For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS 9164/1

PAPER 1

NOVEMBER 2012 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

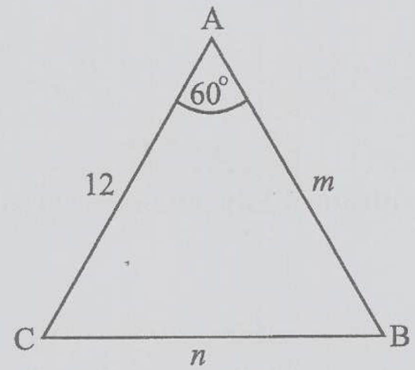
INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 7 printed pages and 1 blank page.

[Turn over

1



The diagram shows triangle ABC with AB = m cm, BC = n cm, AC = 12 cm and angle $B\hat{A}C = 60°$.

It is given that the area of triangle ABC is $\sqrt{675}$ cm^2.

Find the exact values of m and n. [4]

2 (i) Show that the equation $6\sin(\pi + x^2) = 3$ has a root between 1.9 and 2. [3]

(ii) Starting with $x_1 = 1.9$ apply the Newton-Raphson method once to find the second estimate for the root, giving your answer correct to 3 significant figures. [3]

3 (a) The function $y = f(x)$ is given by $y = e^{\sin 3x}$.

Find expressions for

(i) $\frac{dy}{dx}$ and

(ii) $\frac{d^2y}{dx^2}$. [3]

(b) Hence write down the Maclaurin series of $f(x)$ up to the term in x^2. [3]

4 (a) A function is represented parametrically by the equations

$y = e^{t^2}$ and $x = e^{t^2} - 1$, for $1 \le t \le 2$.

Use the chain rule to find $\frac{dy}{dx}$ and show that it is independent of t. [4]

(b) Describe briefly stating the exact coordinates of the end points, the shape of the graph of y against x. [2]

5 The 6th term of a geometric progression is $\frac{1}{81}$ and the 3rd term is $-\frac{1}{3}$.

Find

(a) the value of the

(i) common ratio,

(ii) first term, [4]

(b) the sum of the first n terms of the progression, [2]

(c) the value of the sum to infinity of the progression. [1]

6 The curve C has equation given as $y = p + e^{qx}$, where p and q are real constants.

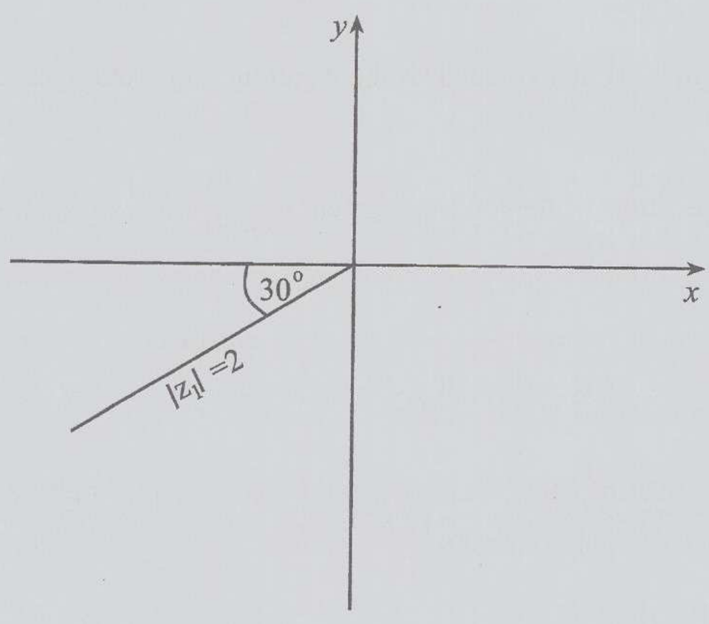
It crosses the y-axis at point A $\left(0; \frac{3}{2}\right)$ and has gradient -2 at point A.

(i) Find the values of p and q. [4]

(ii) State the equation of the asympotote. [1]

(iii) Sketch the curve C and its tangent at point A. [3]

7



A complex number z_1 has modulus 2 and is positioned as shown in the Argand diagram above.

(i) State the principal argument of z_1 and write z_1 in the form $a + ib$ where a and b are exact real numbers. [3]

(ii) Find exactly in the form $a + ib$, the complex number w, given that

$$w = \frac{\left(-8\sqrt{3}\right)i}{z_1}.$$ [2]

(iii) Show a sketch of w in an Argand diagram, labelling the modulus and argument values in your diagram. [3]

8 The function $f(x)$ has equation $f(x) = 2x^3 + bx^2 + x - 2$.

Given that $2x - 1$ is a factor of $f(x)$,

(i) find the value of b, [2]

(ii) factorise $f(x)$ completely. [3]

(iii) Sketch the curve of $y = f(x)$, showing clearly the points of intersection of the curve with the axes without necessarily finding the coordinates of the turning points. [2]

Hence, or otherwise, find the values of x which satisfy the inequality

$$f(x) < 0.$$ [2]

