ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS 9164/1

PAPER 1

JUNE 2012 SESSION 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

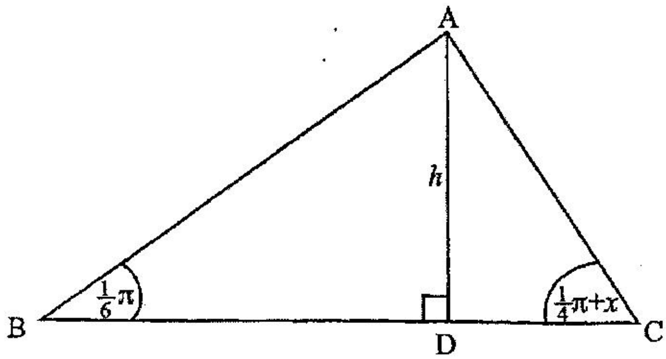
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

1 Given that $-4 < x < 2$ is the solution of the inequality $|x+a| < b$, calculate the value of a and the value of b. [3]

2 Use the trapezium rule with 4 equal intervals to estimate the value $\int_0^{0.2} \frac{\cos x^2}{e^{x^2}} dx$ giving your answer correct to 4 decimal places. [4]

3



The diagram shows a triangle ABC with height h units, angle ABD $= \frac{1}{6}\pi$ and angle ACD $= \frac{1}{4}\pi + x$. Given that x is sufficiently small for x^2 and higher powers of x to be neglected, show that

$BC = h\left[1 + \sqrt{3} - 2x + 2x^2\right]$. [5]

4 If $x^3y^2 = 1$, find the approximate percentage decrease in y given that x increases by 0.5%. [5]

5 Express $\frac{x^4}{x^4 - 1}$ in partial fractions. [5]

6 If the expansion of $(1+x)^{\frac{1}{3}}$ and of $\frac{1+px}{1+qx}$ in ascending powers of x are identical up to and including the term in x^2, calculate the value of p and the value of q. [7]

7 (i) An arithmetic progression has 10 terms. It has first term a and common difference d where $d > 0$.

Given that its first, second and fifth terms are consecutive terms of a geometric progression, find a in terms of d. [3]

(ii) Given further that the difference between the first and the last term is 36, find the sum of the terms. [4]

8 The complex number $w = \frac{4+3i}{3-2i}$.

(a) Express w in the form $x + iy$ where x and y are real. [2]

(b) Find

(i) modulus of w,

(ii) argument of w. [5]

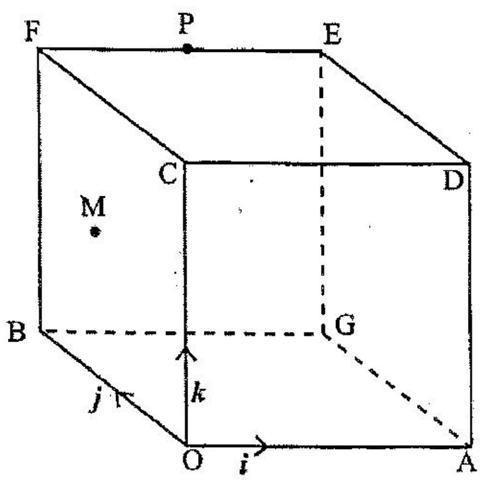
9 Find the equation of the circle which passes through the origin and touches the straight line $4x - 3y = 5$ at a point (2; 1). [7]

9 Solve the following equations

(a) $3(7^x) - 2(7^{-x}) = 5$, [5]

(b) $\log_{10}(2x^3 + 1) - 3\log_{10} x = 1$. [3]

11



The diagram shows a cube of length 8 units. The unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively. M is the point of intersection of $\overrightarrow{OF}$ and $\overrightarrow{BC}$. P is the midpoint of $\overrightarrow{EF}$.

Find

(i) a unit vector parallel to $\overrightarrow{MP}$, [4]

(ii) angle DMP. [4]

[Turn over

12 (i) Prove that $\operatorname{cosec} 2x - \cot 2x \equiv \tan x$. [3]

(ii) Hence or otherwise, solve the equation $\operatorname{cosec} 2x - \cot 2x + 5 \equiv 3\sec^2 x$ for $0 < x \leq 360^\circ$. [5]

13 An open rectangular box has length $\frac{32}{x^2}$ cm and width x cm.

(i) Given that its volume is $128\ \text{cm}^3$,

find in terms of x, the depth, d cm, and the total surface area, $A\ \text{cm}^2$, of the box. [4]

(ii) Find the dimensions of the box correct to 3 decimal places such that the surface area is a minimum. Justify that the surface area is a minimum. [6]

14 (a) Find the coordinates of the turning points on the graph of $y = x^3 - 12x - 12$. Determine the nature of the turning points. Hence, sketch the graph of y against x. [6]

(b) Use your graph to write down the set of values of k for which the equation $x^3 - 12x - 12 - k = 0$ has more than one real root. [1]

(c) Taking 3,9 as the first approximation to the root of the equation $x^3 - 12x - 12 = 0$, apply Newton Raphson method to approximate the root correct to 3 decimal places. [4]

15 (a) Express $9x - x^2$ in the form $a + b(x + c)^2$. Hence or otherwise, sketch the graph of $y = 9x - x^2$ stating the coordinates of the turning point. [4]

(b) The region R is bounded by the curve $9x - x^2$ and the x axis. Calculate

(i) the area of region R,

(ii) the volume generated when R is rotated through four right angles about the x axis. [8]

16 (a) Solve the differential equation $(4 - x)\frac{dy}{dx} = y$, given that $y = 4$ when $x=1$, expressing y in terms of x. [5]

(b) A liquid is kept in a cold room. It is assumed that the rate of decrease in temperature of the liquid, is proportional to θ where θ°C is the temperature of the liquid at time t minutes . When $t = 0$, $\theta = 60^\circ$ and after 5 minutes $\theta = 40^\circ$C.

(i) Form a differential equation relating to θ and t.

(ii) Solve the differential equation and find the temperature of the liquid after a further 5 minutes. [8]