ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

MATHEMATICS
PAPER 1

9164/1

JUNE 2010 SESSION

3 hours

Additional materials:
  Answer paper
  Graph paper
  List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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1. Given that $\cos\theta = -\frac{1}{5}$ and $-180° < \theta < -90°$, find the exact value of $\cot\theta$. [3]
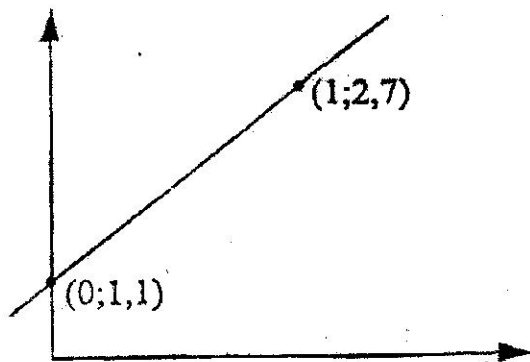
2. Differentiate with respect to t;

   (i) $e^{-2t}\sin t$,

   (ii) $\sec^2(3t - 100)$. [4]

3. Solve the inequality $|3x + 1| \geq 2|x - 2|$. [4]

4. Express $\dfrac{3x + 8}{(2x + 1)(x^2 + 3)}$ in partial fractions. [4]

5. A mathematician working with an exponential relation $y = ab^x$ reduced it to linear form and came out with the graph shown in the diagram below.

   (i) State the label on each of the axes [2]

   (ii) Calculate the value of a and the value of b. [3]

6. Find the value of a for which $(x - 2)$ is a factor of $3x^3 + ax^2 + x - 2$. [2]

   Show that for this value of a, the cubic equation $3x^3 + ax^2 + x - 2 = 0$ has only one real root. [3]

7. Write down the equation of a circle with centre $(-3; 2)$ and radius $\sqrt{10}$. [1]

   Show that the point A $(-2; -1)$ lies on the circle, and find the coordinates of B, the other end of the diameter through A. [4]

8. Solve $\dfrac{dy}{dx} = xy$ given $x = 0$ when $y = 1$.

   Use the series expansion for e^x to write down the first two terms of Maclaurin series for the solution. [5]

Given that $x = \sin^2 t$ and $y = \cos 2t$, find $\dfrac{dy}{dx}$ in its simplest form. [4]

Hence or otherwise describe the shape of the graph of y against x. [1]

10   The complex number $z_1 = 1 - 2i$ and the complex number z_2 is such that

$$z_1 z_2 = -10i.$$

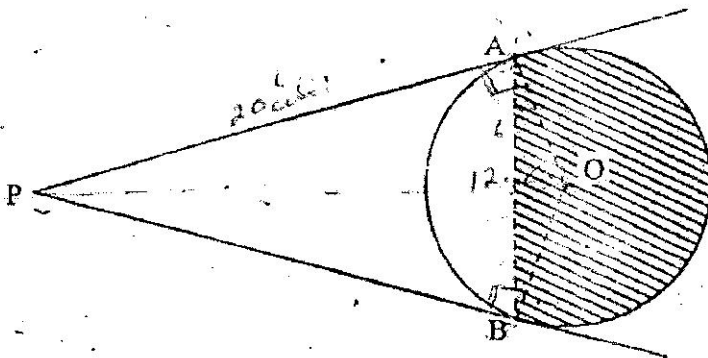Find z_2 in the form $a + ib$ and sketch it on an Argand diagram. [5]

11   (a)   Find the term independent of x in the expansion of

$$\left(x^2 + \frac{3}{x}\right)^6.$$ [2]

(b)   Find the series expansion of $\left(4 + x^2\right)^{\frac{1}{2}}$ up to and including the term in x^6. [4]

12   Use the substitution $x = a\sin\theta$ to evaluate $\displaystyle\int_0^a \sqrt{a^2 - x^2}\,dx$. [6]

13   The diagram shows a circle centre O and two tangents AP and BP drawn from a point P.

Given that AP = 20 cm and AB = 12 cm,

(i)   show that the obtuse angle AOB = 2,532 radians, [3]

(ii)   calculate the radius of the circle, [2]

(iii)   calculate the area of the shaded segment AB. [3]

14 The tangent at point $P\left(x_n; x_n^2 - 2\right)$ to the curve $y = x^2 - 2$ meets the x-axis at point $Q\left(x_{n+1}; 0\right)$.

If $x_n > 0$, show that $x_{n+1} = \dfrac{x_n^2 + 2}{2x_n}$ [4]

Starting with $x_1 = 2$, use this iterative formula to find successive approximations x_2, x_3 for the positive root of the equation $x^2 - 2 = 0$.

Hence show that $x_4 = \dfrac{577}{408}$. [3]

Find the error, to 1 significant figure, in using $\dfrac{577}{408}$ as an approximation to $\sqrt{2}$. [2]

15 The function f is defined as

$$f : x \to \frac{2-x}{x+1}, \quad x \neq -1.$$

(a) (i) Express $f(x)$ in the form $a + \dfrac{b}{x+1}$, where a and b are constants. [2]

(ii) Hence, give a sequence of three transformations which take the graph of $y = \dfrac{1}{x}$ onto the graph of $y = f(x)$. [3]

(iii) State the range of f. [1]

(b) (i) Form the composite function $f\left(f(x)\right)$. [2]

(ii) Hence, or otherwise, obtain an expression for $f^{-1}(x)$. [1]

16. (a) An established insurance company is planning to launch a new policy package with initial expected sales of 2000 policies by the end of the first year. Policies are expected to increase by 600 per year. Assuming success for such a programme, find the year after starting in which more than 6000 policies are sold. [4]

(b) Mr Moyo bought a new car for seven hundred million dollars and the car depreciates at 10% per year. Find the number of years the car can be used before its resale value is less than five hundred million dollars. [6]

17 The curve C has equation $y^2 - 4y = 4x - 12$.

    (a) Find the coordinates of point A at which C meets the x-axis. [2]

    (b) By completing the square or otherwise, show that the equation of the curve [2]
         can be written in the form $(y - 2)^2 = 4(x - 2)$

    (c) Show that a point Q with coordinates $(2 + t^2; 2 + 2t)$, where t is any real [2]
         number, lies on C.

    (d) Find the equation of the tangent at A. [6]

18 (a) Use the trapezium rule with four equally spaced ordinates, to estimate the [3]
         value of $\int_0^{\frac{\pi}{6}} \sin^4 x \, dx$, giving your answer to 3 significant figures.

    (b) Show that $\sin^4 x = \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$. [4]

         Hence, or otherwise,

         (i) evaluate exactly $\int_0^{\frac{\pi}{6}} \sin^4 x \, dx$, [4]

         (ii) solve the equation $\cos 4x - 4\cos 2x + 1 = 0$ for $0 \le x \le 2\pi$. [4]