ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

MATHEMATICS
PAPER 4

9164/4

NOVEMBER 2010 SESSION

3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81 \text{ ms}^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 7 printed pages and 1 blank page.

[Turn over

Section (a): Statistics

1   The probability density function for a discrete random variable X, is given as follows:

$$P(X = x) = \frac{x-1}{16} \text{ for } x = 2,3,4,5;$$

$$P(X = x) = \frac{9-x}{16} \text{ for } x = 6,7,8.$$

(a)   Draw up a table showing the probability distribution of X.   [1]

(b)   Calculate E(X).   [2]

State the nature of the distribution.   [1]

2   In a large batch of bolts, the probability that a randomly chosen bolt is defective is $\frac{1}{6}$. A random sample of 42 bolts is taken, find the probability that more than two bolts are defective.   [5]

3   A continuous random variable X has a probability density function,

$$f(x) = \begin{cases} kx, & 0 \le x \le 3 \\ 3k(4-x), & 3 \le x \le 4 \\ 0, & \text{otherwise,} \end{cases}$$

where k is a constant.

(a)   Find the value of k, and sketch the graph of $f(x)$.   [4]

(b)   Find the probability that $x > 2$.   [3]

4   X and Y are two independent random variables whose means and variances are shown in the table below.

	Mean	Variance
X	20	9
Y	10	4

The random variable Z defined by $aX + bY$, where a and b are integers, has a mean of 20 and a variance of 145.

Find the values of a and b.   [7]

5   Three players A, B and C, in that order, throw a fair cubical die. The first to throw a 6 wins. The game is continued indefinitely until one of the players wins.

(a)   Find the probability that A wins

   (i)   on his first throw,   [1]

   (ii)   on his second throw,   [2]

   (iii) the game.   [3]

(b)   Given that the probability that B wins is $\frac{30}{91}$, find the probability that C wins.   [2]

6   The following data have been collected for a sample from a population that is normally distributed.

5, 10, 8, 11, 12, 6, 15, 13

(a)   Calculate the unbiased estimate of the population mean and the standard deviation.   [3]

(b)   Find a 95% confidence interval for the population mean.   [5]

7   The table below shows examination marks in two papers for seven A-level Chemistry students.

Student	A	B	C	D	E	F	G
Chemistry Theory (x)	68	74	37	75	52	75	44
Chemistry Practical (y)	80	63	49	62	65	80	56

$$\sum x = 425 \quad \sum x^2 = 27\,359 \quad \sum y = 455$$

$$\sum y^2 = 30\,375 \quad \sum xy = 28\,409$$

(a)   Find the equation of the regression line y on x.   [4]

(b)   Use your equation to estimate the Chemistry Practical mark for a student who gets 58 in the theory paper.   [1]

(c)   Find the product moment correlation coefficient and comment on your result.   [3]

[Turn over

8   The General Mechanical Engineering Department has 5 buses which it hires
    out on a weekly basis. The weekly demand for these buses is known to have
    a poison distribution with a mean of 3.

    Find the probabilities, correct to 3 decimal places that in a given week,

    (a)   no bus is hired out,   [2]

    (b)   at least 2 buses will be hired out,   [3]

    (c)   the demand for the buses exceeds the available number of buses.   [4]

9   The following table gives the frequency distribution of the number of computers
    sold during the past month at all different computer stores in Harare.

Computers sold	50 - 52	53 - 55	56 - 58	59 - 61	62 - 64
	51	54	57	60	63
Number of stores	5	10	21	8	6

    (a)   State the number of computer stores that sold the computers.   [1]

    (b)   Calculate the mean.   [3]

    (c)   Draw the cumulative frequency curve and hence estimate the

          (i)   median,

          (ii)   quartiles.   [6]

    (d)   Comment on the nature of the distribution.   [1]

10 (a)   In an election held in 2007, 60% of the voters voted for Party A. In
          a poll of opinion conducted last week, 250 potential voters were asked
          how they would vote if there was an election now. 135 of the voters
          said they would vote for Party A.

          Investigate at 5% level of significance whether the proportion of the
          voters in favour of Party A has decreased significantly.   [6]

    (b)   Ambulance Service claims it takes an average of 8.9 minutes to respond
          to emergency calls. To verify this claim, the Agency which licences
          ambulance services timed 50 responses to emergency calls. The observed
          data gave a mean of 9.3 minutes and a standard deviation of 1.8 minutes.

