \dots \\
 (+10) \times (+2) &= \dots \\
 (+10) \times (+1) &= \dots \\
 (+10) \times 0 &= \dots \\
 (+10) \times (-1) &= \dots \\
 (+10) \times (-2) &= \dots
 \end{aligned}$$

3 Simplify the following.

(a) $(+6) \times (+3)$	(b) $(+5) \times (+10)$
(c) $(+5) \times (-8)$	(d) $(+4) \times (-7)$
(e) $(+17) \times (+2)$	(f) $(+20) \times (+6)$
(g) $(+10) \times (-10)$	(h) $(+15) \times (-4)$
(i) $(+3) \times (+2\frac{1}{2})$	(j) $(+1\frac{1}{4}) \times (+\frac{1}{3})$
(k) $(+2) \times (-3\frac{1}{2})$	(l) $(+1) \times (-\frac{2}{3})$
(m) $(+1.5) \times (+2)$	(n) $(+5.1) \times (+3)$
(o) $(+4) \times (-1.2)$	(p) $(+3.2) \times (-5)$
(q) $8 \times (+7)$	(r) $8 \times (-7)$
(s) $7 \times (+8)$	(t) $7 \times (-8)$

Negative multipliers

Work through Exercise 19b very carefully.

Exercise 19b (Oral or written)

1 Find the next four terms in each of the following patterns.

(a) $+15, +12, +9, +6, \dots, \dots, \dots$
 (b) $(+5) \times (+3), (+4) \times (+3),$
 $(+3) \times (+3), (+2) \times (+3),$

(c) $\dots \times \dots = +15$
 $(+4) \times (+3) = +12$
 $(+3) \times (+3) = +9$
 $(+2) \times (+3) = +6$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$

2 Find the next four terms in each of the following patterns.

(a) $-15, -12, -9, -6, \dots, \dots, \dots$
 (b) $(+5) \times (-3), (+4) \times (-3),$
 $(+3) \times (-3), (+2) \times (-3),$
 $\dots \times \dots = -15$
 $(+4) \times (-3) = -12$

$$\begin{aligned}
 (+3) \times (-3) &= -9 \\
 (+2) \times (-3) &= -6 \\
 \dots \times \dots &= \dots \\
 \dots \times \dots &= \dots \\
 \dots \times \dots &= \dots \\
 \dots \times \dots &= \dots
 \end{aligned}$$

Look at your results in question 1 of Exercise 19b. The last row of 1(c) is: $(-2) \times (+3) = -6$. (-2) is the multiplier. It is negative. The number being multiplied, $(+3)$, is positive. The result is negative. This result may not be surprising. We already know that $(+3) \times (-2) = -6$. Thus we would expect that $(-2) \times (+3) = -6$.

Fig. 19.3 shows $1 \times (+3)$ and $(-2) \times (+3)$ as movements on the number line. The movements are in opposite directions from 0.

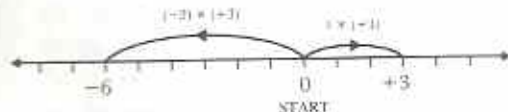


Fig. 19.3

Look at your results in question 2 of Exercise 19b. The last row of 2(c) is: $(-2) \times (-3) = +6$. (-2) is the multiplier. It is negative. The number being multiplied, (-3) , is also negative. The result is *positive*. This result is quite surprising. However, on the number line the movements are again in opposite directions from 0. Fig. 19.4 shows $1 \times (-3)$ and $(-2) \times (-3)$ as movements on the number line.

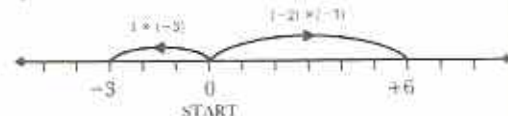


Fig. 19.4

In general, $(-a) \times (+b) = -(a \times b)$
 $(-a) \times (-b) = +(a \times b)$

Example 2

Simplify the following.

(a) $(-7) \times (+4)$ (b) $(-5) \times (-18)$
 (c) $(-\frac{1}{3}) \times (+\frac{2}{5})$ (d) $(-4) \times (-2.2)$
 (a) $(-7) \times (+4) = -(7 \times 4) = -28$
 (b) $(-5) \times (-18) = +(5 \times 18) = +90$
 (c) $(-\frac{1}{3}) \times (+\frac{2}{5}) = -(\frac{1}{3} \times \frac{2}{5}) = -\frac{2}{15}$
 (d) $(-4) \times (-2.2) = +(4 \times 2.2) = +8.8$

Exercise 19c

1 Continue the following number patterns as far as the 7th term.

(a) $+15, +10, +5, 0, \dots$

(b) $-6, -4, -2, \dots$

(c) $-30, -20, -10, \dots$

2 Complete the following.

(a) $(+3) \times (+5) = \dots$

$(+2) \times (+5) = \dots$

$(+1) \times (+5) = \dots$

$0 \times (+5) = \dots$

$(-1) \times (+5) = \dots$

$(-2) \times (+5) = \dots$

$(-3) \times (+5) = \dots$

(b) $(+3) \times (-2) = \dots$

$(+2) \times (-2) = \dots$

$(+1) \times (-2) = \dots$

$0 \times (-2) = \dots$

$(-1) \times (-2) = \dots$

$(-2) \times (-2) = \dots$

$(-3) \times (-2) = \dots$

(c) $(+3) \times (-10) = \dots$

$(+2) \times (-10) = \dots$

$(+1) \times (-10) = \dots$

$0 \times (-10) = \dots$

$(-1) \times (-10) = \dots$

$(-2) \times (-10) = \dots$

3 Simplify the following.

(a) $(-6) \times (+3)$ (b) $(+5) \times (+10)$

(c) $(-5) \times (-8)$ (d) $(-4) \times (-7)$

(e) $(-16) \times (+2)$ (f) $(-20) \times (+4)$

(g) $(-8) \times (-8)$ (h) $(-13) \times (-5)$

(i) $(-2) \times (+1\frac{1}{2})$ (j) $(-1\frac{1}{3}) \times (+1)$

(k) $(-\frac{1}{3}) \times (-3)$ (l) $(-4) \times (-\frac{1}{2})$

(m) $(-5) \times (+1,2)$ (n) $(-2,4) \times (+3)$

(o) $(-0,3) \times (-9)$ (p) $(-6) \times (-3,1)$

(q) -8×6 (r) $8 \times (-6)$

(s) $-8 \times (-6)$ (t) $-6 \times (-8)$

Division with directed numbers

When directed numbers are multiplied together,

two like signs give a positive result;

two unlike signs give a negative result.

For example,

$$\begin{array}{l} (+3) \times (+8) = +24 \\ (-3) \times (-8) = +24 \\ (+3) \times (-8) = -24 \\ (-3) \times (+8) = -24 \end{array}$$

The same rule is true for division. For example

$$(+24) \div (+3) = (+8)$$

$$(-24) \div (-3) = (+8)$$

$$(+24) \div (-3) = (-8)$$

$$(-24) \div (+3) = (-8)$$

Example 3

Divide (a) -36 by 9 , (b) -4 by -12 .

$$(a) \quad -36 \div 9 = \frac{-36}{+9} = -\left(\frac{36}{9}\right) = -4$$

$$(b) \quad -4 \div (-12) = \frac{-4}{-12} = +\left(\frac{4}{12}\right) = +\frac{1}{3}$$

Example 4

Simplify $\frac{(-6) \times (-5)}{-10}$

$$\frac{(-6) \times (-5)}{-10} = \frac{+30}{-10} = -\left(\frac{30}{10}\right) = -3$$

Exercise 19d

1 Divide:

(a) -18 by 3 , (b) -18 by -3 ,

(c) 18 by -3 , (d) 36 by $+4$,

(e) -20 by -4 , (f) -8 by -1 ,

(g) -22 by -11 , (h) 24 by -8 ,

(i) 33 by -3 , (j) -3 by -18 ,

(k) -6 by 12 , (l) -5 by -15 .

2 Simplify the following.

(a) $\frac{(-2) \times (+12)}{-6}$ (b) $\frac{(-6) \times (-10)}{-4}$

(c) $\frac{36}{(-2) \times (-9)}$ (d) $\frac{(-3) \times (-15)}{9}$

(e) $\frac{9 \times 20}{-3}$ (f) $\frac{-28 \times (-3)}{21}$

(g) $\frac{30}{(-5) \times (-4)}$ (h) $\frac{(-1) \times (-5)}{-10}$

(i) $\frac{4 \times (-3)}{-24}$

Chapter summary

- Directed numbers can be multiplied and divided.
- When two numbers of the same sign are multiplied together, the result is positive.
- When two numbers of different sign are multiplied together, the result is negative.
- When two numbers of the same sign are divided, one by the other, the result is positive.
- When two numbers of different sign are divided, one by the other, the result is negative.

Exercise 19e (Practice)

1 Write down the next four terms of the following patterns

- $+6, +4, +2, 0, -2, \dots$
- $2 \times (+3), 2 \times (+2), 2 \times (+1), 2 \times 0, 2 \times (-1), \dots$
- $2 \times (+3) = +6$
 $2 \times (+2) = +4$
 $2 \times (+1) = +2$
 $2 \times 0 = 0$
 $2 \times (-1) = -2$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
- $+16, +12, +8, +4, 0, \dots$
- $(+4) \times 4, (+3) \times 4, (+2) \times 4, (+1) \times 4, 0 \times 4, \dots$
- $(+4) \times 4 = +16$
 $(+3) \times 4 = +12$
 $(+2) \times 4 = +8$
 $(+1) \times 4 = +4$
 $0 \times 4 = 0$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
- $-15, -10, -5, 0, +5, \dots$
- $(+3) \times (-5), (+2) \times (-5), (+1) \times (-5), 0 \times (-5), (-1) \times (-5), \dots$

- $(+3) \times (-5) = -15$
 $(+2) \times (-5) = -10$
 $(+1) \times (-5) = -5$
 $0 \times (-5) = 0$
 $(-1) \times (-5) = +5$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$
 $\dots \times \dots = \dots$

2 Copy and complete the multiplication table in Fig. 19.5.

\times	-6	-4	-2	0	+2	+4	+6
+6							
+4							
+2							
0							
-2							
-4							
-6							

Fig. 19.5

3 Simplify the following.

- $(-3) \times 4$
- $(-4) \times (-7)$
- $5 \times (-3)$
- $(-6) \times (-1)$
- $24 \times (-2)$
- $(-8) \times (-7)$
- $(-1) \times (-1)$
- $0 \times (-3)$
- $(-1) \times 1$

4 The bottom of a water well is -6 metres from ground level. An oil well is 15 times deeper. Find the distance of the bottom of the oil well from ground level.

5 What must be multiplied by:

- 5 to make 40,
- 1 to make -7 ,
- -2 to make 12,
- -3 to make -30 ,
- 4 to make -36 ,
- -9 to make 27?

6 Simplify the following.

- $(+20) \div (-10)$
- $(-26) \div (+2)$
- $(-6) \div (-6)$
- $(+18) \div (-3)$
- $(-27) \div (+3)$
- $(+30) \div (+15)$
- $\frac{-14}{-7}$
- $\frac{+50}{-25}$
- $\frac{-60}{+12}$

7 What must be divided by:

- (a) 6 to make 2, (b) 6 to make -2 ,
(c) -3 to make 5, (d) 8 to make -3 ,
(e) -2 to make 11, (f) -3 to make 30?

8 Simplify the following.

- (a) $(-7) \times (-3) \times (-2)$
(b) $(-1) \times (-1) \times (-1)$

(c) $\frac{(-8) \times (+5)}{-10}$ (d) $\frac{+60}{(-3) \times (-5)}$

(e) $(-3) \times (-2)^2$

(f) $\frac{(-2) \times (-12)}{(+6)^2}$

9 Simplify the following.

(a) $(-1\frac{1}{2}) \times 3\frac{2}{3}$ (b) $(-2,8) \times (-0,2)$

(c) $\frac{4}{9}$ of $(-2\frac{3}{4})$ (d) $7 \times (-6,2)$

10 Simplify the following.

(a) $1\frac{3}{4} \div (-2\frac{1}{4})$ (b) $(-4,8) \div (-6)$

(c) $-7\frac{1}{5} \div (-9\frac{3}{5})$ (d) $-8,4 \div 7$

Geometrical constructions (1)

Triangles, parallel and perpendicular lines

Syllabus Reference 5.7.5

In geometry, to **construct** a figure means to draw it accurately. Accurate construction depends on using measuring instruments properly. Make sure that you have a pencil, a ruler, a pair of compasses, a protractor and a set square before beginning work on this chapter.

Freehand sketching

Before starting any construction you should make a rough sketch of what you are going to draw. This does not need to be accurate. Do not use a ruler. Make a **freehand sketch** and show the dimensions (i.e. measurements) on the sketch.

Constructing triangles

Constructing a triangle, given the lengths of all three sides

Work through the following example.

Example 1

Construct triangle ABC so that $AB = 6$ cm, $AC = 5$ cm and $BC = 4$ cm.

- (a) Make a freehand sketch (Fig. 20.1). This shows what you have to do.

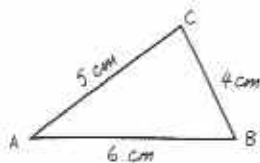


Fig. 20.1

- (b) Draw a straight line AB of length 6 cm (Fig. 20.2).



Fig. 20.2

- (c) Open a pair of compasses to 5 cm. Put the point of the compasses on A. Draw an arc above AB. Every point on this arc is 5 cm from A (Fig. 20.3).

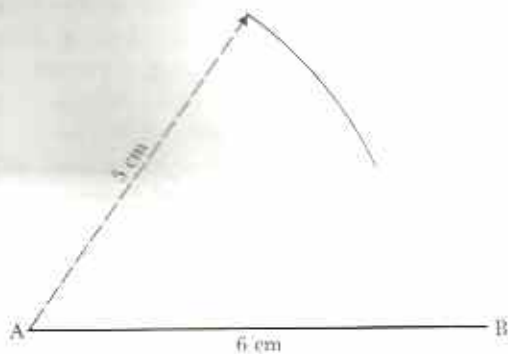


Fig. 20.3

- (d) Now open the compasses to 4 cm. Put the point of the compasses on B. Draw a second arc to cut the first arc (Fig. 20.4).
- (e) The point where the arcs intersect is 5 cm from A and 4 cm from B. This is the point C. Join AC and BC to complete the triangle ABC (Fig. 20.5).

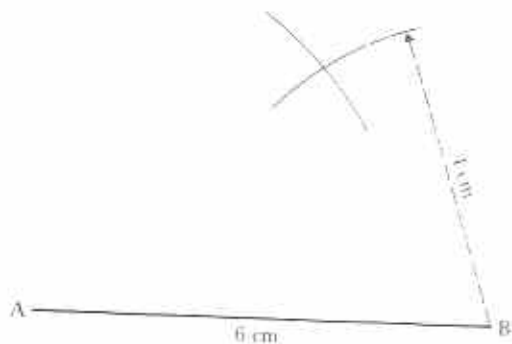


Fig. 20.4

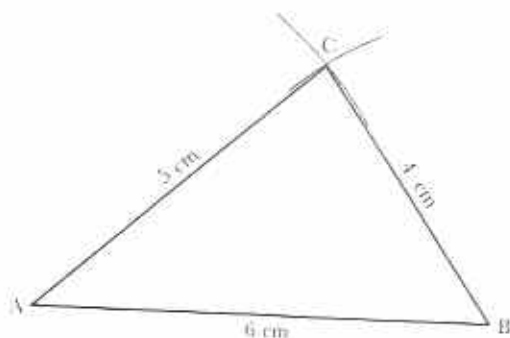


Fig. 20.5

Note: Always leave construction lines, such as arcs, on your drawing.

Exercise 20a

Use ruler and compasses in this exercise.

- Construct triangles with sides of the lengths given in the sketches in Fig. 20.6.

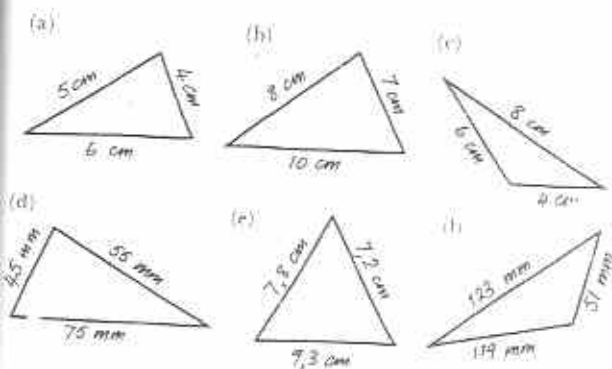


Fig. 20.6

- Construct triangles with sides of the lengths given below. Make rough sketches first.

- 8 cm, 7 cm, 6 cm
- 9 cm, 5 cm, 8 cm
- 10.5 cm, 7.5 cm, 7.5 cm
- 105 mm, 95 mm, 45 mm
- 133 mm, 59 mm, 114 mm
- 16.6 cm, 6.8 cm, 8.1 cm

- Try to construct a triangle with sides 10 cm, 3 cm and 6 cm. What happens? Complete this sentence: The sum of the lengths of any two sides of a triangle must be ... than the length of the third side.

- Make an accurate construction of the diagram in Fig. 20.7. *Hint:* Start by drawing BD.

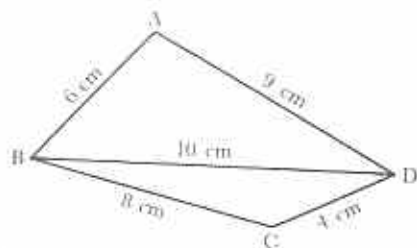


Fig. 20.7

When your drawing is complete, draw the line AC. Measure AC.

- Draw triangle ABC in which $AB = 10$ cm, $BC = 6$ cm and $CA = 11$ cm. On AB mark a point D such that $AD = 4.5$ cm. Draw CD. Measure the length of CD.
 - Draw triangle PQR in which $PQ = 7.9$ cm, $QR = 7.5$ cm and $PR = 12.6$ cm. With centre R and radius 10.5 cm draw an arc to cut PQ at X. Measure the length of PX.
- Note:* Draw a freehand sketch first.

Constructing a triangle, given two angles and one side

Work through the following example.

Example 2

Construct $\triangle ABC$ such that $BC = 5$ cm, $\hat{ABC} = 30^\circ$ and $\hat{ACB} = 70^\circ$.

- Make a freehand sketch (Fig. 20.8).

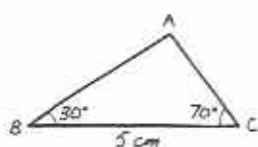


Fig. 20.8

- (b) Draw line BC 5 cm long (Fig. 20.9).

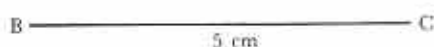


Fig. 20.9

- (c) Place the centre of a protractor over B. Draw a line from B at an angle of 30° to BC (Fig. 20.10).

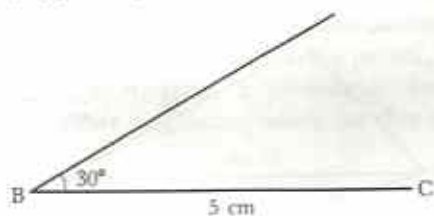


Fig. 20.10

- (d) Now place the centre of the protractor over C. Construct a line from C at 70° to CB. A is the point where the lines intersect above BC (Fig. 20.11).

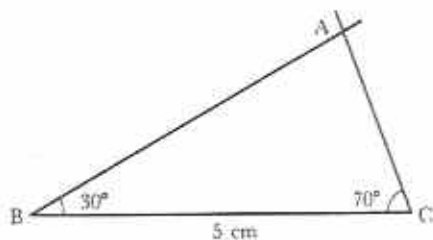


Fig. 20.11

Notice that you need to know the two angles adjacent to the given line. If two angles of a triangle are known, it is always possible to find the third angle.

Example 3

Sketch $\triangle PQR$ in which $PQ = 10$ cm, $\hat{PQR} = 40^\circ$ and $\hat{PRQ} = 75^\circ$.

Calculate the third angle of the triangle.

Rough sketch (Fig. 20.12).

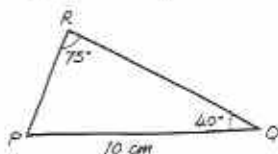


Fig. 20.12

$$\begin{aligned} \hat{RPQ} &= 180^\circ - (75^\circ + 40^\circ) = 180^\circ - 115^\circ \\ &= 65^\circ \end{aligned}$$

It would now be possible to construct triangle PQR accurately. This shows the value of making a rough sketch.

Exercise 20b

- 1 Construct the triangles in the sketches in Fig. 20.13.

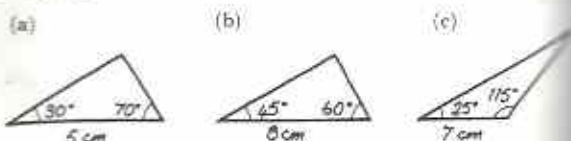


Fig. 20.13

- 2 Calculate the third angle in each of the triangles sketched in Fig. 20.14. Make a rough sketch and construct each triangle accurately.

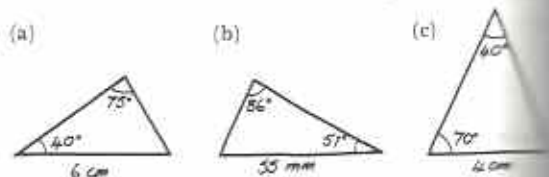


Fig. 20.14

- 3 Construct the following triangles accurately. In each case make a freehand sketch before drawing accurately.

(a) $\triangle ABC$ such that $AB = 5$ cm, $\hat{CAB} = 50^\circ$, $\hat{ABC} = 70^\circ$

(b) $\triangle PQR$ such that $QR = 8$ cm, $\hat{PQR} = 62^\circ$, $\hat{QRP} = 33^\circ$

(c) $\triangle DEF$ such that $DF = 6.5$ cm, $\hat{EDF} = 55^\circ$, $\hat{EFD} = 48^\circ$

(d) $\triangle XYZ$ such that $XY = 6$ cm, $\hat{XYZ} = 40^\circ$, $\hat{XZY} = 60^\circ$

- 4 Construct $\triangle ABC$ such that $\hat{ABC} = 20^\circ$, $BC = 4$ cm and $\hat{BCA} = 125^\circ$. M is the midpoint of BC. Measure AM.

Constructing a triangle, given two sides and the angle between them

Example 4

Construct $\triangle ABC$ in which

$AB = 4$ cm, $BC = 5$ cm and $\angle ABC = 50^\circ$.

(a) Draw a freehand sketch (Fig. 20.15).

