ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

MATHEMATICS 9164/1
PAPER 1

NOVEMBER 2013 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

[Turn over

1 Solve the equation

$$5^{x-1}+5^{x-2}=30.$$ [3]

2 Solve the inequality

$$\frac{3x+1}{9-x^2}\geq -1.$$

[4]

3 A circular plank is cut into 12 sectors whose areas are in arithmetic progression. If the area of the largest sector is twice that of the smallest, find the angle in terms of π between the straight edges of the smallest sector. [4]

4 The position vectors of M and N relative to the origin O are

$(2p-2)\mathrm{i}+(1-p)\,\mathrm{j}+(p-2)\,\mathrm{k}$ and $(p+2)\,\mathrm{i}+p\mathrm{j}+2p\mathrm{k}$ respectively.

Find the values of p when

(i) $|\overrightarrow{OM}|=|\overrightarrow{ON}|$, [2]

(ii) $M\hat{O}N=90^{\circ}$. [3]

5 Given that $(x+k)$ is a factor of x^3+2x^2-3x-6, where $k>0$,

Find

(i) the value of k, [2]

(ii) the exact roots of the equation $x^3+2x^2-3x-6=0$. [3]

6 Given that $\frac{x^2}{25}+\frac{y^2}{16}=1$ is an equation of a curve, find

(a) $\frac{dy}{dx}$, [2]

(b) the equation of the tangent to the curve at the point $\left(3;\frac{16}{5}\right)$. [3]

7 Solve the differential equation $\frac{dy}{dx}=1-y$, giving the general solution in the form $y=Ae^{-x}+C$, where A is an arbitrary constant and C is a constant.

Find the particular solution when the y-intercept is 3. [6]

8 If $Z_1 = -1+i$ and $Z_2 = -1-\sqrt{3}i$,

find

(i) the modulus and argument of Z_2 [2]

(ii) (a) $Z_1 Z_2$,

(b) $\frac{Z_1}{Z_2}$. [4]

9 (i) Use the trapezium rule with 5 ordinates to evaluate

$$\int_0^1 \frac{4}{1+x^2}\,dx$$ correct to 4 decimal places. [4]

(ii) (a) By using the substitution $x = \tan\theta$, find

$$\int_0^1 \frac{4}{1+x^2}\,dx .$$

(b) Hence find, correct to 2 decimal places the percentage error in using the trapezium rule as an approximation to the integral. [5]

10 (i) If $y = \left(1+3e^{-x}\right)^{\frac{1}{2}}$, find

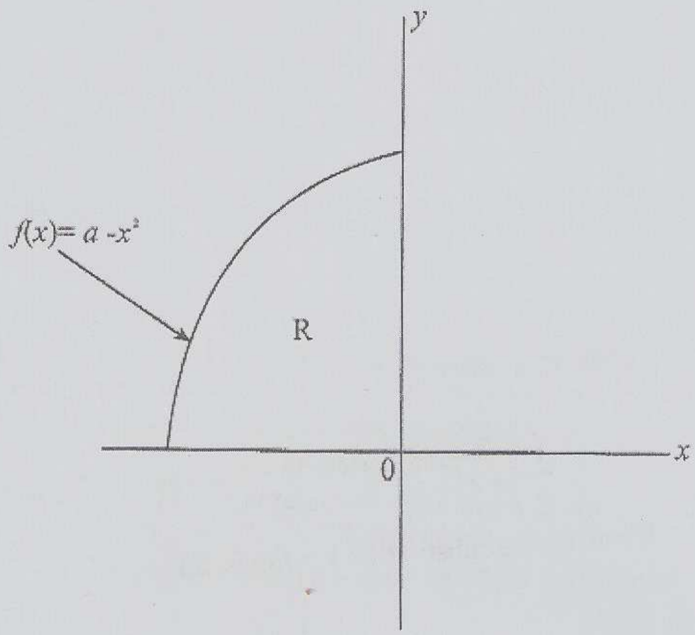
$$\frac{dy}{dx} \text{ at } x = 0.$$ [3]

(ii) Show that $2y\frac{dy}{dx} = 1 - y^2$. [2]

(iii) By further differentiating, use Maclaurin's theorem to find the series expansion for y in ascending powers of x, up to and including the term in x^2. [4]

[Turn over

11



The region R is bounded by the curve $f(x) = a - x^2$ and the axes (see diagram).

(a) Given that the area of R is 18, find the value of a. [4]

(b) Calculate the exact volume of revolution when R is rotated completely about the x- axis. [5]

12 (a) Prove the identity

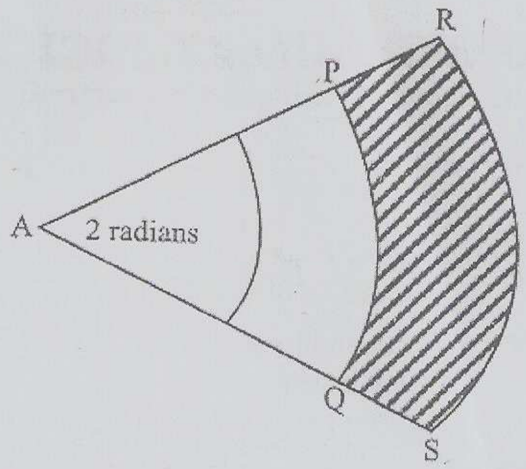
$$\frac{1+\sin 2\theta}{1-\sin 2\theta} \equiv \frac{(\tan\theta+1)^2}{(\tan\theta-1)^2}.$$ [4]

(b) Solve the equation

$$\tan x \cos 2x = \sin x \text{ for } 0 \leq x \leq 360.$$ [6]

13 (a) Calculate the centre and radius of the circle which passes through the points P(0; 0), Q(1; 7) and R(7; -1). [5]

(b)



In the diagram above the shaded region is bounded by sectors APQ and ARS. It is given that the angle PAQ is 2 radians and the perimeter of PQSR is 32 cm.

(i) Calculate the length of AS.

(ii) If the area of PQSR is 28 cm^2, find the length of AQ. [5]

14 (a) Functions f and h are defined as follows:

$$f: x \to (x-2)(x+3) \quad x \in \text{IR} \ x \geq -\frac{1}{2}$$

$$h: x \to 4x^2 + 1 \quad x \in \text{IR}$$

Find

(i) the exact values of x for which $fh(x) = 0$,

(ii) the inverse of $f(x)$ and state its domain. [6]

(b) Find the series expansion of $\frac{(4-x)^{\frac{1}{2}}}{2x^2-1}$ up to and including the term in x^2.

State the range of values of x for which the expansion is valid. [5]

15 (a) A closed tin of oil is in the shape of a right circular cylinder. Given that its capacity is 300 ml,

(i) write down an expression for its total surface area, A in terms of r,

(ii) calculate the radius and height of the tin that minimises A. [5]

(b) Given that $x - y \ln x = \ln y$,

(i) find $\frac{dy}{dx}$ in terms of x and y,

(ii) find an expression for the approximate value of y when $x = 1 + h$ where h is a small increase in x. [6]

16 (a) Find the stationary points of the curve $y = 2\sin 2x + 1$ and determine their nature for $0 \le x \le \pi$. [6]

(b) (i) Sketch on the same axes, the graphs of $y = e^x$ and $y = \frac{2}{1+x}$.

(ii) State the number of real roots of the equation $e^x(1+x) = 2$.

(iii) Taking $x_1 = 0{,}5$ as a first approximation to the root, use the Newton-Raphson method twice to find the root of the equation correct to 3 decimal places. [7]