ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS

9164/1

PAPER 1

NOVEMBER 2011 SESSION — 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

1 Sketch on a single diagram the graphs of $y = |3x+2|$ and $y = -x+1$. Hence or otherwise, find the set of values of x which satisfy $|3x+2| < -x+1$. [5]

2 The cubic polynomial $f(x)$ is given by $f(x) = 2x^3 + 6x^2 + kx + 12$, where k is a constant. The function $f(x)$ leaves a remainder of 6 when divided by $x+2$.

Find

(a) the value of k, [2]

(b) the solutions of the equation $f(x) = 9$, given that it has only one real root. [5]

3 (a) Describe the transformations needed to transform the graph of $y = \sin x$ onto the graph of $y = 6\sin(2x) - \pi$. [3]

(b) A curve has equation $y = \dfrac{x^2 - 4}{x+1}$.

Find the equation of the normal to the curve at P(2; 0) in the form $ax + by + c = 0$, where a, b and c are integers. [4]

4 At time $t = 0$, the area of a circular pond is 36π cm^2. The radius of the pond is increasing at a constant rate of 1.1 cm s^{-1}.

Find the rate at which the area is increasing at the instant when:

(a) the area is 144 cm^2, [3]

(b) $t = 10$ sec. [4]

5 (a) Express the function $\dfrac{3}{x^2(x+3)}$ in partial fractions. [3]

(b) Hence evaluate the integral $\int_2^4 \dfrac{3}{x^2(x+3)}dx$ leaving your answer in exact form. [4]

6 The points O, P, Q and R form a parallelogram, where O is the origin. The position vectors of P and R are $\mathbf{p} = \mathbf{i} + \lambda\mathbf{j} + \mathbf{k}$ and $\mathbf{r} = 4\mathbf{i} + 2\mathbf{k}$ respectively; where λ is a positive constant. The angle $O\hat{P}R$ is a right angle.

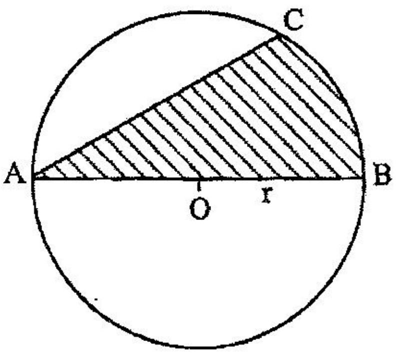
Find

(a) the value of λ, [3]

(b) the position vector of Q, [1]

(c) the exact area of the parallelogram OPQR. [3]

7



The diagram above shows a circle of radius r cm and centre O. The shaded region is bounded by the chord AC, the diameter AB and the arc BC, and the angle $B\hat{A}C = \theta$ radians.

(a) Find an exact expression in terms of r and θ for the perimeter of the shaded region. [3]

(b) Given that $\theta = \frac{1}{6}\pi$, calculate the exact area of the shaded region in terms of r. [4]

8 The function $h(x) = 2x^2 - 6x + 11,\ x \in \mathbb{R}$.

Express

(a) $h(x)$ in the form $a(x+b)^2 + c$ where a, b and c are constants. [2]

(b) Write down the range of h. [1]

(c) Explain why h does not have an inverse. [1]

(d) Given that $h(x)$ is defined for $x \in A$, where $A \subset \mathbb{R}$, find

(i) the largest element of A for which $h(x)$ has an inverse,

(ii) $h^{-1}(x)$ for this set of elements of A. [4]

9 It is given that $f(x) = \frac{1}{(1+x)^2} + \sqrt{9+x}$.

(a) Expand $f(x)$ up to and including the term in x^2 simplifying coefficients. [7]

(b) State the values of x for which the expansion is valid. [1]

10 (a) The complex number u is such that $(-4+3i)u = 5-3i$.

Find

(i) the modulus of u,

(ii) the argument of u. [4]

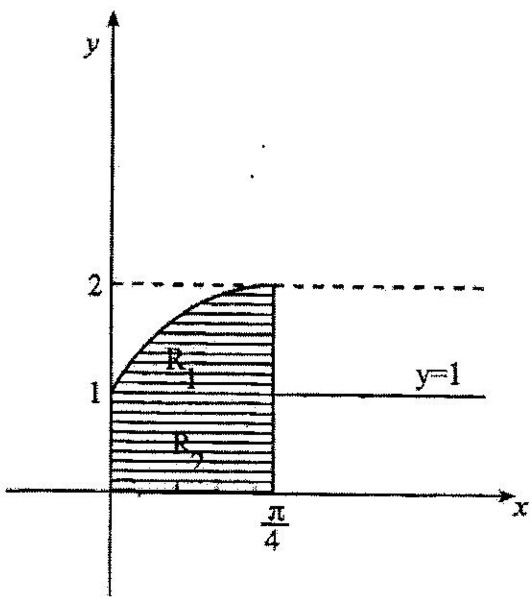
(b) Given that the complex number w is $2i$.

Find in the form $a + ib$

(i) $\frac{u}{w}$,

(ii) uw. [4]

11



The diagram shows the regions R_1 and R_2 enclosed by the curve $y = 1 + \sin 2x$, the line $x = \frac{1}{4}\pi$ and the axes.

The line $y = 1$ divides the two regions.

(a) Use the trapezium rule with 4 equal intervals, to find an approximate total area of $R_1 + R_2$, correct to 3 decimal places. [3]

(b) Find in terms of π, the volume of the solid of revolution obtained when R_1 is rotated through 2π radians about the x-axis. [6]

12 (a) Sketch, on the same axes, the graphs of $y = 2x$ and $y = \tan x$, where $0 \le x \le \frac{3}{2}\pi$. [2]

(b) Show that the smallest positive root of $2x - \tan x = 0$, lies between $x = 1$ and $x = 1;5$. [4]

(c) Starting with $x_0 = 1$ use Newton-Raphson method to find the smallest positive root of $2x - \tan x = 0$, giving your answer correct to 3 decimal places. [4]

[Turn over

13 (a) Show that the equation $3\sin x + 5\cos x \cot x - 4 = 0$ may be written in the form $a\sin^2 x + b\sin x + c = 0$, where a, b and c are constants. [2]

(b) Hence or otherwise solve the equation $3\sin x + 5\cos x \cot x - 4 = 0$, where $0 \le x \le 360°$. [4]

(c) (i) Express $5\cos x + 6\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

Hence solve the equation $5\cos x + 6\sin x = 4$ for $0 \le x \le 360°$.

(ii) State the maximum and minimum values of $5\cos x + 6\sin x$. [7]

14 (a) Find the solution of the differential equation $x\frac{dy}{dx} = y + yx$, given that the curve passes through the point (2; 4). [4]

(b) A water tank has the shape of a cuboid with base area 4 m^2 and height 3 m and is initially empty. Water is poured into the tank at a constant rate of 0.05 m^3 per minute. There is a small hole at the bottom of the tank through which water leaks out. The depth of the water in the tank is h metres when water has been poured in for t minutes.

(i) In a simple model it is assumed that water leaks out of the tank at a constant rate of 0.025 m^3 per minute.

1. Show that the variable h satisfies the differential equation $\frac{dh}{dt} = \frac{1}{160}$.

2. Hence or otherwise, find the time when the tank starts to overflow.

(ii) In a more refined model, the variable, h, satisfies the differential equation $160\frac{dh}{dt} = 2 - h$.

1. Solve the differential equation, expressing h in terms of t.

2. Hence sketch the graph of h against t. [13]