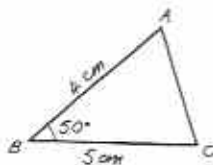


Fig. 20.15

(b) Draw a straight line BC of length 5 cm (Fig. 20.16).

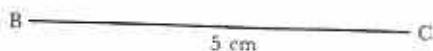


Fig. 20.16

(c) Put the centre of a protractor over B . Draw a line from B at an angle of 50° to BC (Fig. 20.17).

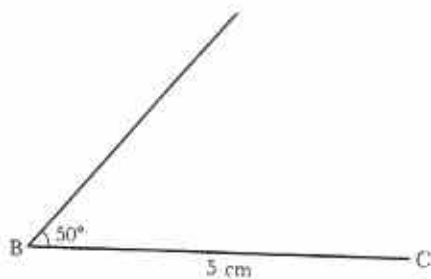


Fig. 20.17

(d) Mark a point A on this line such that $BA = 4$ cm. Join AC to complete $\triangle ABC$ (Fig. 20.18).

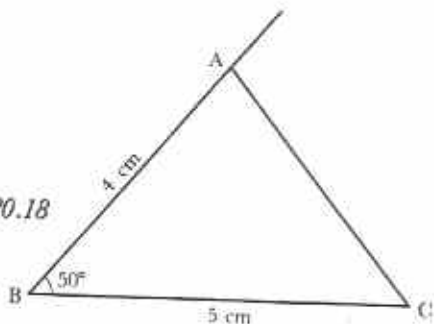


Fig. 20.18

Exercise 20c

1 Construct the triangles sketched in Fig. 20.19.

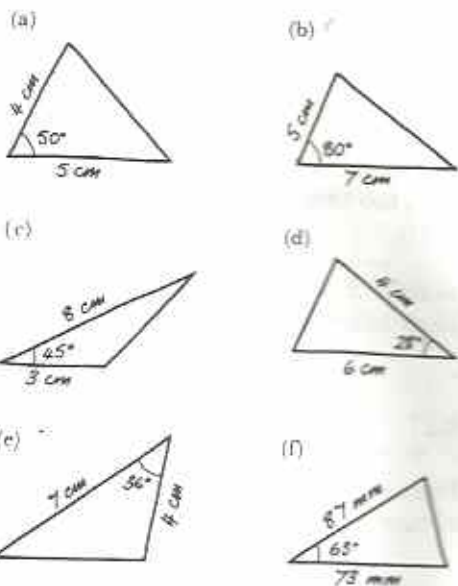


Fig. 20.19

2 In each of the following, two sides of a triangle and the angle between them are given. Construct each triangle and measure the length of the third side.

- 6 cm, 90° , 8 cm
- 8 cm, 60° , 5 cm
- 6.2 cm, 42° , 7.9 cm
- 47 mm, 56° , 74 mm
- 56 mm, 60° , 35 mm
- 5.4 cm, 43° , 4.8 cm

3 Construct the figure shown in Fig. 20.20.
Hint: start by constructing $\triangle ABC$.

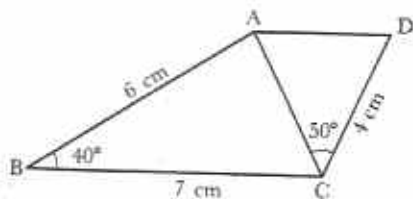


Fig. 20.20

When the drawing is complete, measure the length of AD .

Parallel lines

Constructing parallel lines using ruler and set squares only

Follow Example 5 by copying the work on to a sheet of blank paper.

Example 5

Construct a line through P so that it is parallel to AB (Fig. 20.21).



Fig. 20.21

- (a) Place a set square so that one edge is accurately along AB (Fig. 20.22).

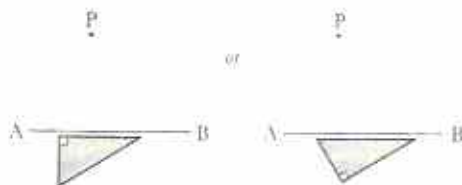


Fig. 20.22

- (b) Place a ruler along one of the other edges of the set square (Fig. 20.23). (Use the left-hand edge if you are right handed.)

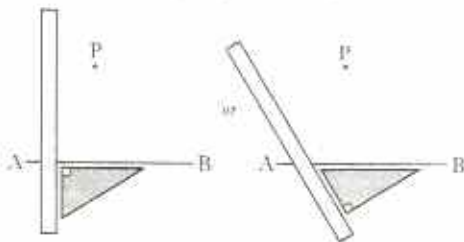


Fig. 20.23

- (c) Hold the ruler firmly. Slide the set square along the ruler towards P. Stop when the edge that was on AB reaches P. Draw a line along this edge of the set square through P (Fig. 20.24).

Notice, in Example 5, that since corresponding angles are equal, the line through P and the line AB are parallel.

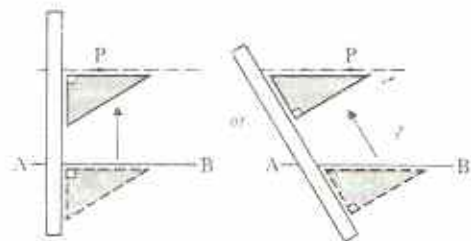


Fig. 20.24

Exercise 20d

Work on blank (i.e. unruled) paper.

- 1 Draw a straight line AB on a piece of unruled paper. Mark a point P which is not on the line. Use a ruler and set square to draw a line through P which is parallel to AB.
- 2 Use a ruler and set square to draw four lines which are parallel to each other.
- 3 Draw any triangle near the centre of a sheet of paper. Use a ruler and set square to draw another triangle with sides parallel to those of the first triangle.
- 4 Draw angle $ABC = 70^\circ$ so that the arms BA and BC are 3 cm and 5 cm long. See the sketch in Fig. 20.24. Draw a line through B parallel to BC. Draw a line through C parallel to BA to make parallelogram ABCD.

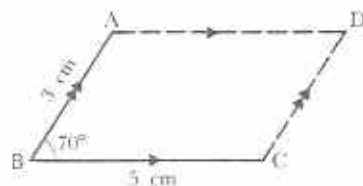


Fig. 20.25

Measure the diagonals AC and BD.

- 5 Construct a parallelogram ABCD with $AB = 3\text{ cm}$, $BC = 4\text{ cm}$ and $\hat{B} = 75^\circ$. Draw the diagonals AC and BD to intersect at point X. Measure XA and XC. Measure XB and XD.
- 6 Construct a rhombus so that one of its acute angles is 65° and each side is 4 cm long. Measure the lengths of its diagonals.
- 7 Construct the trapezium ABCD shown in Fig. 20.26

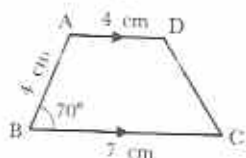


Fig. 20.26

Measure DC and the three unknown angles.

- 8 Construct trapezium ABCD so that $BC = 6\text{ cm}$, $AB = 3\text{ cm}$, $B = 80^\circ$ and $C = 70^\circ$. Measure BD.

Perpendicular lines

When two lines meet at right angles we say that they are **perpendicular** to each other.

In Fig. 20.27, XY is perpendicular to AB and AB is perpendicular to XY.

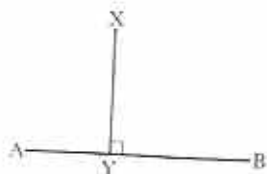


Fig. 20.27

Lines which meet perpendicularly are very common. Adjacent edges of a door frame are perpendicular to each other. The margin in an exercise book is perpendicular to the ruled lines. There are many other examples.

The perpendicular distance of a point from a line

In Fig. 20.28 P is a point and l is any line.

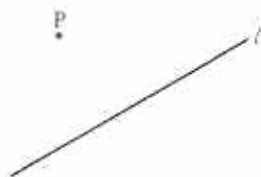


Fig. 20.28

P is different distances from different points $D_1, D_2, D_3, D_4, \dots$ on the line (Fig. 20.29).

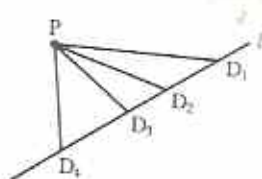


Fig. 20.29

One point on the line nearest to P. This is the point D such that PD is perpendicular to l (Fig. 20.30).

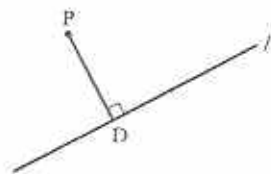


Fig. 20.30

The length PD is the **perpendicular distance** of P from line l .

To construct a perpendicular from a point to a line using ruler and set square

- (a) Place a ruler along the given line (Fig. 20.31).

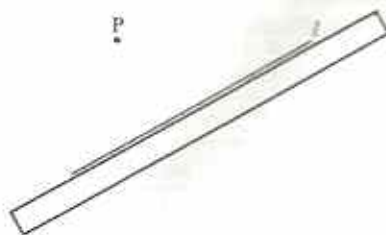


Fig. 20.31

- (b) Use the two edges of a set square which are the arms of its right angle. Place one of these edges along the ruler. Slide the set square along the ruler until the outer edge reaches P (Fig. 20.32).

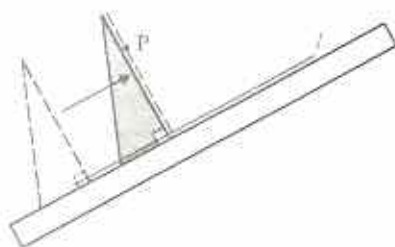


Fig. 20.32

- (c) Hold the set square firmly. Draw the line through P to meet the line perpendicularly (Fig. 20.33).



Fig. 20.33

Exercise 20e

- 1 Mark a point P near the middle of a page in your exercise book. Mark P between two of the ruled lines in your exercise book. Use ruler and set square to draw and measure the perpendicular distance of P from
 - (a) the line which is four lines above P,
 - (b) the line which is six lines below P,
 - (c) the left-hand margin.
- 2 Trace the diagram, Fig. 20.34, into your exercise book. Draw and measure the perpendicular distances of P from the five sides of the pentagon.

perpendicular distances of P from the five sides of the pentagon.

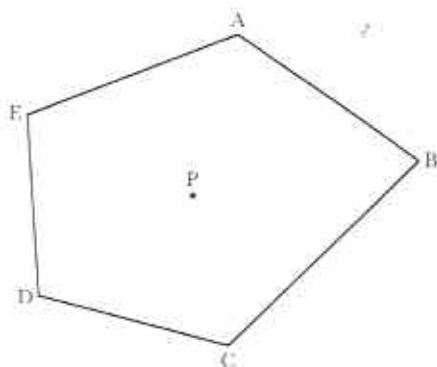


Fig. 20.34

- 3 Use ruler and set square to construct the following.
 - (a) a square of side 3 cm
 - (b) a square of side 5 cm
 - (c) a rectangle 3 cm by 6 cm
 - (d) a rectangle 4 cm by 5 cm
- 4 Use ruler and set square to construct a pair of parallel lines which are
 - (a) 3 cm apart,
 - (b) 5 cm apart,
 - (c) 4.2 cm apart,
 - (d) 57 mm apart.
- 5 Turn back to the parallelograms you constructed in Exercise 20d, questions 4 and 5. In each case, measure the perpendicular distances between (a) parallel lines AD and BC, (b) parallel lines AB and DC. Hence calculate the area of each parallelogram in two ways.

Simplifying algebraic expressions (2)

Brackets, factors, fractions

Syllabus Reference 5.6.2

Remember that in algebra, letters stand for numbers. The numbers can be whole or fractional, positive or negative.

Directed algebraic terms

1 Just as $5a$ is short for $5 \times a$, so $-5a$ is short for $(-5) \times a$.

2 Just as m is short for $1 \times m$, so $-m$ is short for $(-1) \times m$.

3 Algebraic terms and numbers can be multiplied together. For example,

$$\begin{aligned} 4 \times (-3x) &= (+4) \times (-3) \times x \\ &= -(4 \times 3) \times x = -12 \times x \\ &= -12x \end{aligned}$$

$$\begin{aligned} (-2y) \times (-8y) &= (-2) \times y \times (-8) \times y \\ &= (-2) \times (-8) \times y \times y \\ &= +(2 \times 8) \times y^2 \\ &= +16y^2 \text{ or just } 16y^2 \end{aligned}$$

4 Division with directed numbers is also possible. For example,

$$\begin{aligned} 18a \div (-6) &= \frac{(+18) \times a}{(-6)} = -\left(\frac{18}{6}\right) \times a \\ &= (-3) \times a = -3a \end{aligned}$$

$$\frac{-33x^2}{-3x} = \frac{(-33) \times x \times x}{(-3) \times x} = +\left(\frac{33}{3}\right) \times x = 11x$$

Read example 1 carefully.

Example 1

	simplify	working	result
(a)	$-5 \times 2y$	$= (-5) \times (+2) \times y$ $= -(5 \times 2) \times y$	$= -10y$
(b)	$-6 \times -4x$	$= (-6) \times (-4) \times x$ $= +(6 \times 4) \times x$	$= 24x$
(c)	$-3a \times 7$	$= (-3) \times a \times (+7)$ $= (-3) \times (+7) \times a$	$= -21a$
(d)	$4y \times -5$	$= (+4) \times y \times (-5)$ $= (+4) \times (-5) \times y$	$= -20y$
(e)	$-3a \times -6b$	$= (-3) \times a \times (-6) \times b$ $= (-3) \times (-6) \times a \times b$	$= 18ab$
(f)	$\frac{-14a}{7}$	$= \frac{(-14) \times a}{(+7)}$ $= -\left(\frac{14}{7}\right) \times a$ $= -2 \times a$	$= -2a$
(g)	$-\frac{1}{3}$ of $36x^2$	$= \frac{(+36) \times x^2}{(-3)}$ $= -\left(\frac{36}{3}\right) \times x^2$	$= -12x^2$

Exercise 21a (Oral)

Simplify the following.

1 $(-5) \times a$

2 $x \times (-4)$

3 $(-x) \times 3$

4 $3 \times (-c)$

5 $(-6) \times (-x)$

6 $(-y) \times (-9)$

- | | | | | |
|-------------------------|-------------------------|---|--------------|--------------|
| 7 $3 \times (-2a)$ | 8 $(-3) \times (-2a)$ | 19 $ac - b$ | 20 $a - bc$ | 21 $2a + b$ |
| 9 $(-3) \times 2a$ | 10 $(-9) \times 4x$ | 22 $3b + 2c$ | 23 $7a - 2b$ | 24 $4c - 3b$ |
| 11 $(-5x) \times -8$ | 12 $(-5x) \times 8$ | Example 3 | | |
| 13 $7a \times (-6)$ | 14 $2 \times (-11y)$ | What is the value of $\frac{p-q}{p}$ when $p = -5$ and | | |
| 15 $4 \times (-4c)$ | 16 $(-10) \times (-3y)$ | $q = +10$? | | |
| 17 $(-2x) \times (-9y)$ | 18 $3a \times (-7b)$ | Notice that $\frac{p-q}{p}$ is the same as $\frac{(p-q)}{p}$. | | |
| 19 $(-8x) \times (-3x)$ | 20 $6d \times (-3d)$ | Simplify the top line <i>before</i> dividing. | | |
| 21 $21a \div (-7)$ | 22 $(-6x) \div (-2)$ | When $p = -5$ and $q = +10$, | | |
| 23 $20y \div (-10)$ | 24 $18z \div (-3)$ | $\frac{p-q}{p} = \frac{(-5) - (+10)}{(-5)} = \frac{-15}{-5} = +\left(\frac{15}{5}\right) = +$ | | |

- | | |
|---------------------------------|--------------------------------|
| 25 $\frac{-16x}{8}$ | 26 $\frac{28x}{-4}$ |
| 27 $\frac{17x}{-17}$ | 28 $\frac{-49x}{-7}$ |
| 29 $\frac{1}{8}$ of $(-45y)$ | 30 $(-\frac{1}{10})$ of $100z$ |
| 31 $(-\frac{1}{6})$ of $(-48t)$ | 32 $\frac{1}{4}$ of $(-36n)$ |
| 33 $\frac{-36x^2}{-12x}$ | 34 $\frac{22ab}{-2a}$ |
| 35 $\frac{-39xy}{3y}$ | 36 $\frac{-63m^2}{-21m}$ |

Substitution

Example 2

Find the value of (a) $4x$, (b) $xy - 5y$ when $x = 2$ and $y = 3$.

(a) Substitute the value 2 for x , i.e. use the value 2 instead of x .

$$\begin{aligned} \text{When } x = 2, 4x &= 4 \times x \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

(b) $xy - 5y = x \times y - 5 \times y$

$$\begin{aligned} \text{When } x = 2 \text{ and } y = 3, \\ xy - 5y &= 2 \times 3 - 5 \times 3 \\ &= 6 - 15 \\ &= -9 \end{aligned}$$

Exercise 21b (Oral)

Find the value of the following when $a = 1$, $b = 2$ and $c = 3$.

- | | | |
|------------|------------|------------|
| 1 $5a$ | 2 $6b$ | 3 $9c$ |
| 4 $6a$ | 5 $7b$ | 6 $2c$ |
| 7 $-4a$ | 8 $-b$ | 9 $-3c$ |
| 10 $a + b$ | 11 $a + c$ | 12 $b + c$ |
| 13 $b - a$ | 14 $a - b$ | 15 $c - a$ |
| 16 $a - c$ | 17 ab | 18 bc |

Exercise 21c (Oral)

Find the value of the following when $x = 4$, $y = -5$ and $z = -3$.

- | | | |
|----------------------|---------------------|--------------------|
| 1 $2x$ | 2 $4y$ | 3 $10z$ |
| 4 $-8z$ | 5 $-y$ | 6 $-2z$ |
| 7 xy | 8 xz | 9 yz |
| 10 x^2 | 11 y^2 | 12 z^2 |
| 13 xyz | 14 $x + y$ | 15 $x - y$ |
| 16 $x + z$ | 17 $z - x$ | 18 $x - z$ |
| 19 $\frac{y+z}{x}$ | 20 $\frac{x-y}{z}$ | 21 $\frac{x+y}{z}$ |
| 22 $\frac{x+z}{x+y}$ | 23 $\frac{y+5z}{x}$ | 24 $\frac{x}{z-y}$ |

Removing brackets

$3 \times (7 + 5)$ means first add 7 and 5, then multiply the result by 3. Suppose a pencil costs 7 cents and a rubber costs 5 cents.

$$\begin{aligned} \text{Cost of a pencil and rubber} \\ &= 7 \text{ cents} + 5 \text{ cents} \\ &= (7 + 5) \text{ cents} = 12 \text{ cents} \end{aligned}$$

If 3 students each buy a pencil and rubber, then, total cost = $3 \times (7 + 5)$ cents = 3×12 cents = 36 cents

There is another way to find the total cost. 3 pencils cost 3×7 cents. 3 rubbers cost 3×5 cents. Altogether,

$$\begin{aligned} \text{total cost} &= 3 \times 7 \text{ cents} + 3 \times 5 \text{ cents} \\ &= 21 \text{ cents} + 15 \text{ cents} \\ &= 36 \text{ cents} \end{aligned}$$

Thus, $3 \times (7 + 5) = 3 \times 7 + 3 \times 5$.

This shows that brackets can be removed by multiplying the 3 into both the 7 and the 5.

Usually we do not write the multiplication sign. We just write $3(7 + 5)$. $3(7 + 5)$ is short for $3 \times (7 + 5)$. Say $3(7 + 5)$ as '3 multiplied into $(7 + 5)$ ' or just '3 into $(7 + 5)$ '.

Thus, $3(7 + 5) = 3 \times 7 + 3 \times 5$

In general terms, using letters for numbers,

$$a(x + y) = ax + ay$$

Notice also that

$$3(7 - 5) = 3 \times 2 = 6$$

$$\text{and } 3 \times 7 - 3 \times 5 = 21 - 15 = 6$$

$$\text{Thus, } 3(7 - 5) = 3 \times 7 - 3 \times 5$$

Again, using letters for numbers,

$$a(x - y) = ax - ay$$

Exercise 21d

Simplify each of the following in two ways.

Question 1 shows you how to do this.

1 $9(7 - 3)$ (a) $9(7 - 3) = 9 \times 4 = 36$

(b) $9(7 - 3) = 9 \times 7 - 9 \times 3$
 $= 63 - 27 = 36$

2 $3(7 + 2)$ 3 $10(6 - 4)$ 4 $9(4 + 3)$

5 $6(4 - 2)$ 6 $7(1 + 9)$ 7 $3(8 - 2)$

8 $3(8 + 2)$ 9 $4(11 - 3)$ 10 $5(10 + 2)$

Example 4

Remove brackets from the following.

(a) $8(2c + 3d)$ (b) $4y(3x - 5)$

(c) $(7a - 2b)3a$

(a) $8(2c + 3d) = 8 \times 2c + 8 \times 3d$
 $= 16c + 24d$

(b) $4y(3x - 5) = 4y \times 3x - 4y \times 5$
 $= 12xy - 20y$

(c) $(7a - 2b)3a$

Notice that we can multiply into the bracket from the right.

$$(7a - 2b)3a = 7a \times 3a - 2b \times 3a$$

$$= 21a^2 - 6ab$$

Exercise 21e (Oral)

Remove brackets from the following.

1 $3(x + y)$ 2 $4(9 - n)$ 3 $2(a - 7)$

4 $5(2p - q)$ 5 $6(3a + 4b)$ 6 $7(7x + 2y)$

7 $2a(4 + 7c)$ 8 $4y(6x - 8)$ 9 $3x(7 - 8p)$

10 $3q(-p + q)$ 11 $2a(-a - b)$

12 $5d(-b - 2c)$ 13 $(a + 2b)2$

14 $(u - v)5$

16 $(5 - 3d)3a$

18 $(4x + t)6x$

15 $(3a + b)3$

17 $(7c - 2)7c$

The multiplier outside the bracket can be a directed number.

Example 5

Remove brackets from the following. (a) $-7(2n + 3)$

(b) $5z(x - 9y)$ (c) $(3 - 5a)(-2a)$

(a) $-7(2n + 3)$

$$= (-7) \times (+2) \times n + (-7) \times (+3)$$

$$= (-14n) + (-21)$$

$$= -14n - 21$$

Notice that $+(-21) = -(+21) = -21$. Adding a negative number is equivalent to subtracting a positive number of the same value.

(b) $-5z(x - 9y)$

$$= (-5z) \times (+1) \times x - (-5z) \times$$

$$(+9) \times y$$

$$= (-5xz) - (-45yz)$$

$$= -5xz + 45yz = 45yz - 5xz$$

Notice that $-(-45yz) = +(45yz) = +45yz$.

