1 A triangle ABC is such that $AB = 8\sqrt{3}$ cm, $AC = 16$ cm and $B\hat{A}C = 30°$.

(i) Calculate the length BC. [2]

(ii) 1. Calculate the angle ACB.

2. Hence or otherwise find the radius of the circle passing through A, B and C. [3]

2 (i) The expansion of $(1+ax)^n$ up to and including the term in x^2 is $1-6x+\frac{81}{4}x^2$.

Find the values of a and n. [5]

(ii) Hence state the values of x for which the expansion is valid. [1]

3 (i) Show that the equation $x^3 = e^{3\sin x}$ has a root between $x = 2$ and $x = 2.5$. [3]

(ii) Starting with $x_1 = 2$ as the first approximation, use the Newton – Raphson method **once** to estimate the root to two decimal places. [3]

4 (i) Express the function $2\cos 2x + 2\sqrt{3}\sin 2x$ in the form $R\cos(2x-\alpha)$, where $R > 0$ and $0 < \alpha < 90°$. [2]

(ii) Hence or otherwise, solve the equation $\cos 2x + \sqrt{3}\sin 2x = \sqrt{2}$ for $0 \le x \le 360°$. [4]

5 (a) Given that $f(x) = -x^3 + 2x^2 + 3x - 6$,

(i) factorise $f(x)$ completely,

(ii) sketch the curve $y = f(x)$ showing all the intersections with the axes. [6]

[You need not find the turning points]

(b) Hence or otherwise, write down the solution of the inequality $f(x) > 0$. [2]

6 The points A, B and C have position vectors $a = 3i - pj - k$, $b = 5i + 2j - k$ and $c = 7i + \left(2 + \sqrt{5}\right)j - k$ respectively.

(i) Find the exact value of p if

1. $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$,

2. $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{BC}$. [5]

(ii) 1. Find the unit vector in the direction of $\overrightarrow{BC}$.

2. Hence write down a vector parallel to $\overrightarrow{BC}$ with modulus 15. [3]

7 The complex number $w = 3 - 4i$ and u is such that $\frac{w}{u} = \frac{2}{13} + \frac{3}{13}i$.

(a) Find

(i) u in the form $x + iy$,

(ii) 1. $|u|$,

2. $\arg u$. [7]

(b) Sketch u on an argand diagram showing clearly the $|u|$ and $\arg u$. [2]

8 A curve has parametric equations $y = 1 + \cos\left(\frac{\pi}{3}e^{3\theta}\right)$ and $x = 2 - \sin\left(\frac{\pi}{3}e^{3\theta}\right)$ for $0 \le \theta \le 2\pi$.

Find

(i) $\frac{dy}{dx}$ in terms of θ in its simplest form and state the exact value of

$\frac{dy}{dx}$ when $\theta = 0$, [4]

(ii) the cartesian equation of the curve and describe fully what it represents geometrically. [5]

9 The function $f(x) = \sin\left(x - \frac{\pi}{4}\right)$ for $0 \leq x \leq 2\pi$.

(i) Sketch on separate diagrams, showing clearly the intercepts with the x-axis (if any), the graphs of

