

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

MATHEMATICS

PAPER 4

9164/4

Monday 10 NOVEMBER 2003

Morning

3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae

Start 8:10
Finish 11:10

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81 \text{ms}^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 10 printed pages and 2 blank pages.

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Section (A): Statistics

- 1 A Domestic Workers Union claims that the average hourly rate paid to domestic workers is \$15.85. The house-wives league in this country wishes to test this claim. They conducted a survey amongst a sample of 1 225 domestic workers throughout the country. They found the sample mean hourly rate to be \$16.03. Assume that the population standard deviation of hourly rates paid to domestic workers is \$2.87. Test the hypothesis at the 2% significance level that the average hourly rate paid to domestic workers in this country is more than \$15.85. [6]

- 2 Two players A and B take turns to toss a tetrahedral die until a 4 appears. A person who first throws a 4 wins the game. Assuming that A throws first, find

(i) the average number of tosses required before the game is decided, [2]

(ii) the probability that A wins the game. A_1, A_2 [5]

- 3 It is given that about 95% of values of a standard normal distribution lie between a and b, where $a < b$ and $P(a < x < b) = 0.95$.

(i) Show that $b = \mu + 1.96\sigma$ and

$$a = \mu - 1.96\sigma.$$

A_1

[5]

(ii) Hence show that $\mu = \frac{a+b}{2}$.

[2]

- 4 A random sample of men and women indicated their views on adopting a national dress as summarised below.

	In favour	Opposed	Undecided	Total
Women	118	62	25	205
Men	84	78	37	199
Total	202	140	62	404

At the 1% level of significance, test the hypothesis that there is no difference in opinion between men and women in as far as this survey is concerned.

[7]

$9 \times 25 \times 2$
 3×7

- 5 On any day, the amount of time measured in hours, that a viewer spends watching television is a continuous random variable T , with a cumulative distribution function given by

$$F(t) = \begin{cases} 0 & t \leq 0 \\ 1 - k(15 - t)^2, & 0 \leq t \leq 15 \\ 1 & t \geq 15 \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{225}$.

$$1 - k(15)^2 = 0$$

$$1 - 225k = 0$$

[2]

- (ii) Show that for $0 \leq t \leq 15$, the probability density function of T is given by

$$f(t) = \frac{2}{15} - \frac{2}{225}t.$$

[2]

- (iii) Find the median of T .

[3]

- 6 Twenty-three people in a random sample were asked to record the number of kilometres they travelled by bus in a given week. The distances to the nearest kilometre are shown below.

67	76	85	42	93	48	93	46	52	63
70	72	44	66	87	78	47	66	50	72
82	56	58							

- (a) Construct a stem and leaf diagram to represent this data.

[2]

- (b) Using a scale of 2 cm to represent 10 km, draw a box and whisker plot to represent this data.

[4]

- (c) Give **one** advantage of using

- (i) a stem and leaf diagram,

[1]

- (ii) a box and whisker plot.

[1]

- 7 (i) In a large city, one person in 65 dies of AIDS. If a random sample of 250 people is taken, find the probability that the sample includes at most two people who die of AIDS. [4]

- (ii) Calculate the number of people who must be taken at random in order that the probability of including at least one person who dies of AIDS is at least 0.95. [4]

- 8 A random sample of 12 values is taken from a normal distribution whose mean, μ , and variance, σ^2 , are unknown such that

$$\sum x = 5472,$$

$$\sum (x - 456)^2 = 1\,236.$$

- (i) Explain what is meant by an unbiased estimate of a population parameter. [1]
- (ii) Calculate unbiased estimates of μ and σ^2 . [3]
- (iii) Hence find a 97% confidence interval for μ . [4]
- 9 Biscuits are produced with weight W grams where $W \sim N(10; 4)$ and are packed at random into boxes consisting of 25 biscuits.

Find the probability that

- (a) a biscuit chosen at random weighs less than 9.5 g, [2]
- (b) the contents of a box weigh between 247 g and 253 g, [3]
- (c) the mean weight of the biscuits in the box is greater than 10.2 g. [3]

10

Gamma Engineering company manufactures bolts which are sent to customers in batches of 5 000. The company operates a sampling scheme whereby a random sample of size 8 is taken from each batch ready for despatch. A batch is accepted only if the number of defective bolts in the sample is less than 2, otherwise the batch is rejected and re-processed.