(c) $(3 - 5a)(-2a)$

$$= (+3) \times (-2a) - (+5) \times (-2a) \times a$$

$$= (-6a) - (-10a^2)$$

$$= -6a + 10a^2 = 10a^2 - 6a$$

Example 5 shows that when the multiplier is negative, the signs inside the bracket are changed when the bracket is removed.

Exercise 21f (Oral)

Remove brackets from the following.

1 $-3(m + n)$

2 $-2(u + v)$

3 $-4(a + b)$

4 $-5(a - b)$

5 $-8(x - y)$

6 $-9(p - q)$

7 $-4n(3m + 2)$

8 $-7y(3 - 5y)$

9 $-a(4a + 6)$

10 $-2a(-a - 3b)$

11 $-5x(11x - 2y)$

12 $-p(p - 5q)$

13 $(2c + 8d)(-2)$

14 $(-5m + 2n)(-9m)$

15 $(10 + 3t)(-7t)$

16 $-x(2x - 11y)$

17 $-3(12a - 5)$

18 $-5a(-5x - 7y)$

Example 6

Remove brackets and simplify the following.

(a) $3(6a + 3b) + 5(2a - b)$

(b) $2(3x - y) - 3(2x - 3y)$

(c) $x(x - 7) + 4(x - 7)$

$$\begin{aligned}
 \text{(a)} \quad & 3(6a + 3b) + 5(2a - b) \\
 & = 18a + 9b + 10a - 5b \\
 & = 18a + 10a + 9b - 5b \\
 & = 28a + 4b
 \end{aligned}$$

Notice that like terms are grouped.

$$\begin{aligned}
 \text{(b)} \quad & 2(3x - y) - 3(2x - 3y) \\
 & = (+2)(3x - y) + (-3)(2x - 3y)^* \\
 & = 6x - 2y + (-6x) + (+9y)^* \\
 & = 6x - 2y - 6x + 9y \\
 & = 6x - 6x - 2y + 9y \\
 & = 7y
 \end{aligned}$$

* You will be able to leave out the first two steps after trying a few exercises.

$$\begin{aligned}
 \text{(c)} \quad & x(x - 7) + 4(x - 7) \\
 & = x \times x - x \times 7 + 4x - 28 \\
 & = x^2 - 7x + 4x - 28 \\
 & = x^2 - 3x - 28
 \end{aligned}$$

Exercise 21g

Remove brackets and simplify the following.

- | | |
|-----------------------------|----------------------|
| 1 $3a + 2(a + 2b)$ | 2 $6x + 3(2y - x)$ |
| 3 $7p + 5(p - q)$ | 4 $5c + 3(1 + 2c)$ |
| 5 $8a + 5(2a - b)$ | 6 $11x + 3(3x + 2y)$ |
| 7 $12x - 2(4x + 5)$ | 8 $9r - 4(3 + r)$ |
| 9 $14a - 3(2b + 5a)$ | 10 $4a - 5(a - 2)$ |
| 11 $3 - 7(5 - 4x)$ | 12 $10t - 8(3 - t)$ |
| 13 $5(a + 2) + 4(a + 1)$ | |
| 14 $2(5x + 8y) + 3(2x - y)$ | |
| 15 $2(3x - y) + 3(x + 2y)$ | |
| 16 $2(a - 3b) + 3(a - b)$ | |
| 17 $6(4x + y) - 7(3x + 5y)$ | |
| 18 $4(x - 2y) - 3(2x - y)$ | |
| 19 $7(a - b) - 8(a - 2b)$ | |
| 20 $6(a - 2b) - 3(2a + b)$ | |
| 21 $x(x - 2) + 3(x + 2)$ | |
| 22 $x(x + 4) + 5(x + 4)$ | |
| 23 $x(x - 2) + 7(x - 2)$ | |
| 24 $x(x - 8) + 4(x - 8)$ | |
| 25 $a(a + 5) - 2(a + 5)$ | |
| 26 $a(a + 2) - 9(a + 2)$ | |
| 27 $y(y - 3) - 6(y - 3)$ | |
| 28 $z(z - 1) - 10(z - 1)$ | |
| 29 $x(x + a) + a(x + a)$ | |
| 30 $a(a + b) - b(a + b)$ | |

Expanding algebraic expressions

The expression $(a + 2)(b - 5)$ means $(a + 2) \times (b - 5)$. The terms in the first bracket, $(a + 2)$, multiply each term in the second bracket, $(b - 5)$. Just as

$$x(b - 5) = bx - 5x$$

so, writing $(a + 2)$ instead of x ,

$$(a + 2)(b - 5) = b(a + 2) - 5(a + 2)$$

The brackets on the right-hand side can now be removed.

$$\begin{aligned}
 (a + 2)(b - 5) & = b(a + 2) - 5(a + 2) \\
 & = ab + 2b - 5a - 10
 \end{aligned}$$

$ab + 2b - 5a - 10$ is the product of $(a + 2)(b - 5)$.

We often say that the **expansion** of $(a + 2)(b - 5)$ is $ab + 2b - 5a - 10$.

Example 7

Expand the following. (a) $(a + b)(c + d)$
 (b) $(6 - x)(3 + y)$ (c) $(2p - 3q)(5p - 4)$

$$\text{(a)} \quad (a + b)(c + d) = c(a + b) + d(a + b) = ac + bc + ad + bd$$

$$\text{(b)} \quad (6 - x)(3 + y) = 3(6 - x) + y(6 - x) = 18 - 3x + 6y - xy$$

$$\begin{aligned}
 \text{(c)} \quad (2p - 3q)(5p - 4) & = 5p(2p - 3q) - 4(2p - 3q) \\
 & = 10p^2 - 15pq - 8p + 12q
 \end{aligned}$$

When expanding brackets it is most important to be careful with the signs of the terms in the final product.

Exercise 21h

Expand the following.

- | | |
|-----------------------|-----------------------|
| 1 $(p + q)(r + s)$ | 2 $(x + 8)(y + 3)$ |
| 3 $(4 + 5a)(3b + a)$ | 4 $(a - b)(c + d)$ |
| 5 $(x - 7)(y + 1)$ | 6 $(2 - p)(3p + q)$ |
| 7 $(w + x)(y - z)$ | 8 $(a + 6)(b - 9)$ |
| 9 $(2m + 5n)(p - 3q)$ | 10 $(a - b)(c - d)$ |
| 11 $(x - 4)(y - 5)$ | 12 $(5x - y)(x - 3y)$ |

It is often possible to simplify terms in the final product.

Example 8

Expand $(x + 8)(x + 5)$.

$$\begin{aligned}
 (x + 8)(x + 5) & = x(x + 8) + 5(x + 8) \\
 & = x^2 + 8x + 5x + 40
 \end{aligned}$$

Notice that the middle two terms are both terms in x . They can be collected together.

$$(x + 8)(x + 5) = x^2 + 13x + 40$$

Example 9

Expand the following.

$$(a) (6 + x)(3 - x) \quad (b) (2a - 3b)(a + 4b)$$

$$\begin{aligned} (a) (6 + x)(3 - x) &= 3(6 + x) - x(6 + x) \\ &= 18 + 3x - 6x - x^2 \\ &= 18 - 3x - x^2 \end{aligned}$$

$$\begin{aligned} (b) (2a - 3b)(a + 4b) &= a(2a - 3b) + 4b(2a - 3b) \\ &= 2a^2 - 3ab + 8ab - 12b^2 \\ &= 2a^2 + 5ab - 12b^2 \end{aligned}$$

Example 10

Expand $(x - 3)^2$.

$$\begin{aligned} (x - 3)^2 &= (x - 3)(x - 3) \\ &= x(x - 3) - 3(x - 3) \\ &= x^2 - 3x - 3x + 9 \\ &= x^2 - 6x + 9 \end{aligned}$$

Exercise 21i

Expand the following.

- | | |
|-------------------------|-------------------------|
| 1 $(a + 3)(a + 4)$ | 2 $(b + 2)(b + 5)$ |
| 3 $(m + 3)(m - 2)$ | 4 $(n - 7)(n + 2)$ |
| 5 $(x + 2)^2$ | 6 $(y + 1)(y - 4)$ |
| 7 $(c - 2)(c + 5)$ | 8 $(d - 3)(d - 4)$ |
| 9 $(p - 2)(p - 5)$ | 10 $(x - 4)^2$ |
| 11 $(y + 1)(y + 7)$ | 12 $(a - 4)(a + 6)$ |
| 13 $(b - 3)(b - 7)$ | 14 $(c + 5)(c - 1)$ |
| 15 $(3 + d)(2 + d)$ | 16 $(5 - x)(2 + x)$ |
| 17 $(3 - y)(4 - y)$ | 18 $(m + 2n)(m + 3n)$ |
| 19 $(a - 3b)(a + 2b)$ | 20 $(x - 4y)(x - 3y)$ |
| 21 $(p + 2q)^2$ | 22 $(m + 5n)(m - 3n)$ |
| 23 $(a + 5)(2a - 3)$ | 24 $(3x + 4)(x - 2)$ |
| 25 $(2h - k)(3h + 2k)$ | 26 $(5x + 2y)(3x + 4y)$ |
| 27 $(3a - 2b)^2$ | 28 $(5h + k)^2$ |
| 29 $(5a - 2b)(2a - 3b)$ | 30 $(7m - 5n)(5m + 3n)$ |

Factors of algebraic terms

All numbers, other than 1, have two or more factors. For example, the factors of 42 are:

1, 2, 3, 6, 7, 14, 21, 42.

In the same way, algebraic terms have two or more factors. For example, the expression $6ab$ has 16 factors: 1, 2, 3, 6, a , $2a$, $3a$, $6a$, b , $2b$, $3b$, $6b$, ab , $2ab$, $3ab$, $6ab$. Each factor divides exactly into $6ab$. For example,

$$6ab \div 3a = \frac{2 \times 3 \times a \times b}{3 \times a} = 2b$$

$$6ab \div 2ab = \frac{2 \times 3 \times a \times b}{2 \times a \times b} = 3$$

Example 11

Write down all the factors of $5a^2x$.

Expand $5a^2x$ as a product of separate terms:

$$5a^2x = 5 \times 1 \times a \times a \times x$$

The factors will contain:

numerical terms: 1, 5

terms in a : a , $5a$

terms in x : x , $5x$

terms in a^2 : a^2 , $5a^2$

terms in ax : ax , $5ax$

terms in a^2x : a^2x , $5a^2x$

The factors of $5a^2x$ are: 1, 5, a , $5a$, x , $5x$, a^2 , $5a^2$, ax , $5ax$, a^2x , $5a^2x$.

Notice that 1 and the term itself are always factors of an algebraic term.

Exercise 21j (Oral)

State the factors of the following.

- | | | |
|----------|--------------|-------------|
| 1 $3x$ | 2 ab | 3 x^2 |
| 4 a^2b | 5 $6a$ | 6 $5ab$ |
| 7 xyz | 8 $10m^2$ | 9 $25pq$ |
| 10 $8de$ | 11 $2a^2b^2$ | 12 $14ab^2$ |

Highest common factor

Algebraic expressions may have common factors.

Example 12

Find the HCF of $12ab^2$ and $30a^2b$.

$$12ab^2 = 2 \times 2 \times 3 \times a \times b \times b$$

$$30a^2b = 2 \times 3 \times 5 \times a \times a \times b$$

The highest product of factors that is contained in both expressions is $2 \times 3 \times a \times b = 6ab$.

$6ab$ is the HCF of $12ab^2$ and $30a^2b$.

With practice the HCF can be found without expanding each expression.

Exercise 21k (Oral)

Find the HCF of the following.

- | | |
|----------------------------|-------------------------|
| 1 ax and ay | 2 $3d$ and $3e$ |
| 3 ax and bx | 4 $3m$ and $5m$ |
| 5 $2m$ and $2n$ | 6 gp and gq |
| 7 $2xy$ and $7xz$ | 8 $4r$ and $12s$ |
| 9 abz and xyz | 10 $3mx$ and $10nx$ |
| 11 $4pq$ and $4pr$ | 12 abc and abd |
| 13 axy and bxy | 14 $9pq$ and $11pq$ |
| 15 $4ax$ and $10bx$ | 16 amx and anx |
| 17 x^2 and $5x$ | 18 $2x^2$ and $8x$ |
| 19 $5x^2$ and $10x$ | 20 $3a^2$ and $12a$ |
| 21 mn^2 and m^2n | 22 $8de$ and $6de$ |
| 23 $10m^2$ and $15m^2$ | 24 $9ab^2$ and $12a^2b$ |
| 25 $18xy$ and $6xy$ | 26 $2lad$ and $14bd$ |
| 27 $5a^2$ and $20a^2b$ | 28 $16xy^2$ and $4xy$ |
| 29 $18a^2y$ and $27x^2y^2$ | 30 $17ax^2$ and $3by$ |

Factorisation

Example 13

Complete the brackets in the statement

$$15ax + 10a = 5a(\quad).$$

$$15ax + 10a = 5a(\quad)$$

The contents of the bracket multiplied by $5a$ should give $15ax + 10a$. See **Removing brackets** earlier. Thus, divide each term in $15ax + 10a$ by $5a$ to find the contents of the bracket.

$$\begin{aligned} \text{Contents of bracket} &= \frac{15ax}{5a} + \frac{10a}{5a} \\ &= 3x + 2 \end{aligned}$$

$$\text{Thus } 15ax + 10a = 5a(3x + 2)$$

$$\text{Check: RHS} = 5a(3x + 2) = 5a \times 3x + 5a \times 2 = 15ax + 10a = \text{LHS}$$

In the above example, $5a$ is the HCF of $15ax$ and $10a$. $5a(3x + 2)$ is the factorised form of $15ax + 10a$. $5a$ and $(3x + 2)$ are factors of $15ax + 10a$. **Factorisation** means writing an expression in terms of its factors. Think of factorisation as the inverse of removing brackets.

Example 14

Factorise the following.

(a) $12y + 8z$

(b) $4n^2 - 2n$

(c) $24pq - 16p^2$

(a) $12y + 8z$

The HCF of $12y$ and $8z$ is 4.

$$\begin{aligned} 12y + 8z &= 4\left(\frac{12y}{4} + \frac{8z}{4}\right) \\ &= 4(3y + 2z) \end{aligned}$$

(b) $4n^2 - 2n$

The HCF of $4n^2$ and $2n$ is $2n$.

$$\begin{aligned} 4n^2 - 2n &= 2n\left(\frac{4n^2}{2n} - \frac{2n}{2n}\right) \\ &= 2n(2n - 1) \end{aligned}$$

(c) $24pq - 16p^2$

The HCF of $24pq$ and $16p^2$ is $8p$.

$$\begin{aligned} 24pq - 16p^2 &= 8p\left(\frac{24pq}{8p} - \frac{16p^2}{8p}\right) \\ &= 8p(3q - 2p) \end{aligned}$$

* With practice the first line of working can be left out. The results of factorization can be checked by removing the brackets.

Exercise 21l

1 Copy the following statements and complete the brackets.

(a) $9x + 3y = 3(\quad)$

(b) $5a - 15b = 5(\quad)$

(c) $ax + ay = a(\quad)$

(d) $px + qx = x(\quad)$

(e) $8am - 8bm = 8m(\quad)$

(f) $a - ay = a(\quad)$

(g) $2s - rs = s(\quad)$

(h) $3ab + 5ax = a(\quad)$

(i) $3abx + 5adx = ax(\quad)$

(j) $9xy - 3xz = 3x(\quad)$

(k) $12cm + 16dm = 4m(\quad)$

(l) $15x^2 - 10x = 5x(\quad)$

(m) $18ax + 9x = 9x(\quad)$

(n) $4m^2 - 2m = 2m(\quad)$

(o) $2pq - 6q^2 = 2q(\quad)$

(p) $5a^2 + 2ax = a(\quad)$

2 Factorise the following.

(a) $12c + 6d$

(b) $4a - 8b$

(c) $6z - 3$

(d) $9x + 12y$

(e) $xy + xz$

(f) $bc + dc$

- (g) $9x + ax$ (h) $abc + abd$
 (i) $xyz - ayz$ (j) $pmq + amb$
 (k) $3ab - 6ac$ (l) $12ax + 8bx$
 (m) $3x^2 - x$ (n) $6m^2 - 2$
 (o) $2abx + 7acx$ (p) $3d^2e + 5d^2$
 (q) $4am^2 - 6am$ (r) $-15x^2 - 10$
 (s) $-18fg - 12g$ (t) $-5xy + 10y$

Lowest common multiple

Example 15

Find the LCM of the following.

- (a) $8a$ and $6a$ (b) $2x$ and $3y$

(a) $8a = 2 \times 2 \times 2 \times a$

$6a = 2 \times 3 \times a$

LCM = $2 \times 2 \times 2 \times 3 \times a = 24a$

(b) $2x = 2 \times x$

$3y = 3 \times y$

LCM = $2 \times 3 \times x \times y = 6xy$

Exercise 21m (Oral)

Find the LCM of the following.

- | | |
|------------------------|---------------------|
| 1 a and b | 2 x and 5 |
| 3 $2a$ and 3 | 4 $3a$ and $4b$ |
| 5 $2x$ and $5y$ | 6 $9a$ and a |
| 7 x and $3x$ | 8 ab and bc |
| 9 xy and yz | 10 $3b$ and $2b$ |
| 11 x^2 and x | 12 $3a$ and a^2 |
| 13 $3m$ and m^2n | 14 $2a^2$ and $9ab$ |
| 15 $3x^2y$ and $2xy^2$ | 16 $6ab$ and $5b^2$ |

Algebraic fractions

Equivalent fractions

Equivalent fractions can be made by multiplying or dividing the numerator and denominator of a fraction by the same quantity.

multiplication

(a) $\frac{a}{2} = \frac{a \times 3}{2 \times 3} = \frac{3a}{6}$

(b) $\frac{a}{2} = \frac{a \times a}{2 \times a} = \frac{a^2}{2a}$

(c) $\frac{3}{d} = \frac{3 \times 2b}{d \times 2b} = \frac{6b}{2bd}$

division

(d) $\frac{4x}{6y} = \frac{4x \div 2}{6y \div 2} = \frac{2x}{3y}$

(e) $\frac{5ab}{10b} = \frac{5ab \div 5b}{10b \div 5b} = \frac{a}{2}$

Example 16

Fill the blanks in the following.

(a) $\frac{3a}{2} = \frac{\quad}{10}$ (b) $\frac{5ab}{12a} = \frac{\quad}{12}$ (c) $\frac{9bc}{12b} = \frac{3c}{\quad}$

- (a) Compare the two denominators.
 $2 \times 5 = 10$

The denominator of the first fraction has been multiplied by 5. The numerator must also be multiplied by 5.

$\frac{3a}{2} = \frac{3a \times 5}{2 \times 5} = \frac{15a}{10}$

- (b) The denominator of the first fraction has been divided by a . The numerator must also be divided by a .

$\frac{5ab}{12a} = \frac{5ab \div a}{12a \div a} = \frac{5b}{12}$

- (c) Divide both numerator and denominator by $3b$.

$\frac{9bc}{12b} = \frac{9bc \div 3b}{12b \div 3b} = \frac{3c}{4}$

Exercise 21n (Oral)

Fill the blanks in the following.

1 $\frac{8b}{5} = \frac{\quad}{15}$ 2 $\frac{a}{b} = \frac{ax}{\quad}$ 3 $\frac{2x}{5} = \frac{\quad}{10}$

4 $\frac{3a}{8} = \frac{6a}{\quad}$ 5 $\frac{3}{12a} = \frac{\quad}{4a}$ 6 $\frac{9c}{21d} = \frac{\quad}{7d}$

7 $\frac{9ah}{6ak} = \frac{3h}{\quad}$ 8 $\frac{bm}{my} = \frac{\quad}{y}$ 9 $\frac{\quad}{3a} = \frac{1}{a}$

10 $\frac{\quad}{8yz} = \frac{3x}{2y}$ 11 $\frac{2c}{a} = \frac{6c^2}{\quad}$ 12 $\frac{3m}{1} = \frac{\quad}{2d}$

Adding and subtracting algebraic fractions

As with common numerical fractions, algebraic

fractions must have common denominators before they can be added or subtracted.