1. $y = f(x)$,

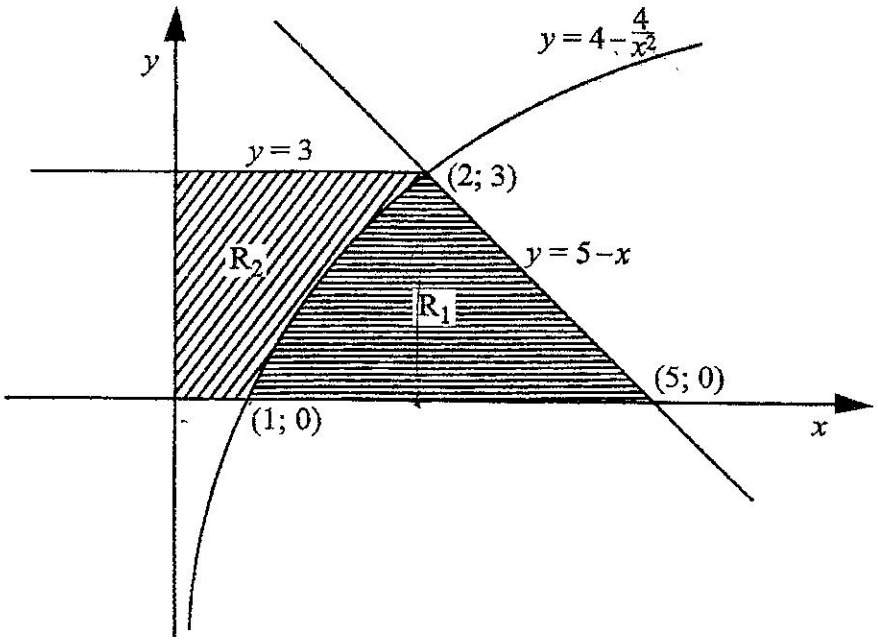
2. $y = f(2x)$,

3. $y = 2 + f\left(x + \frac{\pi}{4}\right)$.

[5]

(ii) Describe fully the geometric transformations which the graph of $y = f(x)$ undergoes to obtain the graphs in part **(i)** 2 and part **(i)** 3. [4]

10



In the diagram, the area, R_1, is bounded by the curve $y = 4 - \frac{4}{x^2}$, the x-axis and the line $y = 5 - x$.

The area, R_2, is bounded by the curve $y = 4 - \frac{4}{x^2}$, the x-axis, the y-axis and the line $y = 3$.

The line $y = 5 - x$ crosses the curve at (2; 3) and x–axis at (5; 0). The curve crosses the x-axis at (1; 0).

Calculate

(i) the area R_1, [5]

(ii) the exact volume generated, when the area, R_2, is rotated about the y-axis through 360°. [5]

11 The rate of increase of mass, m kg, of a particular plant is inversely proportional to $(t + 3)$, where t is the time in years.

(i) Write a differential equation to represent this information. [1]

(ii) Solve the differential equation, given that when $t = 0$, $m = ln9$ and when $t = 24$, $m = 3\ ln9$. [5]

(iii) Find

1. the value of m when $t = 100$ years,
2. the time taken for the plant to increase by 10 kg.

[4]

12 (a) Sketch the graph of the function $y = -ln\, x$. [1]

(b) Use the trapezium rule with 3 ordinates to estimate the area bounded by the curve $y = -ln\, x$, the lines $x = 2$ and $x = 3$, giving your answer correct to four decimal places. [3]

(c) (i) Evaluate $\int_2^3 -ln\, x\, dx$, correct to four decimal places.

(ii) 1. Hence calculate the percentage error in using the trapezium rule to estimate the area.

2. Explain why the trapezium rule gives an underestimate of the area.

[7]

13 (a) The 4th term of an arithmetic progression is 42 and the sum of the first three terms of the series is 12.

Find the

(i) first term and the common difference,

(ii) sum of the first twenty terms.

[6]

(b) The 3rd term of a geometric progression is 36 and the 5th term is 16.

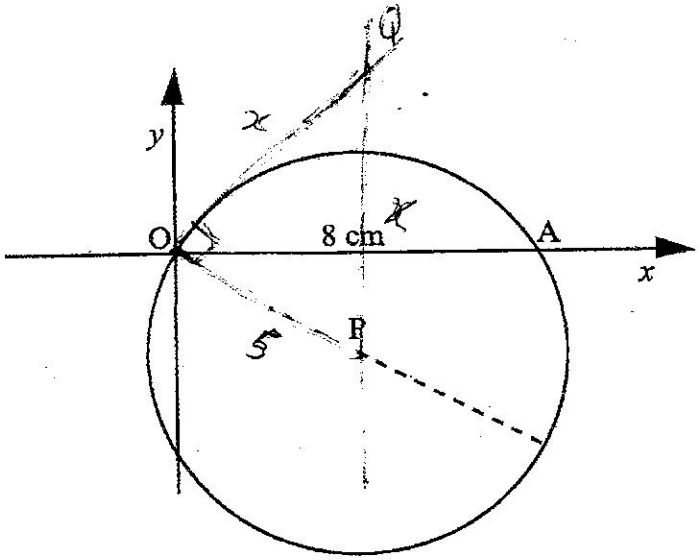
Find

(i) the first term and the common ratio, r, given that $r < 0$,

(ii) the sum to infinity of the series.

[5]

14



The diagram shows a circle of radius 5 cm and centre P. The chord OA on the x-axis is 8 cm long.

(a) Find the

(i) coordinates of P,

(ii) equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants to be found.

(iii) equation of the tangent to the circle at O. [7]

(b) Given that the tangent at O cuts the vertical line through P at Q, calculate the area of triangle OPQ. [5]