‘O’ LEVEL MATHS

QUESTIONS AND ANSWERS

VOLUME 1

TURN- UP COLLEGE

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Foreword

I had the opportunity of discussing this book with several educationists, teachers and students when it was in the process of making, and I felt at once that it was likely to prove unusually useful. It gathers together a great deal of information which must otherwise be delved for in many books and all this is arranged judiciously and on practical lines. The authors' outlook might be described as one of liberal commonsense clarity, simplicity of expression, and examination - skills - focused. Our study packs are there to offer a canvas for Zimbabweans to showcase their best ideas to help transform the country into a knowledge- based society where citizens are free to express their creativity, knowledge and ingenuity. We have set challenging objectives, but we believe that only by striving to achieve the highest, can we elevate ourselves above the elements which tend to hold our country back. However, if your see anything where you feel we may have failed to deliver, and where we may have failed on issues such as content, depth, relevance and usability, please let us know by using the contact numbers (09) 61226/61247, 0773 247 358; or Box 2759 Byo; email at turnupcollege@yahoo.com. We are here to listen and improve.

In my days as a teacher and as a student I should have welcomed this book warmly because:

(i) It approaches the syllabus wholistically
(ii) It uses simplified expression
(iii) It has an in-depth coverage of content
(iv) It provides examination skills at the earliest stage of studying
(v) It provides local, international and commonplace examples; illustrations and case studies.
(vi) It provides intelligent questions and answers of the examination type on a chapter by chapter basis
(vii) Last but not least, it provides a clear platform for self-evaluation as one prepares for the final examinations.

I have no doubt that learners and educators would as well find this book to be the best. It is certainly a manual for success. Every one would find it worthy to have his own copy. I should not be surprised if the Turn-up College Study Pack became the best resource in school and out of school.

Sam Madzingira
Director

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PREFACE

The Turn-up College Mathematics O-Level questions and answers study pack has been designed to primarily complement the content edition. Its main objective is to provide questions that give a student of O-Level Mathematics adequate revision for all topics covered by the syllabus. Raw memorizing of answers is not advised but students and educational practitioners should use this as an operational framework for enhanced mastery of content and examination skill practice. This book is therefore a worthwhile addition to the multiplicity to Mathematics materials already on the market.

CHAPTER 1

Arithmetic

Question 1

1. Simplify the following

(a) $\frac{1}{2}-\frac{3}{4}+\frac{2}{5}$

(b) $2\frac{3}{4}\times\frac{2}{3}\div\frac{8}{12}$

c) $\left(\frac{2}{3}of\,1\frac{2}{7}\right)\div\left(\frac{2}{5}of\,3\frac{1}{3}\right)$

Solution

NB: When two or more operations are given in a problem, use

B - Brackets
O - Of
D - Division
M - Multiplication
A - Addition
S - Subtraction

M-Multiplication

(a) $\frac{1}{2}-\frac{3}{4}+\frac{2}{5} = \frac{1}{2}+\frac{2}{5}-\frac{3}{4}$

$= \frac{10+8-15}{20}$

$= \frac{18-15}{20}$

$= \frac{3}{20}$

b) $2\frac{3}{4}\times\frac{2}{3}\div\frac{8}{12}$

$= \frac{11}{4}\times\frac{2}{3}\times\frac{12}{8}$

$= \frac{11}{4}$

$= 2\frac{3}{4}$

c) $\left(\frac{2}{3}of\,1\frac{2}{7}\right)\div\left(\frac{2}{5}of\,3\frac{1}{3}\right)$

$= \left(\frac{2}{3}\times\frac{9}{7}\right)\div\left(\frac{2}{5}\times\frac{10}{3}\right)$

$= \frac{6}{7}\div\frac{4}{3}$

$= \frac{6}{7}\times\frac{3}{4}=\frac{18}{28}=\frac{9}{14}$

Question 2

a) Express 0,475 as a fraction in its lowest terms

b) Express $\frac{2}{3},\frac{4}{7}$ and $\frac{5}{9}$ as decimals, correct to 2d.p and list the fractions in order of size with the smallest first.

Solution

a) $0{,}475 = \frac{475}{1000}$

$= \frac{95}{200}$

$= \frac{19}{40}$

b) $\frac{2}{3}$ = 0, 666 = 0,67

$\frac{4}{7}$ = 0,571 = 0,57

$\frac{5}{9}$ = 0,555 = 0,56

Ascending order: $\frac{5}{9},\frac{4}{7},\frac{2}{3}$

Question 3

A novel was 504 pages and Linda had completed reading 308 of them. What fraction of the novel was left?

Solution

Pages left:
504-308 =196

$= \frac{196}{504}$

$= \frac{49}{126}$

$= \underline{\underline{\frac{7}{18}}}$

Question 4:
Simplify the following:

a) $\left(3\frac{4}{7}-1\frac{1}{2}\right)\div\left(5\frac{3}{4}+2\frac{6}{7}\right)$

b) $\dfrac{1\frac{1}{5}+\frac{1}{4}}{3\frac{1}{6}-2\frac{1}{3}}$

Solution

a) $\left(3\frac{4}{7}-1\frac{1}{2}\right)\div\left(5\frac{3}{4}+2\frac{6}{7}\right)$

$=\left(\frac{25}{7}-\frac{3}{2}\right)\div\left(\frac{23}{4}+\frac{20}{7}\right)$

$=\left(\frac{50-21}{14}\right)\div\left(\frac{161+80}{28}\right)$

$=\frac{29}{14}\div\frac{241}{28}$

$=\frac{29}{14}\times\frac{28}{241}$

$=\underline{\underline{\frac{58}{241}}}$

b) $\dfrac{1\frac{1}{5}+\frac{1}{4}}{3\frac{1}{6}-2\frac{1}{3}} = \dfrac{\frac{6}{5}+\frac{1}{4}}{\frac{19}{6}-\frac{7}{3}}$

$= \frac{29}{20}\div\frac{5}{6}$

$= \frac{29}{20}\times\frac{6}{5}$

$= \frac{87}{50}$

$= \underline{\underline{1\frac{37}{50}}}$

Question 5

a) How many pieces of wood each $6\frac{2}{3}$ m long can be cut from a log, measuring $46\frac{2}{3}$ m long?

b) Find $\frac{1}{8}$ of 7,24km giving your answer in metres.

Solution

(a) $$\frac{46\frac{2}{3}}{6\frac{2}{3}} = \frac{140}{3} \div \frac{20}{3}$$

$$= \frac{140}{3} \times \frac{3}{20}$$

=7 pieces

b) $\frac{1}{8}$ of 7,24km = $\frac{1}{8} \times 7,24$

= 0,905

= **905m**

Question 6

Calculate the value of the following

a) $\frac{3}{8}$ of \$6

b) $\frac{2}{3}$ of 3,93m

c) $\frac{5}{12}$ of 3hrs 36min in hours and minutes

Solutions

a) \$6

$$\frac{3}{\cancel{8}_{4}} \times \$\cancel{6}^{3}$$

$$= \frac{9}{4}$$

=\$2, 25

b) $\frac{2}{3}$ of 3,93m

$$= \frac{2}{3} \times 3.93$$

$$= 2.62m$$

c) $\frac{5}{12}$ of 3hrs 36min 3hrs 36mins = $(3 \times 60 + 36) = 96$ mins

$$= \frac{5}{12} \times 96 \text{ mins}$$

= 60mins

= 1hr 30mins

Question 7

A college has an enrolment figure of 645 students $\frac{7}{12}$ of them are females. How many males are there?

Solution

$$\frac{7}{12} \times 645$$

= 301

= 645-301

There are 344 males.

Question 8

Simplify

(a) 0,34-5,2 + 62,7

(b) 0,22 +3,21 × 5,2

(c) 0,65 :- 13 × 0,02

Solution

a) 0,34 :- 5,2 + 62,7

0,34-5,2 + 62,7

0,34 + 62, 7-5,2

63,04-5,2

57,84

b) 0,22 + 3,21 × 5,2

0,22 + (3,21 × 5,2)

= 0,22 + 16, 692

= 16, 912

c) (0,65 :- 13) × 0,02

0,05 × 0,02

0,001

Question 9:

Change the following fractions into decimals

a) $\frac{13}{20}$ b) $\frac{1}{8}$ c) $\frac{11}{50}$

d) $\frac{3}{40}$

a) $\frac{13}{20}$ = 0,65 b) $\frac{1}{8}$ = 0,125

c) $\frac{11}{50}$ = 0,22

50

d) $\frac{3}{40} =$ 0,075

Question 10

Change the following decimals into fractions giving your answer in its lowest terms.

a) 0, 35 b) 0, 15 c) 0, 26 d) 0,0025

Solution

a) 0, 35 $= \frac{35}{100}$ $= \frac{7}{20}$

b) 0,15 $= \frac{\overset{3}{\cancel{15}}}{\underset{20}{\cancel{100}}}$ $\frac{3}{20}$

c) 0,26 $= \frac{\overset{13}{\cancel{26}}}{\underset{50}{\cancel{100}}}$ $= \frac{13}{50}$

d) 0,0025 $= \frac{\overset{5}{\cancel{25}}}{\underset{2000}{\cancel{10000}}}$ $= \frac{5}{200}$ $\frac{1}{400}$

Question 11

Evaluate the Following

a) 8,5 ÷ 5
b) 34 ÷0,04
c) 20÷0,02
d) 0,042 ÷ 0,7
e) 0,125÷0,025

Solution

a) $\frac{8,5\times10}{5\times10} = \frac{85}{100} = 0,85$

b) $\frac{34}{0,04} \times 100 = \frac{3400}{4}$

$\frac{\overset{850}{\cancel{3400}}}{4}$

$= \underline{850}$

c) $\frac{20\times100}{0.02\times100} = \frac{200}{2} = 1000$

d) $\frac{0,042\times1000}{0,7\times1000} = \frac{42}{700}$

$= \frac{6}{100}$

$= \underline{0,06}$

e) $\frac{0,125\times1000}{0,025\times1000} = \frac{125}{25} = 5$

Question 12

An Ice Cream costs $2, 40. Jane has $40 and buys as many ice creams as possible.

Find

a) The number of Ice creams she buys

b) The change that she receives

Solution

a) $= \frac{40\times100}{2,40\times100}$

$= \frac{4000}{240}$

$= 16,66$

She buys 16 Ice Creams

b) $2,40$

$\times \quad 16$

2400

$+ 1400$

$38,40 =$ $38,40

$40- $38,40 = $1.60

Question 13

By how much is the product of 0,25 and 55 more than 12?

Solution

0, 25 × 55

0, 25
× 55
1250
+125
13,75

13,75- 12 =1,75

It is more by 1,75

Question 14
How many pencils can Sekai buy at $0, 65 each for $14,95. If she sells them at $0, 80 each, find her total profit.

Solution

$$\frac{\$14,95}{0,65}\times\frac{100}{100}=\frac{\cancel{1495}^{299}}{\cancel{65}_{13}}$$

She can buy 23 pencils.
$0,80
× 23
1600
+240
18, 40 = $18,40
$18,40
- $14, 95
$ 3, 45
Sekai's total profit is $3,45

Question 15
A pack of A4 exercise books weighs10,98kg. If each exercise book weighs 54, 99, how many exercise books are there?

Solution
First change kg to grammes 10,98kg × 1000 = 10 980g

$$\frac{10980}{54,9}\times\frac{10}{10}$$

$$= \frac{109800}{549}$$

= 200 Exercise Books

Question 16

Express

a) 1269 to 3s.f
b) 3,017 to 3 s.f
c) 0,032016 to 3s.f

Solution

a) 1269 = 1270 to 3.s.f
b) 3,017 = 3,02 to 3 s.f
c) 0,0032016 = 0,0320 to 3s.f

Question 17

Express the following numbers to two significant figures

a) 3269
b) 4,027
c) 0,065037

a) 3269 = 3300 to 2 s.f

b) 4,027 = 4,0 to 2 s.f

c) 0,065037 = 0,065 to 2 s.f

Question 18

Express the following to 1 significant figure

a) 6289
b) 273
c) 0,058
d) 0,0062

Solution

a) 6289 = 6000 to 1 s.f

b) 273 = 300 to l s.f

c) 0,058 = 0,06 to I s.f

d) 0,0062 = 0,006 to I s.f

Question 19

Turn-up College "O" Level Mathematics Questions and answers

Give the following correct to 3 decimal places.

a) 13,6731
b) 0,2869
c) 0,0486

Solutions

a) 13,6731 = 13,673 to 3d.p
b) 0,2869 = 0,287 to 3.d.p
c) 0,0486 = 0,049 to 3.d.p

Question 20

Express 0,006287
to:

a) 3. s.f
b) 3 d.p
c) 2 s.f
d) 2 d.p

Solution

a) 0,006287 = 0,00629 to 3s.f

b) 0,006287 = 0,006 to 3d.p

c) 0,006287 = 0,0063 to 2.s.f

d) 0,006287 = 0,01 to 2 d.p

Question 21

State the number of significant figures in each of the following

a) 0,029 cm

b) 8,04g

c) 28000km

Solution

a) 0,029cm = 2 s.f
b) 8,04g = 3 s.f
c) 28 000km = 2 s.f

Question 22

Find the H.C.F of the following
a) 36, 54 and 60
b) 216 and 168

Solution

NB: H.C.F is the product of the common prime factors

a) $36 = 2 \times 2 \times 3 \times 3$
$54 = 2 \times 3 \times 3 \times 3$
$60 = 2 \times 2 \times 3 \times 5$
$H.C.F = 2 \times 3 = 6$
$= 6$

b) $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$168 = 2 \times 2 \times 2 \times 3 \times 7$
$H.C.F = 2 \times 2 \times 2 \times 3$
$= \underline{24}$

Question 23

What is the H.C.F of 324, 432 and 540

Solution

$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$H.C.F = 2 \times 2 \times 3 \times 3 \times 3$
$= \underline{108}$

Question 24

Find the L.C.M of the following
a) 12, 15 and 18
b) 36,45 and 60

Solution

NB: To Find the L.C.M, you have to express each number as a product of its prime factors. Also the highest power of each prime factor must be in the L.C.M.

a) $12 = 2 \times 2 \times 3$
$15 = 3 \times 5$
$18 = 2 \times 3 \times 3$

L.C.M = $2^2 \times 3^2 \times 5$
= 180

b) 36 = $2 \times 2 \times 3 \times 3$
45 = $3 \times 3 \times 5$
60 = $2 \times 2 \times 3 \times 5$

L.C.M = $2^2 \times 3^2 \times 5$
= 180

Question 25
Write the following in standard form
a) 650
b) 37000

Solution
Any positive number can be expressed in the form $a \times 10^n$ where n is a positive or negative integer and a any number between 1 and 10

$1 \leq a < 10$
a) 650 = $6{,}5 \times 100$
= $6{,}5 \times 10^2$

b) 37 00 = $3{,}7 \times 10\ 00$
= $3{,}7 \times 10^3$

Question 26

Express the following numbers in standard form
a) 5 000
b) 708 000

a) 5 000 = $5{,}0 \times 1000$
= 5×10^3
b) 708 000 = $7{,}08 \times 100\ 000$
= $7{,}08 \times 10^5$

Question 27
Write the following numbers in the form a $\times$ 10n where $1 \leq$ a $<$ 10 and n is a positive or negative integer.