Example 17

Simplify the following.

$$(a) \frac{5a}{8} - \frac{3a}{8} \quad (b) \frac{5}{2d} + \frac{7}{2d} \quad (c) \frac{1}{x} - \frac{1}{3x}$$

$$(d) \frac{4}{a} + b \quad (e) \frac{1}{u} + \frac{1}{v} \quad (f) \frac{5}{4c} - \frac{4}{3d}$$

$$(a) \frac{5a}{8} - \frac{3a}{8} = \frac{5a - 3a}{8} = \frac{2a}{8} = \frac{2a \div 2}{8 \div 2} = \frac{a}{4}$$

$$(b) \frac{5}{2d} + \frac{7}{2d} = \frac{5 + 7}{2d} = \frac{12}{2d} = \frac{12 \div 2}{2d \div 2} = \frac{6}{d}$$

$$(c) \frac{1}{x} - \frac{1}{3x}$$

The LCM of x and $3x$ is $3x$.

$$\begin{aligned} \frac{1}{x} - \frac{1}{3x} &= \frac{3 \times 1}{3 \times x} - \frac{1}{3x} = \frac{3}{3x} - \frac{1}{3x} \\ &= \frac{3 - 1}{3x} = \frac{2}{3x} \end{aligned}$$

$$(d) \frac{4}{a} + b = \frac{4}{a} + \frac{b}{1}$$

The LCM of a and 1 is a .

$$\begin{aligned} \frac{4}{a} + \frac{b}{1} &= \frac{4}{a} + \frac{a \times b}{a} = \frac{4}{a} + \frac{ab}{a} \\ &= \frac{4 + ab}{a} \end{aligned}$$

This does not simplify further.

$$(e) \frac{1}{u} + \frac{1}{v}$$

The LCM of u and v is uv .

$$\begin{aligned} \frac{1}{u} + \frac{1}{v} &= \frac{1 \times v}{uv} + \frac{1 \times u}{uv} \\ &= \frac{v}{uv} + \frac{u}{uv} \\ &= \frac{v + u}{uv} \end{aligned}$$

This does not simplify further.

$$(f) \frac{5}{4c} - \frac{4}{3d}$$

The LCM of $4c$ and $3d$ is $12cd$.

$$\begin{aligned} \frac{5}{4c} - \frac{4}{3d} &= \frac{5 \times 3d}{12cd} - \frac{4 \times 4c}{12cd} \\ &= \frac{15d}{12cd} - \frac{16c}{12cd} \\ &= \frac{15d - 16c}{12cd} \end{aligned}$$

Exercise 21a

Simplify the following.

$$1 \quad \frac{7a}{5} - \frac{4a}{5} \quad 2 \quad \frac{9b}{2} - \frac{3b}{2} \quad 3 \quad \frac{4x}{9} + \frac{2x}{9}$$

$$4 \quad \frac{x}{2} + \frac{x}{3} \quad 5 \quad \frac{a}{4} - \frac{a}{20} \quad 6 \quad \frac{6x}{5} - \frac{3x}{5}$$

$$7 \quad \frac{2}{a} + \frac{5}{a} \quad 8 \quad \frac{3}{5y} - \frac{1}{5y} \quad 9 \quad \frac{5}{2x} + \frac{1}{2x}$$

$$10 \quad \frac{4}{3a} - \frac{1}{3a} \quad 11 \quad \frac{7}{8y} + \frac{9}{8y} \quad 12 \quad \frac{1}{x} + \frac{1}{x}$$

$$13 \quad \frac{1}{3x} + \frac{1}{x} \quad 14 \quad \frac{1}{a} - \frac{1}{5a} \quad 15 \quad \frac{1}{z} - \frac{1}{z}$$

$$16 \quad \frac{1}{4a} - \frac{1}{6a} \quad 17 \quad \frac{1}{3x} + \frac{1}{5x} \quad 18 \quad \frac{3}{2a} - \frac{1}{2a}$$

$$19 \quad \frac{3}{x} + 1 \quad 20 \quad 2 - \frac{b}{a} \quad 21 \quad 3 + \frac{1}{x}$$

$$22 \quad \frac{9}{x} + y \quad 23 \quad 2b + \frac{3}{4} \quad 24 \quad p - \frac{1}{x}$$

$$25 \quad \frac{1}{a} + \frac{1}{b} \quad 26 \quad \frac{1}{x} - \frac{1}{y} \quad 27 \quad \frac{3}{4} + \frac{1}{m}$$

$$28 \quad \frac{3}{c} - \frac{2}{d} \quad 29 \quad \frac{5}{4a} - \frac{2}{3b} \quad 30 \quad \frac{2}{5x} + \frac{1}{3y}$$

Fractions with brackets

$$\frac{x+6}{3} \text{ is a short way of writing } \frac{(x+6)}{3}$$

$$\frac{1}{3}(x+6).$$

Notice that all of the terms of the numerator are divided by 3.

$$\frac{x+6}{3} = \frac{(x+6)}{3} = \frac{1}{3}(x+6) = \frac{1}{3}x + 2$$

Example 18

Simplify (a) $\frac{x+3}{5} + \frac{4x-2}{5}$,

(b) $\frac{7a-3}{6} - \frac{3a-5}{4}$.

$$\begin{aligned} \text{(a)} \quad \frac{x+3}{5} + \frac{4x-2}{5} &= \frac{(x+3) + (4x-2)}{5} \\ &= \frac{x+3+4x-2}{5} \\ &= \frac{5x+1}{5} \end{aligned}$$

(b) The LCM of 6 and 4 is 12.

$$\begin{aligned} \frac{7a-3}{6} - \frac{3a+5}{4} &= \frac{2(7a-3)}{2 \times 6} - \frac{3(3a+5)}{3 \times 4} \\ &= \frac{2(7a-3) - 3(3a+5)}{12} \end{aligned}$$

removing brackets $= \frac{14a - 6 - 9a - 15}{12}$

collecting like terms $= \frac{5a - 21}{12}$

The next example shows that further simplification is sometimes possible after collecting like terms.

Example 19

Simplify $\frac{4x+1}{3} - \frac{x-5}{12}$.

The LCM of 3 and 12 is 12.

$$\begin{aligned} \frac{4x+1}{3} - \frac{x-5}{12} &= \frac{4(4x+1)}{12} - \frac{x-5}{12} \\ &= \frac{4(4x+1) - (x-5)}{12} \end{aligned}$$

removing brackets $= \frac{16x+4-x+5}{12}$

collecting like terms $= \frac{15x+9}{12}$

factorizing the numerator $= \frac{3(5x+3)}{12}$

dividing numerator and denominator by 3 $= \frac{5x+3}{4}$

Example 21 p

1 Simplify the following.

(a) $\frac{2a-3}{2} + \frac{a+4}{2}$ (b) $\frac{3b+4}{3} + \frac{2b-5}{3}$

(c) $\frac{4c-3}{5} - \frac{2c+1}{5}$ (d) $\frac{3x+4}{4} - \frac{x-3}{4}$

(e) $\frac{z+5}{4} + \frac{3z-5}{2}$ (f) $\frac{2n+7}{3} - \frac{5n+6}{4}$

(g) $\frac{3a-2}{4} + 2a$ (h) $3b - \frac{5b-1}{2}$

(i) $\frac{5u-3}{4} + \frac{u-3}{6}$ (j) $\frac{m-1}{6} - \frac{m+1}{8}$

(k) $\frac{3c-2d}{10} + \frac{2c-3d}{15}$ (l) $\frac{2a+3b}{9} + \frac{a-4b}{6}$

2 Simplify the following as far as possible.

(a) $\frac{3x+1}{4} - \frac{x-5}{4}$ (b) $\frac{7x-14}{10} - \frac{2x+1}{10}$

(c) $\frac{6h+5}{7} - \frac{4h-6}{21}$ (d) $\frac{a+3b}{3} - \frac{5a-3b}{6}$

Statistics (1)

Syllabus Reference 5.8.1

Statistical information

Suppose a stranger asks you for **information** about yourself. You could tell him or her a lot of things. For example: your name; the town you live in; the school you go to; what you ate last night; the things you like; the things you don't like; etc.

You might also use numbers. For example: I am 15 years old; I have 4 brothers and 2 sisters; I am 171 cm tall and my mass is 48 kg; I wear size 8 shoes; my village is 5 km from the school; etc.

When numbers are used, the information is called **statistics**. Some statistics about two football teams are given in Table 22.1.

Table 22.1

	games played	won	lost
Giantkillers	18	10	5
Locomotives	15	2	8
	drawn	goals for	goals against
Giantkillers	3	60	21
Locomotives	5	19	37

The statistics above give a lot of information about the two teams. Giantkillers seem to be more successful than Locomotives. A good player, looking at the statistics, might prefer to play for Giantkillers than to play for Locomotives. Thus statistics can help when making decisions.

Exercise 22a (Oral)

1 Refer to the statistics in Table 22.1 about the Giantkillers and Locomotives.

- Which team has played more games?
- Which team has drawn more games?
- Do the games won, lost and drawn add up to the games played?
- How many goals have the two teams scored altogether?
- How many goals have been scored against the two teams altogether?
- How many games have the two teams lost altogether?
- For every game that Giantkillers have lost, how many have they won?
- For every game that Locomotives have lost, how many have they won?
- For every goal that Locomotives have scored, approximately how many goals have been scored against them?
- If 'goal average' means goals for divided by goals against, find, to the nearest whole number, the goal average for Giantkillers.
- If a team gets 2 points for a win and 1 point for a draw, how many points do Giantkillers have?
- Similarly, how many points do Locomotives have?

2 Table 22.2 gives the statistics for the numbers of students at a Secondary School for the years 1987 to 1992.

Table 22.2

	1987	1988	1989	1990	1991	1992
number of students	361	364	399	435	470	506

- (a) Is the school growing in size?
 (b) What is the difference in the number of students in 1992 and 1987?
 (c) If there are about 36 students in each class, how many classes did the school have in 1987 and 1988?
 (d) In one year the school started a new Form 1 class. Which year?
 (e) In which year will that Form 1 class be a Form 5 class?
 (f) Estimate the number of students that the school will have in 1993.
- 3 A girl made a note of the first 100 vehicles that passed her on a road. The numbers of each type of vehicle are given in Table 22.3.

Table 22.3

	lorry	bus	car	taxi	motor bike	bicycle
number	7	0	28	2	15	48

- (a) Nearly half of the vehicles were of one kind. What were they?
 (b) How many vehicles had only two wheels?
 (c) How many cars were there for every one lorry?
 (d) Which was the third most common type of vehicle?
 (e) How many buses did the girl see?
 (f) Is it true to say that buses never go on the road?
 (g) Is this road more likely to be in a big city or in a small village? Give reasons for your answer.
- 4 Table 22.4 shows how much a family spends on food, rent and entertainment for each of four months.

Table 22.4

	month 1	month 2	month 3	month 4
food	\$ 178	\$ 150	\$ 204	\$ 168
rent	\$ 96	\$ 96	\$ 96	\$ 96
entertainment	\$ 72	\$ 80	\$ 58	\$ 68

- (a) Which item always costs the same?
 (b) Which item is always the most expensive?

Table 22.5

	killed	injured	total
1st year	8 000	30 023	
2nd year	9 252	28 854	

- (a) Find the totals for each year.
 (b) Which year was safer?
 (c) How many people were killed in the two years?
 (d) One of the four numbers looks like an estimate. Which one?

Collecting data

It would be impossible to give statistics unless **data** were collected beforehand. Data means basic information, usually in number form. To be able to collect data, you need to be able to count. You also need to be able to write down, or **record**, the data clearly. The examples in Fig. 22.1 show the same data collected in two different ways by two students.

The first student tried to write down every vehicle as it came by. When two bicycles came by he did not have time to write them down properly. It is easy to make mistakes when counting this student's totals.

The second student spent some time before beginning to record. She wrote down all the vehicles she could think of in a column. When

(a) bus, car, car, car, lorry, bicycle,
bicycle, car, car, lorry, bicycle, car

(b) Vehicles	Tally	Total
Car		4
Bus		1
Lorry		2
Taxi		
Bicycle		3
Motorbike		

Fig. 22.1 (a) First student (b) Second student

a vehicle came by she made a tally. It is easy to count her totals.

Work through the assignments in Exercise 22b; they show how to collect data clearly.

Exercise 22b

Keep the data you collect in the following assignments. You will use the data in Exercise 22d.

1 Class statistics

Make a large chart showing the full name of everyone in your class. The chart should contain the columns given in Fig. 22.2. Pin the chart on the classroom wall. Find your name and enter your personal statistics under the column headings.

2 Traffic survey: Types of vehicle

Make a table in your exercise book as in Table 22.6. Go to a place where traffic passes. Make a tally of the different kinds of vehicle that pass in 1 hour. (The time may be

longer or shorter. Try to get between 50 and 100 vehicles.)

Table 22.6

Name:		
Place:		Date:
Direction of traffic: towards.....		
Vehicle	Tally	Totals
Car		
Bicycle		
Motorbike		
Lorry		
Taxi		
Bus		
Others		

3 Traffic survey: Traffic density

Make out a page of your exercise book as shown in Table 22.7.

Go to a place where traffic passes. Make a tally of the number of vehicles that pass during each hour for the 12 hours from 8 am to 8 pm. Work in groups. Change groups each hour.

Note: If a vehicle passes at, say, 1100 hours, count it in the 1000 to 1100 period.

Note to teacher: The times available for the Survey will depend on the school day. It may be necessary to obtain permission for groups to leave other lessons. At boarding schools weekend days could be used for the Survey.

Class:	Date:				
Name	Age (years)	Height (cm)	Mass (kg)	Number of brothers	Number of sisters
Alice Mandora	14	169	47	1	4

Fig. 22.2

Table 22.7

Name:		Date:
Place:		
Time	Number of vehicles (tally)	Totals
0800-0900		
0900-1000		
1000-1100		
⋮		
1900-2000		

4 Nature study

Mark out a small plot of land (about 1 m²). Count all the things you find growing, living or lying in the plot. Record the numbers of things you find. Some examples are given in Table 22.8, but you may find others.

Table 22.8

Things	Tally	Things	Tally
Stones (> 2 cm)		Insects	
Stick (> 2 cm)			
Seeds			
Flowers			
Bits of paper			

5 Questionnaire

A **questionnaire** is a set of questions. Make a copy of the questionnaire in Table 22.9. Leave enough spaces for tally marks. Ask the questions of 10 or 20 working adults, e.g. shop or market traders, office workers, farmers, mechanics, etc. Be sure that each person does not mind answering the questions. Be polite.

Presentation of data

Data should be **presented** clearly. Good presentation makes statistical data easy to read and understand. The following example is used to

Table 22.9

Questionnaire	Date:			
How many languages can you speak?	1	2	3	4
	5	6	More than 6	
Which schools (if any) did you go to?	Primary	Secondary	Other	None
How many children do you have?	0	1	2	3
	4	5	6	>6
Which vehicles have you travelled in?	Car	Lorry	Train	Aeroplane
Have you ever been to Hazare?	YES			
	NO			
Have you ever been to Bulawayo?	YES			
	NO			
Do you own any cattle, sheep or goats?	YES			
	NO			

show different kinds of presentation:

An English teacher gave an essay to 15 students. She graded the essays from A (very good), through B, C, D, E to F (very poor). The grades of the students were:
B, C, A, B, A, D, F, E, C, C, A, B, B, E, B

Rank order list

Rank order means in order from highest to lowest. The 15 grades are given in rank order below:

A, A, A, B, B, B, B, B, C, C, C, D, E, E, F
Notice that *all* the grades are put in the list even although most of them appear more than once.

The advantages of the rank order list, in this case, are that we can easily find the following: the highest and lowest grades; the number of students who got each grade; the most common grade; the number of students above and below each grade; and so on.

Frequency table

Frequency means the number of times something happens. For example, 3 students got grade A. The frequency of Grade A is 3. Table 22.10, a **frequency table**, gives the frequency of each grade.

Table 22.10

grade	A	B	C	D	E	F
frequency	3	5	3	1	2	1

In most cases, a picture will show the meaning of statistical data more clearly than a list or table of numbers. The following methods of presentation give the data of the example in picture, or **graph**, form.

Pictogram

A **pictogram** uses pictures or drawings to give a quick and easy meaning to statistical data. In the pictogram, in Fig. 22.3, each pin figure represents a student who gets the grade shown.

Grade A	人 人 人
Grade B	人 人 人 人 人
Grade C	人 人 人
Grade D	人
Grade E	人 人
Grade F	人

Fig. 22.3

Bar chart

A **bar chart** is very like a pictogram. The number of students who get each grade is represented by a bar instead of a picture. The height of each bar in Fig. 22.4 represents the frequency of that grade.

The scale at the left-hand side of the bar chart shows the frequency. Each bar is the same width.

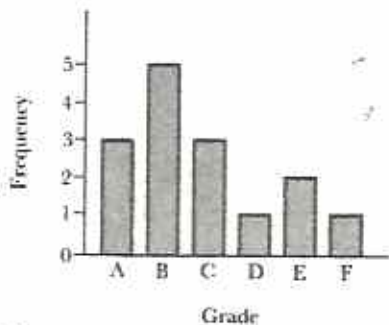


Fig. 22.4

Pie chart

A **pie chart** is a graph in the shape of a circular 'pie'. In the example in Fig. 22.5, the total number of students make up the whole pie. Each piece of the pie is a sector of the circle. The size of each sector represents the number of students who get the grade shown in that sector.

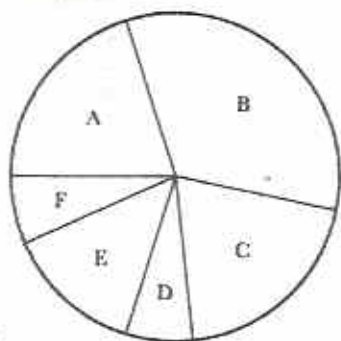


Fig. 22.5

Usually there are *no* numbers on a pie chart. The sizes of the sectors give a quick comparison between the numbers of students getting each grade.

Exercise 22c (Oral)

- The following is a rank order list of an exam result: 87, 82, 78, 76, 75, 70, 66, 64, 59, 59, 59, 51, 49, 48, 41
 - How many students took the exam?
 - What was the highest mark?
 - What the lowest mark?
 - What is the mark of the student who came 6th?
 - What is the position of the student who got 76 marks?

- (f) Three students got 59 marks. What is their position?
- (g) What is the position of the student who got 51 marks?
- (h) How many students got less than 75 marks?
- (i) How many students got more than 45 marks?
- (j) If 45 is the pass mark, how many students failed?
- (k) What is the mark of the student in the middle of the rank order?
- 2 Frequency Table 22.11 shows a tally of types of vehicles that were wrecked in serious accidents during a month on a busy road.

Table 22.11

vehicle	car	lorry	bus	taxi	others
frequency	###	### 			

- (a) Read the frequencies as numbers instead of tallies.
- (b) Which type of vehicle had the biggest number of serious accidents?
- (c) How many vehicles were wrecked altogether?
- (d) Name some types of vehicles that might be included in 'others'.
- (e) Is it true that cars and lorries together had nearly 3 times as many serious accidents as all the other vehicles?
- 3 The pictogram in Fig. 22.6 shows the manpower of the army, navy and airforce of a country.

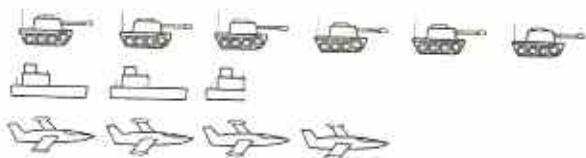


Fig. 22.6

- (a) The pictogram shows tanks, ships and aircraft. Which of these represents the navy?
- (b) Which has the most men, the army, the navy or the airforce?

- (c) Which has the least men?
- (d) Does the fact that there are 6 tanks mean that the army has only 6 men?
- (e) Each tank, ship and aircraft represents 10 000 men. How many men are in the army?
- (f) How many men are in the airforce?
- (g) Approximately how many ships are there?
- (h) Approximately how many men are in the navy?
- 4 The bar chart in Fig. 22.7 shows the rainfall in mm at Chirundu for each month in a year.

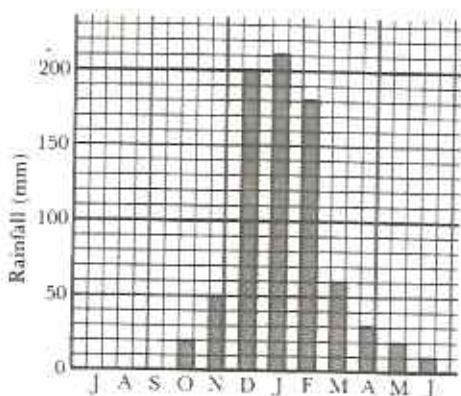


Fig. 22.7

- (a) Which month had most rainfall?
- (b) How many mm of rain fell in the wettest month?
- (c) Which months had no rainfall?
- (d) Which months had less than 50 mm of rain.
- (e) List the six wettest months in rank order.
- (f) Find, in mm, the total rainfall for the year.
- (g) Is it true that over half the rainfall for the year fell in just two months?
- (h) Give a reason for the order of the months in Fig. 22.7.
- 5 The pie chart in Fig. 22.8 shows the division of money that a government spends on Universities, Teacher Training Colleges, Secondary Schools and Primary Schools.

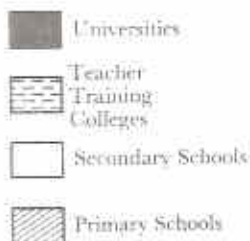
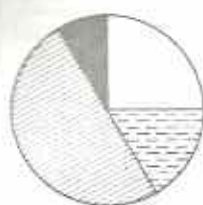


Fig. 22.8

- Does the pie chart tell you how much money the government spent?
- Which of the four gets least money?
- Which of the four gets most money?
- Can you think of reasons for your answers to (b) and (c)?
- What fraction of the money is spent on Primary Schools?
- What fraction of the money is spent on Secondary Schools?
- Approximately what fraction of the money is spent on Teacher Training Colleges?

Exercise 22d

Use squared paper to draw pictograms and bar charts.

- A History test was graded from A to D. The results of ten students are given below:
B, A, C, C, B, C, D, B, A, B
 - List the grades in rank order.
 - Which grade did most students get?
 - How many students got above grade C?
 - Make a frequency table of the results.
 - Draw a pictogram to show the results of the test.
- The dress sizes of 20 women are given in frequency table 22.12.

Table 22.12

size	10	12	14	16	18
frequency	1	6	8	4	1

- Draw a bar chart to show the frequencies of the dress sizes.
- If you were a trader selling dresses, which three sizes of dress would you order most of?

- A transport company has 6 lorries, 4 vans and 2 cars.
 - How many vehicles does the transport company have?
 - Draw a pie chart to show how the vehicles are divided.
- 15 people were asked to name their favourite colour. Their answers are shown in the frequency table 22.13.

Table 22.13

colour	blue	red	green	yellow	black
frequency	3	4	5	1	2

- Which is the most popular colour?
 - Draw a bar chart to show the results in the table. If possible, use the given colours to colour the bars.
- Every 800 g of dried fish contains about 300 g of water, 100 g of fats, 300 g of protein and 100 g of other substances.
 - What is 100 g as a fraction of 800 g?
 - What is 300 g as a fraction of 800 g?
 - What is $\frac{1}{8}$ of 360° ?
 - What is $\frac{3}{8}$ of 360° ?
 - Draw a pie chart to show the contents of dried fish.
 - Use the data you collected for Exercise 22b (Class statistics), question 1.
 - Copy and complete the frequency table (Table 22.14) of ages of students.

Table 22.14

age	12 years and under	13 years	14 years	15 years	16 years and over
frequency					

- What is the most common age?
 - Draw a bar chart to show the frequency of ages of students in your class.
- Use the data you collected for Exercise 22b (Types of vehicle), question 4. Either:
 - Draw a pictogram to represent the types and numbers of vehicles, *or*
 - draw a bar chart to show the types and frequencies of vehicles.

- 8 Use the data you collected for Exercise 22b (Traffic density), question 3.
- (a) Copy and complete the frequency table (Table 22.15). (Note that times are in 2-hour intervals.)

Table 22.15

time of day	0800-1000 hours	1000-1200 hours	1200-1400 hours
number of vehicles			
time of day	1400-1600 hours	1600-1800 hours	1800-2000 hours
number of vehicles			

- (b) Draw a bar chart to show the numbers of vehicles at different times of day.
- (c) Which are the busiest times of the day for traffic?
- 9 Use the data you collected for Exercise 22b (Nature study), question 4. Draw a pictogram to represent the number of things in your plot.
- Note:* if the numbers are high, let each picture represent 10 things. For example, 30 ants, 45 stones and 12 flowers can be shown as in Fig. 22.9.

