MATHEMATICS

9164/4

PAPER 4

NOVEMBER 2013 SESSION

3 hours

Additional materials:
    Answer paper
    Graph paper
    List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81 \text{ ms}^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 7 printed pages and 1 blank page.

Section (a): Statistics

1   In an Olympiad Quiz Examination paper, there are 100 questions. Each question has 5 suggested answers and a candidate has to choose the correct one.

Given that Mary is equally likely to choose any of the 5 answers in each question since she was guessing, use a suitable approximation to find the probability that she gets at least 27 correct answers.   [4]

2   In a certain factory, there are two machines producing the same brand of fuses. The first machine produces 10% and the second machine produces 90% of the fuses. It is known that the probability that the first machine produces a defective fuse is 1% and the probability that the second machine produces a defective fuse is 5%.

(i)   Find the probability that a fuse drawn at random from the production line is defective.   [2]

(ii)   Given that a fuse is defective, find the probability that it was produced by the first machine.   [3]

3   The times, to the nearest second, taken by 200 students to solve a Maths problem are summarized in the table below.

time (seconds)	41-50	51-55	56-60	61-65	66-70	71-75
frequency	20	60	56	35	19	10

(a)   Draw a cumulative frequency curve for the distribution using a horizontal scale of 2 cm to represent 5 seconds and 2 cm to represent 20 students on the horizontal axis and vertical axis respectively.   [3]

(b)   The time taken by the slowest and the fastest student to solve the problem were 72 and 46 seconds respectively. Using this information and your answer in (a), draw a box and whisker plot for the data.   [4]

4   The probability density function of the lifespan, X months, of a bulb is given by

$$f(x) = \begin{cases} \dfrac{k}{x(4-x)} & 1 \le x \le 3 \\ \\ 0, & \text{otherwise.} \end{cases}$$

(a)   Find the exact value of k.   [3]

(b)   Given that $\mathrm{E}(X) = 2$, show that $Var(X) = 4 - \dfrac{4}{\ln 3}$.   [3]

(c)   Find the probability that a bulb chosen at random will have a lifespan exceeding 2 months.   [2]

5   The number of people joining a queue in a supermarket between 6.30 am and 7.00 am on a week day follows a Poisson distribution with mean of 2 people joining the queue per minute.

Find the probability that

(a)   five people join the queue in one minute,   [2]

(b)   more than four people join the queue in one minute,   [3]

(c)   less than four people join the queue in a two minute interval.   [4]

6   A milling company claims that a bag of flour it packs weighs 10 kg each on average. A random sample of 50 bags is examined and the mass, x kg, of the contents of each bag is recorded. It is found that

$$\sum(x-10) = -12.3 \text{ and } \sum(x-10)^2 = 37.7.$$

(a)   Estimate the population mean and variance of the mass of the contents of a bag.   [4]

(b)   Test, at 10% level of significance, whether the milling company is overstating the average mass of the contents of each bag.   [6]

7   The length and height of a brick are independent normal variables with means and standard deviations as shown in the table.

	length (mm)	height (mm)
mean	198	98
standard deviation	1	1

(a)   Find the probability that

(i)   the sum of the length of five randomly chosen bricks exceeds 994 mm,

(ii)   the height of a randomly chosen brick is less than one half of the length.   [7]

(b)   L denotes the sum of the lengths of 40 randomly chosen bricks and H denotes the sum of the heights of 75 randomly chosen bricks.

Find the mean and variance of L – H.   [4]

8   Every year a local cellular network provider holds a competition. The proportion
     that a dollar spent on airtime wins a prize is 1 in 110.

   (a)   Show that the probability that a subscriber who spent $50 on airtime wins
          at least one prize is 0.365 correct to 3 significant figures.   [3]

   (b)   Find the probability that in a group of

          (i)   10 subscribers each spending $50 on airtime, 3 or more win at
                 least one prize,

          (ii)   100 subscribers each spending $50 on airtime, 40 or more win at
                 least one price.   [8]

9   The values of y, length of a spring in cm, were measured for preselected values
      of x, the load in Newtons, and are shown in table:

x newtons	y cm
1	10.7
2	11.3
3	12.0
4	12.4
5	13.0
6	13.7
7	14.5
8	15.1
9	15.6
10	16.0

   (a)   Draw a scatter diagram to represent the data in the table.   [3]

