ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS

PAPER 1

9164/1

NOVEMBER 2010 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

1 If a and b are positive real numbers, $a \neq b$ and $\log_a b + \dfrac{2}{\log_a b} = 3$, express b in terms of a. [4]

2 (a) Find the product of $x^2 - 3x - 5$ and $x^2 + 3x - 2$. [1]

(b) Solve the inequality

$$2x^2 - 3x < 5.$$ [3]

3 Express $z = \dfrac{1+i}{3+4i}$ in the form $a + bi$, where a and b are real. [3]

Hence or otherwise find $|z|$ in the form $c\sqrt{d}$ where d is a prime number. [2]

4 The first and last terms of an arithmetic progression are -10 and 25.

(a) Find the number of terms of the progression for which the sum of the terms first exceeds 300. [3]

(b) Calculate the difference between consecutive terms in this progression for the number of terms above. [2]

5 The position vectors of points A and B with respect to the origin O, are given by

$$\overrightarrow{OA} = i + 3j + 3k,$$
$$\overrightarrow{OB} = -4i + 5j + 3k.$$

Show that $\cos\left(A\hat{O}B\right) = \dfrac{4}{\sqrt{38}}$. [2]

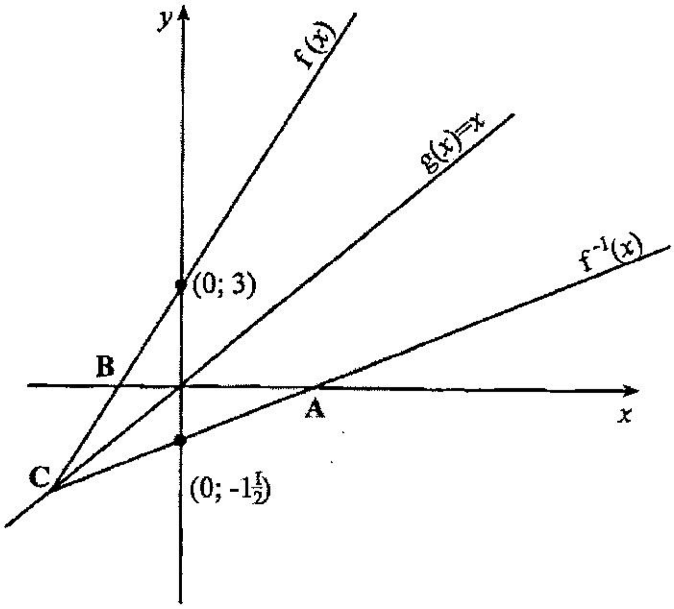
Hence, or otherwise, find the position vector of the point P on OB such that AP is perpendicular to OB. [4]

6 By putting $10^x = m$, show that the solution to the equation

$\dfrac{10^x + 10^{-x}}{10^x - 10^{-x}} = k$ is $x = \dfrac{1}{2} lg\left(\dfrac{k+1}{k-1}\right)$.

Hence show that $x = lg2 + \dfrac{1}{2}lg3 - \dfrac{1}{2}$ when $k = 11$. [6]

7 The diagram below shows the linear graphs of $f(x)$, $g(x)=x$ and $f^{-1}(x)$ which intersect at **C**. The graph $f^{-1}(x)$ intersects the x-axis at point **A** and $f(x)$ intersects x-axis at **B**.



(a) State the name given to the function $g(x)$ in relation to the functions $f(x)$ and $f^{-1}(x)$. [1]

(b) Write down the coordinates of the points **A** and **B**. [3]

(c) Calculate the coordinates of the point **C**. [3]

8 **(a)** By means of the substitution $u = \sin x$ or otherwise, find the exact value of $\int_0^{\frac{\pi}{2}} \cos x\sqrt{\sin x}\ dx$. [3]

(b) Solve the equation $\cos 3y = -\frac{1}{\sqrt{2}}$ for $0° < y < 270°$. [4]