a) 76,25
b) 0,0062

Solution
a) 76,25 = $7{,}625 \times 10$

$= \quad 7{,}625 \times 10$

b) $0{,}0062 = \dfrac{6,2}{1000} = \dfrac{6,2}{10^3}$

$= \quad 6{,}2 \times 10^{-3}$

Question 28

Express the following numbers in standard form
a) 0,526
b) 0,00309

Solution

a) $0{,}526 = \dfrac{5.26}{10} = \dfrac{5.26}{10}$

$= \quad 5{,}26 \times \dfrac{1}{10^1}$

$= \quad 5{,}26 \times 10^{-1}$

b) $0{,}00309 = \dfrac{3.09}{1000}$

$= \dfrac{3.09}{103}$

$= \dfrac{3,09}{10^3}$

$= \quad 3{,}09 \times 10^{-3}$

Question 29

Evaluate the following giving your answers in standard form

a) $(4{,}4 \times 10^{6}) \div (2 \times 10^{2})$

b) $(3{,}5 \times 10^{7}) \div (5 \times 10^{4})$

Solution

a) $\frac{4.4\times10^{6}}{2\times10^{2}}$

$= \frac{4.4}{2} \times \frac{10^{6}}{10^{2}}$

$= 2{,}2 \times 10^{4}$

b) $\frac{3.5\times10^{7}}{5\times10^{4}} = \frac{3.5}{5} \times \frac{10^{7}}{10^{4}}$

$= 0{,}7 \times 10^{3}$

$= 7 \times 10^{-1} \times 10^{3}$

$= 7\times 10^{2}$

Question 30

Simplify, leaving your answer in standard form

a) $(9{,}6 \times 10^{5}) \div (3 \times 10^{3})$

b) $(1{,}2 \times 10^{9}) \div (3\times x\, 10^{5})$

Solution

a) $\frac{9,6}{3}\times\frac{10^{5}}{10^{3}} = \frac{9,6}{3}\times\frac{10^{5}}{10^{3}}$

$= 3{,}2 \times 10^{2}$

b) $(1{,}2 \times 10^{9}) \div (3 \times 10^{5})$

$\frac{1.2\times10^{9}}{3\times105} = \frac{1.2}{3} \times \frac{10^{9}}{10^{5}}$

$= 0{,}4 \times 10^{4}$

= $4 \times 10^{-1} \times 10^{4}$
= 4×10^{3}

Question 31

Evaluate giving your answer in standard form

a) $\left(6\times10^{-4}\right) \times \left(2\times10^{2}\right)$

b) $\left(5\times10^{-1}\right) \times \left(3\times10^{-4}\right)$

Solution

a) $\left(6\times10^{-4}\right) \times \left(2\times10^{-2}\right)$

= $\left(6\times2\right) \times \left(10^{-4}\times10^{-2}\right)$

= 12 x 10^{-6}

= $1{,}2 \times 10^{1} \times 10^{-6}$

= $1{,}2 \times 10^{-5}$

b) $\left(5\times10^{-1}\right) \times \left(3\times10^{-4}\right)$

= $\left(5\times3\right) \times \left(10^{-1}\times10^{-4}\right)$

= 15×10^{5}

= $1{,}5 \times 10^{1} \times 10^{-5}$

= $1{,}5 \times 10^{-4}$

Question 32

Express the first quantity as a percentage of the second

a) \$26 of \$200

b) 1,2 litres of 6 litres

c) 45 cents of \$3

Solution

a) $\frac{\$26}{\$200} \times \frac{100\%}{1}$

= 13%

c) $\frac{45}{300} \times \frac{100\%}{1}$

$\frac{1200}{6000} \times \frac{100\%}{1}$ = 15%

= 12%

b) $\frac{1,2}{6} \times 100$ **% =** $\frac{12}{60} \times 100$ **% =20%**

Question 33

25% of a sum of money is $45. Find the sum of money

Solution

25% = $45
100% = more

$= \frac{100}{25} \times 45$

$= 180$

The sum of money = $135 + $45 = $180

Question 34

What is 34% of R45?

Solution

~~34~~
~~100~~ × R45
~~20~~
10

1,7 x 9 = 15,3
= R 15, 30

Question 35

Find the value of the following correct to the nearest cent.

a) 9% of $6,40
b) 13% of $8,20

a) $\frac{9}{100} \times 640c$

= $\frac{9 \times 320}{50} = \frac{288}{50}$

= 5,76c

= $0,58

b) $\frac{13}{100} \times$ $8,20

= 0,13 x 8,20
=1,066
=$1,07

Question 36

Express the following percentages as fractions in their lowest form.
a) 20%
b) 55%

Solution

a) $\frac{\cancel{20}}{\cancel{100}} = \frac{1}{5}$

b) 55% = $\frac{\cancel{55}}{\cancel{100}} = \frac{11}{20}$

Question 37

Express the following as Decimals
a) 10%
b) 3%
c) $66\frac{2}{3}$

Solution

a) 10% = $\frac{10}{100}$ =0,1

= $\underline{0,1}$

b) 3% = $\frac{3^{\circ}}{100}$

= 0,03

c) $66\frac{2}{3}$ = $\frac{200\%}{3}$

$\underline{200}$:- $\frac{100\%}{1}$

= $\underline{200}$ x $\frac{1}{100}$

= 0, 6666…
= $\underline{0,67}$ to 2.d.p

Question 38
A man buys a TV set at $ 3200. He pays a 15% deposit. How much is the deposit?

Solution
$\frac{15}{100} \times \frac{3200}{1}$

$15 \times 32 = 480$

The deposit is $480

Question 39
A vendor reduced all his prices by 12%. What will be the price of a blanket originally marked at $40?

Solution
$\frac{12}{100} \times \$40$

$\frac{24}{5} = 4,8$

$4,80

$40- $4,80 = $35,20

The new price = $35, 20

Question 40

The price of a suit is $320
4 months later it is $335. What was the percentage increase?

Solution
NB: Percentage increase/Decrease is calculated using the formular:-

Increase/ Decrease $\times$ 100%
Original value

Increase = $335- $320
= $15

$$\frac{15}{320} \times 100\%$$

$$\frac{15\times10}{32} = \frac{150}{32}$$

= 4, 6875
= 4, 70%

CHAPTER 2

Number systems, Number Bases

Convert the following numbers into the bases shown on:

a) 57_{10} to base 5

b) 39_{10} to base 2

Solution

1a)

5	57
5	11 r 2
5	2 r 1
	0 r 2

$\therefore 57_{10} = 212_5$

2	39
2	19 r 1
2	9 r 1
2	4 r 1
2	2 r 0
2	1 r 0
	0 r 1

$\therefore 39_{10} = 100111_2$

2. Convert the following numbers into base 10

a) $210\ 3_5$ b) $10\ 110_2$ c) $14\ 3_5$

Solution

a) $2 \times 5^3 + 1 \times 5^2 + 0 \times 5^1 + 3 \times 5^0$

$= 2 \times 125 + 1 \times 25 + 0 \times 5 + 3 \times 1$
$= 250 + 25 + 3$
$= 278_{10}$

b) $1011\ 0_2$
$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
$= 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 1$
$= 16 + 4 + 2$
$= 22_{10}$

c) $1 \times 5^2 + 34 \times 5^1 + 3 \times 5^0$
$= 1 \times 25 + 20 + 3$
$= 25 + 23$
$= 48_{10}$

3. Evaluate the following, giving your answers in base 2.

a) $1011_2 + 111_2$ (b) $101_2 + 11_2$

Solution

When adding in any base, remember to 'carry', borrow' or add equality' in the base

a)

	1011_2
+	111_2
	$10010_2 = 10010_2$

b)

	101_2
+	11_2
	$1000_2 = 1000_2$

NB: when adding 1 +1 = 2 then you divide by 2 (The Base) you get 1 r 0, you now write the remainder and carry the 1.

4. Given that:

$\xi = \{2; 3; 4; 5; 6; 8; 9; ....; 16\}$

A = $\{ x : x$ is a perfect square$\}$

B= $\{x : x \text{ is a factor of } 36\}$

C= $\{x : x \text{ is a prime number}\}$

a) List the members of

i) A (ii) $A \cap B$

b) $n(C^1)$

Solution

Note: You must list the member of the three sets first

A = $\{4;9;16\}$

B = $\{2;3;4;6;9;12\}$

C = $\{2;3;5;7;11;13\}$

a) i) A= $\{4;9;16\}$

ii) $A \cap B = \{4;9\}$

b) $C' = \{4;6;8;9;10;12;14;15;16\}$

$\therefore n(C') = 9$

5)a) Solve the inequality $2x + 5 \geq 4$

b) Write down the smallest value of x given that

i) x is an integer

ii) x is a natural number

iii) x is a Prime number

iv) x is a rational number

v) x is a real number

Solution

5a) $2x + 5 \geq 4$

$2x \geq 4-5$

$2x \geq -1$

$x \geq -1/2$

B (i) 0

ii) 1
iii) 2
iv) -1/2
v) -1/2

6) Find the prime factors of 70, 42 and 105 and state their HCF and L CM.

Solution

2	70
5	35
7	7
	1

70 = $2 \times 5 \times 7$

2	42
3	21
7	7
	1

$42 = 2 \times 3 \times 7$

3	105
5	35
7	7
	1

$105 = 3 \times 5 \times 7$

70 = $2 \times 5 \times 7$
42 = $2 \times 3 \times 7$
105 = $3 \times 5 \times 7$
:- H.C.F = 7
L.C.M = $2 \times 3 \times 5 \times 7 = 210$

7a) Find the value of $\left(1\frac{1}{2}+\frac{2}{3}\right)\times 1\frac{1}{5}$
Giving your answer as a fraction in its lowest terms
b) Evaluate 5,4 + 4 × 0,3
c) Calculate 6% of £5450
d) Express 42cm as a percentage of 1,05m.

Solution

Apply the BOMDAS method

1 B—— Brackets first

2. ODM —— Of Multiplication, Division next

3. AS ——— addition & Subtraction Last

a) $\left(1\frac{1}{2}+\frac{2}{3}\right)\times 1\frac{1}{5}$

$=\left(\frac{3}{2}+\frac{2}{3}\right)\times\frac{6}{5}$

$=\frac{9+4}{6}\times\frac{6}{5}$

$=\frac{13}{6}\times\frac{6}{5}$

$=\frac{13}{5}$

$=2\frac{3}{5}$

b) $\frac{6}{100}\times 5450$

= £ 327

c) 5,4 +4 x 0,3

5,4 + 1,2

= 6,6

d) $\frac{\cancel{42}^{2}}{\cancel{105}}\times\frac{\cancel{100}^{20}}{}\%$

= 40%

CHAPTER 3

Quadratic Expressions

1. Expand the Following

a) $(2a- 5)\ (a + 7)$

b) $(3x - 7)(5x +1) - (x +2)(x +5)$

Solution

$(2a-5)\ (a+7)$

$2a\ (a+7) -5)\ (a+7)$

$2a^2 + 14a – 5a -35$

$2a^2 + 9a -35$

b) $(3x -7) - (x +2)(x +5)$

$= 3x\ (5x +1)-7\ (5x +1)$ (i)

$= 15x^2 + 3x - 35x -7$

$= 15x^2 -32x -7$

And $(x +2)(x +5) = x\ (x +5) + 2\ (x +5)$

$= x^2 + 5x +2x + 10$

$= x^2 +7x +10$ (ii)

:- $(15x^2 - 32x - 7) - (x^2 +7x +10)$

$15x^2 - x^2 -32x -7 - x - 7 -10$

$14x^2 - 39x -17$

2. Fractorise the following

a) $ax – bx + by - any$

b) $x^2 - x - 72$

c) $4x^2 - 9y^2$

Solution

a) $ax –bx + by - ay$

$= ax –ay + by - bx$

$= a(x -y) +b\ (y - x)$

$= a\ (x -y) –b\ (x -y)$

$= (a-b)\ (x -y)$

b) $x^2 - x - 72 =$ **(x -9) (x +8)**

c) $4x^2 - 9y^2$

Note:- $a^2 - b^2 = (a-b)(a+b)$

$50\ 4x^2 - 9y^2 = (2x)^2 - (3y)^2$

$= (2x - 3y)(2x + 3y)$

3. Solve the following quadratic equations

a) $x^2 - 3x - 10 = 0$

b) $8x^2 - 2x - 15 = 0$

c) $7x^2 - 3x = 0$

Solution

a) $x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

Either $x - 5 = 0$ or $x + 2 = 0$

:- $x =$ **5 or -2**

b) $8x^2 - 2x - 15 = 0$

$(2x - 3)(4x + 5) = 0$

either $2x - 3 = 0$ or $4x + 5 = 0$

:- $\frac{2x}{2} = \frac{3}{2}$ or $\frac{4x}{4} = -\frac{-5}{4}$

$\therefore \quad x = 1\frac{1}{2}$ **or** $-1\frac{1}{4}$

c) $7x^2 - 3x = 0$

$x(7x - 3) = 0$

either $x = 0$ or $7x - 3 = 0$

$= \quad x = 0$ or $\frac{7}{7}x = \frac{3}{7}$

$\therefore \quad x = 0$ or $\frac{3}{7}$

Question 4 Factorise the following

a) $2ax - 6a + bx - 3b$

b) $4 - 9m^2$

Solution

a) $2a\,x - 6a + b\,x - 3b$

$2a\,(x-3) + b\,(x-3)1$

$1(2a + b)\,(x-3)$

b) $4 - 9m^2$

Note: Difference of two squares

$a^2 - b^2 = (a-b)(a+b)$

$4-9m^2 = (2)^2 - (3m)^2$

$=(2-3m)(2 + 3m)$

Question 5 **Factorise the following expressions**

a) $16\,x^2-1$

b) $\bar{x}r^2 + 2\bar{x}rh + \bar{x}rhl$

c) $\bar{x}^2 - 2x - 15$

Solution

a) $16\,x^2 - 1 \quad = (4\,x)^2-(1)^2$

$= \quad (4\,x-1)(4\,x+1)$

b) $\bar{x}r^2 + 2\,\bar{x}\,rh + \bar{x}rl$

$= \quad \bar{x}r\,(r + 2h + l)$

c) $x^2 - 2\,x - 15$

Factors of $-15\,x^2$	sum of factors
$-5\,x$ and $+3\,x$	$-2\,x$
$+5\,x$ and $-3\,x$	$2\,x$
$+15\,x$ and $-x$	$+14\,x$
Replace $-2\,x$ by	$-5\,x$ & $+3\,x$

$x^2 - 5\,x + 3\,x - 15$

$x\,(x-5) + 3\,(x-5)$

$(x+3)(x-5)$

Question 6

Use the formulae $x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ to solve $3x^2 - 4x + 1 = 0$

Solution

Note: a = The coefficient of x^2

b = The coefficient of x

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ c = The constant

x =+3, b = -4, c= +1

x = - $\frac{-(-4) \pm \sqrt{(-4)^2 - 4(3 \times 1)}}{2 \times 3}$

x = $\frac{4 \pm \sqrt{4}}{6}$

x = $\frac{4 + \sqrt{4}}{6}$

x = $\frac{4 \pm \sqrt{4}}{6}$

= $\frac{6}{6}$ - or $\frac{2}{6}$

:- x = 1 or $\frac{1}{3}$

Question 7

Solve $5x^2 - 3x - 2 = 0$

Solution

a = + 5, b = -3, c= -2

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

x = $\frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times (5 \times (-2)}}{2 \times 5}$

Question 8 Solve

$55x^2 - 6x - 3 = 0$

Solution

a= 5, b=-6, c= -3

x= - $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

x = $\frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 5 \times 3}}{2 \times 5}$

x = $\frac{6 \pm \sqrt{96}}{10}$

$x = \frac{-3 \pm \sqrt{9+40}}{10}$

$= \frac{3 \pm \sqrt{49}}{10}$

$x = \frac{-6 \pm 9,798}{10}$

$= \frac{3+7}{} \text{ or } \frac{3-7}{}$

$= \frac{-15,798}{10} \text{ or } -\frac{-3798}{10}$

$= \frac{10}{10} \text{ or } \frac{-4}{10}$

$\therefore$ x= 1,5798 or -03798

$\therefore$ x = 1 or $\frac{-2}{5}$

x = 1,58 or -0,38 to 2.dp.