$P(X \geq 2)$
reject

- (a) If 5% of all the bolts produced are known to be defective, find the proportion of batches that will be rejected. [4]
- (b) The company replaces all its bolt producing machines causing the proportion of defective bolts to drop to 0.5%. It now accepts batches only if there are no defective bolts in a sample of 8. Calculate the change in proportion after the replacement of the machines. [5]

11 (a) If E_1 and E_2 are any two events, explain what is meant by

- (i) E_1 and E_2 are independent. [1]
- (ii) E_1 and E_2 are mutually exclusive. [1]

(b) In the **who, what** or **where** game, three contestants each chooses one of the three categories of a question. Assuming that the contestants choose independently and that they are equally likely to select any of the categories, find the probability that

- (i) all will choose different categories. [3]
- (ii) two will be alike and the third different. [4]

X

$A \rightarrow who$ or $A - white$ $A B C$
 $B - what$ $D - who$ A
 $C - where$ $G - where$

$A -$ $3C_2$

$A - who$	$A - what$	$A - where$	$B - who$
$B - what$	$D - where$	$A - what$	
$C - who$	$C - who$	$C - who$	

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[Turn over

- 12 The pressure P and volume V of a fixed mass of gas are related by an equation of the form

$$PV^\alpha = k,$$

where k and α are constants.

From this equation obtain a linear equation, $y = mx + c$, where $x = \ln P$ and $y = \ln V$. [2]

In six experiments on the fixed mass of a gas, in each of which P was controlled and V measured, the results satisfied:

$$\sum x = 2.420, \quad \sum y = -1.708,$$

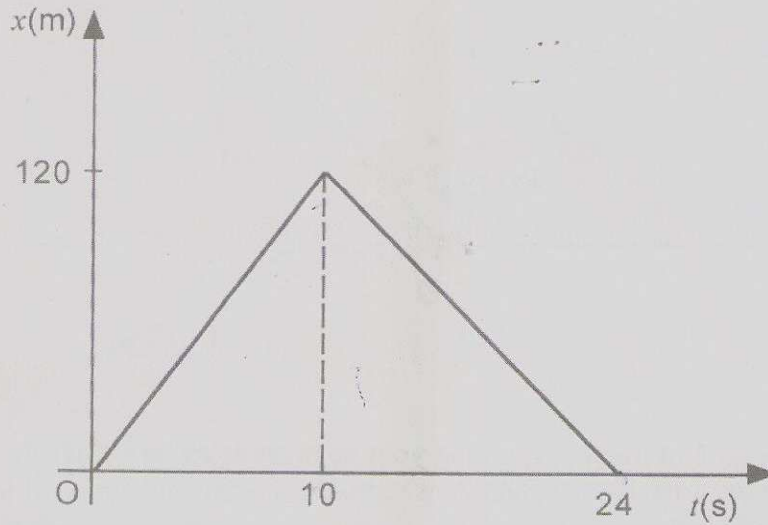
$$\sum x^2 = 3.171, \quad \sum y^2 = 1.561,$$

$$\sum xy = -2.224.$$

- (a) (i) Calculate the equation of the line of regression of y on x . [6]
(ii) Hence estimate, to 2 decimal places, the value of α . [2]
- (b) Calculate the value of V correct to 3 significant figures when $P = 0.75$. [2]

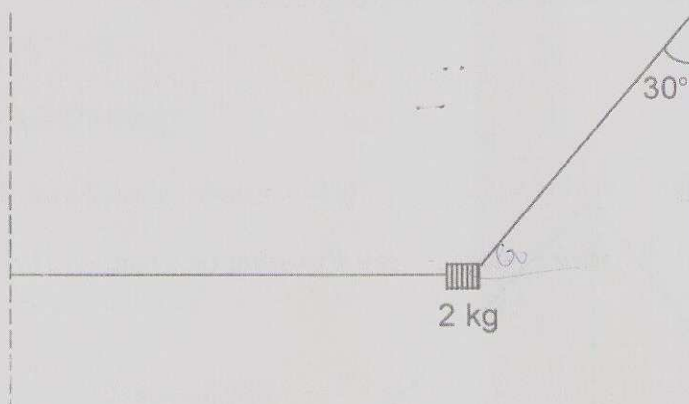
Section (B): Mechanics

- 13 An athlete's motion is represented by a (t, x) graph below.