   (b)   Calculate the equation of the regression line Y on X.   [4]

   (c)   Fit the line on the scatter diagram and use it to predict the length for a
          load of 6.4 Newtons.   [4]

   (d)   Calculate the product-moment correlation and comment on the
          relationship between X and Y.   [4]

10   The table gives the frequency distribution of the number of boys in
   100 families each with 4 children.

number of boys	number of families
0	10
1	24
2	35
3	22
4	9

(a)   Calculate the mean number of boys in a family.   [2]

(b)   Test, at 5% level of significance, whether the observed frequencies
       follow a Binomial distribution.   [14]

[Turn over

Section (b): Mechanics

11 **(a)** A particle starts with a velocity of 2 ms^{-1} and travels along a straight line, covering a distance of 20 metres in 2.5 seconds.

Find the constant acceleration of the particle. [2]

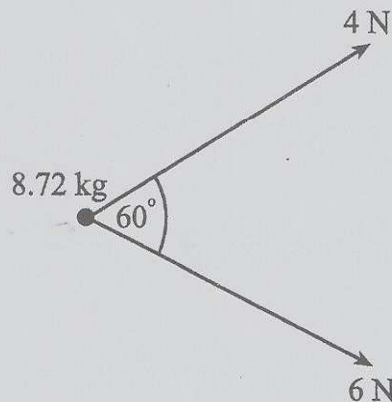**(b)** Draw a velocity-time graph for this motion. [2]

12
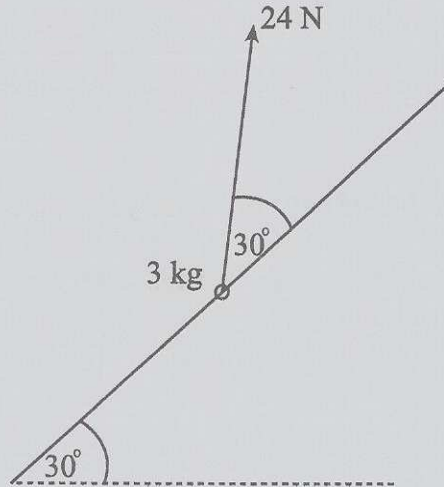
A ring of mass 3 kg is threaded on a rough wire inclined at 30° to the horizontal. The ring is about to slip down the wire when a force of 24N inclined at 30° above the wire is applied to it, (see diagram).

(i) Find the value of the normal reaction exerted on the wire. [3]

(ii) Calculate the coefficient of friction. [2]

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A particle of mass 8.72 kg is pulled horizontally at a constant speed of 5 ms^{-1} along a rough straight road by two horizontal forces of 4 N and 6 N inclined to each other at 60°, (see diagram).

(i) Find the magnitude of the resistance to the motion of the particle. [3]

(ii) The pulling forces are removed instantly.

Find the distance travelled by the particle after the removal of the forces. [4]

14   (a)
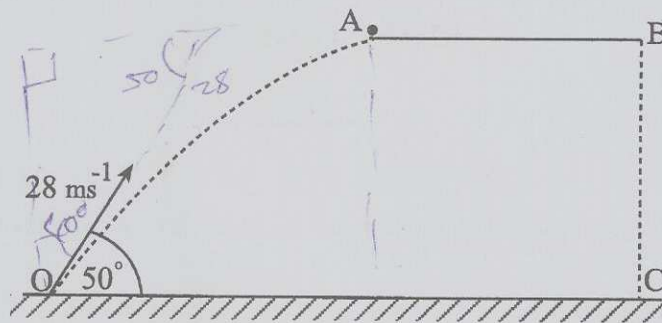
A particle of mass m kg is projected with a velocity of 28 ms⁻¹ from the point O on level ground at an angle of 50° to the horizontal. It reaches its highest point at A on the edge of a rough horizontal plank AB, (see diagram).

Giving your answers correct to two decimal places,

calculate

(i)   the height of A above O,

(ii)   the speed of the particle at the point A.   [4]

(b)   When it reaches the point A, the particle slides along the plank and just manages to reach the point B and drops to the point C which is vertically below B, and on the same level as O. The points O, A, B and C are on the same vertical plane. The coefficient of friction between the plank and the particle is 0.8.

(i)   Calculate the retardation of the particle between A and B.

(ii)   Hence find the length of the plank AB.   [4]