Question: 9

Solve the equation $5x^2 + 8x - 2 = 0$
And leave your answer to d.p.

Question 10

Find the roots of the equation $3x^2 + 7x + 3 = 0$

Solution

a= +5, b= +8, c= -2

$x = -\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{8^2 - 4(5 \times -2)}}{2 \times 5}$

$x = \frac{-8 \pm \sqrt{64+40}}{10}$

$x = \frac{-8 \pm \sqrt{104}}{10}$

$x = -\frac{-8 \pm 10,2}{10}$

Solution

a= +3, b= +7, c +3

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = -\frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times 3}}{2 \times 3}$

$x = -\frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times 3}}{2 \times 3}$

$x = \frac{-7 \pm \sqrt{49-36}}{6}$

$x = \frac{-7+3,606}{6}$ or $\frac{-7-3,606}{6}$

$x = \frac{-2,2}{10}$ or $\frac{-18}{10}$

$x = -\frac{-3,394}{6}$ or $\frac{-10,606}{6}$

x=- 0,57 or -1,77 to 2.dp

∴ ∴ x = 0,22 or -1,82

:-

Question 11:

Find the roots of the equation 3x12x + 10 -0
2 -

Solution

a= +3, b =-12, c= +10

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 3 \times 10}}{2 \times 3}$

$x = \frac{-12 \pm \sqrt{144-120}}{6}$

6

$x = \frac{12 \pm \sqrt{24}}{6}$

$x = \frac{12 \pm 4,899}{6}$

$x = \frac{12-4,899}{6}$ or $\frac{12+4,899}{6}$

$= \frac{16,899}{6}$ or $\frac{7,101}{6}$

2,8165 / 1 and 1,1835 / 1

:-x= 2,82 or 1,18 to 2.d.p

Question 12:

Turn-up College “O” Level Mathematics Questions and answers

Solve $3x^2 - 8x + 2 = 0$

Solution

a= +3, b=-8, c=+2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

b= +3, c=-3, a=+5

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$= \frac{8 \pm \sqrt{64 - 24}}{6}$$

$$= \frac{8 \pm \sqrt{40}}{6}$$

$$= \frac{8 \pm 6,325}{6}$$

$$= \frac{8 + 6,325}{6} \; or \; \frac{8 - 6,325}{6}$$

2,3875 0,27916

$$= \frac{14,325}{6} \; or \; \frac{1,675}{6}$$

:- x= 2,39 or 0, 28 to 2.dp.

Question 13: Find the solution of the equation $5x^2 + 3x - 3 = 0$.

Solution

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(+3) \pm \sqrt{32 - 4(5x - 3)}}{2 \times 5}$$

$$= \frac{-3 \pm \sqrt{9 + 60}}{10}$$

$$= \frac{-3 + \sqrt{69}}{10}$$

$$= \frac{-3 \pm \sqrt{69}}{10}$$

$$= \frac{-3 \pm 8,307}{10}$$

$$= \frac{-3 + 8.307}{10} \; or \; \frac{-3 - 8.307}{10}$$

0,5307 -11,307

:-x= 0, 53 or -1, 13 to 2.d.p

Question 14: Solve the quadratic equation $x^2 + 3x + 1 = 0$ leaving your answer to 2.d.p.

Solution

a= +1, b=+3, c= +1

$$x = -\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -\frac{+3 \pm \sqrt{3^2 - 4(1 \times 1)}}{2 \times 1}$$

$=- \frac{3 \pm \sqrt{9-4}}{2}$

$=- \frac{3 \pm \sqrt{5}}{2}$

$= \frac{-3 \pm 2,236}{2} or \frac{-3-3236}{2}$

0,382 -2,618

$\frac{0,764}{2} or \frac{5,236}{2}$

x= -0,38 or -2,62 to 2.d.p

Quotation 15. Find the roots of the equation x^2- 2x- 4=0 and leave your answer to 2.d.p.

Solution

a= +1, b=-2, c= -4

$x=- \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-2) \pm \sqrt{(-2)^2-4(1\times-4)}}{2\times1}$

$= \frac{2 \pm \sqrt{4+6}}{2}$

$= \frac{2 \pm 4,472}{2}$

$= \frac{2-4,472}{2}$ or $\frac{2-4,472}{2}$

3,236 -1,236

$=\frac{6,472}{2}$ or $\frac{-2,472}{2}$

x=3, 24 or -1, 24 to 2.d.p

Question 16

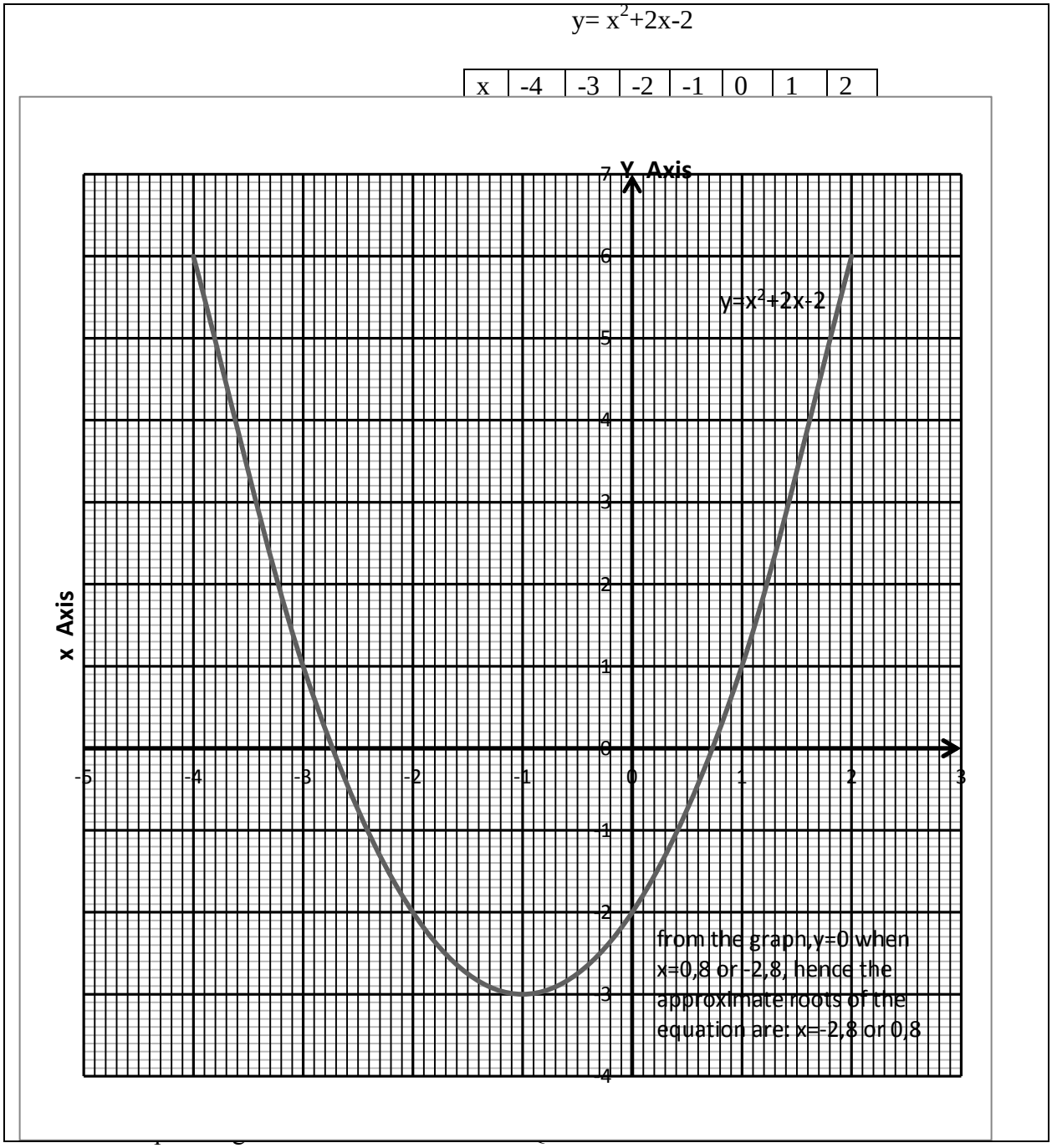
a) Draw the graph of the function $x^2+ 2x-2$ from x= -4 to x= +2

b) Hence find the approximate roots of the equation.
$x^2 + 2x -2 = 0$

Solution

$y= x^2+2x-2$

x	-4	-3	-2	-1	0	1	2



Q 17

a)Draw the graph of the function

$11 + 8x - 2x^2$ from x= -2 to x = +6

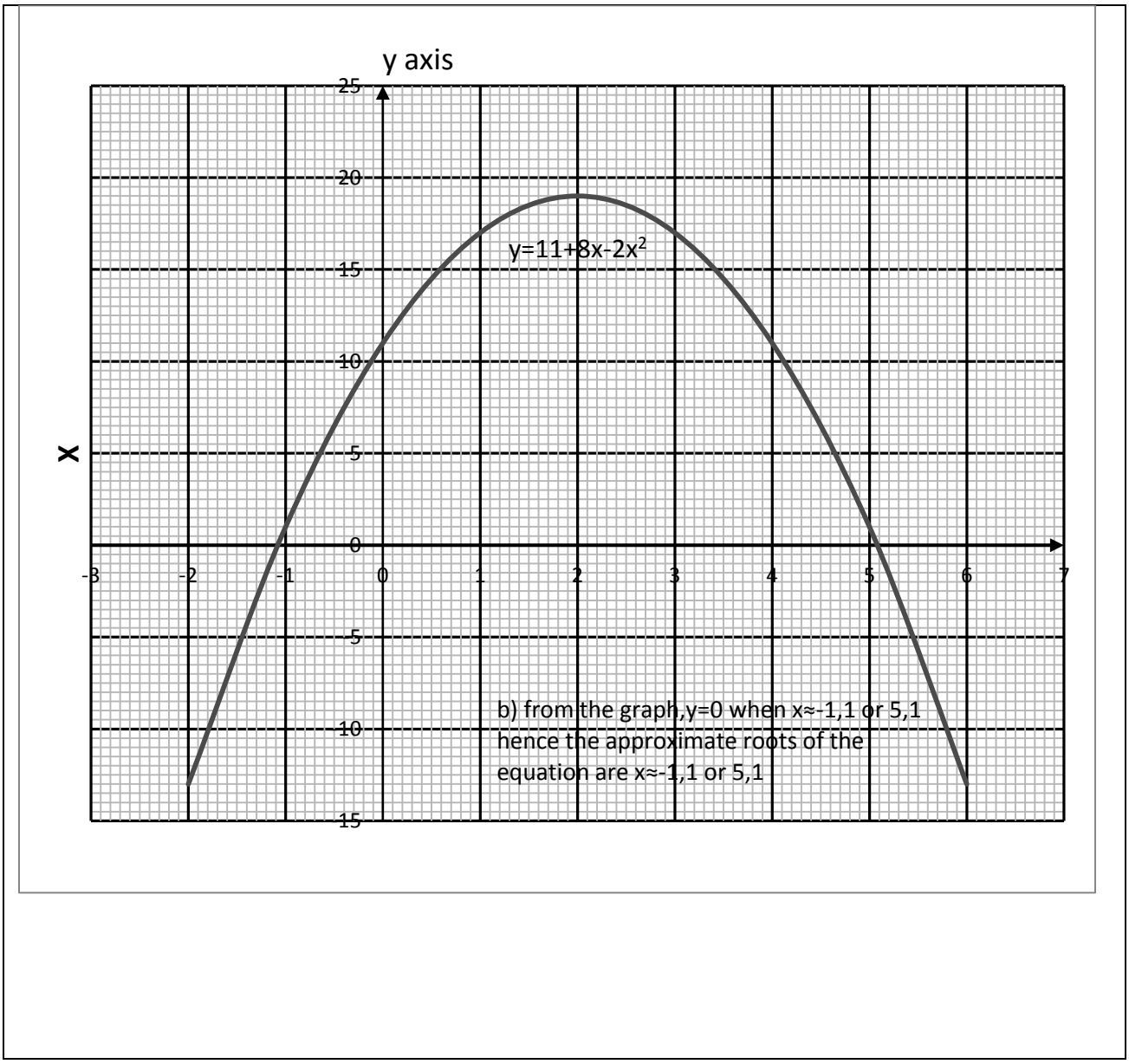
b) Hence find the approximate roots of the equation

$2x^2-8x-11 =0$

Solution

$y = 11+8x-2x^2$

x	-2	-1	0	1	2	3	4	5	6
y	-13	1	11	17	19	17	11	1	13



CHAPTER 4

Inequalities

1. Illustrate the following inequalities on the number line and give integral values of x which satisfy the inequalities

a) $x > 4$

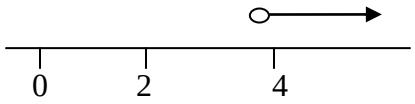
b) $x \leq -2$

c) $-3 < x < 4$

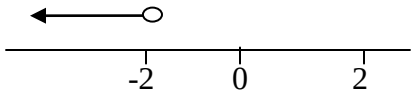
d) $-1 < 2x - 3 \leq 5$

Solution

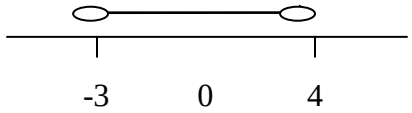
Turn-up College "O" Level Mathematics Questions and answers

a) 

x - Can assume any integer value greater than but not 4

b) 

x can assume any integer value less than or equal to-2

c) 

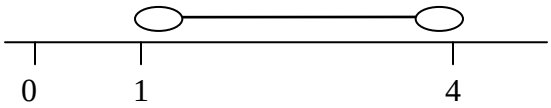
x can assume any integer value less than 4 and greater than -3 but not 4 or -3 ie x= -2,-1, 0,1,2 0r 3.

d) $-1 < 2x - 3 < 5$

$-1 + 3 < 2x \leq 5 + 3$

$\frac{2}{2} < \frac{2}{2} \leq \frac{8}{2}$

$1 < x \leq < 4$



x can assume any integer value less than or equal to 4 but greater than 1 i.e. $x = 2, 3,$ or 4

2. **Write the inequalities**

$x + 1 < 6 < 2x + 4$ in the form

$A < x < B$, stating the values of A and required

Solution

$x + 1 < 6 < 2x + 4$

Note: Take each inequality separately.