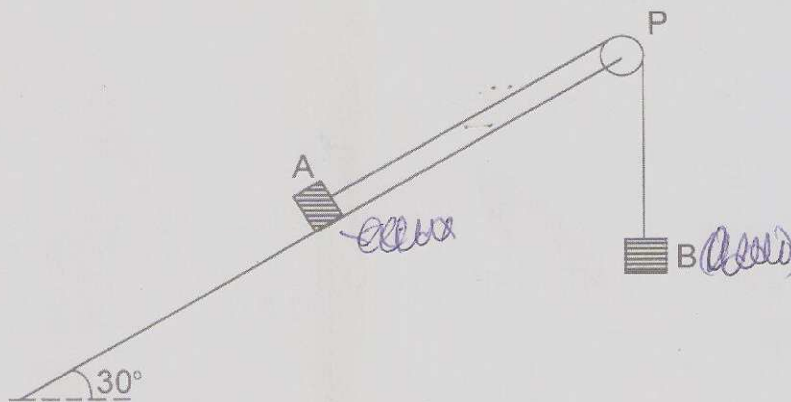
- (a) Sketch a (t, v) graph for the motion of the athlete during the 24 seconds. [3]
- (b) Calculate the average speed during the 24 seconds. [2]

- 14 A brick of mass 2 kg is held at rest by two ropes. One rope is horizontal and the other makes an angle of 30° with the vertical (see diagram).



- (i) Find the tension in each of the ropes, giving your answers as exact values in terms of g . [4]
- (ii) State the underlying assumptions made in answering (i). [1]

- 15 The diagram above shows a particle A of mass $4m$ on a smooth plane inclined at 30° to the horizontal.



A light inextensible string is attached to A and passes over a small smooth pulley P fixed at the top of the inclined plane. To the end of the string is attached a particle B of mass m which hangs freely.

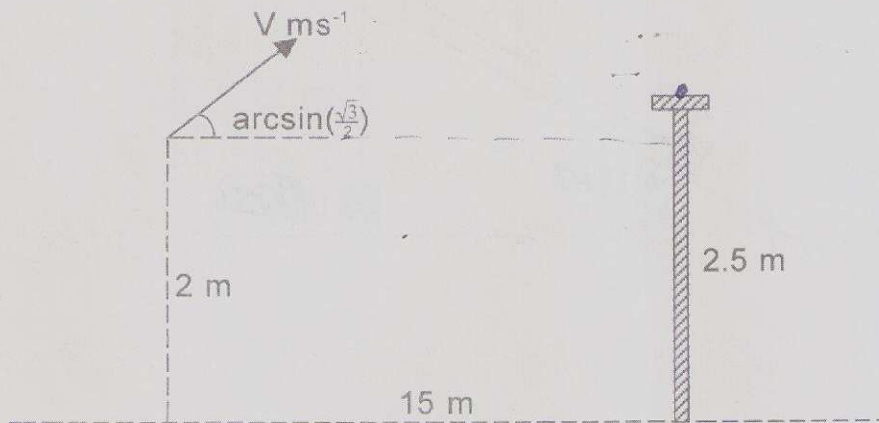
The particles are released from rest with the string taut and with the portion AP parallel to a line of greatest slope of the inclined plane.

Calculate, in terms of m and g ,

- (a) the acceleration of particle A down the plane, [4]
- (b) the magnitude of the tension in the string. [2]

16

A child throws a ball from a height 2 m above the ground level, aiming at a target. The target is on top of a vertical pole of height 2.5 m. The distance of the child from the pole is 15 m (see diagram).



The initial speed of the ball is $V \text{ ms}^{-1}$ at an angle of elevation $\arcsin\left(\frac{\sqrt{3}}{2}\right)$, and the ball moves freely under gravity.

Calculate

- (a) the value of V for which the ball hits the target, [4]
- (b) the direction of the ball as it hits the target. [4]

$$V^2 = u^2 - 2gh$$