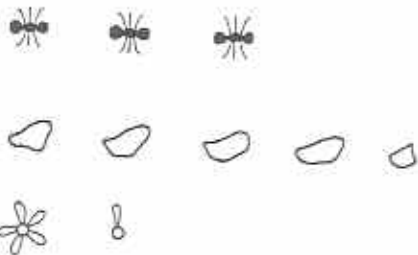


Fig. 22.9

- 10 Use the data you collected for Exercise 22b (Questionnaire), question 5. In the questionnaire, 10 people replied to the question, 'Which schools did you go to?'
- (a) Combine your results with those of your classmates to get the replies of about 300 people.
- (b) Make a frequency table as shown in Table 22.16.

Table 22.16

schools	Primary	Secondary	other	none
frequency				

- (c) Draw a bar chart to show the schools attended by your sample of people.
- 11 A farmer has 215 cattle, 53 sheep and 92 goats.
- (a) How many animals has he altogether?
- (b) Use a protractor to draw a pie chart showing the animals that the farmer has.
- 12 The following are the sizes of shoes worn by 20 people: 7, 9, 6, 10, 8, 8, 9, 11, 8, 7, 9, 6, 8, 10, 9, 8, 7, 7, 8, 9.
- (a) Copy and complete the frequency table (Table 22.17).

Table 22.17

size	frequency
6	
7	
8	
9	
10	
11	

- (b) Draw a bar chart showing the frequency of shoe size.
- (c) A trader sells shoes. Which sizes do you think she sells most of?
- 13 A hospital has 300 patients. Table 22.18 shows the sex and type of treatment of the patients, by number.

Table 22.18

	male	female
surgical	25	75
medical	100	100

Draw a pie chart to represent the data in Table 22.18.

14 Table 22.19 gives the population of Zimbabwe in 10-year intervals from 1940 to 1990.

Table 22.19

year	1940	1950	1960	1970	1980	1990
population (millions)	1,9	2,7	3,8	5,3	7,4	9,4

Show this information on a bar chart.

Plane shapes (2) Symmetry

Syllabus Reference 5.7.6

Line symmetry

A **line of symmetry** divides a shape into two parts, each part a reflection of the other. A shape which has one or more lines of symmetry is said to be **symmetrical**. Fig. 23.1 shows some of the symmetrical shapes found in Chapter 12.

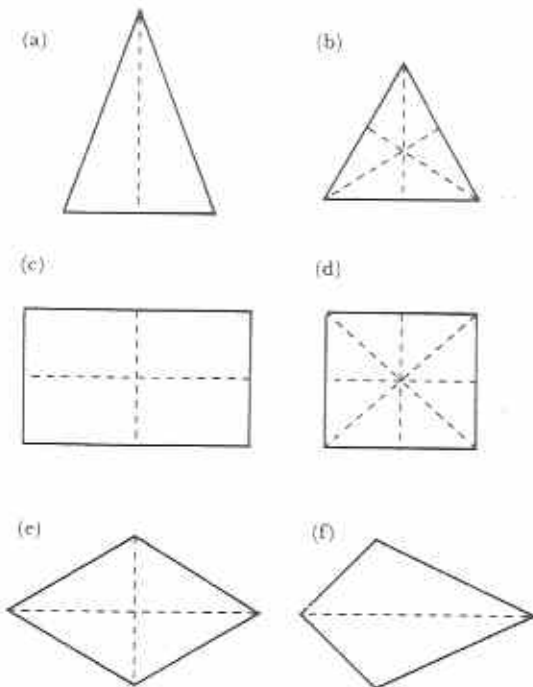


Fig. 23.1 Lines of symmetry: (a) isosceles triangle
(b) equilateral triangle (c) rectangle
(d) square (e) rhombus (f) kite

Fig. 23.2 shows how to make a symmetrical shape by folding and cutting a piece of paper. The symmetrical shapes in Fig. 23.3 were

made by folding pieces of paper which had wet paint on them.

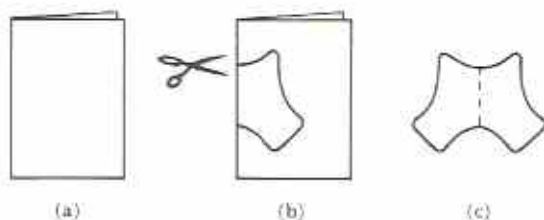


Fig. 23.2 (a) Fold a sheet of paper
(b) Cut out any shape through both thicknesses of paper
(c) Unfold the part which has been cut out, the result is symmetrical

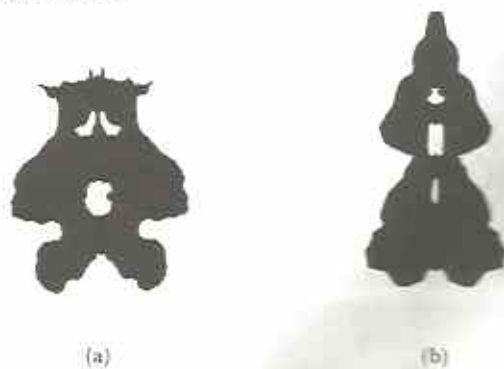


Fig. 23.3

Since one side of a symmetrical shape is a reflection of the other, line symmetry is often called **mirror symmetry** or **bi-lateral symmetry**. *Bi-lateral* means *two-sided*.

Exercise 23a

- Use the methods shown in Figs 23.2 and 23.3 to make some symmetrical shapes and designs.
- Fold a sheet of paper twice as in Fig. 23.4. Cut through all four thicknesses of paper.



Fig. 23.4

Unfold the shape you have cut out. How many lines of symmetry does it have?

3 Which of the shapes in Fig. 23.5 have lines of symmetry?

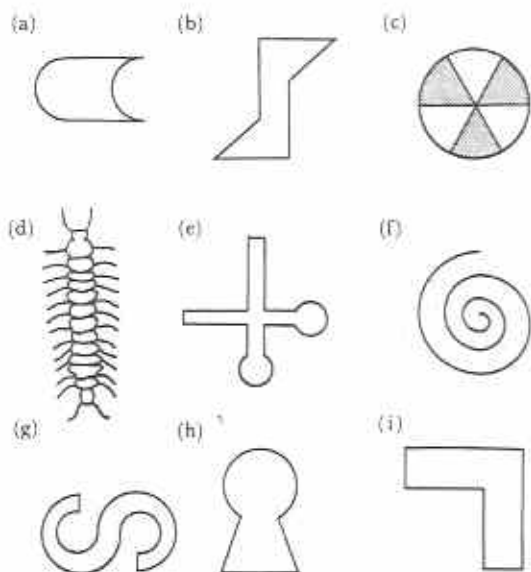


Fig. 23.5

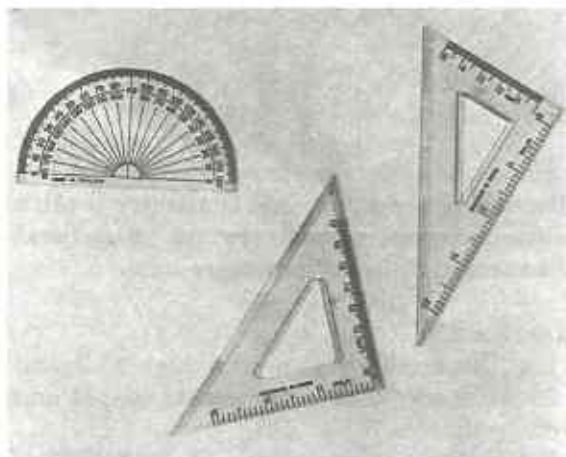


Fig. 23.6

4 Which of the mathematical instruments in Fig. 23.6 have bi-lateral symmetry? (Neglect any writing on the instruments.)

5 Make a copy of each shape in Fig. 23.7. Draw any lines of symmetry.

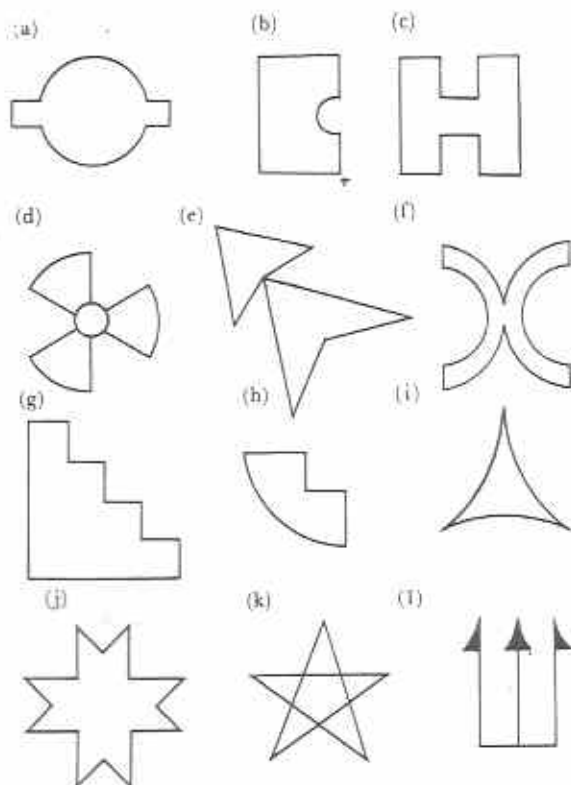


Fig. 23.7

6 Fold a sheet of paper three times as in Fig. 23.8.

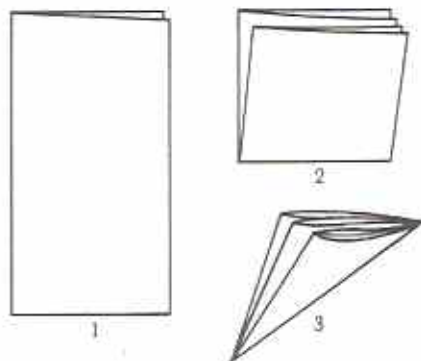


Fig. 23.8

Cut through all eight thicknesses of paper. Unfold the shape you have cut out. How many lines of symmetry does it have?

- 7 How would you cut a piece of paper folded as in Fig. 23.8 to give the following shapes? (Experiment by folding and cutting.)

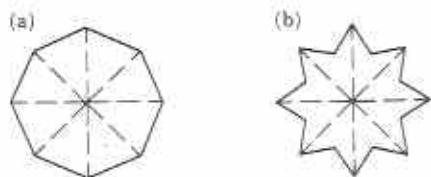


Fig. 23.9 (a) Regular octagon
(b) Eight-pointed star

- 8 Copy each of the shapes in Fig. 23.10 onto a graph paper. Complete the symmetry about the lines given on the figures.

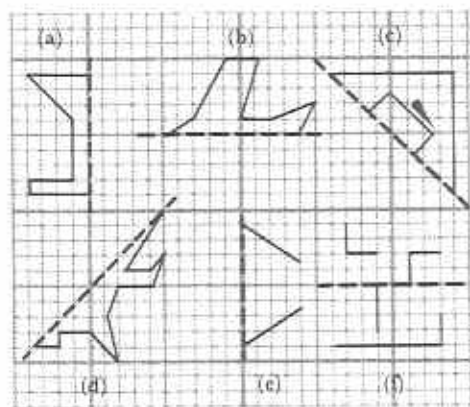


Fig. 23.10

Properties of symmetrical shapes

In Fig. 23.11, m is the line of symmetry of the given figure. The left-hand side is the mirror image of the right-hand side. Hence,

$$\text{angle } a' = \text{angle } a$$

$$\text{length } OP' = \text{length } OP$$

We say that a' is the **image** of a in the line m . Similarly P' is the image of P in the line m .

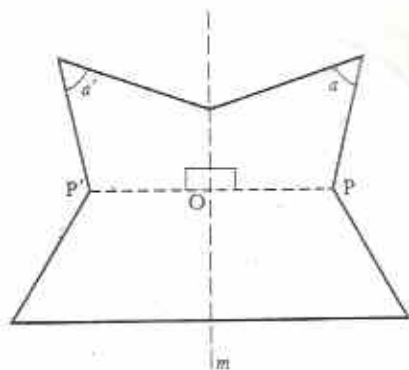


Fig. 23.11

Isosceles triangle

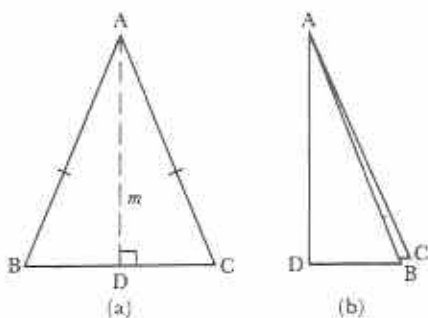


Fig. 23.12

In Fig. 23.12 $\triangle ABC$ is isosceles with $AB = AC$. m is a line through A perpendicular to BC . If $\triangle ABC$ is folded along AD , as in Fig. 23.12(b), the points B and C meet as shown. Hence, by folding, $\hat{B} = \hat{C}$, $BD = DC$ and $\hat{BAD} = \hat{CAD}$. This leads to some important results:

- 1 The base angles of an isosceles triangle are equal.
- 2 The perpendicular from the vertex of an isosceles triangle to its base bisects the base and the vertical angle.

Circle

In Fig. 23.13(a) AB is a chord of a paper circle, centre O .

In Fig. 23.13(b) the paper circle is folded so that A meets B .

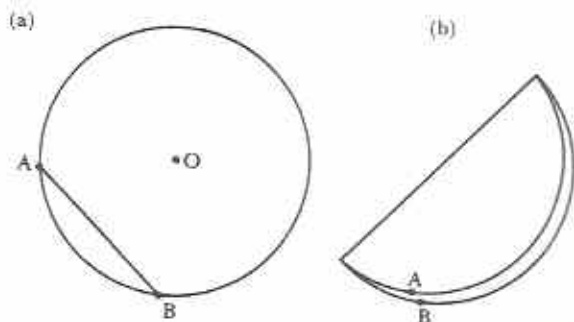


Fig. 23.13

When the circle is opened out as in Fig. 23.14 it can be seen that the line of the fold is a diameter of the circle.

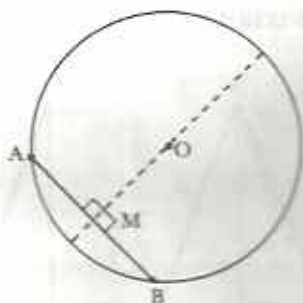


Fig. 23.14

Any diameter of a circle is a line of symmetry. It follows that $\angle OMA = \angle OMB = 90^\circ$ and $AM = MB$. Hence:

- 1 The perpendicular from the centre of a circle to a chord bisects the chord;
- or, conversely,
- 2 the perpendicular bisector of a chord passes through the centre of the circle.

The symmetrical properties of quadrilaterals are discussed in Chapter 12.

Exercise 23b

1 In Fig. 23.15 m is the line of symmetry of the given figure.

- (a) What is the image of the point B?
- (b) What is the image of line AE?
- (c) What is the image of \widehat{ACB} ?
- (d) Name two isosceles triangles.
- (e) Name four right-angled triangles.
- (f) If $\widehat{EAC} = 40^\circ$, what is \widehat{ACE} ?
- (g) If $\widehat{CED} = 28^\circ$, what is \widehat{CDE} ?

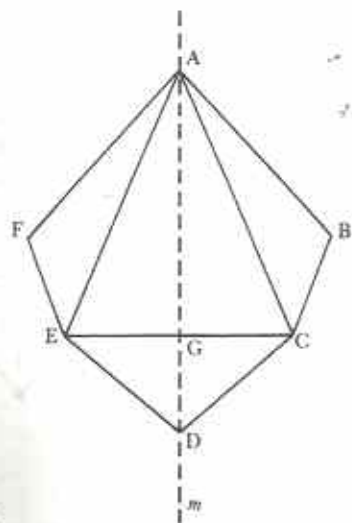


Fig. 23.15

- (h) What kind of quadrilateral is ACDE?
- (i) Name two points equidistant from m .
- (j) If $EG = 2.5$ cm and $AC = 7$ cm, what is the perimeter of $\triangle ACE$?

2

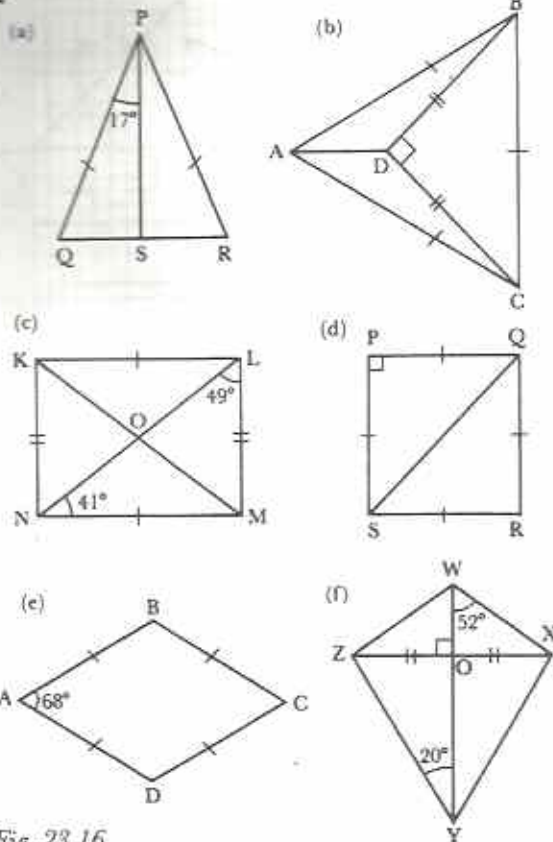


Fig. 23.16

- 2 Use angle and symmetry properties to fill in as many angles as possible in each part of Fig. 23.16.
- 3 m is a line of symmetry in Fig. 23.17. O is the centre of the circle.

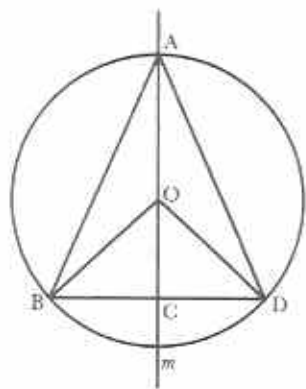


Fig. 23.17

- (a) Name as many isosceles triangles as possible.
- (b) Name as many right-angled triangles as possible.
- (c) If $BD = 9$ cm, what is BC ?
- (d) If $CD = x$ cm, what is BD in terms of x ?
- 4 In Fig. 23.18, PQRS is a circle with centre O . QOS is a diameter which meets PR perpendicularly at T .

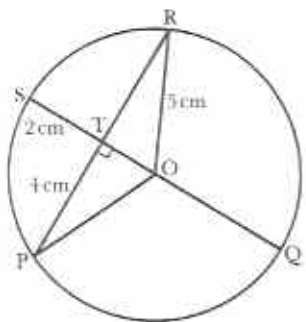


Fig. 23.18

If $OR = 5$ cm, $PT = 4$ cm and $ST = 2$ cm, state the lengths of the following.

- (a) TR (b) PR (c) OP
 (d) OS (e) OT (f) OQ
 (g) QT (h) QS
- 5 In Fig. 23.19, O is the centre of circle AQBM and Q is the centre of circle AOBN.

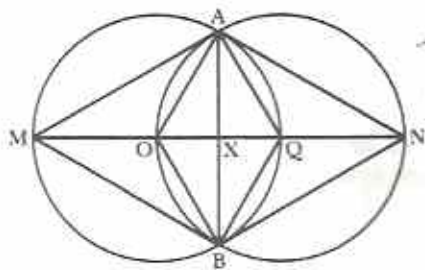


Fig. 23.19

- (a) Name two lines of symmetry.
- (b) Name two rhombuses.
- (c) What kind of quadrilateral is MAQB?
- (d) What kind of triangle is $\triangle AOQ$?
- (e) What is the size of \widehat{AOM} ?
- (f) What is the size of \widehat{MAO} ?
- (g) What kind of triangle is $\triangle NAB$?
- (h) If $OX = 2$ cm, what is MN ?

Point symmetry (optional)

Fig. 23.20 shows a rectangular sheet of paper pinned to a notice board. The pin is at the centre of the rectangle.



Fig. 23.20

If the rectangle is given a half-turn it will take up the same position as before (Fig. 23.21).

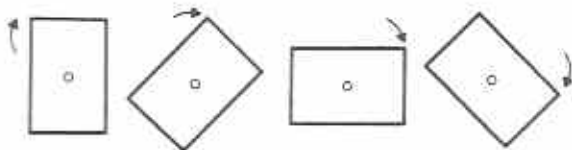


Fig. 23.21

This is an example of **point symmetry** or **rotational symmetry**. If the rectangle is now given another half-turn it will return to its original position. We say that the rectangle has point symmetry of **order 2** about its centre, i.e. if the rectangle is given one complete revolution

about its centre there are 2 positions in which it will appear as it started: one after turning through 180° , the other after turning through 360° .

Fig. 23.22 shows that a parallelogram also has point symmetry of order 2 about its centre.

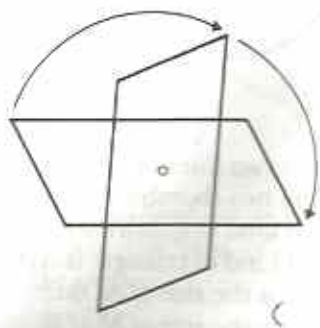


Fig. 23.22

The equilateral triangle in Fig. 23.23 has point symmetry of order 3 about its 'centre'.

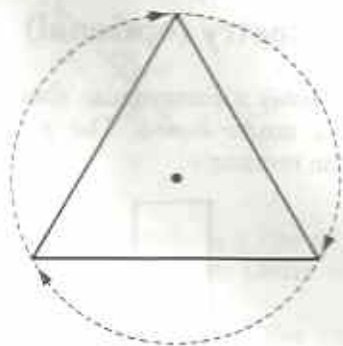


Fig. 23.23

Fig. 23.24 shows that any shape will return to its original position after 1 revolution about a point.

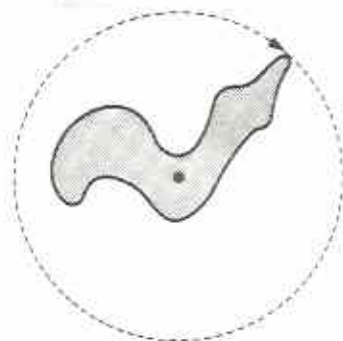





Fig. 23.24

Hence plane figures are only considered to have point symmetry in cases where the order is 2 or more.

There is often a connection between bilateral symmetry and rotational symmetry. However, as Table 23.1 shows, some shapes, such as the parallelogram, may have point symmetry without having line symmetry.