9 A thin string of length 220 cm is divided (without cutting it up), into n parts in the ratio 1 : 2: 3: …n.

(a) Show that the length of the longest segment is $\frac{440}{n+1}$ cm. [3]

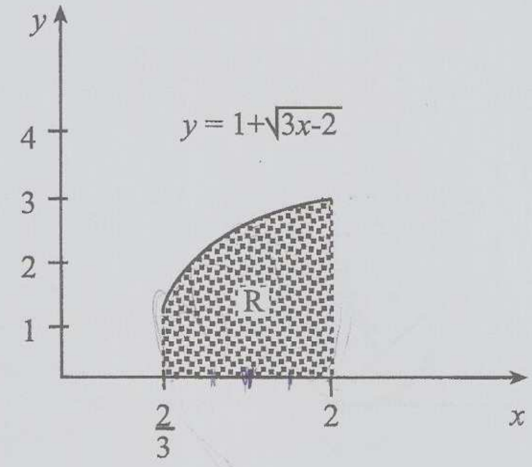
(b) The string is made to form a circle so that the ends just touch each other.

(i) Find the radius of this circle.

(ii) Show that the angle subtended at the centre by the largest segment in part **(a)** is approximately $\frac{12.57}{n+1}$ radians.

(iii) Hence find the smallest value of n for which this angle is less than $\frac{\pi}{12}$ radians. [6]

10



The diagram shows the graph of the curve $y = 1 + \sqrt{3x-2}$ for $\frac{2}{3} \le x \le 2$.

The region R is enclosed between the curve, the x-axis and the lines $x = \frac{2}{3}$ and $x = 2$.

Calculate the

(i) area of the region R, [3]

(ii) volume generated when the region R is rotated completely about the x-axis. [6]

11 The points A, B and C have position vectors $8j - 4k; pi + 3j + 4k$ and $-2i + 3j + 5k$ respectively.

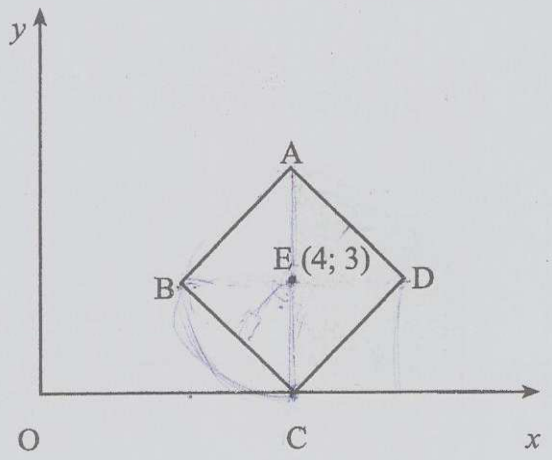
(a) Calculate the

(i) unit vector parallel to $\overrightarrow{AC}$,

(ii) positive value of p such that BA is perpendicular to BC. [7]

(b) Hence or otherwise, find the area of triangle ABC for the value of p in **(a)(ii)**. [3]

12



The diagram shows a square whose perimeter is $12\sqrt{2}$ cm and has centre at E(4; 3)

(a) **(i)** Show that the radius of the circle which touches all the four corners of the square is 3 cm.

(ii) Find the equation of this circle giving your answer in the form

$$x^2 + y^2 + ax + by + c = 0.$$

(iii) Find the area of the minor segment enclosed between this circle and the line AD.

[8]

(b) It is given that the diagonal line AEC of the square is parallel to the y-axis.

Write the

(i) gradients of the lines BC and BA,

(ii) equation of the tangent to the circle at point A.

[3]

13 A colony of ants brings food to its hive at a constant rate of 500 grammes per day. It consumes food in the hive at a rate of $\frac{1}{2}x$ grammes per day, where x is the amount of food present after time t days.

(i) Taking x and t as continuous variables, write down a differential equation satisfied by x and t, to show the rate at which food is increasing in the hive. [2]

(ii) Solve the differential equation leaving x in terms of t, given that when $t = 0$, $x = 400$ grammes. [6]

(iii) Find correct to 3 significant figures, the time at which the amount of food in the hive is 980 grammes. [3]

(iv) Find the maximum amount of food which the hive can have.
Explain why this amount of food cannot be exceeded. [2]

14 **(a)** **(i)** Write down the function $3\sqrt{2}(\cos x + \sin x)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(ii) Hence or otherwise, solve the equation $\sqrt{2}(\cos x + \sin x) = \sqrt{3}$ for $0° \leq x \leq 360^0$. [6]

(b) **(i)** State the maximum and minimum values of the function
$y = 6 + 3\sqrt{2}\ (\cos x + \sin x)$.

(ii) Sketch the graph of y against x for $0^0 \leq x \leq 360^0$.

(iii) Describe the basic transformations which the graph of $y = \cos x$ undergoes to obtain the graph of $6 + 3\sqrt{2}(\cos x + \sin x)$.

[8]