$x + 1 < 6$

$x < 6 - 1$

$x < 5$

$6 < 2x + 4$

$6 - 4 < 2x$

$\frac{2}{2} < \frac{2x}{2}$

$1 < x$

$1 < x < 5$

: - A= 1 and B= 5

3. If x is a prime number
List the values of x which satisfy $x > 9$ and $2x - 5 < 39$.
$2x < 39 + 5$

$$\frac{\cancel{2}x}{\cancel{2}} < \frac{44}{\cancel{2}}$$

$x < 22$

Solution
$2x < 39 + 5$

$$\frac{2x}{1} < \frac{44}{2}$$

$\underline{x < 22}$

:- $9 < x < 22$
Thus combining the two inequalities

The values of x are 11, 13, 17, 19

4) Find the true set of
2y- 1 > 2y +5 and illustrate
O > 6 contradiction! no true set

Solution

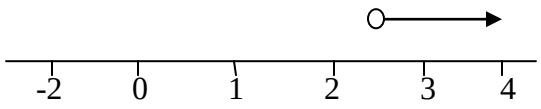
$$\frac{2y-1}{2} > \frac{2y+5}{5}$$ Remove fractions first

5(2y-1) > 2 (2y +5)
10y-5 >4y + 10
10y -4y > 10 +5

$$\frac{6}{6}y > \frac{15}{6}$$

$$y > 2\frac{1}{2}$$

The truth set is y: y > 2 ½



5. List the integer values which satisfy: $1 \ x \leq 10 - 2x < 7$

Solution

Note: Split the inequality into two

$1 - x \leq 10 - 2x$ $\qquad$ $10 - 2x < 7$

$-x + 2x \leq 10 - 1$ $\qquad$ $10 - 7 < 2x$

$x \leq 9$ $\qquad$ $\frac{3}{2} < \frac{2x}{2}$

$x > 1\frac{1}{2}$

:- $1\frac{1}{2} < x \leq 9$

The integer values which satisfy this are:- 2, 3,4,5,6,7,8,9.

6. Solve the following inequalities and show them on a number line

a) $2x - 3 < 5$

b) $-2 < 3 - x \leq 5$

c) $-3 \leq 2x - 1 \leq 7$

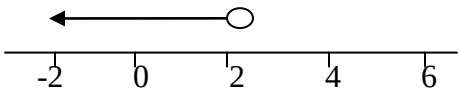
Solution

a) $2x - 3 < 5$

$2x < 5 + 3$

$\frac{2x}{2} < \frac{8}{2}$

$x < 4$



b) $-2 < 3 - x \leq 5$

$-2 < 3 - x$

$-2 - 3 < -x$

$\frac{-5}{-1} < \frac{-x}{-1}$

$5 > x$

$3 - x < 5$

$3 - 5 \leq x$

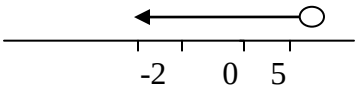
$-2 \leq x$

c) $-3 \leq 2x - 1$

$-3 + 1 \leq 2x$

$-2 \leq 2x$

$-2 \leq x < 5$



$2 \quad 2$

$-1 \leq x$

$2x - 1 \leq 7$

$2x \leq 7 + 1$

$\frac{\cancel{2}x}{\cancel{2}} < \frac{8}{2}$

$x \leq 4$

$-1 \leq x \leq 47.$

Illustrate the inequalities

On the number line and give integral values which satisfy each inequality

a) $-5 \leq 3x + 1 < 10$

b) $6 < 5x - 1 \leq 29$

Solution

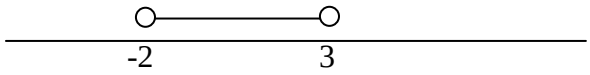
a) $-5 \leq 3x + 1$ $\qquad$ $3x + 1 < 10$

$-5 - 1 \leq 3x$ $\qquad$ $3x < 10 - 1$

$\frac{-6}{3} \leq \frac{\cancel{3}x}{\cancel{3}}$ $\qquad$ $\frac{\cancel{3}x}{\cancel{3}} < \frac{9}{3}$

$-2 \leq x$ $\qquad$ $x < 3$

$-2 \leq x < 3$



$x = -2, -1, 0, 1, 2$

b) $6 < 5x - 1 \leq 29$ $\qquad$ $5x - 1 \leq 29$

$6 < 5x - 1$ $\qquad$ $5x \leq 29 + 1$

$6 + 1 < 5x$ $\qquad$ $\frac{5x}{5} \leq \frac{30}{5}$

$\frac{7}{5} < \frac{5x}{5}$

$\frac{7}{5} < x$ $\qquad$ $x \leq 6$

$\frac{7}{5} < x \leq 6$

x = 2, 3, 4, 5, 6

CHAPTER 5

Variation

1. If $x \propto$ y and x = 5 when y = 4, Find x when y =6 and y when x = 10,5.

Solution

x α y

x = ky

$5 = k\frac{4}{4}$

$x = \frac{5y}{4}$

When y =6

$x = \frac{5y}{4}$

$x = \frac{5y}{4} \times 6$

$x = \frac{30}{4}$

$x = 7\frac{1}{2}$

When x = 10,5

10, 5= $10\ \frac{1}{2} = \frac{21}{2}$

$x = \frac{5y}{4}$

$\frac{21}{2} = \frac{5}{4}y$

$\frac{84}{10} = \frac{10y}{10}$

y= $8\frac{2}{5}$

2. Given that $F\alpha\frac{v^2}{r}$ and

That F= 50 when v= 5 and r= 2, Find the value of f
When v= 9 and r = 6

Solution

$F\alpha\frac{v^2}{r}$

r

F= $F = \frac{kv^2}{r}$

$$50=\frac{5^2 k}{2}$$

$$50=\frac{25k}{2}$$

$$\frac{100}{25}=\frac{25k}{25}$$

$4 = k \quad \text{-}\rightarrow \quad k = 4$

$$F=\frac{4v^2}{r}$$

$$F=\frac{4\times v^2}{r}$$

$$F=\frac{4\times 81}{6}$$

$$F=\frac{324}{6}$$

:-F= 54

3. $x\alpha \frac{1}{y}$ and x= 20 when y = $\frac{1}{2}$

a) Find the relationship between x and y.
b) Find x when y= 4
c) Find y when x = 15

Solution

a) $x \alpha \frac{1}{y}$

$x = \frac{1}{y} k$

$$20=\frac{k}{\frac{1}{2}}$$

10= k = k= 10

: $x = \frac{10}{y}$

b) $x = \frac{10}{y}$

$x = \frac{10}{y}$

x = $2\frac{1}{2}$

c) $x = \frac{10}{y}$

15= $\frac{10}{y}$

15y= $\frac{10}{y}$

y = $\frac{2}{3}$

4. t varies inversely as r and t = 3.5 when r = 10
a) Find the law connecting t and r
b) Find t when r= 10, 5
c) Find r when t= 28

Solution

a) $t\alpha \frac{1}{r}$

$t = \frac{k}{r}$

3,5 = $\frac{k}{10}$

k= 35

c) t= $\frac{k}{r}$

b) t = $\frac{35}{r}$

t= $\frac{35}{r}$

$t = \frac{35}{10,5}5$

$\therefore t = 3\frac{1}{3}$

$$28 = \frac{35}{r}$$

$$28 = \frac{35}{28}$$

$$\therefore r = 1\frac{1}{4}$$

Question 5

The kinetic energy E Joules of a moving object varies jointly as the mass making of the object and the square of its velocity vm/s.

a) Write down an equation connecting E.M, V and A constant K.

b) Given that E= 150 joules when the mass of the object M= 12kg and the velocity v= 5m/s, find the value of k.

c) Calculate the kinetic energy E of an object of mass 24kg which is moving with a velocity of 7m/s.

Solution

a) $E \alpha mv^2$

$E= kmv^2$

b) $E= kmx^2$

$150= k \times 12 \times 5^2$

$150= k \times 12 \times 25$

$$\frac{150}{\cancel{300}} = k\frac{\cancel{300}}{\cancel{300}}$$

$½ = k \quad \therefore \quad k= ½$

c) $E= ½ mv^2$

$E= ½ \times 24 \times 7^2$

$E= ½ \times 24 \times 49$

$\underline{E= 588J}$

.Question 6 P is partly constant and partly varies as Q.P= 120 when Q = 5 and P = 100 when Q = 3 Find :-

a) P when Q = 15

b) Q when P = 360

Solution

$P= a + kQ$

$120= a + 5k$ (i)

$(100= a + 3k)$ (ii)

Subtract (ii) from (i)

$$\frac{\cancel{20}}{\cancel{2}} = \frac{\cancel{k2}}{\cancel{2}}$$

$10= \quad k \quad \therefore \quad \underline{k = 10}$

$120 = a + 10\ (5)$

$120-50 = a$

$70 = a \quad \therefore \quad \underline{a = 70}$

a) $\underline{p= 70 + 10q}$

$P= 70 +10Q$

$P=70 + 10\ (15)$

$P= 70 + 150$

P= 220

b) P= 70 + 10Q
360= 70 + 10Q
360-70 = 10Q
$\frac{290}{10} = \frac{10Q}{10}$

29 = Q
:- **Q= 29**

Question 7
x α y and x = 7 when y= 20
a) Find x when y = 60
b) Find y when x = 21

Solution

x α y

x= ky

$\frac{7}{} = k\frac{20}{20}$

$k= \frac{7}{20}$

a) $x= \frac{7y}{20}$

$x= \frac{7\times 60}{20}$

x= 21

b) $x= \frac{7y}{20}$

$21= \frac{7\times y}{20}$

$\frac{\cancel{420}^{60}}{\cancel{7}_{1}} = \frac{7y}{7}$

7

60= y

y= 60

Question 7.
Given that c varies directly as d and c = 25 when d= 4
a) Find the law connecting c and d.
b) Find d when c= 75

Solution
c α d

a) C= kd

c = kd

$25 = \frac{k4}{4}$

k= $\frac{25}{4}$

b) C= $\frac{25d}{4}$

$75= \frac{25d}{4}$

$\frac{\overset{12}{\cancel{300}}}{\cancel{25}} = \frac{25d}{25}$

12= d

∴ d= 12

Question 8:

Given that r varies directly as the square of t and r= 5 when t = $\frac{1}{3}$

a) Find the relationship between r and t
b) Find t when r = 405
b) Find v when r =5
c) Then r when v= 32

Solution

(a) r α t^2

$r= kt^2$

$5= k(\frac{1}{3})^2$

$5= k(\frac{1}{9})$

45= k

:- k = 45

a) $r= 45t^2$
b) $r= 45t^2$

$\frac{405}{45} = \frac{45t}{45}$

Solution

v α r^3

$V = kr^3$

$108 = 3^3k$

$\frac{\overset{4}{\cancel{108}}}{\cancel{27}} = \frac{\cancel{27}k}{\cancel{27}}$

4= k

:- k= 4

a) v= kr^3

$v = 4r^3$

$\sqrt{9}$ = $\sqrt{t^2}$

+3 = t

:- t = -3 or 3

b) $v= 4r^3$

$v=4 (5^3)$

v= 4 x 125

v= 500

Question 9:

Given that V varies as the cube of r and that v = 108 when r = 3

a) Find the law connecting v and r
b) Find v when r= 5
c) Find r when v= 32

Solution

$V \alpha r^3$

$V= k r^3$

$108 = k3^3$

$\frac{108}{2727} = k\frac{27}{27}$

$4 = k$

$\therefore k= 4$

a) $V= kr^3$

$V= 4r^3$

b) $V= 4r^3$

$V= 4 (5)^3$

V= 4 x 125

V= 500

c) $V= 4r^3$

$\frac{32}{4} = \frac{4r^3}{4}$

$3\sqrt{8}= 3\sqrt{r^3}$

2= r

$\therefore$ r= 2

Question 10: Given y α x and y = 54 when x =9

a) Find the relationship between y and x

b) Find y when x = $1\frac{7}{9}$

c) Find x when y = 72

Solution

y α x

y = kx

$\frac{54}{9} = \frac{k9}{9}$

6= k $\Rightarrow$ k= 6

a) y = kx

y = 6x

b) y = 6x

$y = 6^2 \times \frac{16}{9^3}$

$y = \frac{64}{81}^{10}$

c) y = 6x

$\frac{72}{6} = \frac{6x}{6}$

12= x

:- x = 12

Question 11: Given that q varies as the cube root of P and q= 7 when p = 27

a) Find the law connecting q and p.

b) Find p when q= 14

c) Find q when p = $\frac{18}{125}$

Solution

q α $\sqrt[3]{p}$

q = k $\sqrt[3]{p}$

7= k $\sqrt[3]{27}$

$\frac{7}{3} = k\frac{3}{3}$ $\Rightarrow$ k= $\frac{7}{3}$

a) q = k $\sqrt[3]{p}$

q=$\frac{7}{}$ $\sqrt[3]{p}$

b) q= $\frac{3}{7}$ $\sqrt[3]{}$p

c) q = $\frac{7}{3}$ $\sqrt[3]{}$p

$14 = 7\sqrt[3]{p}$

$14 = \underline{7}\sqrt[3]{p}$

$\frac{\cancel{42}^{6}}{\cancel{7}_{1}} = \frac{7\times\sqrt[3]{p}}{7}$

$(6)^3 = (3\sqrt{p})^3$

$216 = p$

:- $p = \underline{216}$

$q = \frac{7}{3}\times\sqrt[3]{\frac{18}{125}}$

$q = \frac{14}{3}$

Question 12:

Given that A varies directly as the square of x and A= 10 when x = 2

a) Find the law connecting A and x
b) Find A when x = 6
c) Find x when A= 50, leaving your answer in surd form

Solution

$A \alpha x^2$

$A = kx^2$

$10 = k2^2$

$\underline{10} = \underline{4k}$
$4 \quad 4$

$\frac{k}{2} = \frac{5}{2}$

a) $A = kx^2$

$A = \frac{5x^2}{2}$

b) $A = \frac{5}{2}x^2$

$A = \frac{5}{2}\times 6^2$

$A = \frac{5}{2}\times 36$

$A = 90$

c) $A = \frac{5}{2}x^2$

$50 = \frac{5x^2}{2}$

$\frac{\overset{20}{\cancel{100}}}{\cancel{5}} = \frac{5x^2}{\cancel{5}}$

$\sqrt{20} = \sqrt{x^2}$

$2\sqrt{5} = x$

$\therefore -x = \underline{2\sqrt{5}}$

Question 13

x α y and x = $17\frac{1}{2}$ when

y= 10 ½

a) Find the equation which connects x and y

b Find x when y = 12

Solution

x α y

x = ky

$\frac{35}{2} = k\frac{21}{2}$

$\frac{\overset{5}{\cancel{70}}}{\cancel{42}} = \frac{k\cancel{42}}{42}$

$\frac{5}{3} = k \quad \Rightarrow \quad k = \frac{5}{3}$

a) x = ky

x= 5y

$x = \frac{5y}{3}$

b) $x = \frac{5y}{3}$

$x = 5 \times 12^4$

x = 20

Question 14: If y varies directly as the square root of x (x, y) always positive) and y = 9 when x =9, Find x when y =15.

Solution

y α √x

y= k√x

9= k√9

$\frac{\overset{3}{\cancel{9}}}{\cancel{3}} = \frac{k\cancel{3}}{\cancel{3}}$ (the value of x)

3 = k k = 3

$y = k\sqrt{x}$

$y = 3\sqrt{x}$

$$\frac{\overset{3}{\cancel{15}}}{3} = \frac{3\sqrt{x}}{3}$$

$(5)^2 = (\sqrt{x})^2$

$25 = x$

$\therefore \ x = 25$

Question 15:

For circular wires of a given length, the electrical resistance varies as the square of their diameters. If the resistance is 0,718 OHMS when the diameter is 2,4mm, Find the Resistance when the diameter is 1, 8 mm (correct to 3 significant figures)

Solution

R= Resistance
D= Diameter

$R \alpha d^2$

$R = kd^2$

$0,718 = k\,(2,4)^2$

$$\frac{0,718}{5,76} = \frac{\cancel{k}5,76}{\cancel{5.76}}$$

$0,12465 = k$ $\quad k = 0,12465$

$R = 0,12465d^2$

$R = 0,12465\,(1,8)^2$

$= 0,12465 \times 3,24$

$= 0,403866$

:-R= 0,404 to 3. S.F

Question 16:

If y varies inversely as x, and y = 2 when x = 3, Find y when x = 6.