Table 23.1

shape	number of lines of symmetry	order of point symmetry
rectangle 	2	2
parallelogram 	0	2
equilateral triangle 	3	3

Exercise 23c

1 Which of the shapes in Fig. 23.25 have point symmetry?

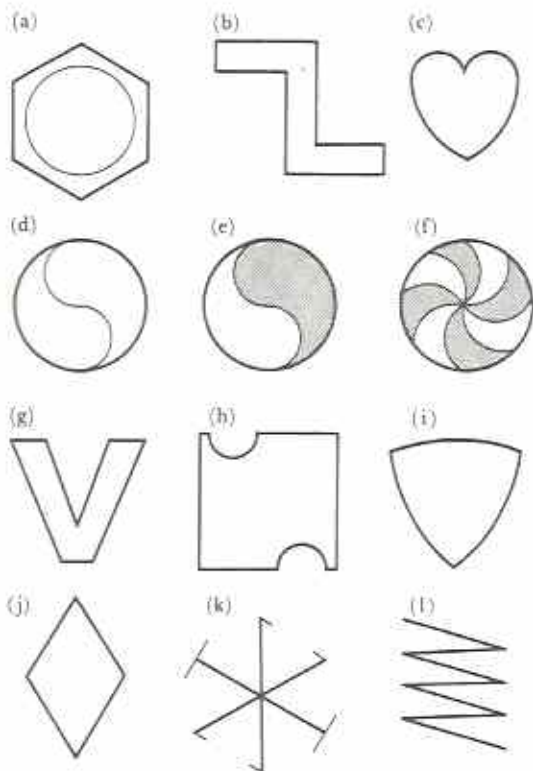


Fig. 23.25

2 Make a table like that of Table 23.1 for the shapes in Fig. 23.25.

3 Complete Table 23.2 by making a tick in the correct column.

Table 23.2

shape	has point symmetry only	has line symmetry only	both	neither
isosceles Δ				
equilateral Δ				
scalene Δ				
rectangle				
square				
parallelogram				
rhombus				
kite				
trapezium				
circle				

4 Do any of the mathematical instruments in Fig. 23.6 have rotational symmetry?

5 Each shape in Fig. 23.26 is part of a point symmetrical figure. Copy the shapes and complete the symmetry using the given orders and points.

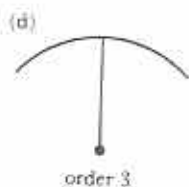


Fig. 23.26

Properties of point symmetrical shapes (optional)

Rotation of shapes

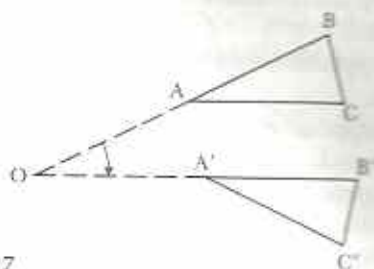


Fig. 23.27

In Fig. 23.27 ΔABC is rotated about point O to the position shown. $\Delta A'B'C'$ is the image of ΔABC after rotation about O . Notice that only the position of ΔABC changes. All other properties, such as area, length and angle, stay the same.

Parallelogram

In Fig. 23.28 ABC is any triangle in which the mid-point of AC , M , is joined to B .

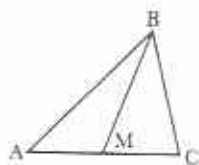


Fig. 23.28

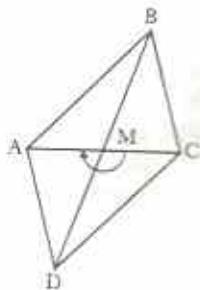


Fig. 23.29

If the triangle is rotated through 180° about M it combines with its image to give a quadrilateral $ABCD$ (Fig. 23.29). Notice the following:

- (1) Since AB was rotated through 180° to give its image CD , $AB \parallel DC$. Similarly $CB \parallel DA$. Hence $ABCD$ is a parallelogram.
- (2) $AB = CD$ and $CB = AD$ since CD and AD are the images of AB and CB respectively.
- (3) $\hat{B} = \hat{D}$ and $\hat{BAD} = \hat{BCD}$.
- (4) $\Delta ABC \equiv \Delta CDA$ and $\Delta ABD \equiv \Delta CDB$, where \equiv means 'is identical to'.

- (5) $MD = MB$ since D is the image of B .
Also $MA = MC$.

These properties can be stated as follows:

- 1 The opposite sides of a parallelogram are parallel.
- 2 The opposite sides of a parallelogram are equal.
- 3 The opposite angles of a parallelogram are equal.
- 4 A diagonal bisects a parallelogram.
- 5 The diagonals of a parallelogram bisect one another.

Rhombus

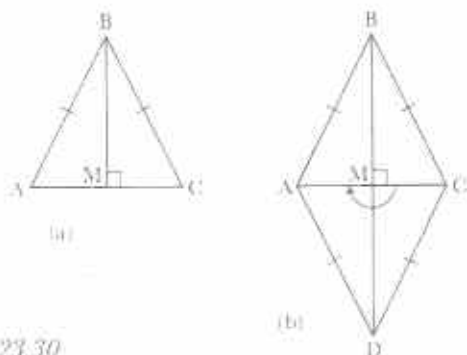


Fig. 23.30

Fig. 23.30(a) is an isosceles ΔABC in which BM is the perpendicular from B to M , the midpoint of AC . In Fig. 23.30(b) ΔABC has been rotated through 180° about M to produce a rhombus $ABCD$.

Since a rhombus is a special parallelogram all the properties given for the parallelogram above are also true for rhombuses. In addition, the symmetry of Fig. 23.30(b) shows that:

- 1 All four sides of a rhombus are equal.
- 2 The diagonals of a rhombus bisect one another at right angles.
- 3 The diagonals of a rhombus bisect its angles.

Exercise 23d

- 1 In Fig. 23.30(b) name the point symmetrical image of

(a) point A	(b) point B
(c) point C	(d) point M
(e) line BC	(f) line AC
(g) MBC	(h) MAD

- 2 Fig. 23.31 has point symmetry of order 2.

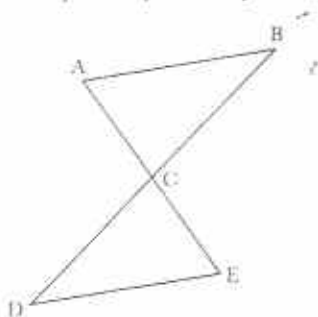


Fig. 23.31

- (a) Which point is the centre of symmetry?
 - (b) Name two lines which are parallel.
 - (c) What is the image of point A ?
 - (d) If points A, B, E, D are joined, what kind of quadrilateral is formed?
 - (e) If $\angle CAB = 83^\circ$, name another angle which is 83° .
 - (f) If $DB = 11$ cm, what is the length of CD ?
- 3 Use angle and point symmetrical properties to fill in as many missing angles as possible in each part of Fig. 23.32. (In each figure, M is the centre of symmetry.)

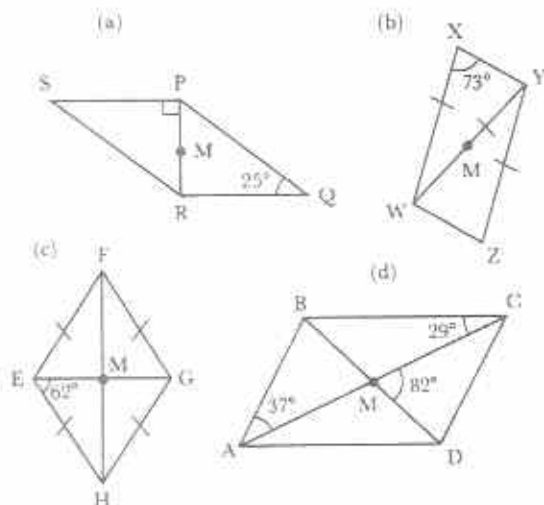


Fig. 23.32

- 4 In Fig. 23.33 AB is a chord of the circle centre O . The circle is rotated about O so that $A'B'$ is the image of AB .

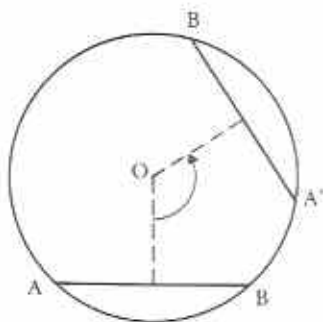


Fig. 23.33

- (a) Name a line equal in length to AB .
 (b) If AB is a distance d from O , what is the distance of $A'B'$ from O ?
 (c) Complete the following statement:

In a circle, equal chords are the same _____ from the centre.

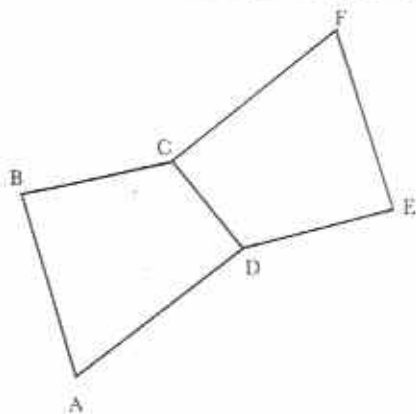


Fig. 23.34

- 5 (a) What is the order of symmetry of Fig. 23.34?
 (b) Where is the centre of symmetry?
 (c) If the angles of quadrilateral $ABCD$ are 59° , 102° , 113° , 86° respectively, what are the sizes of the angles of quadrilateral $CDEF$?
- 6 Fig. 23.35 is an extension of Fig. 23.34.

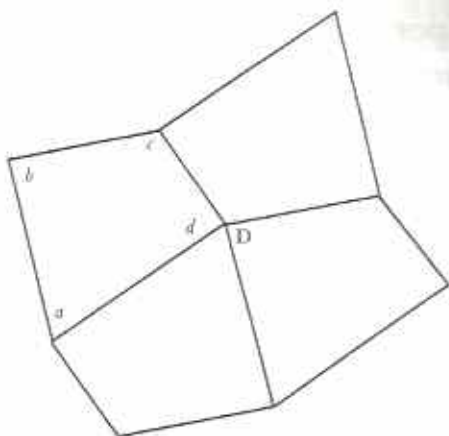


Fig. 23.35

- (a) Express the four angles at D in terms of a , b , c , d .
 (b) What is the sum of the angles at a point?
 (c) Complete the following statement:
The sum of angles of a quadrilateral is _____.

Chapter 24

Proportion, ratio, rate

Syllabus Reference 5.1.3

Proportion

Unitary method

Read Examples 1 and 2 carefully.

Example 1

A man is paid \$90 for 10 days of work. Find his pay for (a) 3 days, (b) 24 days, (c) x days.

We are to find money. Money comes last in every line of working.

For 10 days the man gets \$90.

For 1 day the man gets $\$90 \div 10 = \9 .

- (a) For 3 days the man gets $3 \times \$9 = \27 .
(b) For 24 days the man gets $24 \times \$9 = \216 .
(c) For x days the man gets $x \times \$9 = \$9x$.

Notice the following.

- 1 Each line of working is a complete sentence.
- 2 The quantity to be found comes last in each sentence.
- 3 The first sentence states the given facts.
- 4 The second sentence gives the pay for 1 day, a unit.

This is an example of **direct proportion**. The less the man works (3 days) the less he is paid (\$27). The more the man works (24 days) the more he is paid (\$216).

Example 2

7 workers dig a piece of ground in 10 days. How long would 5 workers take?

We are to find time. Time comes last in every line of working.

7 workers take 10 days

1 worker takes 10×7 days = 70 days

5 workers take $70 \div 5$ days = 14 days

Notice the following.

- 1 We assume all workers work at the same rate.

- 2 The second sentence gives the time for 1 unit, a worker.

This is an example of **inverse proportion**, or indirect proportion. Less workers (5) take a longer time (14 days). More workers (7) take a shorter time (10 days).

When solving problems by unitary method, always:

- 1 write sentences with the quantity to be found at the end;
- 2 decide whether the problem is an example of direct or inverse proportion;
- 3 find the rate for 1 unit before answering the problem. This is where **unitary method** gets its name from.

Many problems cannot be answered by unitary method. For example, 'If a girl is 1 metre tall when she is 6 years old, how tall will she be when she is 18?' We cannot say. She will certainly *not* be 3 metres tall!

Exercise 24a (Oral)

In this exercise (i) give the first sentence of the working, (ii) state whether the problem is an example of *direct proportion* or *inverse proportion* or *neither*, (iii) answer part (a) only.

- 1 A worker is paid \$100 for 5 days. Find her pay for (a) 1 day, (b) 2 days, (c) 22 days.
- 2 6 notebooks cost 90c. Find the cost of (a) 1 notebook, (b) 5 notebooks, (c) 44 notebooks.
- 3 5 workers build a wall in 10 days. How long will it take (a) 1 worker, (b) 10 workers, (c) 25 workers?
- 4 A 2-year-old boy has 6 sisters. (a) How many sisters did he have when he was 1 year old? How many sisters will he have when he is (b) 3 years old, (c) 21 years old?

- 5 A car travels 84 km in 2 hours. At the same rate, how far does it travel in (a) 1 hour, (b) 3 hours, (c) x hours?
- 6 A piece of land has enough grass to feed 15 cows for 4 days. How long would it last (a) 1 cow, (b) 6 cows, (c) y cows?
- 7 The temperature of 6 litres of liquid is 30°C . Find the temperature of (a) 1 litre, (b) 8 litres, (c) 400 ml.
- 8 9 equal bottles hold $4\frac{1}{2}$ litres of water altogether. How much water does (a) 1 bottle, (b) 5 bottles, (c) x bottles hold?
- 9 A container has enough water to last 9 people for 4 days. How long would it last (a) 1 person, (b) 8 people, (c) z people?
- 10 A car uses 10 litres of petrol in 80 km. How far will it go on (a) 1 litre, (b) 5 litres, (c) x litres?

Exercise 24b

- 1 Where possible, work out parts (b) and (c) of the questions in Exercise 24a.
- 2 A worker gets \$160 for 5 days' work. How much does he get for 14 days?
- 3 It takes 4 people 3 days to dig a small field. How long would it take 3 people to do the same work?
- 4 A girl buys 7 pens for 77c. How much would 10 pens cost?
- 5 A 3-tonne lorry makes 10 journeys to move a pile of earth. How many journeys would a 5-tonne lorry make?
- 6 A 14-year-old boy runs 100 m in 14 seconds. How long would it take a 6-year-old boy?
- 7 A bag of corn can feed 100 chickens for 12 days. How long would the same bag feed 80 chickens?
- 8 It takes 21 9-litre buckets to fill a drum with water. How many 7-litre buckets would it take?
- 9 A car travels at 60 km/h and takes 5 hours for a journey. How long would the car take if it travelled at 90 km/h?
- 10 A 480-page book is 2.4 cm thick (not counting the thickness of the covers). Find the thickness of 380 pages of the book.

Ratio

Suppose the prices of two chairs are \$60 and \$80. The **ratio** of the prices is 60 : 80, 'sixty to eighty'.

Ratios behave in the same way as fractions. For example,

$$\frac{60}{80} = \frac{30}{40} = \frac{120}{160} = \frac{3}{4}$$

and $60 : 80 = 30 : 40 = 120 : 160 = 3 : 4$

Hence, both parts of a ratio may be multiplied or divided by the same number. It is usual to express ratios as whole numbers in their lowest terms.

Example 3

Express the ratio of 8 cm to 3.5 cm as simply as possible.

$$\begin{aligned} 8 \text{ cm to } 3.5 \text{ cm} &= 8 : 3.5 \\ &= 2 \times 8 : 2 \times 3.5 \\ &= 16 : 7 \end{aligned}$$

Notice that we do not give units in a ratio.

Example 4

Express the ratio 96c : \$1.20 as simply as possible.

Express both sums of money in cents.

$$\frac{96\text{c}}{\$1.20} = \frac{96\text{c}}{120\text{c}} = \frac{96}{120} = \frac{96 \div 24}{120 \div 24} = \frac{4}{5}$$

The ratio is 4:5.

Notice that quantities must be in the same units before they can be given as a ratio.

Exercise 24c

Express the following ratios as simply as possible.

- | | |
|---------------------------------|----------------------------|
| 1 3 : 6 | 2 20 to 30 |
| 3 15 : 12 | 4 28 to 21 |
| 5 \$10 to \$15 | 6 1 h to 15 min |
| 7 1 mm to 1 cm | 8 400 kg : 1 tonne |
| 9 80 s to 2 min | 10 18 boys to 12 boys |
| 11 \$200 to \$150 | 12 \$1.25 : 75c |
| 13 2 days : 1 week | 14 3 days to 3 weeks |
| 15 1 h 30 min : 2 h | 16 5 cm to 5 mm |
| 17 60c : 84c | 18 5 m to $3\frac{1}{2}$ m |
| 19 35° to 90° | 20 90 km/h : 120 km/h |

Example 5

Fill the gap in the ratio $2 : 7 = \underline{\hspace{2cm}} : 28$.

Let the missing number be a .

$$\text{Then } 2 : 7 = a : 28$$

$$\text{or } \frac{2}{7} = \frac{a}{28}$$

$$\Leftrightarrow a = \frac{2 \times 28}{7} = 8$$

The missing number is 8.

Exercise 24d

Fill the gaps in the following.

1 $\frac{4}{5} = \frac{\quad}{10}$

2 $4 : 5 = \underline{\hspace{1cm}} : 10$

3 $\frac{3}{8} = \frac{9}{\quad}$

4 $3 : 8 = 9 : \underline{\hspace{1cm}}$

5 $6 : 9 = \underline{\hspace{1cm}} : 15$

6 $10 : 12 = 25 : \underline{\hspace{1cm}}$

7 $8 : 12 = \underline{\hspace{1cm}} : 9$

8 $\underline{\hspace{1cm}} : 9 = 16 : 24$

9 $120 : \underline{\hspace{1cm}} = 84 : 56$

10 $22 : 18 = 33 : \underline{\hspace{1cm}}$

Sharing

To divide a quantity into two parts which are in the ratio $2 : 5$, first divide it into 7 equal shares (since $2 + 5 = 7$). The required parts will then be 2 shares and 5 shares, i.e. $\frac{2}{7}$ and $\frac{5}{7}$ of the given quantity.

Example 6

Two children share 35 oranges in the ratio $2 : 3$. How many oranges does each child get?

One child gets 2 shares and the other gets 3 shares. There are 5 shares altogether.

$$\text{Number of oranges in 5 shares} = 35$$

$$\text{Number of oranges in 1 share} = \frac{35}{5} = 7$$

$$\text{Number of oranges in 2 shares} = 2 \times 7 = 14$$

$$\text{Number of oranges in 3 shares} = 3 \times 7 = 21$$

One child gets 14 oranges and the other gets 21 oranges.

$$\text{Check: } 14 + 21 = 35$$

Example 7

Divide \$1,17 between Chipo and Rudo so that their shares are in the ratio $8 : 5$.

$$8 + 5 = 13$$

$$\frac{1}{13} \text{ of } \$1,17 = \frac{117}{13} c = 9c$$

$$\text{Chipo's share} = 8 \times 9c = 72c$$

$$\text{Rudo's share} = 5 \times 9c = 45c$$

$$\text{Check: } 72c + 45c = \$1,17$$

Example 8

X, Y, Z share \$85 so that for every \$1 that X gets, Y gets \$3 and for every \$2 that Y gets, Z gets \$3. Find Y's share.

If X gets 1 share, then Y gets 3 shares. Z gets $1\frac{1}{2}$ times as much as Y. Hence Z gets $4\frac{1}{2}$ shares. X, Y, Z receive money in the ratio $1 : 3 : 4\frac{1}{2}$.

$$1 : 3 : 4\frac{1}{2} = 2 : 6 : 9 \text{ (multiplying each by 2)}$$

$$2 + 6 + 9 = 17$$

$$\frac{1}{17} \text{ of } \$85 = \$\frac{85}{17} = \$5$$

$$\text{Y gets } \frac{6}{17} \text{ of } \$85.$$

$$\frac{6}{17} \text{ of } \$85 = \$5 \times 6$$

$$= \$30$$

Y's share is \$30.

Exercise 24e

1 (Oral) Share the following quantities in the given ratios.

(a) \$15 in the ratio $1 : 2$

(b) 26 kg in the ratio $5 : 8$

(c) 20 cm in the ratio $4 : 1$

(d) \$30 in the ratio $2 : 3$

(e) 80 ml in the ratio $3 : 7$

(f) 22 oranges in the ratio $9 : 2$

2 Divide the following quantities in the given ratios.

(a) 98c 5:9 (b) \$5,76 1:3:5

(c) 96 m 3:4:5 (d) 56 kg $4\frac{1}{3} : 2\frac{3}{5}$

(e) 153 5:2:6:4 (f) \$33,76 2:5:9

(g) 10,8 kg 11:7 (h) 22,95 m 5:12

(i) \$5,39 $1\frac{1}{3} : \frac{1}{2} : 2\frac{1}{4}$ (j) 28,6 kg 4:5:6:7

3 Two women share 5 dozen eggs in the ratio $1 : 2$. How many eggs does each receive?

4 Ruth is 12 years old and Gono is 8 years

4. They share 15 mangoes in the ratio of their ages. How many does each get?

Ben, Sam and Zodiwa are aged 12, 14 and 20 respectively. They share \$6,90 in the ratio of their ages. How much does each get?

A cooperative farmer divides 240 cattle between his three children in the ratio 5:4:3. How many cattle does each get?

Two builders share 11 tonnes of bricks so that one gets $1\frac{1}{2}$ times as much as the other. The total cost is \$227,50. How much does each pay?

It costs \$2 320 to produce a machine. The cost arises from materials, labour and overheads in the ratio 7:9:2. Calculate the cost of labour for producing 32 such machines.

X, Y, Z share \$68 so that for every \$1 that Z gets, Y gets \$2 and for every \$3 that Y gets, X gets \$4. How much does Y get?

A and B contribute \$1 400 and \$1 800 respectively to a business partnership. Of the profit, A receives 20% as manager. The rest is shared in the ratio of their investments. Find the ratio of A's total share of the profits to B's.

Ratio and proportion

Example 9

What is the result of increasing \$3,50 in the ratio

6:5? Multiply \$3,50 by $\frac{6}{5}$.

$$\begin{aligned} 3,50 \times \frac{6}{5} &= \$0,70 \times 6 \\ &= \$4,20 \end{aligned}$$

Example 10

These 3 litres in the ratio 5:12.

$$\begin{aligned} \text{per } \frac{5}{12} \times 3\ell &= \frac{5}{4}\ell \\ &= 1,25\ell \end{aligned}$$

$$\frac{\text{new amount}}{3 \text{ litres}} = \frac{5}{12}$$

$$\begin{aligned} \text{new amount} &= \frac{5}{12} \times 3\ell \\ &= 1,25\ell \end{aligned}$$

Exercise 24f

Find the result of

- 1 increasing 20 in the ratio 3:2.
- 2 decreasing 15 in the ratio 2:5.
- 3 increasing \$2 in the ratio 5:4.
- 4 decreasing \$2 in the ratio 4:5.
- 5 decreasing 1 kg in the ratio 3:4.
- 6 increasing 100 m in the ratio 7:5.
- 7 increasing 2 km in the ratio 7:4.
- 8 decreasing 7,5 litres in the ratio 3:5.
- 9 increasing $2\frac{1}{2}$ h in the ratio 6:5.
- 10 decreasing \$1,80 in the ratio 7:12.

