ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS
PAPER 4

9164/4

NOVEMBER 2012 SESSION   3 hours

Additional materials:
  Answer paper
  Graph paper
  List of Formulae

TIME   3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

Section (a): Statistics

1   A group of 50 children raised money for charity. The amount raised by each child to the nearest dollar ($) are recorded in the table.

amount ($)	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30
number of children	20	10	5	6	8	1

(i)   State the smallest possible amount which may have been raised by one child.   [1]

(ii)   Find

    (a)   the mean amount,

    (b)   standard deviation.   [4]

2   A discrete random variable X, has a probability distribution given in the table.

x	0	1	2
$P(X = x)$	P_0	P_1	P_2

Given that $E(X) = 1.2$ and $Var(X) = 0.36$.

find the values of P_0, P_1 and P_2.   [6]

3   A student wishes to be enrolled at one of the three Zimbabwean universities BUSE, NUST and MSU. It is equally likely that the student will apply to any one of these three universities. The probability that the student will be admitted at BUSE is 60 % while the probability that the student will be admitted at NUST and MSU are 45 % and 35 % respectively.

Find the probability that

(i)   the student will not be admitted at any one of the universities.   [2]

(ii)   the student will apply to NUST given that the student will not be admitted into any one of the universities.   [4]

4   The masses of passengers on a flight are normally distributed with mean μ and standard deviation σ. 1.7 % of the passengers had masses greater than 75 kg and 98 % had masses more than 45 kg.

Find the value of μ and the value of σ.   [6]

5   A continuous variable X has a probability density function given by

$$f(x) = \begin{cases} ax^2 & 0 \le x \le 1 \\ \frac{1}{4}(7-3x) & \text{for } 1 \le x \le \frac{7}{3} \\ 0 & \text{otherwise} \end{cases}$$

where a is a constant.

(a)   Show that $a = 1$.   [2]

(b)   Find the

(i)   mean,

(ii)   median.   [5]

6   20 % of the employees of an organisation joined a Housing Scheme.

(a)   From a random sample of 8 employees of the organisation, find the probability that

(i)   only 2 employees joined the scheme,

(ii)   less than 3 employees joined the scheme.   [4]

(b)   A larger random sample of 100 employees of the organisation was taken, calculate the probability that less than 25 employees joined the scheme.   [4]

7   A transport company has 2 vans available for daily hire to ferry vegetables to a market. The daily demand for these vans may be assumed to have a Poison distribution with mean 2.

(a)   Show that demands for one and 2 vans on any day are equally probable.   [3]

(b)   Calculate the probability that

(i)   on a particular day, the transporter may not be able to meet the demand for these vans,

(ii)   exactly 4 hirings will occur in two consecutive days.   [6]

[Turn over

8   A truck is carrying boxes of bathing soap and washing powder. The average mass of a box of bathing soap is 65 kg with variance 8 kg. A box of washing powder has an average mass of 50 kg with variance 7 kg.

Find the probability that

(i)   a box of bathing soap weighs less than 62 kg,   [2]

(ii)   2 randomly chosen boxes of bathing soap weigh a total of more than 140 kg,   [4]

(iii) the mass of a box of bath soap is greater than $\dfrac{4}{3}$ of the mass of a box of washing powder.   [4]

9   The table shows a random sample of employees classified by educational qualifications and earnings.

qualifications	earnings		
	low	medium	high
'A' level	115	293	242
diploma	171	415	364
degree	684	992	864

Test, at 5 % level of significance, if there is any association between qualification and earnings.   [11]

10   In a region, the best 'O' level mathematics student can score between 70 % to 100 %. In experiments carried out to test this claim the percentage scored, x, was recorded for 250 students after writing a Mathematics examination and the data is summarised by

$$\sum(x - 80) = 700 \text{ and}$$

$$\sum(x - 80)^2 = 25\ 142.$$

The population mean and variance of X are denoted by μ and σ^2 respectively.

(i)   Show that the unbiased estimate of σ^2 is 93.1, correct to one decimal place.   [3]

(ii)   Test the hypothesis that $\mu = 85$ against the alternative $\mu < 85$ using a 5 % significant level.   [7]

(iii) Calculate a symmetric 95 % confidence interval for the mean score.   [3]

11   A farmer supplies potatoes daily to a hypermarket. The mass of potatoes supplied depend on the day's demand. The masses of potatoes supplied and the fuel consumed by the delivery truck per trip were recorded for eight such trips.

mass in tonnes	0.7	1.8	2.5	3.2	4.1	5.4	6.3	6.9
volume of fuel (in litres)	10.0	11.2	12.2	13.0	13.0	14.2	15.6	15.8

(a)   Plot a scatter diagram on a graph paper showing the masses of potatoes on the horizontal, x-axis, and the volume of fuel consumed on the vertical, y-axis.   [3]

(b)   The equation of the regression line of Y on X is of the form $Y = a + bX$.

Find the values of the constants a and b.   [4]

(c)   Fit the regression line on the graph and use it to estimate the maximum mass of potatoes that may be transported by 14 litres of fuel.   [4]

(d)   Find the product moment correlation coefficient and comment on the relationship between fuel consumption and the mass of potatoes.   [4]

[Turn over

Section (b): Mechanics

12 Two concurrent coplanar forces of magnitude 4N and 9N act on a particle.
Given that the angle between the two forces is 40°, find the magnitude of their
resultant.   [3]

13 A particle of mass 2 kg is held at rest on a rough horizontal table connected by a light
inextensible string which passes over a smooth pulley at the end of the table to a
particle of mass 5 kg which hangs freely. The coefficient of friction between the
particle and the table is $\frac{1}{4}$. The particles are released from rest.

Find, in terms of g,

(i)   (a)   the acceleration of the particles,

(b)   the tension in the string,

(ii)   the force exerted by the string on the pulley.   [6]

14 A stone is thrown vertically upwards from a point T, 1 m above ground level with
a speed of 12 ms⁻¹. It hits an obstacle and comes to an instantaneous rest 0.6 m
before hitting the ground.

(i)   Find

(a)   the greatest height above its initial position,

(b)   the speed with which it hits the obstacle.   [4]

(ii)   Sketch the velocity – time graph for the motion.   [3]

15 A ball is projected at an angle of 30° above the horizontal with a velocity of 36 ms⁻¹
from a point which is 1.5 metres above the level ground.

Calculate

(i)   the time taken by the ball to reach the ground,   [4]

(ii)   the magnitude of the velocity with which the ball hits the ground.   [4]