9 **(a)** Show, by sketching two appropriate graphs, that the equation $x^3 + 3x - 3 = 0$ has only one real root. [2]

(b) Show, by calculation, that the root of the equation in **(a)** lies between $x = 0{,}8$ and $x = 1$. [1]

(c) Obtain approximations to the root of the equation in (a), to 3 significant figures

(i) by performing one application of the Newton-Raphson procedure using $x = 0{,}8$ as a first approximation, [2]

(ii) by performing two iterations, using $x_{n+1} = \dfrac{3 - x_n^3}{3}$ and starting with $x = 0{,}8$. [2]

10 (i) Write down the equation of a circle with centre (4; –3) and radius 5. [1]

(ii) State the condition satisfied by the point $(x; y)$ inside this circle. [1]

(iii) Sketch this circle and the line $2x + y = 3$ on the same diagram with the line intersecting the circle at 2 points. [2]

(iv) Find the range of values of x such that the point inside the circle lies on the line $2x + y = 3$. [3]

11 Given that $x = ln(3 + 2t)$ and $y = e^{3t^2}$,

(i) find $\dfrac{dy}{dx}$ in terms of t. [3]

(ii) show that the curve has only one turning point and write down the coordinates of the turning point. [4]

12 One root of the equation $x^4 + ax^3 + bx^2 + 16x - 12 = 0$ is 2 and the other root is –2.

Find

(a) the values of a and b, [4]

(b) the other two roots using the values of a and b found in (a) above. [4]

13 (a) In a triangle ABC, AB = 23,2 cm, BC = 15,3 cm and angle ABC = 121°.

Find, by calculation, angle BAC, giving your answer to the nearest 0,1°. [5]

(b) In a triangle PQR, PQ = $2x$ units, PR = $(x - 2)$ units and angle P is almost a right angle, such that angle P = $\left(\dfrac{\pi}{2} - x\right)$ radians, where x is small.

Show that $QR^2 \approx -4x^3 + 13x^2 - 4x + 4$. [3]

14 **(i)** Show that $2x^3 - x^2 + 8x - 4 = (2x-1)(x^2+4)$. [2]

(ii) Express $\dfrac{x^2+2x+20}{2x^3-x^2+8x-4}$ in partial fractions. [4]

(iii) Hence evaluate $\int_1^3 \dfrac{x^2+2x+20}{2x^3-x^2+8x-4}dx$, and give your answer correct to three significant figures. [4]

15 **(i)** Express $12x - 4x^2$ in the form $c-(ax+b)^2$, where a, b and c are constants to be found. [2]

(ii) Find the equation of the tangent to the curve $y = 12x - 4x^2$, at the point $y = 8$ and $x < \frac{3}{2}$. [6]

(iii) A finite region R is bounded by the y-axis, the tangent calculated in **(ii)** and the curve $y = 12x - 4x^2$.

Illustrate, by shading, the region R on a sketch graph. [2]

(iv) Calculate in terms of π, the exact volume when R is rotated through 360° about the x-axis. [3]

16 **(a)** Some juice is tapped into a cylindrical container at a rate of 100 cm^3 per minute. It is sieved out through a hole at the bottom of the cylinder at a rate of 2,5h cm^3 per minute, where h cm is the height of the juice in the cylinder at time t minutes. The radius of the cylinder is 5 cm.

(i) Show that $\dfrac{dh}{dt} = \dfrac{(40-h)}{10\pi}$ [4]

(ii) Solve the differential equation to find h in terms of t, given that at time $t = 0$, $h = 0$.

Hence or otherwise state the maximum value of h and explain why this height cannot be exceeded. [8]

(b) The delivering tap in part (a) was closed off when juice was at maximum height, and the juice was allowed to drain out. The differential equation satisfied by the drainage process only is $\dfrac{dh}{dt} = -\dfrac{1}{10\pi}h$.

Solve this differential equation to find t in terms of h and show that the time taken to make the juice go down to a height of 0.88 cm is nearly 2 hours. **[4]**