Solution

$y \alpha \frac{1}{x}$

$y = \frac{k}{x}$

$2 = \frac{k}{3}$

$2 \times 3 = k \quad \Rightarrow k = 6$

$y = \frac{6}{6}$

$\underline{y = 1}$

Question 17:
P is inversely proportional to Q and P = 5 when Q = 4.
What is the value of Q when P = 25

Solution

$P \alpha \frac{1}{Q}$

$P = \frac{k}{Q}$

$5 = k/4$

$5 = \frac{k}{4}$

$4 \times 5 = k \longrightarrow k = 20$

$P = \frac{20}{Q}$

$P = \frac{20}{Q}$

$25 = \frac{20}{Q}$

$\frac{\cancel{25}^{q}}{\cancel{25}} = \frac{20^4}{\cancel{25}_5}$

$Q = \frac{4}{5}$

Question 18: If a varies inversely as the square of y, and x= 4 when y = ½, what is y when x is 5?

Solution

$x \alpha \frac{1}{y^2}$

$x = \frac{1}{y^2}$

$4 = \frac{k}{\left(\frac{1}{2}\right)^2}$

$4 = \frac{k}{\frac{1}{4}}$

¼

$4 \times ¼ = k$

$1 = k \quad \underline{k = 1}$

$x = \frac{1}{y^2}$

$5 = \frac{1}{y^2}$

$\frac{\cancel{5}y^2}{\cancel{5}} = \frac{1}{5}$

$\sqrt{y^2} = \sqrt{\frac{1}{3}}$

$y = \underline{\pm\sqrt{\frac{1}{5}}}$

Question 19:

The electrical residence R of a wire varies inversely as the square of the radius r use a constant k to show the relation between R and r.

Solution

$R \alpha \frac{1}{r^2}$

$R = \frac{k}{r^2}$

Question 20:
P. Varies inversely as the square root of v and P = 4,5 when v= 25. Find v when P= 15

Solution

$P \alpha \frac{1}{\sqrt{v}}$

$4,5 = \frac{k}{\sqrt{25}}$

$22, 5 = k$

$P = \frac{22,5}{\sqrt{v}}$

$\frac{15}{15}\sqrt{v} = \frac{22,5}{15}$

$\left(\sqrt{v}\right)^2 = (1,5)^2$

v= 2, 25

Question 21
X varies inversely as the cube root of y and x = 4 when
Y = 125

a) Find the law connecting x and y
b) Find x when y = 64
c) Find y when x = 2,5

Solution

$x \alpha \frac{1}{\sqrt[3]{y}}$

$x = \frac{k}{\sqrt[3]{y}}$

$4 = \frac{k}{\sqrt[3]{125}}$

$4 = \frac{k}{5}$

$\underline{k = 20}$

c) $x = \frac{20}{\sqrt[3]{y}}$

$\frac{2,5}{25} \times \sqrt[3]{y} = \frac{20}{2,5}$

$(\sqrt[3]{y})^3 = (8)^3$

$\underline{y = 512}$

b) $x = \frac{20}{\sqrt[3]{64}}$

$x = \frac{20^5}{4}$

$\underline{x = 5}$

Question 22:

Given $A \alpha \frac{1}{B}$ and A= 5 when $B = \frac{1}{3}$

a) Find the law connecting A and B
b) Find A when B = 7
c) Find B when A= 25

Solution

$A \alpha \frac{1}{B}$

$A = \frac{k}{B}$

$5 = \frac{k}{\frac{1}{3}}$

5x 1/3= k $k = \frac{5}{3}$

b) $A = \frac{5}{3B}$

$A = \frac{5}{3 \times 7}$

$A = \frac{5}{21}$

a) $A = \frac{5}{3B}$

:- $A = \frac{5}{3B}$

c) $A = \frac{5}{3B}$

$25 = \frac{5}{3B}$

$25 \times 3B = 5$

$\frac{75B}{75} = \frac{5}{75}$

$B = \frac{1}{15}$

Question 23:

V varies inversely as the square of W and v= 7.
When W = 3. Find W when v= 1 ¾

$V \propto \frac{1}{w^2}$

$V = \frac{63}{W^2}$

$V = \frac{K}{W^2}$

$7 = \frac{k}{3^2}$

$9 \times 7 = k$ $\quad$ $k = 63$

:- $v = \frac{63}{w^2}$

$\frac{7}{4} = \frac{63}{w^2}$

$7w^2 = 63 \times 4$

$\frac{7w^2}{7} = \frac{252}{7}$

$\sqrt{w^2} = \sqrt{36}$

$\underline{W = 6}$

Question 24:

A varies inversely as the cube root of B and A= 5 when B = 216.
Find A when B = 3, 375

Solution

$A \alpha \frac{1}{\sqrt[3]{B}}$

$A = \frac{k}{\sqrt[3]{B}}$

$5 = \frac{k}{\sqrt[3]{216}}$

$5 = \frac{k}{6}$

$6 \times 5 = k \therefore \quad \underline{k = 30}$

$A = \frac{30}{\sqrt[3]{B}}$

$A = \frac{30}{\sqrt[3]{3,375}}$

$A = \frac{30}{1,5}$

$\therefore \underline{A = 20}$

Question 25

C varies as the cube root of D and D = 125 when C= 2. Find C when D= 8

Solution

$C \ \alpha \ \frac{1}{\sqrt[3]{D}}$

$2 = \frac{k}{\sqrt[3]{D}}$

$2 = \frac{k}{\sqrt[3]{125}}$

$2 = \frac{k}{5}$

$2 \times 5 = k \ \therefore \quad \underline{k = 10}$

$C = \frac{10}{\sqrt[3]{D}}$

$C = \frac{10}{\sqrt[3]{8}}$

$C = \frac{10}{2}$

$\underline{C = 5}$

Question 26: x, y and Z are related quantities such that x varies directly as y and inversely at the square root Z when x = 300 and y = 65, Z = 25. Calculate the value of x when y = 468 and Z = 144.

Solution

$x \alpha y \times \frac{1}{\sqrt{Z}}$

$x = \frac{ky}{\sqrt{Z}}$

$300 = \frac{k \times 65}{\sqrt{25}}$

$300 = \frac{kx\ 65}{5}$

$\therefore 3900 = k$

$x = \frac{3900y}{\sqrt{Z}}$

$x = \frac{3900 \times 468}{\sqrt{144}}$

$x = \frac{1825200}{12}$

$x = 15\ 2100$

Question 27:

V varies directly as the square of D and inversely as H. Given that V= 40 when H= 10 and D = 4.

a) Find V when D = 5 and H= 7

b) Find D when V= 75 and H= 3

Solution

(a) $V \alpha \frac{D^2}{H}$

$V = \frac{KD^2}{H}$

$40 = \frac{k \times 4^2}{10}$

a) $V = \frac{25D^2}{H}$

$V = \frac{25 \times 25}{7}$

$40 = \frac{k \times 16}{10}$

$40 \times 5 = 8k$

$\frac{\cancel{200}^{25}}{\cancel{8}} = 8k$

$25 = k \quad \therefore \quad k = 25$

b) $V = \frac{25D^2}{H}$

$75 = \frac{25 \times D^2}{3}$

$75 \times 3 = 25D^2$

$\frac{225}{25} = \frac{25}{25}D^2$

$\sqrt{9} = \sqrt{D^2}$

$\therefore$ D= -3 or +3

$V = \frac{635}{7}$

V= 89, 29

Question 28: If W varies jointly as the square of x and as y, Find the law that connects W, x and y. Given W= 72, x = 3 and y =4 use the law to Find:-

a) w when x = 13 and y =50
b) x when W= 1350 and y= 3

Solution

a) $W \alpha x^2y$

$W = kx^2y$

$72 = k \times 3^2 \times 4$

$72 = k \times 36$

$\frac{\cancel{72}^{2}}{\cancel{36}} = \frac{k\cancel{36}}{\cancel{36}}$

$2 = k \quad \therefore \quad k = 2$

$W = 2x^2y$

$W = 2 \times 13^2 \times 50$

$W = 2 \times 169 \times 50$

W= 16 900

b) $W = 2x^2y$

$1350 = 2 \times x^2 \times 3$

$$\frac{1350}{6} = \frac{6x^2}{6}$$

$$\sqrt{225} = \sqrt{x^2}$$

$$\pm 15 = x$$

$\therefore$ <u>x= 15 or 15</u>

Question 29:
P Varies Directly as Q and inversely as R. Given that P= 37,5 when Q= 76 and R= 2, Find R when P = 25 and Q = 3,5

<u>Solution</u>

$P \alpha Q \times \frac{1}{R}$ $\qquad$ $P = \frac{KQ}{R}$

$P = \frac{KQ}{R}$ $\qquad$ $37,5 = \frac{k76}{2}$

$$\frac{37,5}{38} = \frac{k\cancel{38}}{\cancel{38}}$$

$0,987 = k$ $\quad \therefore$ $\quad$ <u>k= 0,987</u>

$$P = \frac{0,987Q}{R}$$

$$25 = \frac{0,987}{R} \times 3.5\,25$$

<u>R= 0,138</u>

<u>Question 30:</u>
The mass M of a steel rod of uniform circular cross- section varies jointly as the square of the radius and length ℓ. A rod of radius 2cm has a mass of 420g. Its length is 15cm.

a) Find the mass of a rod of radius 3cm and length 10cm.

b) Find the radius of a rod having a mass of 180g if its length is $10\frac{2}{7}$ cm

Solution

$M \alpha r^2 \ell$

$M = Kr^2 \ell$

$420 = k\ 2^2 \times 15$

$7 = k \ \therefore \ k = 7$

$\frac{420}{60} = \frac{k60}{60}$

a) $M = 7r^2 \ell$

$M = 7 \times 3^2 \times 10$

$M = 7 \times 9 \times 10$

$M = 630g$

b) $M = 7r^2 \ell$

$180 = 7 \times r^2 \times \frac{72}{7}$

$\frac{180}{72} = \frac{72r^2}{72}$

$\sqrt{2,5} = \sqrt{r^2}$

$1,58 = r$

:- r = 1,58cm

Question 30:

The cost of giving a partly is party constant and partly varies as the number of people invited. The cost for 20 people is $90 and the cost for 30 people is $110.

a) Find the cost if there are 40 people.

b) If the person giving the party does not want to spend more than $200, what is the maximum number of guests he should invite.

Solution

$C = a + kn$

$90 = a + k\ 20$ (i)

$(110 = a + k30)$ (ii)

a) $C = 50 + 2N$

$C = 50 + 2\ (40)$

$C = 50 + 80$

$C = \$130$

Subtract (i) from (ii) you get

$\frac{20}{10} = \frac{k10}{10}$

$2 = k$

$k = 2$

$90 = a + 2\ (20)$

b) $C = 50 + 2N$

$200 = 50 + 2N$

$200 - 50 = 2N$

$150 = 2N$

$75 = N$

90 = a + 40
90 – 40 = a
50 = a ∴ a= 50

Maximum number of guests 75

Question 31

It is given that y is partly constant and partly varies directly as x. Given that when x = 2,y = 6 and when x= 6, y= -2. Find an expression for y in terms of x.

Solution

y = a + kx

6 = a + k2 (i)

-2 = a + k6 (ii)

subtract (ii) from (i)

$$\frac{8}{-4} = \frac{-4k}{-4}$$

- = k

6= a + k2
6 = a + -2(2)
6= a -4
6 +4 = a
10 = a
y = a + kx
y= 10- 2x

Question 32

R is partly constant and partly varies as the square of V. Given that R= 100 when V= 3, and R= 80 when V= 2. Find the law that connects R and V. Find V when R= 164.

Solution

$R= a + kv^2$

100= a + k9 (i)

80 = a +k4 (ii)

subtract (ii) from (i)

$$\frac{20}{5} = \frac{5k}{5}$$

4= k

80= a + 4(4)

80- 16 = a

64= a

$R= 64 +4V^2$

$164= 64 + 4V^2$

$$\frac{100}{4} = \frac{4V^2}{4}$$

$\sqrt{25} = \sqrt{V^2}$

∴ V = 5

CHAPTER 6

MATRICES
Simplify the following matrices

a) $\begin{pmatrix} 5 & -1 & 3 \\ -9 & 3 & 0 \\ 8 & 0 & -7 \end{pmatrix} + \begin{pmatrix} 2 & 6 & 1 \\ -6 & -2 & 8 \\ -5 & 5 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 0 & -1 \\ 9 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 3 \\ 5 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 11 \\ -2 & -5 \end{pmatrix}$

Solution

a) $\begin{pmatrix} 5 & -1 & 3 \\ -9 & 3 & 0 \\ 8 & 0 & -7 \end{pmatrix} + \begin{pmatrix} 2 & 6 & 1 \\ -6 & -2 & 8 \\ -5 & 5 & 0 \end{pmatrix}$

$= \begin{pmatrix} 5+2 & -1+6 & 3+1 \\ -9-6 & 3+-2 & 0+8 \\ 8+(-5) & 0+(-5) & -7+0 \end{pmatrix}$

$= \begin{pmatrix} 7 & 5 & 4 \\ -15 & 1 & 8 \\ 3 & -5 & -7 \end{pmatrix}$

b) $\begin{pmatrix} 0 & -1 \\ 9 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 3 \\ 5 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 11 \\ -2 & -5 \end{pmatrix}$

$= \begin{pmatrix} 0-3+4 & -1-3+11 \\ 9-5-2 & 2-4-5 \end{pmatrix}$

$= \begin{pmatrix} 1 & 7 \\ 2 & -7 \end{pmatrix}$

Question 2.

If A= $\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$ and B= -$\begin{pmatrix} -1 & 0 \\ 3 & 1 \end{pmatrix}$

Find 3A-2B

Solution

3A- 2B

$= \quad 3\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} - 2\begin{pmatrix} -1 & 0 \\ 3 & 1 \end{pmatrix}$

$= \quad \begin{pmatrix} 6 & -3 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} -2 & 0 \\ 6 & 2 \end{pmatrix}$

$= \quad \begin{pmatrix} 6-(-2) & -3-0 \\ 0-6 & 3-2 \end{pmatrix}$

$= \quad \begin{pmatrix} 8 & -3 \\ -6 & 1 \end{pmatrix}$

Question 3

If M= $\begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$ and N= $\begin{pmatrix} 5 & -2 \\ -1 & 4 \end{pmatrix}$

Find the value of the following
a) M + N
b) M- N
c) MN
d) NM

Solution

a) M + N

$= \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ -1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1+5 & 5-2 \\ 2+1 & 0+4 \end{pmatrix}$

$= \begin{pmatrix} 5 & -2 \\ -1 & 4 \end{pmatrix}$

b) M- N

$\begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix} - \begin{pmatrix} 5 & -2 \\ -1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1-5 & 3-(-2) \\ 2-(-1) & -4 \end{pmatrix}$

$= \begin{pmatrix} -4 & 5 \\ 3 & -4 \end{pmatrix}$

c) MN

d) NM

$= \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 5 & -2 \\ 2 & 0 \end{pmatrix}$ $= \begin{pmatrix} 5 & -2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$

$= \begin{pmatrix} 1\times5+3\times(-1) & 1\times(-2)+3\times4 \\ 2\times5+0\times(-1) & 2\times(-2)+0\times4 \end{pmatrix}$ $= \begin{pmatrix} 5\times1+(-2)\times2 & 5\times3+(-2)\times0 \\ -1\times1+4\times2 & -1\times3+4\times0 \end{pmatrix}$

$= \begin{pmatrix} 5+(-3) & -2+12 \\ 10+0 & -4+0 \end{pmatrix}$ $= \begin{pmatrix} 5+(-4) & 15+0 \\ -1+8 & -3+0 \end{pmatrix}$

$= \begin{pmatrix} 2 & 10 \\ 10 & -4 \end{pmatrix}$ $= \begin{pmatrix} 1 & 15 \\ 7 & -3 \end{pmatrix}$

Question 4

P$= \begin{pmatrix} -3 & a \\ b & -1 \end{pmatrix}$ and Q $= \begin{pmatrix} -2 & 2 \\ 1 & 3 \end{pmatrix}$

If P + Q = $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

Find The values of a and b.