Example 11

In what ratio is 20,25 m changed if it is decreased to 11,25 m?

$$\begin{aligned} \text{The ratio } \frac{\text{new length}}{\text{old length}} &= \frac{11,25 \text{ m}}{20,25 \text{ m}} = \frac{1125}{2025} \\ &= \frac{125}{225} = \frac{5}{9} \end{aligned}$$

The length is decreased in the ratio 5:9.

Exercise 24g

Find the ratio in which the first quantity must be changed to make the second. State whether it is an increase or a decrease.

- | | |
|-------------------------|---------------------------|
| 1 \$10; \$15 | 2 \$10; \$7,50 |
| 3 40 girls; 25 girls | 4 6 g; 16 g |
| 5 5 g; $3\frac{1}{2}$ g | 6 150 m; 120 m |
| 7 \$2,40; \$3,30 | 8 6 km; $4\frac{1}{2}$ km |
| 9 $2\frac{1}{4}$ m; 3 m | 10 1 h 30 min; 1 h 15 min |

Many problems which can be solved by unitary method can also be solved by using ratios. Read Examples 12, 13 and 14 carefully.

Example 12

If 7 balls cost \$131,25, find the cost of 9 balls.

Since the number of balls is increased in the ratio 9:7, the cost will also be increased in the ratio 9:7.

$$\begin{aligned} 7 \text{ balls cost } & \$131,25 \\ \Leftrightarrow 9 \text{ balls cost } & \$131,25 \times \frac{9}{7} \\ & = \$18,75 \times 9 \\ & = \$168,75 \end{aligned}$$

This is an example of **direct ratio**, or direct proportion.

Example 13

If 5 men dig a drain in 14 days, how long would 7 men take?

The number of men is *increased* in the ratio 7:5.
The time taken is *decreased* in the ratio 5:7.
(More men take less time.)

$$\text{Time taken by 5 men} = 14 \text{ days}$$

$$\Leftrightarrow \text{Time taken by 7 men} = 14 \times \frac{5}{7} \text{ days} \\ = 10 \text{ days}$$

This is an example of **inverse ratio**, or inverse proportion.

Example 14

A 900 g jar of jam costs \$1,10. How much jam can be bought for \$8,25?

Money is *increased* in the ratio 8,25 : 1,10
55c buys 900 g

$$\Leftrightarrow 8,25 \text{ buys } 900 \times \frac{8,25}{1,10} \text{ g} \\ = 900 \times \frac{825}{110} \text{ g} \\ = 900 \times 7,5 \text{ g} \\ = 6\,750 \text{ g} \\ = 6\frac{3}{4} \text{ kg}$$

Exercise 24h

- If 10 ball-point pens cost \$3,60 how much will 3 pens cost?
- A shelf holds 21 books each 5 cm thick. How many books, each 7 cm thick, will the shelf hold?
- If 13 bars of soap cost \$9,36, how much will 17 bars of soap cost?
- It takes 15 monthly instalments of \$64 to pay for a television set. How many monthly instalments of \$40 will it take?
- It costs \$403,20 to stay at a hotel for 7 days. How much does it cost to stay for 3 days?
- If 5 radios cost \$197, how many can be bought for \$315,20?
- A rope can be cut into 18 lengths of 17,5 cm. How many lengths of 15,75 cm can it be cut into?

- If \$40 is equivalent to £27,20, what is \$25 equivalent to in £ sterling?
- A lorry can carry 820 bricks each of mass 1,68 kg. How many bricks of mass 1,64 kg can it carry?
- A cyclist takes 3 h 41 min to cycle to a village at an average speed of 19 km/h. If she travels back the next day and takes 4 h 7 min, what is her average speed?
- A tank of water will last a family for 15 weeks when 15 litres a day are used. How long will it last if 50 litres a day are used?
- Working for 7 hours a day, a man does a job in 24 days. How long would it take him if he worked for 6 hours a day?
- A motorist does a journey in 2 h 24 min at an average speed of 55 km/h. How long would the journey take at an average speed of 60 km/h?
- A woman has enough money to buy $15\frac{3}{4}$ m of cloth at \$10,80 per metre. How many metres at \$8,40 could she buy?
- The rent for 126 ha of land is \$688,50. Find the rent for 196 ha of the same land.

Ratio and percentage

Example 15

\$6,46 is 85% of a sum of money. Find the sum of money.

Either by unitary method:

$$85\% \text{ of the money} = \$6,46$$

$$1\% \text{ of the money} = \frac{\$6,46}{85}$$

$$100\% \text{ of the money} = \frac{\$6,46}{85} \times 100 \\ = \$ \frac{646}{85} = \$ \frac{38}{5} \\ = \$7,60$$

Or by the direct ratio method:

$$85\% \text{ of the money} = \$6,46$$

$$100\% \text{ of the money} = \$6,46 \times \frac{100}{85} \\ = \$7,60$$

Exercise 24i

- Find the quantity of which
 - 8c is 20%.
 - 12 m is 75%.
 - 5 g is $33\frac{1}{3}\%$.
 - \$9 is 30%.
 - \$5.50 is 5%.
 - 4.35 kg is 150%.
 - 39 is 78%.
 - 2.4 m is $66\frac{2}{3}\%$.
 - 50c is 250%.
 - \$2.21 is 65%.
- 85% of a quantity has a mass of 1.19 kg. Find the mass of the quantity.
- 56c is $17\frac{1}{2}\%$ of a sum of money. Find the sum of money.
- 42% of a football pool is given as first prize and 28% as second prize. If the first prize is \$10 860, what is the second prize?
- Find the quantity of which 5.6 kg is 175%.
- A farmer sells 99 ha of land. This represents $37\frac{1}{2}\%$ of the farm. How big is the farm?
- A man gave 42% of his cattle to his younger child and 48% to his older child. If the younger child got 224 cattle, how many did the older child get?
- A school has 675 desks. 54 of them are broken. Express the ratio, *number of broken desks : number of desks altogether* as a fraction in its lowest terms. Hence find the percentage of broken desks and the percentage of desks that are not broken.
- In 1991 the population of a village was 1 015 people. This was an increase of 16% on its 1990 population. What was its 1990 population?
- 65% of a farmer's trees are orange trees. The remainder are apple trees. What is the ratio, *number of orange trees : number of apple trees*? If the farmer has 936 orange trees, how many apple trees are there?

Rate

Ratios compare quantities of the same kind, for example 4 kg : 7 kg. The units may be different, e.g. 1 cm : 5 km, but the quantities are of the same kind and it is possible to express km in cm.

Quantities of *different* kinds may be con-

nected in the form of a **rate**. The following are some examples of rates.

- A worker is paid \$14.40 for an 8-hour day. Her rate of pay is \$1.80 per hour.
- A cyclist travels 28 km in 2 hours. His rate is 14 km per hour. In this case the rate is called **speed**.
- A piece of metal has a volume of 20 cm^3 and a mass of 180 g. Its **density** is 9 g per cm^3 . The density of gases, liquids and solids is a rate giving the mass per unit volume.
- A town of 32 000 people has an area of 40 km^2 . The population density of the town is 800 people/ km^2 . **Population density** is a rate giving the average number of people per unit area.

Notice that km/h is short for kilometres per hour.

Example 16

Find, in km/h, the rate at which a car travels if it goes $38\frac{1}{2}\text{ km}$ in 35 min.

In 35 min the car goes $38\frac{1}{2}\text{ km}$.

In 1 min the car goes $\frac{38\frac{1}{2}}{35}\text{ km}$

$$\begin{aligned}\text{In 60 min the car goes } & \frac{38\frac{1}{2}}{35} \times 60\text{ km} \\ & = \frac{77 \times 60}{2 \times 35}\text{ km} \\ & = \frac{11 \times 60}{2 \times 5}\text{ km} \\ & = 66\text{ km}\end{aligned}$$

The rate (speed) of the car is 66 km/h.

Example 17

A village is roughly square in shape. Its perimeter is about 6 km. If the population density of the village is 1 200 people/ km^2 , find the approximate population of the village.

Perimeter of village = 6 km

$$\Leftrightarrow \text{Side of square} = \frac{6}{4}\text{ km} = 1\frac{1}{2}\text{ km}$$

$$\begin{aligned}\Leftrightarrow \text{Area of village} & = 1\frac{1}{2} \times 1\frac{1}{2}\text{ km} \\ & = 2\frac{1}{4}\text{ km}^2\end{aligned}$$

The population density is 1 200 people/km².

∴ 1 km² contains 1 200 people

∴ 2 $\frac{1}{4}$ km² contains $1\,200 \times 2\frac{1}{4}$ people
 $= \frac{1\,200 \times 9}{4}$ people

$= 2\,700$ people

Population of village = 2 700 people.

Notice in Examples 16 and 17 how unitary method is used to solve problems involving rates.

Exercise 24j

- 1 A worker is paid \$174 for a 40-hour week. Calculate the rate of pay per hour.
- 2 A car travels 153 km in 2 $\frac{1}{4}$ h. Calculate its average speed in km/h.
- 3 A steel beam is 5,2 m long and has a mass of 137,8 kg. Find its mass in kg/m.
- 4 42 cm³ of sea water has a mass of 43,26 g. Find its density in g/cm³.
- 5 A town has an area of 24 km² and a population of about 31 000. Find the population density of the town per km² correct to 2 s.f.
- 6 A shop reduces all its prices at the rate of 15c in the \$. Find the new price of an article which was \$7,40.
- 7 A car uses petrol at the rate of 1 litre for every 6,5 km travelled. How many litres does it use when travelling 117 km?
- 8 A container has a capacity of 20 litres. It is filled with wine of density 0,8 kg/litre. What is the mass of the wine?
- 9 A village is roughly in the shape of a rectangle 1 $\frac{1}{2}$ km by 1 $\frac{1}{4}$ km. What is its population if the average density is 570 people/km²?
- 10 When I travel at 60 km/h it takes me 2 hours for a certain journey. (a) How long is the journey? (b) How long does it take me when I travel at 50 km/h?
- 11 A bridge is 220 m long and has a mass of 11 220 tonnes. Find its average mass in t/m.
- 12 A worker gets \$30,16 for working 14 $\frac{1}{2}$ hours. Find the rate of pay per hour.
- 13 The estimated population of Harare is 975 000 and the city covers an area of 570 km². Find the population density of Harare to 1 s.f.
- 14 Each week a typist works from 8.00 am to 12.30 pm on six days and from 2.00 pm to 5.30 pm on four days. Her rate of pay is \$1,68/hour. What is her total wage?
- 15 How long will it take me to cycle a distance of 12 km at an average rate of 5m/s?
- 16 A car uses petrol at the rate of 1 litre for every 11 km. If petrol costs 38c per litre, find the cost of the petrol for a journey of 891 km.
- 17 Last year a cooperative factory produced 9 324 bicycles. Allowing 2 weeks for holidays and a further 100 days for weekends, find the rate of production in bicycles/day.
- 18 In a town with a population of 53 280 there were 562 deaths in one year. Find the death-rate per 1 000 people correct to 3 s.f.
- 19 In the town in question 18, 613 babies were born in the same year. Find the birth-rate per 1 000 people to 3 s.f.
- 20 A car travels for 80 km at 40 km/h and then for 150 km at 50 km/h. (a) Find the total distance travelled. (b) Find the time taken in each part of the journey. (c) Hence find the total time taken. (d) Use your answers to parts (a) and (c) to calculate the average speed of the car for the whole journey.

Chapters 17-24

Revision exercise 8 (Chapters 17, 20)

- \widehat{AOB} and \widehat{BOC} are adjacent on straight line AOC . If $\widehat{AOB} = 53^\circ$, calculate \widehat{BOC} .
- Two angles of a triangle are 63° and 45° . Calculate the size of the third angle.
- In Fig. R15, find a, b, c .

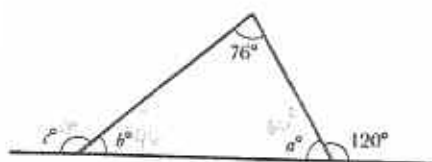


Fig. R15

In Fig. R16, find a, b, c, d, e .

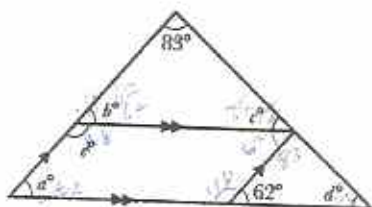


Fig. R16

In Fig. R17, $ABCD$ is a kite with angles as shown. Find x and y .

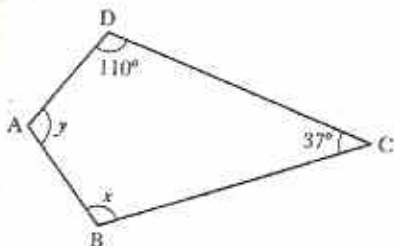


Fig. R17

- In Fig. R18, $AB = AC = CD$ and $\widehat{CAD} = 28^\circ$. If BCD is a straight line, find the sizes of angles a, b, c, d, e .

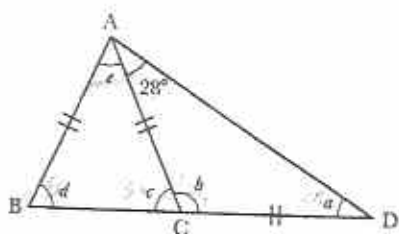


Fig. R18

- Use a ruler and set square to construct a pair of parallel lines which are 57 mm apart.
- Construct the triangle sketched in Fig. R19.

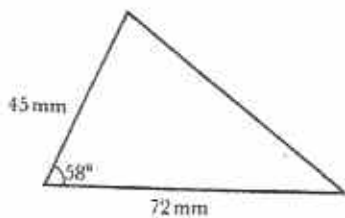


Fig. R19

Measure the third side of the triangle.

- Make a *freehand sketch* of $\triangle ABC$ in which $AB = 6$ cm, $\widehat{BAC} = 54^\circ$ and $\widehat{ACB} = 69^\circ$. Calculate the third angle of the triangle and show this on your sketch. Make an accurate construction of $\triangle ABC$.
- Construct $\triangle ABC$ in which $AB = 3,5$ cm, $BC = 4,5$ cm and $AC = 5,5$ cm. Measure the perpendicular distance of A from BC and hence calculate the area of the triangle.

Revision test 8 (Chapters 17, 20)

1 The following are given as dimensions of triangles:

ΔABC in which $AB = 11$ cm,
 $BC = 3$ cm, $CA = 7$ cm

ΔPQR in which $PQ = 8$ cm,
 $\hat{PQR} = 50^\circ$, $QR = 3$ cm

ΔXYZ in which $XY = 5$ cm,
 $\hat{XYZ} = 100^\circ$, $\hat{XZY} = 30^\circ$

Which of the triangles is (are) impossible to construct?

- A ΔABC only
B ΔPQR only
C ΔXYZ only
D ΔABC and ΔPQR only
E ΔPQR and ΔXYZ only

2 In ΔXYZ , $XY = 5$ cm, $\hat{XYZ} = 40^\circ$ and $\hat{XZY} = 60^\circ$. Which one of the sketches in Fig. R20 shows this information correctly?

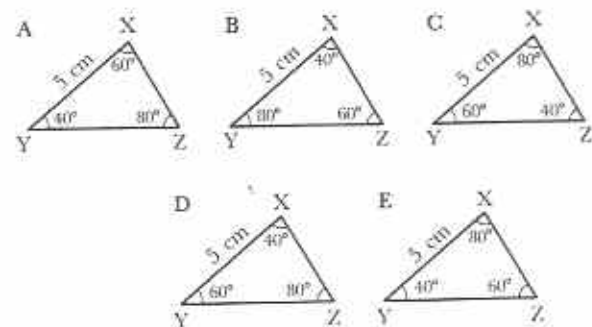


Fig. R20

3 Two angles of ΔABC are 46° and 67° . Calculate the third angle of ΔABC . Hence decide which one of the following kinds of triangle it is.

- A equilateral B isosceles C scalene
D right-angled E obtuse-angled

4 A quadrilateral has angles of 128° , 91° , n° and $2n^\circ$. $n =$

- A 47 B 73 C 89
D 141 E 219

5 In Fig. R21, POQ and SOR are straight lines.

If $\hat{POS} = 40^\circ$ and $\hat{TOQ} = 65^\circ$, calculate \hat{TOR} .

- A 105° B 35° C 30°
D 25° E 15°

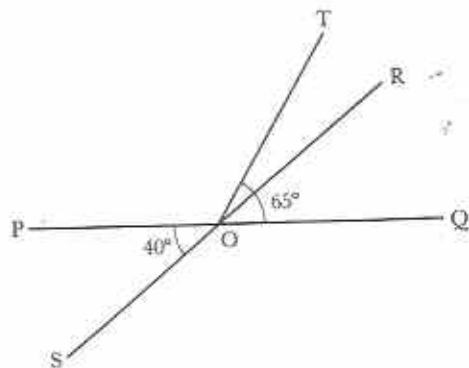


Fig. R21

6 In Fig. R22 find a , b , c .

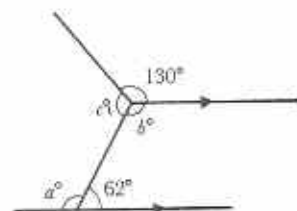


Fig. R22

7 In Fig. R23, find a , b , c , d .

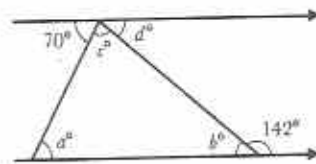


Fig. R23

8 use a ruler and set square to construct a rectangle 6 cm by 2.5 cm. Measure one of its diagonals.

9 Construct ΔABC in which $BC = 4$ cm, $\hat{ABC} = 50^\circ$ and $AB = 6$ cm. Measure AC .

10 Construct a parallelogram $ABCD$ in which $AB = 4$ cm, $\hat{ABC} = 70^\circ$ and $BC = 5$ cm. Measure the distance between one pair of parallels and hence calculate the area of $ABCD$.

Revision exercise 9 (Chapters 18, 23)

- 1 Calculate the volume of a cuboid measuring 12 cm by 10 cm by 6 cm. How many 2 cm by 2 cm by 2 cm cubes would this cuboid contain?
- 2 How many litres of water does a 5 m by 10 m by 2 m tank hold?
- 3 The area of the end-face of a beam is 24 cm². Calculate the volume of a 5 m length of the beam.
- 4 A triangular prism has a volume of 142 cm³. If the prism is 8 cm long, calculate the area of one of its triangular faces.

In Fig. R24, m is a line of symmetry of shape ABCDEFGH.

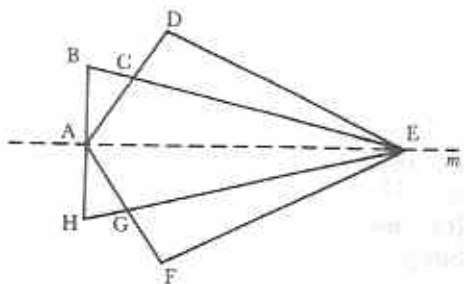


Fig. R24

Use Fig. R24 to answer questions 5–8.

- 5 (a) What is the image of point C in m ?
 (b) What is the image of point E in m ?
 (c) What is the image of line HG in m ?
 (d) What is the image of $\triangle CDE$ in m ?
- 6 (a) Name two points which are equidistant from m .
 (b) Name two right-angled triangles.
 (c) Name an isosceles triangle.
 (d) Name two kites.
- 7 (a) If $\widehat{BEA} = 26^\circ$, what is \widehat{AHE} ?
 (b) If $\widehat{HAG} = 31^\circ$, what is \widehat{FAD} ?
- 8 If $AB = 3,8$ cm and $EH = 6,3$ cm, what is the perimeter of $\triangle EHB$?
- 9 Draw any shape which has two axes of symmetry.
- 10 ABCDE is a regular pentagon with centre O.
 (a) How many lines of symmetry has ABCDE?
 (b) Calculate (i) \widehat{BOA} , (ii) \widehat{BOA} .

Revision test 9 (Chapters 18, 23)

- 1 A sheet of paper is folded 3 times. All 8 thicknesses of paper are cut through to give a shape. If the paper is unfolded how many lines of symmetry does it have?
 A 1 B 2 C 3
 D 4 E 8

In Fig. R25, MN is a line of symmetry and O and Q are the centres of the large and small circles respectively.

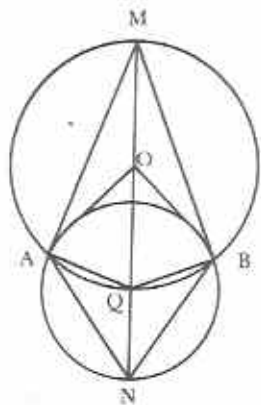


Fig. R25

Use Fig. R25 to answer questions 2 and 3.

- 2 What is the image of $\triangle MAO$ in MN?
 A $\triangle NAQ$ B $\triangle NBQ$ C $\triangle MBO$
 D $\triangle MOA$ E $\triangle AMO$
- 3 If $OA = 5$ cm and $QA = 3$ cm, $MN =$
 A 6 cm B 8 cm C 12 cm
 D 13 cm E 15 cm
- 4 Which of the following has (have) two axes of symmetry?
 I kite, II rhombus, III square
 A I only B II only
 C III only D I and II only
 E I and III only
- 5 The number of cm³ in 1 m³ is
 A 100 B 1 000 C 10 000
 D 100 000 E 1 000 000
- 6 A floor 4 m long by $2\frac{1}{2}$ m wide is concreted to a thickness of 10 cm. Calculate the volume of the concrete.
- 7 1 litre of water is poured into a rectangular container. Find the height that the water will rise to if the area of the base of the container is 80 cm².

- 8 In Fig. R26, O is the centre of the circle and OMXN is a square of side 7 cm. $YM = ZN = 7$ cm.

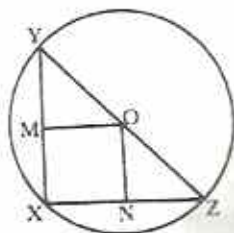


Fig. R26

Calculate the area of $\triangle XYZ$.

