KAMBUZUMA 1 HIGH SCHOOL

MATHEMATICS DEPARTMENT

ADVANCED LEVEL MATHEMATICS PAPER 1

MID-YEAR EXAMS JULY 2013

Additional materials:

- Answer paper
- Graph paper
- List of formulae

TIME: 3 Hours

INSTRUCTIONS TO CANDIDATES

- Write your name and class in the spaces provided on the answer paper/ answer booklet.
- There is no restriction on the number of questions which you may attempt.
- If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers

This question paper consists of 6 printed pages and 2 blank pages.

1 Solve the simultaneous equations

$$2^x + 3^y = 5$$
$$2^{x+2} - 3^{y+1} = 13.$$
[4]

2 Solve the inequality $|2x+1| < 3x+2$. [4]

3 The variable quantities x and y are related by the equation $y = ab^x$ where a and b are constants. When a graph is plotted showing values of $\ln y$ on the vertical axis and values of x on the horizontal axis, the points lie on a straight line having gradient 0.8 and y intercept at 3.3.

Find the values of a and b correct to one decimal place. [4]

4 If 1% error is made in measuring the diameter of a sphere, find approximately the resulting percentage error in

(i) the volume, [2]

(ii) the surface area of the sphere. [2]

$$\left[\begin{array}{l} \textit{volume} \text{ of a sphere } = \frac{4}{3}\pi r^3 \\ \text{surface area of a sphere } = 4\pi r^2 \end{array}\right]$$

5 When a polynomial $f(x) = x^3 + ax^2 + bx + c$ is divided by $x^2 - 4$ the remainder is $2x + 11$. If $x + 1$ is a factor of $f(x)$, find the values of a, b and c. [5]

6 (i) Express $\dfrac{4x+6}{(x+2)(x+1)(x+3)}$ in partial fractions. [3]

(ii) By using series expansion up to and including the term in x^2 show that $\dfrac{4x+6}{(x+2)(x+1)(x+3)}$ can be reduced to $1 - \dfrac{7}{6}x + \dfrac{41}{36}x^2$. [4]

7 Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$ where α is an acute angle and $R > 0$.

Hence, or otherwise

(a) find the solution of the equation

$4\sin\theta - 3\cos\theta = 3$ for $0° \leq \theta \leq 360°$, [5]

(b) determine the value of θ for which $4\sin\theta - 3\cos\theta$ is

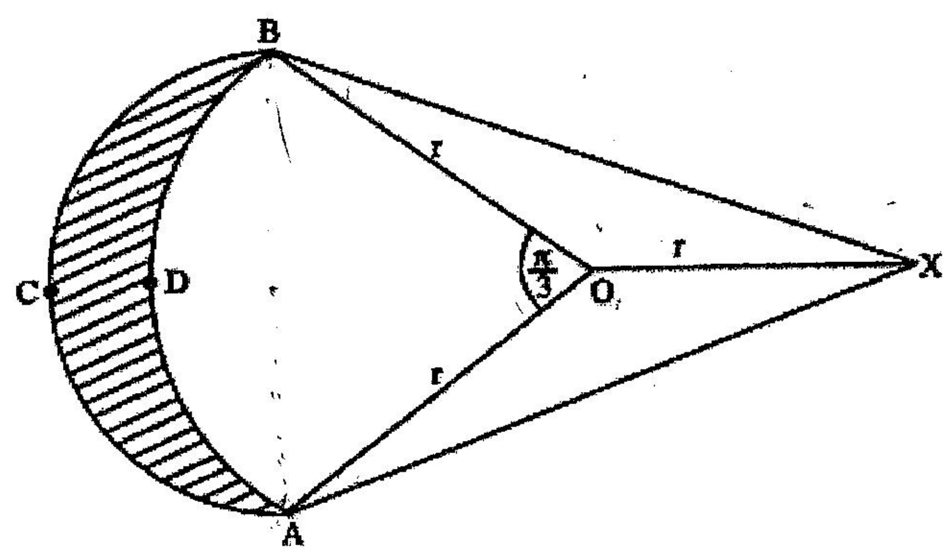
1. maximum,
2. minimum. [2]

8 A curve has parametric equations $x = 1 + t^2$ and $y = 4t - t^3$ where $t > 0$.

Find $\frac{dy}{dx}$ in terms of t, hence show that the coordinates of the turning point are

$\left(\frac{7}{3};\ \frac{16}{3\sqrt{3}}\right)$. [7]

9



The diagram above shows a school logo. The angle AOB = $\frac{\pi}{3}$ radians. O is the centre of the sector OACB and X is the centre of sector XADB
OX = OA = OB = r cm.

