



ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

MATHEMATICS
PAPER 4

9164/4

JUNE 2016 SESSION

3 hours

Additional materials:

Answer paper
Graph paper
List of Formulae
Electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81 \text{ ms}^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 8 printed pages.

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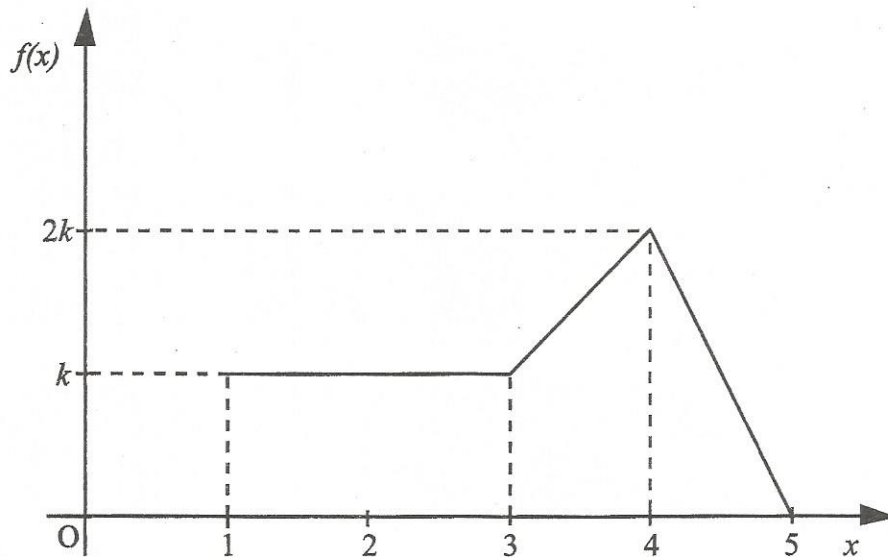
Section (A): Statistics

- 1 The diameters of washers produced by a particular machine follow a Normal distribution with a standard deviation of 0.1 mm.

Find the mean diameter if there is to be a probability of only 3% that the diameter exceeds 2.0 mm.

[4]

- 2 A continuous random variable X has a probability density function $f(x)$ given by $f(x) = 0$ for $x < 1$ and for $x > 5$. For x between 1 and 5, its form is shown on the graph.



- (a) Show that $k = \frac{2}{9}$. [2]
- (b) Construct the probability density function of X . [2]
- 3 Bag A contains 3 red balls and 2 white balls. Bag B contains 2 red balls and 3 white balls. A bag is selected at random and two balls are drawn from it, one after the other without replacement.
- (a) Find the probability that the two balls drawn are red. [2]
- (b) Given that the two balls drawn are red, find the probability that they are from bag A. [3]

- 4 An unbiased tetrahedral die has faces marked 1, 2, 3 and 4. In a game, a player tosses 2 tetrahedral dice. X is the sum of scores on the 2 dice.
- (a) Construct a probability distribution of X . [2]
- (b) In order to play the game, a player pays \$1. If the sum of scores is 2, 3 or 4 the player wins nothing. If the sum is 5, 6 or 7 the player wins \$2 and for a sum greater than 7, the player wins \$4.

Find the expected gain or loss from each game. [3]

- 5 The stem and leaf diagram shows the number of hectares owned by farmers around a small town.

Stem	Leaf								
0	1	4	7						
1	1	3	8	9					
2	0	1	2	4	7	8			
3	0	0	2	3	4	5	7	7	
4	2	3	5	7					

Key 1/3 means 13 hectares.

- (a) Find the
- (i) median,
- (ii) interquartile range. [4]
- (b) Hence draw a box and whisker plot to illustrate the above information. [2]
- 6 (a) A dozen loaves of bread were taken at random from a large batch and weighed. The masses were found to be 741; 701; 834; 829; 808; 660; 659; 739; 472; 865; 851 and 801 grammes.

Assuming that the masses of the loaves are from a Normal distribution, find the unbiased estimate of the population

- (i) mean
- (ii) variance. [3]
- (b) Another batch of 50 loaves gave a mean of 750 grammes.
- Given that the population variance of the loaves is 10 816, construct a 90% confidence interval for the mean mass of the loaves. [4]

- 7 The number of passengers arriving at a taxi rank per hour was found to have a Poisson distribution with mean 2.
- (a) Calculate the probability that in a particular hour there will be no passenger arriving. [2]
- (b) At the beginning of an hour there will be 4 taxis available for hire.
Calculate the probability that this will be an insufficient number for the hour assuming that each taxi allows only one passenger. [3]
- (c) Find the probability that there will be exactly 2 passengers arriving in 2 consecutive hours. [3]

- 8 (a) (i) State the conditions under which the Normal distribution can be used as an approximation to the Binomial distribution.
- (ii) A fair coin is tossed 100 times. By using a suitable approximation, calculate the probability that the number of heads obtained is less than 37. [7]
- (b) 2 000 such coins in (a) (ii) are tossed each 100 times. By using another suitable approximation, find the probability that less than 37 heads are obtained more than 3 times. [5]

- 9 The masses, in grams, of the contents and packaging of a randomly chosen packet of powdered milk of brand M may be taken to have a Normal distribution with mean and standard deviation given in the table.

	mean	standard deviation
contents	500	8
packaging	20	2

- (a) Find the probability that
- (i) a randomly chosen packet of brand M has a total mass exceeding 525 grammes.
- (ii) the total weight of the contents of three randomly chosen packets of brand M exceeds 1 515 grammes. [9]
- (b) The masses of the contents of a randomly chosen packet of another brand N may be taken to have a normal distribution with mean 405 grams and standard deviation 6 grams.
Find the probability that the contents of five randomly chosen packets of brand N weigh more than the contents of four randomly chosen packets of brand M . [5]

