ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

For Performance Measurement

General Certificate of Education Advanced Level

MATHEMATICS

PAPER 1 **9164/1**

NOVEMBER 2015 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae
- Scientific calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

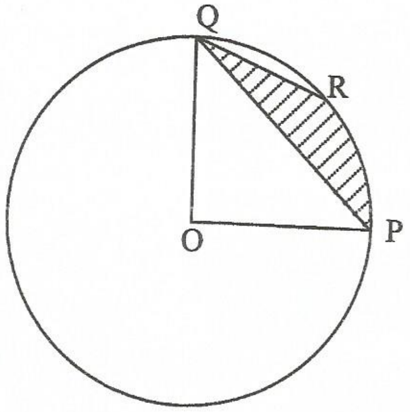
This question paper consists of 6 printed pages and 2 blank pages.

1 When a polynomial f(x) is divided by $(x-3)$ the remainder is -9 and when divided by $(2x-1)$, the remainder is -6.

Find the remainder when f(x) is divided by $(x-3)(2x-1)$. [4]

2 Given that $y = 2x^3 + \frac{8}{x}$, find the percentage increase in y as x increases from 2 to 2.003. [4]

3



The diagram shows a circle centre O and radius r.
A chord PQ subtends a right-angle at the centre 0 of the circle. QR is a chord such that angle PQR is $\frac{\pi}{12}$.
Show that the shaded area bounded by the chords PQ and QR, and the arc PR is

$\frac{1}{2}r^2\left(\frac{\pi}{6}+\frac{1}{2}\sqrt{3}-1\right)$. [5]

4 (i) Sketch on the same diagram the graphs of $y=|3x+4|$ and $y=|2x+1|$.

(ii) Hence or otherwise, solve the inequality $|3x+4| < |2x+1|$. [5]

5 Express $\frac{5x^2-5x+11}{(x-2)(x^2+3)}$ in partial fractions. [5]

6 (i) Expand $(3-2x)^{-3}$ up to and including the term in x^3, simplifying the coefficients.

(ii) Hence state the set of values of x for which the expansion in (i) is valid. [5]

7 The variables x and y satisfy a relationship of the form

$y = ax^2 + bx$, where a and b are constants.

The values obtained in an experiment are shown in the table.

x	1	2	3	4	5
y	1.0	8.4	22.3	39.1	65.8

Plot the graph of $\frac{y}{x}$ against x and determine the values of a and b. [5]

8 The position vectors of points A, B and C are $\begin{pmatrix} n-2 \\ n \\ 3-n \end{pmatrix}$, $\begin{pmatrix} n+1 \\ n-4 \\ 2n \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$

respectively.

(i) Given that $|\overrightarrow{AB}| = 13$ units and $n > 0$, find the value of n. [3]

(ii) Hence find $B\hat{A}C$. [3]

9 Find the equation of a circle whose centre is (5; 4) and touches the line joining the points (0; 5) and (4; 1). [7]

10 (i) Express $6\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$. [2]

(ii) Hence or otherwise, solve the equation

$6\cos\theta - 8\sin\theta = -2.5$ for $-180° \leq \theta \leq 180°$. [3]

(iii) State the minimum and maximum values of

$$\frac{1}{6\cos\theta - 8\sin\theta + 13}.$$ [2]

11 The parametric equations of a circle are

$$x = \operatorname{cosec}\Phi$$

$$y = \cot\Phi, \text{ where } \quad 0 < \Phi < 2\pi.$$

(i) Show that $\frac{dy}{dx} = \sec\Phi$. [3]

(ii) Find the equation of the normal to the curve at the point where $\Phi = \frac{\pi}{6}$ in the form $y = mx + c$.

[4]

12 Given that $\ln y = xy$, where $y > 0$,

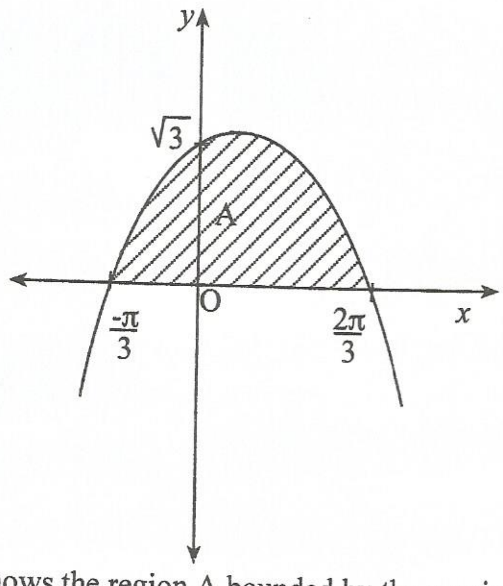
(i) 1. show that $\frac{dy}{dx} = \frac{y^2}{1 - xy}$,

2. find $\frac{d^2y}{dx^2}$.

[5]

(ii) Hence or otherwise, find the Maclaurin expansion of y up to and including the term in x^2. [4]

13



The diagram shows the region A bounded by the x-axis and the curve $y = \sin x + \sqrt{3}\cos x$.

(a) Find the area of region A. [4]

(b) (i) Express $\sin x + \sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$.

(ii) Hence find the volume of the solid formed when A is rotated completely about the x-axis, leaving your answer in terms of π. [5]

14 Two complex numbers $z = x+iy$ and $w = a+ib$ are such that

$z + iw = 2$ and $iz + w = 2+3i$

Find

(i) 1. z,

2. w, [4]

(ii) the modulus of zw, [2]

(iii) the argument of $\frac{z}{w}$. [3]

15 (i) Use the trapezium rule with 4 equal intervals to estimate the value of

$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \operatorname{cosec}^2 x\, dx$, giving your answer correct to 3 decimal places. [4]

(ii) Evaluate exactly $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \operatorname{cosec}^2 x dx$. [4]

(iii) Calculate the percentage error in using the trapezium rule as an approximation to the integral. [2]

16 (i) By sketching two appropriate graphs, show that the equation

$(x-1)e^x = x$ has two real roots. [3]

(ii) Show by calculation, that one of the roots lies between $x = 1.2$ and $x = 1.5$. [3]

(iii) Taking $x_1 = 1.2$ as a first approximation to the root of the equation, use the Newton Raphson method twice to obtain the root correct to 4 decimal places. [4]

17 A rectangular tank has a square base with sides 3 m long and height 5 m.

The tank is initially full of water. The water leaks out through the base and sides of the tank at a rate proportional to the total area in contact with the water. When the depth of the water is 3 m, the level of water is falling at a rate of 0.3 m/h.

If y denotes the depth of water in the tank after t, hours,

(i) show that $\frac{dy}{dt} = \frac{-3}{50}\left(\frac{4y}{3}+1\right)$, [5]

(ii) solve the differential equation in (i), giving y in terms of t, [6]

(iii) find how long it takes for the tank to be half full. [2]