          Test at the 5% significance level whether there is evidence to justify
          Ambulance Service's claim.   [8]

11   The number of electrical faults at a station was observed over a period of 160 days. The following table gives the frequency distribution of the observations.
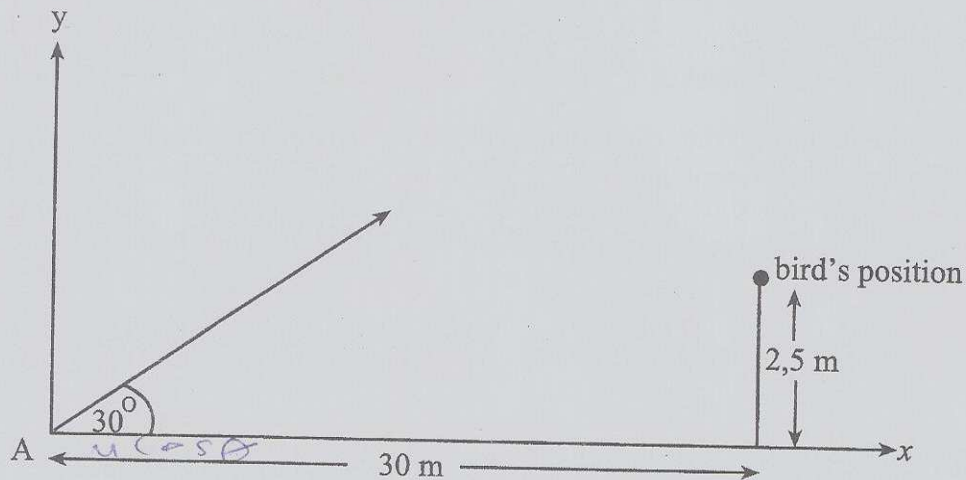
Number of electrical faults	0	1	2	3	4	5	6	7
Number of days	25	35	35	25	20	10	7	3

Apply the χ^2 test at the 5% level of significance to determine if the number of electrical faults follows a Poisson distribution.   [15]

Section (b): Mechanics

12

A stone is projected from point A with a speed of 20 ms^{-1}, at an angle of 30° to the horizontal, towards a bird 30 m away. The bird is standing on a a pole 2,5 m vertically above the level ground (see diagram).

Assuming that the path of the stone is not impeded, determine whether the stone will hit the bird and if not by how far vertically above or below the bird does the stone miss it.   [5]

[Turn over

13
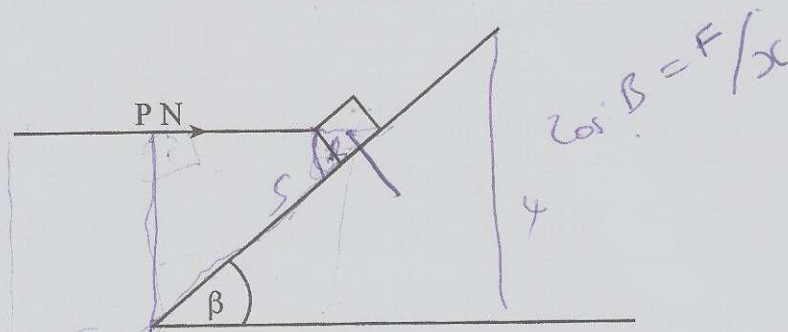
A block of weight 20N is placed on a rough horizontal plane inclined at an angle β to the horizontal, where $\sin \beta = \dfrac{4}{5}$. The block is acted on by a horizontal force of P N and is about to slip downwards. (See diagram)

Given that the coefficient of friction between the block and plane is $\dfrac{1}{4}$, show that the horizontal force P N needed to prevent the block from slipping is 16,25 N. [6]
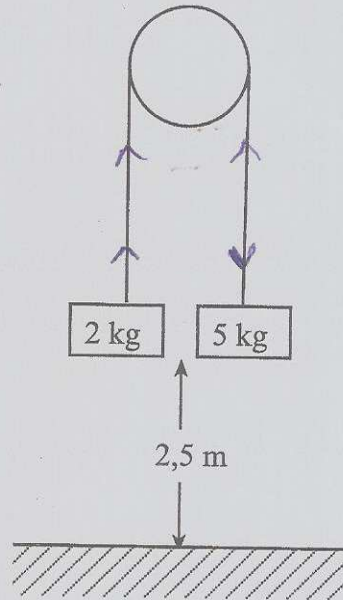
14 A particle starts from rest and moves in a straight line. Its acceleration a ms^{-2} is given by

$$a = \begin{cases} 3 & 0 \le t \le 2 \\ -3 & 2 < t \le 8 \end{cases}$$

when t is time in seconds.

(a) Write down the velocities of the particle when $t = 2$ and $t = 8$. [2]

(b) Hence sketch the $(t;v)$ graph and find the total distance travelled by the particle in the interval $0 \le t \le 8$. [4]

15

Masses of 2 kg and 5 kg are connected by a light inextensible string and hang
vertically on either side of a smooth pulley. The system is released from rest
with both portions of string vertical and both particles at a height of 2,5 metres
above the ground (see diagram).

(a)   Find the acceleration of the particles in terms of g.   [3]

(b)   Find the velocity of the 5 kg mass as it hits the ground.   [2]

(c)   If the 5 kg mass hits the ground and does not rebound, find the greatest
        height reached by the 2 kg mass if it does not hit the pulley.   [2]

2

-2a)

vet +a²