Solution

P + Q $= \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

$\begin{pmatrix} -3 & a \\ b & -1 \end{pmatrix} + \begin{pmatrix} -2 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

$\therefore \begin{pmatrix} 3+-2 & a+2 \\ b+1 & -1+3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

$\begin{pmatrix} 1 & a+2 \\ b+1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

a+2 = 0 ... (i)
b+ 1 = 0 ..(ii)

From (i) a+2 = 0 a -2
(iii) b +1 = 0 b = -1

$\therefore$ a = -2 and b = -1

Question 5

If $\begin{pmatrix} 2 & -1 \\ 0 & a \end{pmatrix}\begin{pmatrix} b & o \\ 1 & 6 \end{pmatrix}=\begin{pmatrix} 3 & c \\ 3 & d \end{pmatrix}$

Find the values of a, b, c and d.

Solution

$$\begin{pmatrix} 2 & -1 \\ 0 & a \end{pmatrix}\begin{pmatrix} b & o \\ 1 & 6 \end{pmatrix}=\begin{pmatrix} 3 & c \\ 3 & d \end{pmatrix}$$

$$=\therefore\begin{pmatrix} 2\times b+1\times 1 & 2\times 0+(-1\times 6) \\ 0\times b+a\times 1 & 0\times 0+a\times 6 \end{pmatrix}=\begin{pmatrix} 3 & c \\ 3 & d \end{pmatrix}$$

$$=\begin{pmatrix} 2b-1 & -6 \\ a & 6a \end{pmatrix}=\begin{pmatrix} 3 & c \\ 3 & d \end{pmatrix}$$

$\therefore$ 2b -1 = 3
a = 3
c = -6
6a = d :-
$\therefore$ b = 2
a = 3
c = -6
d = 18

Question 6.

The matrices A= $\begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix}$ and B=$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$

AB= A + B. Find the values of a, b and c

Solution

AB= $\begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$

$= \begin{pmatrix} 4a+0 & 4b+0 \\ 0+0 & 0+5c \end{pmatrix} = \begin{pmatrix} 4a & 4b \\ 0 & 5c \end{pmatrix}$

A + B $= \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} + \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 4+a & b \\ o & 5+c \end{pmatrix}$

AB = A + B

$\begin{pmatrix} 4a & 4b \\ 0 & 5c \end{pmatrix} = \begin{pmatrix} 4+a & b \\ 0 & 5+c \end{pmatrix}$

$\frac{4c}{4} = \frac{5}{4}$ $\therefore$ c = 5

4a = 4 +a
4a-a= 4
$\frac{3a}{4} = \frac{4}{4}$ $\therefore$ a = 4

4b = b
4b- b= 0
$\frac{3b}{3} = \frac{3}{3}$
$\therefore$ b = 0

Question 7

If A = $\begin{pmatrix} -1 & 5 \\ 2 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} 6 & 0 \\ 4 & -8 \end{pmatrix}$

Find:-

a) 3A b) -2A

c) $\frac{1}{2}$ B d) A- 3B

Solution

3A=3 $\begin{pmatrix} -1 & 5 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 3\times(-1) & 3\times 5 \\ 3\times 2 & 3\times 3 \end{pmatrix} = \begin{pmatrix} -3 & 15 \\ 6 & 9 \end{pmatrix}$

b) -2A = -2$\begin{pmatrix} 6 & 0 \\ 4 & -8 \end{pmatrix}$

$$= \begin{pmatrix} -2\times(-1) & 4\times 5 \\ 3\times 2 & 3\times 3 \end{pmatrix} = \begin{pmatrix} 2 & -10 \\ -4 & -6 \end{pmatrix}$$

c) $\frac{1}{2}$ B

$$= \frac{1}{2}\begin{pmatrix} 6 & 0 \\ 4 & -8 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}\times 6 & \frac{1}{2}\times o \\ \frac{1}{2}\times 4 & \frac{1}{2}(-8) \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 \\ 2 & -4 \end{pmatrix}$$

d) A-3B

$$= \begin{pmatrix} -1 & 5 \\ 2 & 3 \end{pmatrix} - 3\begin{pmatrix} 6 & 0 \\ 4 & -8 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 5 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 18 & 0 \\ 12 & -24 \end{pmatrix}$$

$$= \begin{pmatrix} -1-8 & 5-0 \\ 2-12 & -3(-24) \end{pmatrix}$$

$$= \begin{pmatrix} -19 & 0 \\ -10 & 27 \end{pmatrix}$$

Question 8

If M= $\begin{pmatrix} 4 & -6 \\ -1 & 2 \end{pmatrix}$

a) Find the value of the determinant of M.
b) Hence write down the inverse of M

Solution

a) Det of M = $(2 \times 4) - (-6) \times (-1)$
= 8-6
= 2

b) Inverse of M= $\frac{1}{2}\begin{pmatrix} 4 & 6 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ \frac{1}{2} & 1 \end{pmatrix}$

Question 9

The value of the determinant of the matrix $\begin{pmatrix} 5 & -2 \\ -4 & x \end{pmatrix}$ is 7

a) Find the value of x
b) Hence write down the inverse of the matrix

Solution

a) $5 \times x - (-2) \times (-4) = 7$
$5x - 8 = 7$
$\frac{5x}{5} = \frac{15}{5}$
$\therefore x = 3$ -

b) Inverse of the matrix

= $\frac{1}{7}\begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}$

Question 10

Find the value of k for which the matrix $\begin{pmatrix} 4 & k-2 \\ 8 & 6 \end{pmatrix}$

Does not have an inverse

Solution

Note:
If the matrix does not have. An inverse, its determinant is equal to zero i.e

$\det\begin{pmatrix} 4 & k-2 \\ 8 & 6 \end{pmatrix} = 0$

Det:- $4 \times 6 - 8(k-2) = 0$
$24 - (8k-16) = 0$
$24 - 8k + 16 = 0$

24 + 16 -8k= 0

$\frac{\cancel{40}}{\cancel{8}} = \frac{\cancel{8}}{\cancel{8}}k$

5 = k

$\therefore$ k =5

Question 11

Find a and b if: $\begin{pmatrix} 3 & 7 \\ b & a \end{pmatrix}\begin{pmatrix} a & -7 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ where $\mathrm{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity matrix

Solution

$$\begin{pmatrix} 3 & 7 \\ b & a \end{pmatrix}\begin{pmatrix} a & -7 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3\times a + 7\times(-1) & 3\times(-7) + (7\times 3) \\ b\times a + (a\times(-1) & b\times(-7) + a\times 3 \end{pmatrix} = \mathrm{I}$$

$$= \begin{pmatrix} 3a - 14 & -2 + 21 \\ ab - 2a & -7 + 3a \end{pmatrix} = \mathrm{I}$$

hence

but $\mathrm{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

then

$$\begin{pmatrix} 3a - 14 & -2 + 21 \\ ab - 2a & -7 + 3a \end{pmatrix} = \mathrm{I}$$

3a-14= 1

$\frac{3a}{3} = \frac{15}{3}$ $\therefore$ a = 5

ab- 2a = 0 but a =5

$5\times$b -2 (5) =0

5b -10 = 0

$\frac{5a}{5} = \frac{10}{5}$

$\therefore$ a = 5 and b =2

Question 12

A = $\begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}$ B= $\begin{pmatrix} \frac{1}{4} & k \\ 0 & \frac{1}{3} \end{pmatrix}$ and

C= $\begin{pmatrix} 12 & 4 \\ -9 & M \end{pmatrix}$

a) Evaluate A^2
b) Find the value of k which makes AB the identity matrix
c) Find the value of M which makes the determinant of A equal to the determinant of C.

Solution

a) $A^2 = \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}$

$= \begin{pmatrix} 4\times4+2\times0 & 4\times2+2\times3 \\ 0\times4+3\times0 & 0\times2+3\times3 \end{pmatrix}$

then $\begin{pmatrix} 16+0 & 8+6 \\ 0+0 & 0+9 \end{pmatrix} = \begin{pmatrix} 16 & 14 \\ 0 & 9 \end{pmatrix}$

$\therefore A^2 = \begin{pmatrix} 16 & 14 \\ 0 & 9 \end{pmatrix}$

b) AB = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\therefore \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} k & -k \\ 0 & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 4\times\frac{1}{4}+2\times0 & 4\times k+2\times\frac{1}{3} \\ 0\times\frac{1}{4}+0\times3 & 0\times(-k)+3\times\frac{1}{3} \end{pmatrix}$

$= \begin{pmatrix} 1 & 4k+\frac{2}{3} \\ 0 & 1 \end{pmatrix}$ then

c) Det A = $4\times3-2\times0$

= 12

Det C = 12 x m-4x(-9)

= 12m+36

hence 12m +36 = 12

i.e. m = -2

$$\begin{pmatrix} 1 & 4k+\frac{2}{3} \\ 0 & 1 \end{pmatrix} = \text{I} \quad \text{i.e.}$$

$$\begin{pmatrix} 1 & 4k+\frac{2}{3} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$4\text{k} + \frac{1}{3} = 0$$

$$\frac{4k}{4} = \frac{2}{3} \times \frac{1}{4} - \therefore \text{k} = \frac{1}{6}$$

Question 13

$\text{A} = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$ and $\text{B} = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$

a) Find A + 2B

b) Given that $\text{A}\begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 27 \end{pmatrix}$

Find The value of x and y

Solution

a) A + 2B

$$= \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} + 2\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ -2 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 2+2 & 0+2 \\ 3+(-2) & 1+6 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 4 \\ 1 & 7 \end{pmatrix}$$

b) $\begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 2y \end{pmatrix}$

$$\begin{pmatrix} 2 \times x + 0 \times 2 \\ 3 \times x + 1 \times 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 2y \end{pmatrix}$$

$2x = 8$

$3x+2y = 8$

$x = 4$

hence $3 \times 4 + 2 = 2y$ i.e.

$2y = 14$

$\therefore y = 7$

Question 14

Given that the value of the determinant of the matrix
$\begin{pmatrix} x & -3 \\ -1 & 2 \end{pmatrix}$ is 5
Find the value of x and hence write down the inverse of the matrix
Solution

Det $x \times 2 - (-3) \times (-1) = 5$
$2x - 3 = 5$
$\frac{2x}{2} = \frac{8}{2}$
$\therefore x = 4$

The matrix is $\begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix}$

The inverse = $\frac{1}{5}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

Question 15
Find a, b, c, such that

$$\begin{pmatrix} a & b \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 9 \\ 5 & 0 \end{pmatrix} - \begin{pmatrix} 4 & -6 \\ 3 & 2c \end{pmatrix}$$

Solution

$$\begin{pmatrix} a\times 0 + b\times 1 & a\times 3 + b\times(-1) \\ 0\times 0 + 2\times 1 & 0\times 3 + 2\times(-1) \end{pmatrix} = \begin{pmatrix} 1-4 & 9-(-6) \\ 5-3 & 0-x \end{pmatrix}$$

$$\therefore \begin{pmatrix} b & 3a-b \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} -3 & 15 \\ 2 & -2c \end{pmatrix}$$

hence b=- -3 : 3a-b = 15
-2 = -2c hence

3a –(-3) = 15

3a = 15-3

$3a = 12$

$$\frac{3a}{3} = \frac{12}{3}$$

$a = 4$

$a = 4$, $b = -3$ and $c = 1$

CHAPTER 7

Formulae and Substitution

Question 1:

If $P = 3x^2 - 4x^2 - 4x + 2$ Find the value of P when x = :

a) -2
b) 0
c) 4

Solution

a) $P = 3x^2 - 4x + 2$
$P = 3(-2)^2 - (4(-2) + 2$

b) $P = 3(0)^2 - 4(0) + 2$
$P = 0 - 0 + 2$

P= 12 + 8 +2
P= 22 P= 2

c) $P = 3(4)^2 - 4(4) + 2$
$P = 48 - 16 + 2$
$P = 48 + 2 - 16$
$P = 34$

Question: 2

Given that a= 5, b = -4,

C= $\frac{1}{2}$, d = 2, n = -3

Find the values of: -

a) $2abc - n^2$

b) $2a^2 - 3ab + b^2$

c) $a^d - c^b$

Solution

a) $2abc - n^2$

$= 2 \times 5 \times (-4) \times \frac{1}{2} - (-3)^2$

$= -20 - 9$

$= -29$

b) $2a^2 - 3ab + b^2$

$2(5^2) - 3(5 \times -4) + (-4)^2$

$50 + 60 + 16$

$= 126$

c) $a^d - c^b$

$= 52 - (\frac{1}{2})^{-4}$

$= 25 - 16$

$= 9$

Question 3:

If m= 3, n= -4, c= -2, and d= -1

Find The value of:

a) $\frac{c^2 - n}{m}$ b) $\frac{md}{2n}$

d) $3m^c - 4d$

Solution

a) $\frac{c^2 - n}{m} = \frac{(-2)^2 - (-4)}{3}$

$= \frac{4+4}{3}$

$= \frac{8}{3}$

b) $\frac{md}{2n} = \frac{3\times(-1)}{2\times(-4)}$

$= \frac{-3}{-8}$

$= \frac{3}{8}$

c) $3m^c - 4d = 3(3^{-2}) - 4(-1)$

$= 3(\frac{1}{9}) + 4$

$= \frac{1}{3} + 4$

$= 4\frac{1}{3}$

Question 4:

Make x the subject of the following equations:

a) $\frac{a}{x} + b = c$

b) $m = \sqrt{\frac{1+x}{1-x}}$

Solution

a) $\frac{a}{x} + b = c$

i.e. $\frac{a+bx}{x} = c$

$a + bx = cx$

$a = cx - bx$

$a = x(c - b)$

$\frac{a}{c-b} = \frac{x(c-b)}{c-b}$

$\frac{a}{c-b} = x$

b) $m = \sqrt{\frac{1+x}{1-x}}$

$m^2 = \left(\sqrt{\frac{1+x}{1-x}}\right)^2$

$m^2 = \frac{1+x}{1-x}$

$\therefore \frac{a}{c-b} = x$

$m^2 (1- x)= 1 +x$

$m^2-m^2x= 1 + x$

$\frac{m^2-1}{m^2+1} = \frac{x(m^2+1)}{m^2+1}$

$\therefore x = \frac{m^2-1}{m^2+1}$

Question 5:

Make **a** the subject of the formulae.

