ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS **6042/2**

PAPER 2

NOVEMBER 2021 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF 7
- Scientific calculator (Non-programmable)

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in Section A and any **five** questions from Section B.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (40 marks).

Answer **all** questions in this section.

1 Solve the inequality

$|2x-3| < x-5.$ [4]

2 **(a)** Prove the identity
$cosec\ 2\theta + cot\ 2\theta \equiv cot\ \theta.$ [3]

(b) Hence or otherwise solve the equation
$cosec\ 2\theta + cot\ 2\theta = 0{,}5$, for $0 \le \theta \le 2\pi$, giving the answer to 2d. p [4]

3 **(a)** Find the value of x given that $3^{x+2} = \frac{1}{81}$. [3]

(b) Solve the equation.
$3(2^{2x}) - 7(2^{x}) + 2 = 0$ [5]

4 Given that $-m, n$ and 1 are any consecutive terms of a geometric progression and 1, n and m are the first three terms of an arithmetic progression,

(a) show that $n^2 + 2n = 1$, [5]

(b) hence or otherwise find the exact value of m if n is positive. [4]

5 **(a)** Show that the set $\{0,1,2,3\}$, forms a group under addition modulo 4. [7]

(b) Show that group is abelian. [2]

(c) Write down the subgroups of the group. [2]

(d) State the order of the group. [1]

Section B (80 marks)

Answer any **five** questions from this section. Each question carries **16** marks.

6 Functions h and g are defined by

$h: x \longmapsto \frac{1}{2}x - 4, \qquad x \in \mathbb{R}$
$g: x \longmapsto \frac{32}{4-x^2} \qquad x \in \mathbb{R}\ x \neq k.$

(a) Find

(i) the possible values of k, [3]

(ii) the values of x for which $hg(x) = 0$, [3]

(iii) $h^{-1}(x)$. [3]

(b) On the same axes sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$ showing clearly the relationship between the graphs. [3]

(c) Describe completely the sequence of transformation which map the graph $y = \frac{1}{x^2}$ onto $y = \frac{32}{4-x^2}$. [4]

7 A curve has the equation $y = x^2 - xy$ and passes through the point $P\left(1; \frac{1}{2}\right)$

(a) Find the equation of the tangent to the curve at P. [6]

(b) Hence or otherwise show that the equation of the normal to the curve at P intersects the curve again at $\left(-\frac{11}{14}\,;\,\frac{121}{42}\right)$. [8]

(c) Find the distance between the two points $\left(1\,;\,\frac{1}{2}\right)$ and $\left(-\frac{11}{14}\,;\,\frac{121}{42}\right)$. [2]

8 The vector equations of the lines n and m, and the plane π_1, are

$$\boldsymbol{r} = \begin{pmatrix} 4 \\ -3 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix},$$

$$\boldsymbol{r} = \begin{pmatrix} 0 \\ 10 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \text{ and}$$

$\boldsymbol{r} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = -1$ respectively where μ and λ are parameters.

[Turn over

The line m intersects the plane π_1, at a point P.

(a) Show that the line n lies in the plane π_1, [2]

(b) **(i)** Find the coordinates of point P. [4]

(ii) Hence or otherwise find the vector equation of the plane π_2 passing through point P and perpendicular to the line n. [3]

(c) **(i)** Find the point of intersection of the line n and the plane π_2 [4]

(ii) Hence or otherwise show that the vector equation of the line through point P which lies in plane π_1 and is perpendicular to line n is; $\boldsymbol{r} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ [3]

9 **(a)** **(i)** Find the first 3 terms of the Maclaurin series in the expansion of $ln\,(5+x)$. [3]

(ii) Write down the first 3 terms of the Maclaurin series in the expansion of $ln\,(5-x)$. [1]

(iii) Hence or otherwise show that when x is small $ln\left(\frac{5+x}{5-x}\right) \approx \frac{2x}{5}$. [2]

(b) **(i)** Prove by mathematical induction that

$$\frac{d^n}{dx^n}(x^m) = \frac{m!}{(m-n)!} \cdot x^{m-n}$$

for all $n \in Z^+$ [8]

(ii) Hence find $\frac{d^4}{dx^4}(x^5)$. [2]

10 Given the complex numbers
$z_1 = 2$ and $z_2 = 2 + 2i$,

(a) **(i)** express z_2 in the form $r(Cos\theta + isin\theta)$, [3]

(ii) hence or otherwise, express the complex number $\frac{z_1}{z_2}$ in polar form. [3]

(b) Describe fully the locus represented by $\operatorname{Re}\frac{z+3}{z-3} = 0$ given that z is a complex number $x + iy$. [4]

(c) By using de Moivres theorem or otherwise, find the roots of the equation, $z^4 + 4 = 0$ [6]

11 It is given that $\mathbf{A} = \begin{pmatrix} 3 & -1 & 2 \\ 1 & -1 & 0 \\ -2 & 4 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}$.

(a) A line with the equation $y = 2x - 3$ is transformed under the transformation matrix B. Show that the equation of its image is $y - 6x + 24 = 0$. [5]

(b) Evaluate A^{-1}. [7]

(c) Hence or otherwise solve the following simultaneous equations. [4]

$$3x - y + 2z = 1$$
$$x - y = 2$$
$$-2x + 4y - 3z = 3$$

12 (a) A learner used the value of π as $\frac{22}{7}$ to calculate the volume of a sphere of radius 8 cm.

Find correct to 3 decimal places the

(i) absolute error involved,

(ii) percentage error involved. [5]

(b) (i) Sketch on the same axes the graphs of $y = \ln 2x$ and $y = x - 1$ and state the number of real roots for the equation $\ln 2x = x - 1$.

(ii) Show that a root of the equation $\ln 2x = x - 1$ lies between 0.2 and 0.3.

(iii) Using 0.2 as the first approximation, use the Newton Raphson method twice to obtain an approximation to the root correct to 3 decimal places. [11]