- 9 Use angle and symmetry properties to find the sizes of all the angles in Fig. R27.

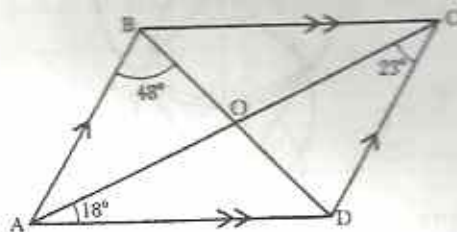


Fig. R27

- 10 The water tank of a house is in the shape of an open cuboid and is made of plastic 1 cm thick. Its internal dimensions are 60 cm long, 40 cm wide and 35 cm deep. Calculate (a) the capacity of the tank in litres, (b) the volume of plastic used in cm^3 .

Revision exercise 10 (Chapters 19, 21)

- 1 Simplify the following.

(a) $(-5) \times 3$ (b) $(-8) \times (-2)$
 (c) $6 \times (-5)$ (d) $(-1) \times (-9)$
 (e) $(-7) \times (-3)$ (f) $29 \times (-2)$

- 2 Simplify the following.

(a) $18 + (-6)$ (b) $(-8) + (-4)$
 (c) $(-30) + (-10)$ (d) $\frac{-19}{-1}$
 (e) $\frac{-100}{25}$ (f) $\frac{-48}{-8}$

- 3 Find the value of the following when $a = 3$, $b = -1$ and $c = 4$.

(a) $-2b$ (b) $\frac{a+b}{c}$ (c) $\frac{c}{a-b}$

- 4 Place the following in order of size, from lowest to highest.

(a) $(-6) \times (+4)$ (b) $(-8) + (-2)$
 (c) $(+25) \times (-1)$ (d) $(+100) \div (-5)$

- 5 Remove brackets and simplify the following.

(a) $3x - 2(2x - y)$
 (b) $3(3x - 7) + (5x - 9)$
 (c) $7(x - 2y) - 5(x - 3y)$
 (d) $8(x + y) - 7(2x - y)$

- 6 Expand the following.

(a) $(a + b)(x + y)$
 (b) $(2p + q)(3r - 5s)$
 (c) $(2c - 5)(c - 3)$
 (d) $(2a - 9b)(4a + 5b)$
 (e) $(b - 5)^2$
 (f) $(2x + 1)^2$

- 7 Factorize the following.

(a) $10 + 15b$ (b) $x^2 - ax$
 (c) $4ab - 2a^2$ (d) $27x^2y - 36xy^2$

- 8 Simplify the following.

(a) $\frac{7x}{m} - \frac{3x}{m}$ (b) $\frac{4x}{3} - \frac{2x}{9}$
 (c) $\frac{5}{a} + \frac{8}{3a}$ (d) $\frac{1}{p} - \frac{1}{q}$

- 9 Simplify the following.

(a) $\frac{3a+1}{2} + \frac{a+2}{3}$ (b) $\frac{2b-3}{3} - \frac{2b-5}{4}$
 (c) $\frac{5a+2}{2} - \frac{2(3a+1)}{3}$

- 10 A girl is x cm tall. Her father is $1\frac{1}{4}$ times her height. Her sister is $\frac{9}{10}$ of her height. (a) Express the heights of the father and sister in terms of x . (b) Write down an expression for the difference in height between the father and sister. (c) Simplify the expression.

Revision test 10 (Chapters 19, 21)

1 If $(-3) + (-24) = y$, then $y =$
 A + 72 B + 8 C $+\frac{1}{8}$ D $-\frac{1}{8}$ E -8

2 Evaluate $\frac{x-y}{x+y}$ if $x = -5$ and $y = -15$.
 A -2 B $-\frac{1}{2}$ C +1 D $+\frac{1}{2}$ E +2

3 Simplify $\frac{-5a^2}{15a}$
 A $\frac{5a}{3}$ B $-\frac{2}{3}$ C -2 D $-\frac{a}{3}$ E $-\frac{a}{5}$

4 Simplify $(a + (-a)) + b$.
 A $2b$ B b C $2a$
 D $2a - b$ E $2a + b$

5 A teacher asks a class to write $\frac{x+8}{4}$

in another way. Three of the answers are:

I $\frac{1}{4}(x+8)$, II $x+2$, III $\frac{1}{4}x+2$.

Which of I, II, III is (are) correct?

A I only

B II only

C III only

D I and II only

E I and III only

6 Simplify the following.

(a) -6×8

(b) $(-2\frac{1}{2}) \times (-2\frac{2}{5})$

(c) $\frac{7}{8}$ of $(-5\frac{1}{3})$

(d) $38 \div (-2)$

(e) $-3.6 \div (-9)$

(f) $-3\frac{2}{3} + 6\frac{2}{3}$

7 Find the value of the following when $x = -2, y = 5, z = -1$.

(a) $x^2 + y^2$

(b) $(x+y)^2$

(c) $\frac{x+y}{y+z}$

(d) $\frac{x}{y} + \frac{z}{z}$

8 Expand the following.

(a) $(v-4)(v-9)$

(b) $(b+4)(3b+2)$

(c) $(5c+2)(2c-3)$

(d) $(4m-n)(3m-2n)$

9 Factorize the following.

(a) $18 + 9c$

(b) $2\pi rh + \pi r^2$

(c) $28x^2y^2 - 21x^3y$

(d) $2\pi r^3 - \frac{1}{3}\pi r^2h$

10 Simplify the following.

(a) $\frac{7-3}{2a-5a}$

(b) $\frac{1}{u} + \frac{1}{v}$

(c) $\frac{9a-5}{5} - \frac{3a-2}{2}$

(d) $\frac{3(y-1)}{4} + \frac{2(5-3y)}{7}$

Revision exercise 11 (Chapters 22, 24)

1 (a) Place the following distances in rank order: 5 km, 1 km, 6 km, 4 km, 9 km, 2 km, 15 km, 8 km, 10 km.

(b) Find the fifth greatest distance.

2 In one year rain fell on 80 days. There was no rainfall on any of the other days. Take a year to be 360 days and draw a pie chart to show this information.

3 At a meeting, half of the people are women, one-third are men and the rest are children. Draw a pie chart to show this information.

4 If there were 60 people at the meeting in question 3, draw a pictogram to show the information of the question. (Let one symbol represent 10 people.)

5 100 people were asked their ages; the results are given in Table R3.

Table R3

age	under 15	15-29	30-44
frequency	43	32	17
age	45-59	60 and over	
frequency	5	3	

(a) What fraction of the people were under 30?

(b) What percentage of the people were between 45 and 59?

6 Draw a bar chart to show the information in question 5.

7 A car uses 20 litres of petrol for a journey of 180 km. How many litres will it use for a journey of 108 km?

8 The number of boys in a school is 120. If the ratio of boys to girls is 2 : 3 find the total number of students in the school.

9 (a) Express 15 as a percentage of 40.

(b) Increase 80 in the ratio 6 : 5.

(c) The price of an article decreased from \$125 to \$100. Express this as a ratio.

10 A factory made 1 500 refrigerators in a 40-hour week. Find its production rate in refrigerators/hour.

Revision test 11 (Chapters 22, 24)

- 1 When recording data, the tally marks
 ### ## # ||| represent the number
 A 13 B 15 C 18
 D 20 E 33

Fig. R28 is a bar chart showing the numbers of students who got grades A, B, C, D, E in an essay.

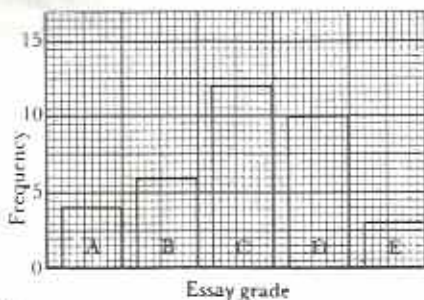


Fig. R28

Use Fig. R28 to answer questions 2, 3 and 4.

- 2 Which grade did most students get?
 A A B B C C
 D D E E
- 3 How many students got grade B?
 A 3 B 4 C 6
 D 10 E 12
- 4 How many students wrote the essay?
 A 5 B 12 C 15
 D 35 E 60
- 5 In 1983 a hi-fi cost \$560. In 1993 the same model cost \$1040. Give the 1983 to the 1993 costs as a ratio in its simplest terms.
 A 1:2 B 7:13 C 28:52
 D 26:14 E 56:104
- 6 A book has 120 pages of drawings, 72 pages of photographs and 168 pages of writing. Show this information on a pie chart.
- 7 Show the information of question 6 on a pictogram. (Let each symbol represent 24 pages.)
- 8 I travelled at 60 km/h and took 2 h for a certain journey. How long would it have taken me if I had travelled at 50 km/h?
- 9 A shop assistant is paid a bonus of 8c in the \$ on the amount of sales she makes. Find how much she gets if she sells 3 skirts at \$39 each, 5 T-shirts at \$18 each and 2 pairs of sandals at \$22 per pair.

- 10 A new candle is 15 cm long. It burns at the rate of 1.2 cm per hour. Calculate the length of the candle after it has been burning for 2 h 36 min.

General revision test C (Chapters 17-24)

- 1 If $m \times (-2) = 12$, then $m =$
 A +24 B +14 C +6
 D -6 E -24
- 2 Simplify $(y + z)^2 - 2yz$.
 A $y^2 + z^2 + 4yz$ B $y^2 - z^2$
 C $y^2 + z^2$ D $y^2 + z^2 - 2yz$
 E $y^2 + z^2 - 4yz$
- 3 Five men can do a piece of work in 6 days. How many days, to the nearest whole number, should 8 men take to do it? Assume that each man works at the same rate.
 A 4 B 5 C 7
 D 8 E 9
- 4 Which of the following are factors of $4x^2y^2$?
 I 2x II xy III 4y
 A I only B II only C III only
 D none of them E all of them
- 5 The LCM of $5ab^2$ and $3a^2b$ is
 A ab B $\frac{3a}{5b}$ C $\frac{5b}{3a}$
 D $15a^2b^2$ E $15a^3b^3$
- 6 In Fig. R29, $a =$
 A 21 B 24 C 42
 D 48 E 69

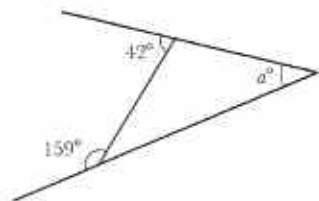


Fig. R29

- 7 In Fig. R30, ABCD is a kite, $\widehat{BAC} = 26^\circ$ and $\widehat{ADC} = 105^\circ$. $\widehat{ACD} =$
 A 26° B 41° C 49°
 D 62° E 64°

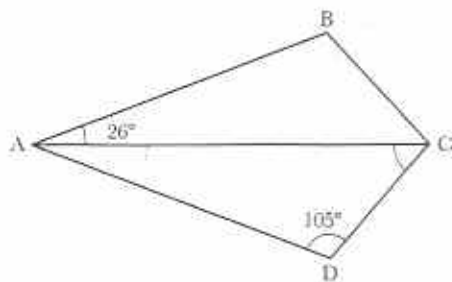


Fig. R30

- 8 If $\frac{5}{6}$ of a certain number is $-6\frac{2}{3}$, the number is
 A $-\frac{1}{8}$ B $-5\frac{1}{3}$ C $-5\frac{5}{9}$
 D $-7\frac{1}{2}$ E -8
- 9 In Fig. R31, ABCDE is a pentagon such that $\triangle ABD$ is equilateral.

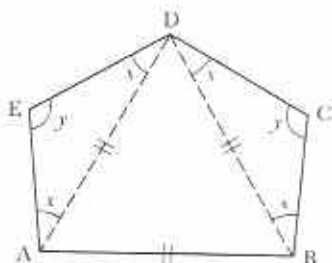


Fig. R31

If $x = 35^\circ$, then $y =$

- A 95° B 110° C 120°
 D 130° E 145°

Fig. R32 is a star shape developed from a regular pentagon.

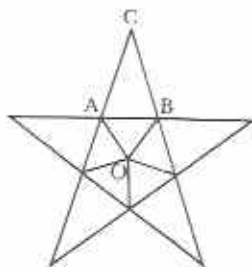


Fig. R32

Use Fig. R32 to answer questions 10 and 11.

- 10 How many axes of symmetry has the shape in Fig. R32?

- A 2 B 3 C 5
 D 10 E 15

- 11 Calculate (a) the size of each angle at O, (b) \widehat{OAB} , (c) \widehat{C} .

- 12 Expand the following.

- (a) $(x - 5)(x - 6)$
 (b) $(2p - 3q)(2p - 5q)$
 (c) $(m + 4)(m - 4)$
 (d) $(t - 8)^2$

- 13 In one year a man spent \$65 on tobacco and \$24 on sweets. Next year he reduced his tobacco spending by 40% and increased his spending on sweets in the ratio 5:3. How much did he save?

- 14 (a) Complete the brackets:

$5x^2 - 10x = 5x(\quad)$

- (b) Factorise: $18ax + 12ay$

(c) Simplify: $\frac{5-t}{5} - \frac{4-t}{4}$

(d) Simplify: $\frac{3(2x-3)}{5} - \frac{1}{3}$

- 15 Simplify the following.

(a) $(-6) \times (+5)$ (b) $(+7) \times (-0.9)$

(c) $(-10)^2$ (d) $(-5,4) \div (+0,6)$

- 16 In Fig. R33 find a, b, c .

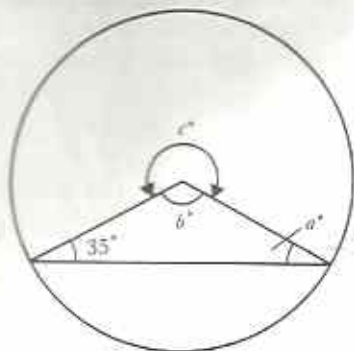


Fig. R33

- 17 Factorise the following.

(a) $8xy + 2x^2$

(b) $mn^2 + m^2n$

- 18 A cylindrical rain barrel has a base area of $0,38 \text{ m}^2$. It contains rainwater to a depth of 91 cm. How many times can a bucket of capacity 14 litres be filled from the barrel?
- 19 A traffic survey gave the results shown in Table R4.

Table R4

vehicle	car	lorry	bus	bicycle
frequency	12	10	5	23

- (a) How many lorries were there for every one bus?
- (b) What percentage of the vehicles were bicycles?
- (c) Represent the data in Table R4 on a bar chart.
- 20 Construct $\triangle ABC$ in which $BC = 6 \text{ cm}$, $\hat{A}BC = 30^\circ$ and $\hat{B}AC = 100^\circ$. Measure the perpendicular distance of A from BC and hence calculate the area of the triangle which you have drawn.

Tables

SI units

Length

The **metre** is the basic unit of length.

unit	abbreviation	basic units
1 kilometre	1 km	1 000 m
1 hectometre	1 hm	100 m
1 decametre	1 dam	10 m
1 metre	1 m	1 m
1 decimetre	1 dm	0,1 m
1 centimetre	1 cm	0,01 m
1 millimetre	1 mm	0,001 m

The most common measures are the millimetre, the metre and the kilometre.

$$1 \text{ m} = 1\,000 \text{ mm}$$

$$1 \text{ km} = 1\,000 \text{ m} = 1\,000\,000 \text{ mm}$$

Area

The **square metre** is the basic unit of area.

Units of area are derived from units of length.

unit	abbreviation	relation to other units of area
square millimetre	mm ²	
square centimetre	cm ²	1 cm ² = 100 mm ²
square metre	m ²	1 m ² = 10 000 cm ²
square kilometre	km ²	1 km ² = 1 000 000 m ²
hectare (for land measure)	ha	1 ha = 10 000 m ²

Mass

The **gramme** is the basic unit of mass.

unit	abbreviation	basic units
1 kilogramme	1 kg	1 000 g
1 hectogramme	1 hg	100 g
1 decagramme	1 dag	10 g
1 gramme	1 g	1 g
1 decigramme	1 dg	0,1 g
1 centigramme	1 cg	0,01 g
1 milligramme	1 mg	0,001 g

The **tonne** (t) is used for large masses. The most common measures of mass are the milligramme, the gramme, the kilogramme and the tonne.

$$1 \text{ g} = 1\,000 \text{ mg}$$

$$1 \text{ kg} = 1\,000 \text{ g} = 1\,000\,000 \text{ mg}$$

$$1 \text{ t} = 1\,000 \text{ kg} = 1\,000\,000 \text{ g}$$

Volume

The **cubic metre** is the basic unit of volume.

Units of volume are derived from units of length.

unit	abbreviation	relation to other units of volume
cubic millimetre	mm ³	
cubic centimetre	cm ³	1 cm ³ = 1 000 mm ³
cubic metre	m ³	1 m ³ = 1 000 000 cm ³

Time

The **second** is the basic unit of time.

unit	abbreviation	basic units
1 second	1s	1s
1 minute	1 min	60 s
1 hour	1 h	3 600 s

Capacity

The **litre** is the basic unit of capacity. 1 litre takes up the same space as 1 000 cm³.

unit	abbreviation	relation to other units of capacity	relation to units of volume
millilitre	mℓ		1 mℓ = 1 cm ³
litre	ℓ	1ℓ = 1 000 mℓ	1ℓ = 1 000 cm ³
kilolitre	kℓ	1 kℓ = 1 000ℓ	1 kℓ = 1 m ³

Money

Some African currencies

Zimbabwe	100 cents (c) = 1 dollar (\$)
Botswana	100 thebe (t) = 1 Pula (P)
Kenya	100 cents (c) = 1 Shilling (Sh)
Malawi	100 tambala (t) = 1 Kwacha (K)
Mozambique	100 centivos (c) = 1 Metical (M)
Nigeria	100 kobo (k) = 1 Naira (₦)
Zambia	100 ngwee (n) = 1 Kwacha (K)

Other currencies

Britain	100 pence (p) = 1 Pound (£)
USA	100 cents (c) = 1 Dollar (\$)

Exchange rates

At the time of going to press, \$1 Zimbabwean was approximately equivalent to the following.

US Dollar	\$0,40
UK sterling	£0,20
Botswana	P0,70
Kenya	Sh9
Mozambique	M360
Zambia	K16

Note: Exchange rates change from day to day. The above rates may only be taken as approximate.

The calendar

Remember this poem:

Thirty days have September,
 April, June and November.
 All the rest have thirty-one,
 Excepting February alone;
 This has twenty-eight days clear,
 And twenty-nine in each Leap year.

For a leap Year, the year date must be divisible by 4.

Thus 1980 was a Leap Year.

Century year dates such as 1900 and 2000 are Leap Years only if they are divisible by 400. Thus 1900 was not a leap year but 2000 will be a Leap Year.

Multiplication table

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Divisibility tests

Any whole number is exactly divisible by

2	if its last digit is even
3	if the sum of its digits is divisible by 3
4	if its last two digits form a number divisible by 4
5	if its last digit is 5 or 0
6	if its last digit is even and the sum of its digits is divisible by 3
8	if its last three digits form a number divisible by 8
9	if the sum of its digits is divisible by 9
10	if its last digit is 0

Mensuration formulae

	perimeter	area	volume
square side s	$4s$	s^2	
rectangle length l , breadth b	$2(l + b)$	lb	
circle radius r	$2\pi r$	πr^2	
triangle base b , height h		$\frac{1}{2}bh$	
parallelogram base b , height h		bh	
cube edge s		$6s^2$	s^3
cuboid length l , breadth b , height h			lbh
right-triangular prism length l , breadth b , height h			$\frac{1}{2}lbh$

Symbols

symbol	meaning
$=$	is equal to
\neq	is not equal to
\approx	is approximately equal to
\Leftrightarrow	is equivalent to
\equiv	is identical to
$>$	is greater than
$<$	is less than
\geq	is greater than or equal to
\leq	is less than or equal to
$^\circ$	degrees (size of angle)
$^\circ\text{C}$	degrees Celsius (temperature)
A, B, C, ...	points
AB	the line joining the point A and the point B or the distance between points A and B
ΔABC	triangle ABC
\widehat{ABC}	the angle ABC
\perp	lines meeting at right angles
π	pi (3.14... ..)
%	per cent
$A = \{p; q; r\}$	A is the set p, q, r.
$B = \{1; 2; 3; \dots\}$	B is the infinite set 1, 2, 3 and so on.
$n(A)$	number of elements in set A
\in	is an element of
\notin	is not an element of
$\{\}$ or \emptyset	the empty set
\mathcal{U}	the universal set
$A \subset B$	A is a subset of B
$A \supset B$	A contains B
\subset, \supset	negations of \subset and \supset
$A \cup B$	union of A and B
$A \cap B$	intersection of A and B

Squares from 1,0 to 9,9

	,0	,1	,2	,3	,4	,5	,6	,7	,8	,9
1	1,00	1,21	1,44	1,69	1,96	2,25	2,56	2,89	3,24	3,61
2	4,00	4,41	4,84	5,29	5,76	6,25	6,76	7,29	7,84	8,41
3	9,00	9,61	10,24	10,89	11,56	12,25	12,96	13,69	14,44	15,21
4	16,00	16,81	17,64	18,49	19,36	20,25	21,16	22,09	23,04	24,01
5	25,00	26,01	27,04	28,09	29,16	30,25	31,36	32,49	33,64	34,81
6	36,00	37,21	38,44	39,69	40,96	42,25	43,56	44,89	46,24	47,61
7	49,00	50,41	51,84	53,29	54,76	56,25	57,76	59,29	60,84	62,41
8	64,00	65,61	67,24	68,89	70,56	72,25	73,96	75,69	77,44	79,21
9	81,00	82,81	84,64	86,49	88,36	90,25	92,16	94,09	96,04	98,01

Square roots from 1 to 99

	0	1	2	3	4	5	6	7	8	9
1	100 316	105 332	110 346	114 361	118 374	122 387	126 400	130 412	134 424	138 436
2	141 447	145 458	148 469	152 480	155 490	158 500	161 510	164 520	167 529	170 539
3	173 548	176 557	179 566	182 574	184 583	187 592	190 600	192 608	195 616	197 624
4	200 632	202 640	205 648	207 656	210 663	212 671	214 678	217 686	219 693	221 700
5	224 707	226 714	228 721	230 728	232 735	235 742	237 748	239 755	241 762	243 768
6	245 775	247 781	249 787	251 794	253 800	255 806	257 812	259 819	261 825	263 831
7	265 837	266 843	268 849	270 854	272 860	274 866	276 872	277 877	279 883	281 889
8	283 894	285 900	286 906	288 911	290 917	292 922	293 927	295 933	297 938	299 943
9	300 949	302 954	303 959	305 964	307 970	308 975	310 980	311 985	313 990	314 995

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