$b= \frac{1}{3} \sqrt{x^2- a^2}$

Solution

$b = \frac{1}{3} \sqrt{x^2- a^2}$

$(3b)^2 = (\sqrt{x^2- a^2})^2$

$9b^2 = x^2 – a^2$

$\sqrt{a^2} = \sqrt{x^2 – 9b^2}$

$a= \sqrt{x^2 -9b^2}$

Question 6:

Make x the subject of

The Formulae, $\sqrt{x + 3} = W$

Solution

$(\sqrt{x +3})^2 = (W)^2$

$x +3 = W^2$

$x= W^2- 3$

Question 7:

Make N the subject of the formulae

$\sqrt{m+\frac{N}{2}} =R$

Solution

$\left(\sqrt{M+\frac{N}{2}}\right)^2 = R^2$

$M+ \frac{N}{2} = R^2$

$2M + N = 2R^2$

$N= 2R^2- 2M$

$N= 2 (R^2- M)$

Question 8: Make P and M the subject of the formula

$M^2 + 3pq = n + p$

Solution

$M^2 + 3pq = n+p$

$M^2 - n = P - 3pq$

$$\frac{M^2 - n}{1-3q} = \frac{p(1-3q)}{1-3q}$$

$$\therefore p = \frac{M^2 - n}{1-3q} :-$$

$M^2 + 3pq = n + P$

$M^2 = n + p - 3pq$

$$M = \sqrt{n + p - 3pq}$$

Question 9:

The sum of the squares of the firsts S_n integers is given by:-

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

Calculate:-

a) S_{30}

b) The sum of the squares from 31 to 50 inclusive

Solution

a) S_{30} means the value of S_n when n= 30

$$S_{30} = \frac{30(03+1)(2\times 30+1)}{6}$$

$$= \frac{30\times 31\times 61}{6}$$

$$= \frac{5673}{6}$$

$$6$$

$$= \underline{9455}$$

b) The sum of squares from 31 to 50:- Sum of squares from 1 to 50- sum of squares from 1 to 30

$$= S_{50} - S_{30}$$

$$S_{50} = \frac{50(50+1)(2\times 50+1)}{6}$$

$$= \frac{50\times 51\times 101}{6}$$

$$= \underline{42925}$$

$\therefore S_{50} - S_{30} =$ 42925- 9455

$=$ 33470

Question 10:

The volume V of A container which consists of a right cylinder base of radius r and height h surmounted by a hemisphere of the same radius is given by $V = xr^2(h + \frac{2}{3r})$

a) Make h in the subject of the formulae
b) Find h if V= 35 9 1/3m^3, r= 3
$\pi = 22$

Solution

$V = xr^2(h + \frac{2}{3r})$

$\frac{V}{\pi r^2} = h + \frac{2r}{3}$

$\frac{V}{\pi r^2} - h = \frac{2r}{3}$

$\therefore h = \frac{3V - 2\pi r^3}{3\pi r^2}$

b) $\therefore h = \frac{3V - 2\pi r^3}{3\pi r^2}$

$= \frac{3(1078)}{3} - \frac{2 \times 22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{1078 - 269,5}{115,5}$

$= \frac{808,5}{115,5}$

$= 7$m

Question 11

The formula F= 9c +32 shows the relationship between temperature in degrees Fahrenheit (F) and degrees Celsius (c). Find
a) F when C= 80
b) Make C the subject of the formulae
c) C when F= 50.

Solution

a) F = $\frac{9c}{5}$ + 32

b) F = $\frac{9c}{5}$ + 32

F = $\frac{9\times80}{5}+32$

F = $\frac{720}{5}+32$

F = 144+32

F = 176

5F = 9c + 32

$\frac{5F-160}{9}=\frac{9c}{9}$

$\therefore c=\frac{5F-160}{9}$

c) $c=\frac{5F-160}{9}$

c = $\frac{5\times50-160}{9}$

c = $\frac{250-160}{9}$

$\therefore c=\frac{90}{9}$

c = 10

Question 12:

Make U the subjects of the formulae:

$A=\frac{1}{2}M(V^2-U^2)$

Solution

$A=\frac{1}{2}M(V^2-U^2)$

$\frac{2A}{M}=\frac{M(V^2-U^2)}{M}$

$U^2=V^2-\frac{2A}{M}$

$\sqrt{U^2}=\sqrt{V^2-\frac{2A}{M}}$

$\therefore U=\sqrt{V^2-\frac{2A^2}{M}}$

Question 13:

Make x the subject of the formula in:-

a) $a\sqrt{x}=b$

b) $\sqrt{x^2+a^2}=3a$

Solution

a) $a\sqrt{x}=b$

$\left(a\sqrt{x}\right)^2=b^2$

$\therefore x=\frac{b^2}{a^2}$

b) $\left(\sqrt{x^2+a^2}\right)^2=(3a)^2$

$x^2+a^2=9a^2$

$x^2=9a^2-a^2$

$\sqrt{x^2}=\sqrt{8a^2}$

$x=\sqrt{8}\times\sqrt{a^2}$

Question 14:

The period of a compound pendulum is given by:

$T = \frac{2\pi\sqrt{h^2 + y^2}}{bh}$ Express

y in terms of T, h and b taking π as 10.

Solution

Tbh = $2\pi\sqrt{x^2 + y^2}$

$(Tbh)^2 = \left(2\pi\sqrt{x^2 + y^2}\right)^2$

$(Tbh)^2 = 4\pi^2(x^2 + y^2)$

$\therefore y^2 = \frac{T^2b^2h^2}{4\pi^2} - x^2$

$y = \sqrt{\frac{T^2b^2h^2}{4\pi^2} - 4\pi^2h^2}$

$y = \frac{1}{40}\sqrt{T^2b^2h^2 - 400h^2}$

Question 15: The formula

$\mathbf{A = P}\left(\mathrm{I} + \frac{RT}{100}\right)$

gives the total money, A, that a a principal P, amounts to in T years at R% simple interest.

Find the amount that a principal of $1500 becomes if invested for 10 years at $6\frac{1}{2}$ % simple interest per annum

Solution

$A = P\left(\mathrm{I} + \frac{RT}{100}\right)$

$= 1500(1 + \frac{13 \times 10}{100})$

$= 1500(1 + \frac{13 \times 10}{100})$

$= 1500(1 + \frac{130}{100})$

$= 1500(1 + \frac{13}{20})$

$= 1500(1,65)$

$= \$2475$

P, amounts to in T years at R% simple interest per annum. Find the amount that

principal of $1500 becomes if invested for 10 years 6 ½ % simple interest per annum.

Solution

$A = P(\frac{1 + RT}{100})$

Question 16: Find the value of

$2\pi\sqrt{\frac{l}{g}}$ when $\pi = 3\frac{1}{7}$

l = 98 and g = 32

Solution

Hence $2 \times \frac{22}{7}\sqrt{\frac{l}{g}}$

$= 2\times\frac{22}{7}\times\sqrt{\frac{49}{16}}$

$= 2\times\frac{22}{7}\times\frac{7}{4} = 11$

Question 17

Given that y = 2πr,

a) Find y when π = 3, 142 and r = 45

b) Make r the subject of the subject of the formula, hence find r when y = 429 and x = $3\ ^{1}/_{7}$

Solution

a) y = 2πr

y = 2 x 3, 142 x 45

y = 282, 78

b) y = 2πr

y = 2πr

$\frac{y}{2\pi} = \frac{2\pi r}{2\pi}$

r $= \frac{y}{2\pi}$

r $= \frac{y}{2\pi}$

r $= \frac{429}{2\times 22/7}$

Question 18

a) Make M the subject of the formula $\frac{m}{a}+\frac{n}{b}=1$

b) Hence, if a= 4, b= 1 and n = -2, evaluate m.

Solution

a) $\frac{m}{a}+\frac{n}{b}=1$

$\frac{mb+an}{ab}=1$

hence mb + an = ab

$= 429 \div \frac{44}{7}$

$= 429 \times \frac{7}{44}$

$= \frac{3003}{44}$

$= 68{,}25$

$\underline{mb} = ab - \underline{an}$

$mb = \underline{ab - an}$

$\therefore\ m = \frac{ab - an}{b}$

b) $m = \frac{ab - an}{b}$

$= \frac{4(1) - 4(-2)}{1} = 4 + 8 = 12$

Question 19

The length of the hypotenuse L in a right angled triangle is given by the formula

$L = \sqrt{x^2 + y^2}$

where x and y are the lengths of the other two sides of the triangle.

a) Make x the subject of this formula

b) Hence find x if L= 68 and y = 32

Question 20

The simple interest I, on a sum of money P after T years at . R% is given by the formula.

$I = \frac{PRT}{100}$

a) Make T the subject of the formula

b) Find T if I = 102, P = 510 and R = 2 ½

Solution

Solution

a) $L = \sqrt{x^2 + y^2}$

$\therefore (L^2) = (x^2 + y^2)^2$

$L^2 = x^2 + y^2$

$L^2 - y^2 = x^2$

$\sqrt{x^2} = \sqrt{L^2 - y^2}$

$\therefore x = \sqrt{L^2 - y^2}$

$x = \sqrt{68 - 32}$

$x = \sqrt{36}$

$\therefore\ \underline{x = 6}$

a) $I = \frac{PRT}{100}$

$100I = PRT$

$\therefore T = \frac{100I}{PR}$

b) $\therefore T = \frac{100\text{I}}{\text{P}\,R}$

$$\text{T=} \quad \frac{100\times102}{510\times{}^{5}\!/_{2}}$$

$$= \frac{2\times100\times102}{510\times5}$$

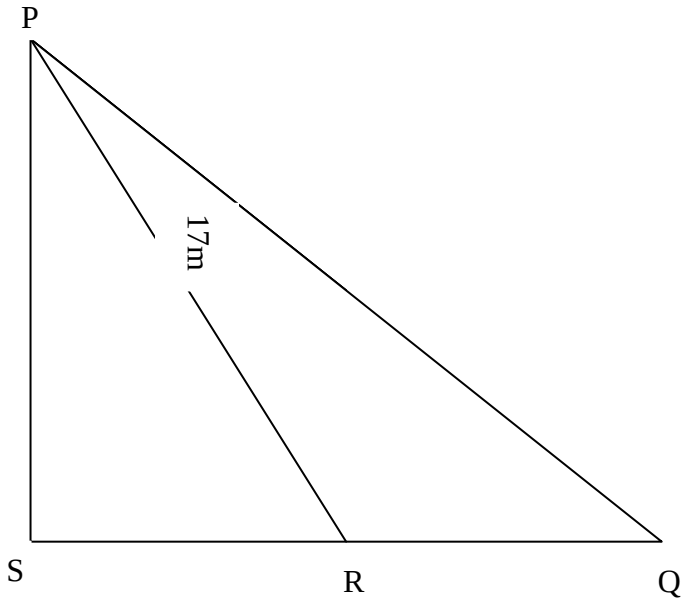
$\therefore$ T= 8 years

CHAPTER 8

Pythagoras' Theorem and Trigonometrical Ratios

Question 1 Calculate the length of PQ

DIAGRAM



Solution

In $\triangle$ PRS:

$PR^2 = PS^2 + SR^2$

$PR^2 = SR^2 = PS^2$

$PS^2 = PR^2 - SR^2$

$PS^2 = 17^2 - 8^2$

$PS^2 = 289 - 64$

$\sqrt{PS^2} = \sqrt{225}$

$PS = 15m$

In $\triangle$ PQS

$PQ^2 = PS^2 + 5Q^2$

$PQ^2 = 152 + (8 + 12)^2$

$PQ^2 = 15^2 + 20^2$

$PQ^2 = 15^2 + 20^2$

$PQ^2 = 225 + 400$

$\sqrt{PQ^2} = \sqrt{625}$

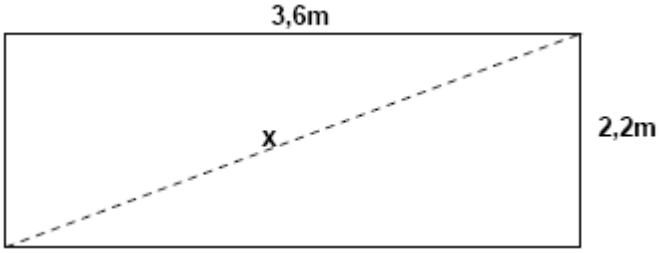
$\therefore PQ = 25m$

Question 2:

Find the length of the longest straight line which can be drawn on a rectangular chalkboard which measures 36m by 2,2m

Solution

The longest straight line is the diagonal

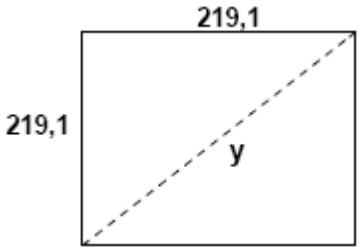


Let the diagonal be x

$x^2 = (3,6)^2 + (2,2)^2$
$x^2 = 12,96 + 4,84$
$\sqrt{x^2} = \sqrt{17,8}$
x= 4,22m to 2.d.p

Question 3:
A garden is square in shape and has an area of 4,8 hectares. Calculate:
a) The Length of a side in metres
b) The length of the diagonal in metres

Solution
a) 1ha $= 10\,000m^2$ b)
Area of A square= S x S
$4,8 \times 10000 = S^2$
$\sqrt{48000} = \sqrt{S^2}$
$S^2 = \sqrt{48\,000}$
S = 219,1m to 1.dp

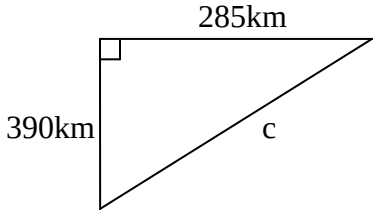


Let the diagonal = y
$y^2 = 219,1^2 + 219,1^2$
$y^2 = 8004,81 + 48004,81$
$\sqrt{y^2} = \sqrt{96009,62}$
y= 309,9m to 1.d.p

Question 4:

A plane flies northwards for 390m. It then flies eastwards for 285km. How far is it from its starting point? (Neglect its height above the ground)

Solution



$C^2 = 390^2 + 285^2$

$C^2 = 152100 + 8\ 1225$

$\sqrt{C^2} = \sqrt{233325}$

C = 483km to 3 S.F

Question 5

Sandra cycled from point A on a bearing of 060^0 for 13km and Roy walked from the same point on a bearing of150^0, for 18km. Find their distance apart.

Solution

A bearing is always measured clockwise from the north.

Diagram

Let x be their distance apart.

$x^2 = 13^2 + 18^2$

$x^2 = 169 + 324$

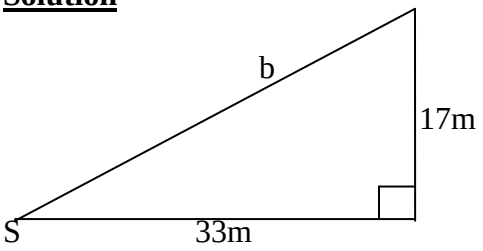
$\sqrt{x^2}^2 = \sqrt{493}$

x = 22, 2km to 1 d.p

Question 6:

Roy walks 33m directly East then 17m directly North. How far is he from the starting point?

Solution



$b^2 = 33^2 + 17^2$

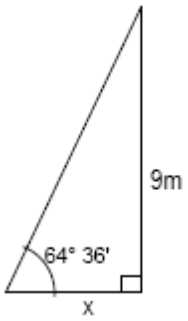
$b^2 = 1089 + 289$

$\sqrt{x^2} = \sqrt{1378}$

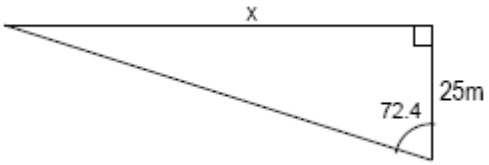
b = 37,1 to 1d.p.