(i) Show that angle AXB = $\frac{\pi}{6}$. [2]

(ii) Calculate the exact area of the shaded region in terms of r. [6]

10 The position vectors of points A, B and C relative to the origin O, are $4\mathbf{i}-9\mathbf{j}-\mathbf{k}$; $\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ and $\lambda\mathbf{i}-\mathbf{j}+3\mathbf{k}$ respectively.

(a) (i) Find the unit vector parallel to $\overrightarrow{AB}$.

(ii) Find the value of λ such that A, B and C are collinear. [5]

(b) Calculate the angle AOB. [3]

11 Given that $p = 5 + i$ and $q = -2+3i$,

(a) (i) show the complex numbers ip and $p+q$ on an argand diagram,

(ii) describe the geometrical transformation which maps ip onto p. [3]

(b) Find

(i) the modulus and argument of p,

(ii) pq,

(iii) $\frac{p}{q}$. [5]

12 In a chemical reaction in which a compound P is formed from a compound Q, the masses of P and Q present at time t, are x and y respectively. The sum of the masses x and y is 10 and at any time the rate at which x is increasing is proportional to the product of the two masses at that time.

(i) Show that $\frac{dx}{dt} = kx(10-x)$ where k is a constant. [2]

(ii) Find the general solution of this differential equation. [2]

Hence find t correct to 3 significant figures when $x = 9.9$, given that $x = 2$ when $t = 0$ and $x = 5$ when $t = \ln 2$. [4]

13 The curve C has parametric equations $y = at$, $x = \frac{a}{t}$ where $t > 0$.

(i) Write down the cartesian equation of C in terms of a. [2]

(ii) Given that point P lies on C, show that the equation of the normal to C at P when $t = 2$, is $8y = 2x+15a$. [4]

(iii) This normal meets C again at point $Q\left(-8a; \frac{-a}{8}\right)$. Given that PQ is the diameter of the circle, show that the equation of this circle is

$$\left(x+\frac{15a}{4}\right)^2+\left(y-\frac{15a}{16}\right)^2=\frac{4913a^2}{256}.$$ [4]

14 (i) Sketch the graph of the function $y = \frac{x}{2-x}$ in the first quadrant. [2]

(i) The area A in the first quadrant bounded by the curve $y = \frac{x}{2-x}$, the line $x = 1$ and the x-axis is rotated completely about the x-axis to form a solid of volume V.

Find in exact form

1. the area A,

2. the volume V. [9]

15 (a) Use trapezium rule with 4 ordinates to find an approximate value of $\int_0^3 xe^{-x}dx$. [4]

(b) Sketch on the same axes the graphs of $f(x) = e^{x/2}$ and $g(x) = x + 2$. [1]

(c) From **(b)**,

1. state the number of roots of the equation $e^{x/2} = x + 2$.

2. show by calculation that one of the roots lies between 3 and 4.

3. taking 4 as the first approximation to one of the roots of this equation, use Newton Raphson method to find the second approximation to the root giving your answer correct to 3 significant figures. [7]

16 (a) The sum of the first 5 terms of a geometric series is 5 and the sum of the fifth to the ninth terms is 80.

Show that the common ratio is ±2 and hence find two possible values of a. [6]

(b) (i) Express $4x^3 - 6x^2 + 2x$ in the form $Ax(x+1)(x+2) + Bx(x+1) + Cx$ where A, B and C are constants. [3]

(ii) Given that $\sum_{x=1}^{x=n} x(x+1)(x+2) = \frac{n}{4}(n+1)(n+2)(n+3)$ and

$\sum_{x=1}^{x=n} x(x+1) = \frac{n}{3}(n+1)(n+2)$, show that

$$\sum_{x=1}^{x=n} 4x^3 - 6x^2 + 2x = n^2(n^2 - 1).$$ [4]