- 10 A taxi operator keeps records of the performance of his cars. The total distance travelled by a car since it was purchased as new is denoted by m and the distance it can travel with one litre of petrol is denoted by d . For 7 cars of the same make and model, the values of d against m for each car are shown in the table below.

m ($\times 1000$ km)	50	100	150	200	300	400	500
d (km)	23	22	20	20	17	16	12

- (a) (i) Draw a scatter plot for the data in the table.
(ii) Comment on the relationship between the total distance travelled and the distance travelled with one litre of petrol. [5]
- (b) Find the equation of the regression line of d on m for the data. [4]
- (c) Fit the regression line on the scatter diagram and use it to estimate the distance travelled per litre by a car which has travelled a total distance of 450 000 km. [5]
- 11 The maximum heights cleared by 100 male athletes who took part in a high jump competition are shown in the table below.

height, h , (cm)	frequency
$161 < h \leq 165$	4
$165 < h \leq 170$	18
$170 < h \leq 175$	37
$175 < h \leq 180$	26
$180 < h \leq 185$	10
$185 < h \leq 190$	5

- (a) Find the
- (i) mean,
(ii) variance.

- (iii) The expected frequencies for a Normal distribution having the same mean and variance as the data in (a) are given in the table.

height, h , (cm)	observed frequency	expected frequency
$161 < h \leq 165$	4	3.73
$165 < h \leq 170$	18	16.00
$170 < h \leq 175$	37	31.11
$175 < h \leq 180$	26	x
$180 < h \leq 185$	10	y
$185 < h \leq 190$	5	z

Calculate the expected frequencies x , y and z .

[7]

- (b) Test the goodness of fit using a 5% level of significance.

[10]

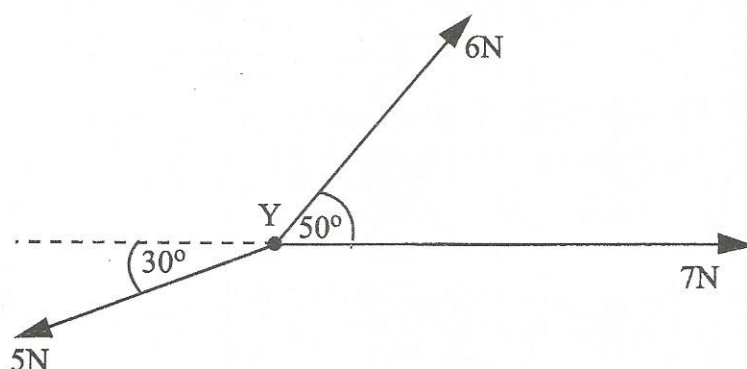
Section (B): Mechanics

- 12 The trajectory of a projectile's motion is described by the equation $y = 2x - 0.01x^2$ where x is the horizontal displacement and y is the vertical displacement from the point of projection.

Find the

- (i) angle of projection, [1]
 (ii) initial velocity of projection. [2]

13



The diagram shows three coplanar forces acting at point Y. The 7 N force is horizontal, the 6 N force is inclined to it at 50° and the 5 N force acts at an angle of 30° below the horizontal.

Find the magnitude and direction of the resultant of the three forces. [5]

- 14 A particle starts from rest and accelerates at 2 ms^{-2} for 3 seconds. It then maintains the attained velocity for 4 seconds and then decelerates at 5 ms^{-2} for 2 seconds.

- (a) Sketch a velocity-time graph for the motion of the particle for the 9 seconds. [4]
 (b) Calculate the
 (i) total distance travelled by the particle in the 9 seconds, [2]
 (ii) displacement of the particle at the end of the 9 seconds. [2]

- 15 Two identical small trays each of mass 0.2 kg, are connected by a light inextensible string which passes over a fixed smooth pulley, as shown in Fig. 15.1. The trays remain balanced.

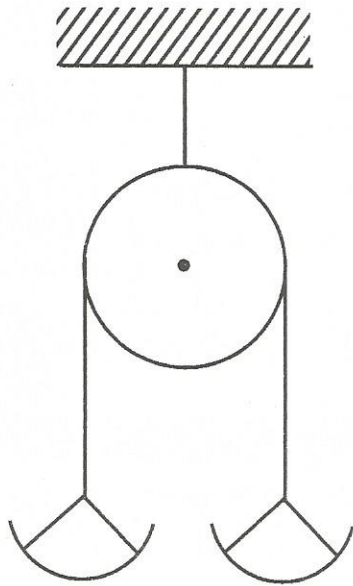


Fig. 15.1

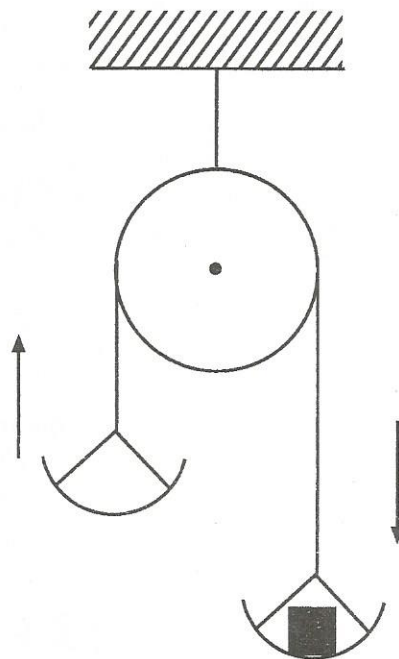


Fig. 15.2

A mass of 80 grammes is then placed on one of the trays, which begins to move downwards, see Fig 15.2.

Calculate the

- (i) acceleration of the trays and the tension in the string, [5]
- (ii) force exerted on the 80 gramme mass by the tray when in motion. [3]