Question 7:
Calculate the lengths marked x

a)



b)



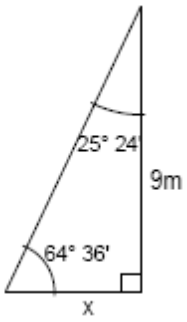
Solution
Use the SOHCAHTOA method

SOH $\Rightarrow$ $\sin\theta = \frac{opp}{Hyp}$

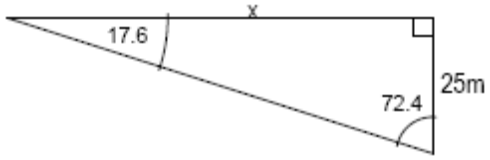
CAH $\Rightarrow$ $\text{Cos}\,\theta = \frac{Adj}{Hyp}$

TOA $\Rightarrow$ $\text{Tan}\,\theta = \frac{opp}{Adj}$

a)



Use the complement of $64^0\ 36^1$ which is $25^0\ 24^1$ b)



Tan θ = $\frac{opp}{adj}$

Tan $25^0\ 24^1$= $\frac{x}{9}$

9 x Tan $25^0 24^1$ = x

9 x 0,4748 = x

x= 4,27m to 3 s.f

Tan θ = $\frac{opp}{adj}$

Tan $72{,}4^0$= $\frac{x}{25}$

25 x Tan $72{,}4^0$ = x

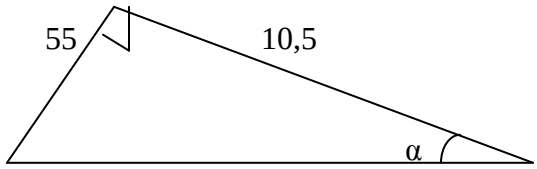
x= 25 x 3, 1524

x= 78,81

x= 78,8m to 3.3.f

Question 8

Calculate the angle marked α in the triangle below



Solution

Tan α = $\frac{opp}{adj}$

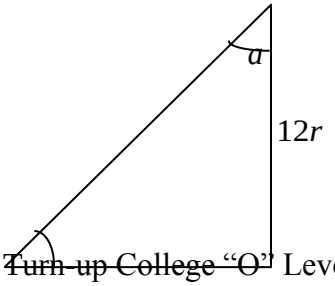
Tan α = $\frac{5,5}{10,5}$

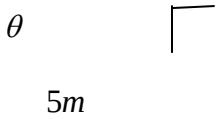
Tan α = 0,5238

From the tables α = $27^0\ 39^1$

Question 9:

Calculate the angles marked θ and α below





Solution

Tan θ = $\frac{12}{5}$

Tan θ = 2,4000

From tables $\theta = 67^0\ 23^1$

$\alpha = 90^0 - 67^0\ 23^1$

but $90^0 = 89^0\ 60^1$

$89^0\ 60^1$

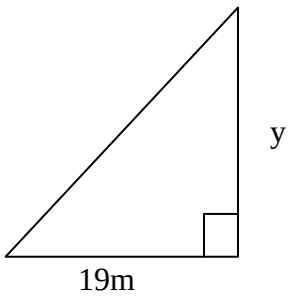
$-\ 67^0\ 23^1$

$\therefore \quad \alpha = 22^0\ 37^1$

Question: 10

From a point 19m from the foot, a man observes the angle of elevation of the top of the building to be 440°. Find the height of the building.

Solution



Let the height of the building be y.

Question 11:

Turn-up College "O" Level Mathematics Questions and answers

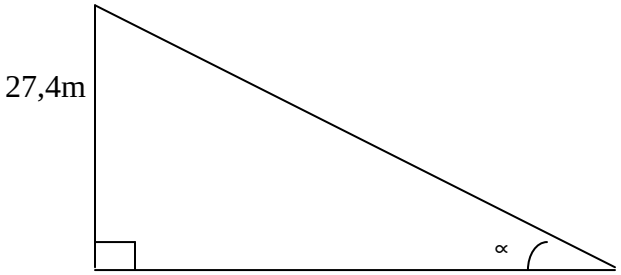
Tan $\theta = \frac{opp}{adj}$

Find the angle of elevation of the top of a flag pole 27, 4m high from a point 39m away of level ground.

Tan $44^0 = \frac{y}{19}$

19 × Tan 440 = y
y= 19 × 0, 9657
y= 18, 3483
∴ y =18, m to 1.d.p

Solution



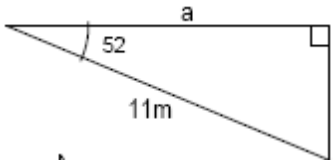
39m

Tan $\alpha = \frac{opp}{adj}$

Tan $\alpha = \frac{27.4}{39}$

Tan α =0,7026
From tables α = 35°5′

Question 12
Find the value of a in the triangles below:
b)



Solution

SOHCAHTOA

a) Sin θ = OPP / Hyp

Sin 25^0= a / 15

b) Cos θ = Adj / Hyp

15 x8m 25^0= a
15 x 0,4226 = a

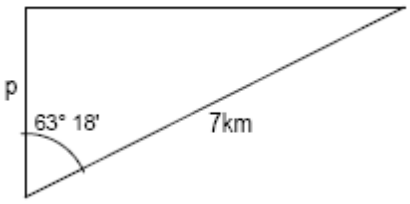
a = 6, 339
∴ a = 6,34cm to 3 s.f

Cos 52 = $\frac{a}{11}$

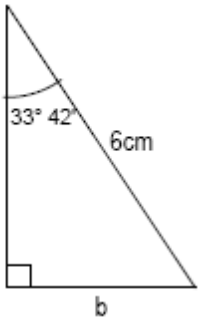
11 x Cos 52= a
11 × 0,6157= a
a= 6,7727
:- a=6,77m to 3. s.f

Question 13:
Find the value of the marked side and give your answers to 3 S.F.

a)



b)



Solution

a) $\text{Cos}63^0 18^1 = \frac{p}{7}$

7 x Cos 63^0 18^1= P
P= 7 x Cos 63^0 18^1
P= 7 x 0,4493
P= 3,1451
P= 3,15km

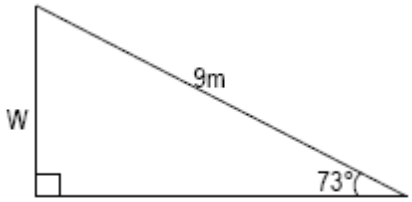
b) Sin 33^0 42^1 = $\frac{b}{6}$

6 x Sin 33^0 42^1=b
b= 6 x Sin 33^0 42^1
b= 6 x 0,5548
b=3,3288
b = 3,33cm

Question 14

A ladder 9m long, leans against a wall so that, it makes an angle of $73^0$with the horizontal ground. Calculate how far up the wall the ladder reaches.

Solution



$\text{Sin } 73^0 = \frac{W}{9}$

$9 \times \text{Sin } 73^0 = W$

$W = 9 \times 0{,}9563$

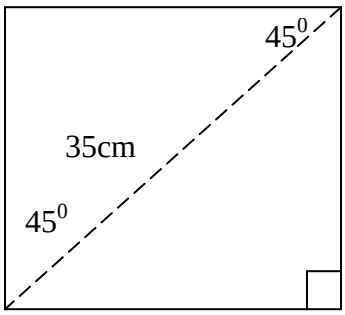
$W = 8{,}6067$

$W = 8{,}61\text{m to 3.s.f}$

Question 15

A diagonal of a square id 35cm long. How long is each side?

Solution



$\text{Sin } 45^0 = \frac{S}{35}$

$35 \times \text{Sin } 45^0 = S$

$S = 35 \times 0{,}7071$

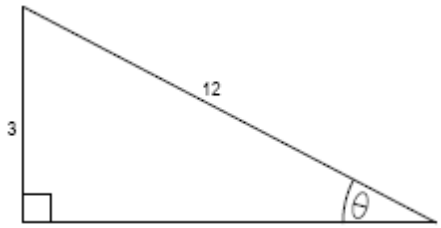
$S = 24{,}7485$

$S = 24, 7\text{cm}$

Each side is = 247cm to 3s.f

Question 16

Calculate the marked angle $\ominus$ and give your answer to the nearest $0{,}1^0$



Solution

Sin ⊖ = OPP / Hyp

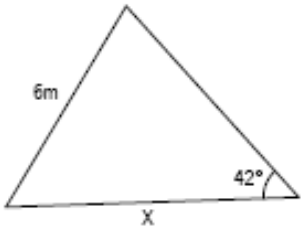
Sin ⊖ = 3/12

Sin ⊖ = 0, 2500

⊖ = **$14^0\ 29^1$**

Question 17

Calculate the length of the hypotenuse.



Solution

$\text{Sin } 42^0 = \frac{6}{x}$ from the reciprocal Tables

$\frac{1}{0,6691} = 1,506$

$\frac{x \times \sin 42^\circ}{\sin 42^\circ} = \frac{6}{\sin 42^\circ}$

$\therefore x = 1,506 \times 6$

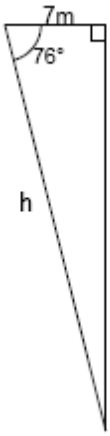
$x = \frac{6}{0,6691}$

$x = 9,036$

$x = 9m$

Question 18

Calculate the length of the hypotenuse



Solution

$\text{Cos } 76^0 = \frac{7}{h}$

$\frac{h \times \text{Cos } 76^0}{\text{Cos } 76^0} = \frac{7}{\text{Cos } 76^0}$

$h = \frac{7}{0{,}2419}$

From the reciprocal tables

$\frac{1}{0{,}2419} = 4, 133$

$h = 4{,}133 \times 7$

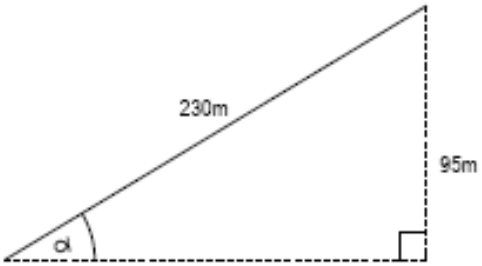
$h = 28{,}931$

$:\text{-}h = 28{,}9\text{m}$

Question 19

A ball rolls 230m down a slope.
As it falls, it drops 95m vertically
Calculate the angle of the slope.

Solution



$\sin \propto = \frac{95}{230}$

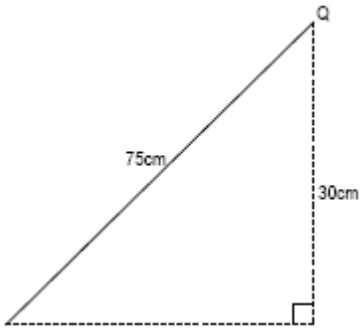
$\sin \propto = 0{,}4130$

From tables $\propto = 24^0\ 23^1$

Question 20

A brick is suspended from a point Q by a piece of string 75cm long. It swings back and forward. Calculate the angle the string makes with the vertical side when the brick is 30cm vertically below Q.

Solution



$\cos\theta = \frac{30\text{cm}}{75\text{cm}}$

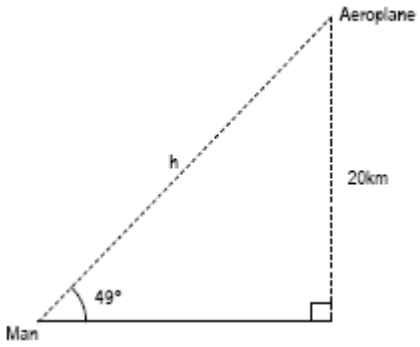
$\cos\theta = 0{,}4000$

From tables $\theta = 66^0\ 25^1$

Question 21

An aeroplane is flying at a height of 20km. Its angle of elevation to a man on the ground is 49^{o}. Calculate the distance of the aeroplane from the man.

Solution



$\sin 49^{0} = \frac{20}{h}$

$h \times \sin 49^{0} = 20$

$h = \frac{20}{\sin 490}$

$h = \frac{20}{0,7547}$

From reciprocal Tables:-

$\frac{1}{0,7547} = 1,325$

$h = 20 \times 1,325$

$h = 26,5km$

CHAPTER 9

Vectors

A vector is any quantity which has direction and size e.g force velocity acceleration,velocity, displacement and force

Question 1:

Find the magnitudes of the following vectors

a) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ b) $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ c) $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$

3 -

Solution

NB: Magnitude of **a** $= |a|$

and $|a| = \sqrt{x^2 + y^2}$

a) $\sqrt{(-4)^2 + 3^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

= 5 units

b) $\sqrt{0^2 + (-2)^2}$

$= \sqrt{0^2 + 4}$

$= \sqrt{4}$

= 2 units

c) $\sqrt{(15)^2 + (-8)^2}$

$= \sqrt{225 + 64}$

$= \sqrt{289}$

=17 units

Question 2

If P **p** = $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$, **q**=$\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ and **r**=$\begin{pmatrix} -3 \\ -2 \end{pmatrix}$

-

Find

a) **p + r**

b) **p-r**

c) $|r|$ to 1 d.p

d) **(p –q) –r**

Solution

a) **p + r** = $\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} 5+(-3) \\ -2+(-7) \end{pmatrix} = \begin{pmatrix} 2 \\ -9 \end{pmatrix}$

-

b) **p- r** = $\begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} --3 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} 5-(-3) \\ -2-(-7) \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$

c)
$$|r| = \sqrt{(-3)^2 + (-7)^2}$$
$$= \sqrt{9 + 49}$$
$$= \sqrt{58}$$
$$= 7,6$$

d) **(p-q)-r** =
$$\begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ 6 \end{pmatrix} - \begin{pmatrix} -3 \\ -7 \end{pmatrix}$$
$$= \begin{pmatrix} 3 \\ -8 \end{pmatrix} - \begin{pmatrix} -3 \\ -7 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

Question 3:

If a = $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and b = $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

State a + b, 3b- a as column vectors
And find $|a + b|$

Solution
a+b =
$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} 3 + (-2) \\ 4 + 5 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ 9 \end{pmatrix}$$

3b-a =

$$3\begin{pmatrix}-2\\5\end{pmatrix}-\begin{pmatrix}3\\4\end{pmatrix}$$
$$=\begin{pmatrix}-6\\15\end{pmatrix}-\begin{pmatrix}3\\4\end{pmatrix}$$
$$=\begin{pmatrix}-6-3\\15-4\end{pmatrix}$$
$$=\begin{pmatrix}-9\\11\end{pmatrix}$$

$$|a+b| = \sqrt{1^2+9^2}$$
$$= \sqrt{1+81}$$
$$= \sqrt{82}$$

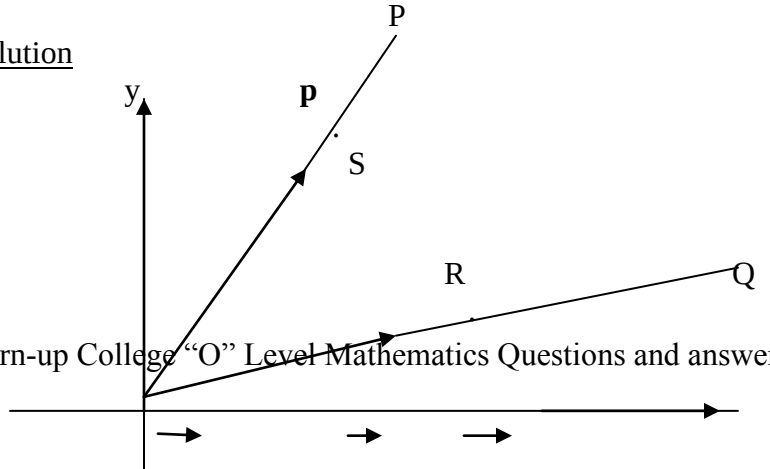
Question 4:

Given that $\overrightarrow{OP} =$ **p**, $\overrightarrow{OQ} =$ **q**

$\overrightarrow{OS} = \frac{2}{3}\overrightarrow{0P}$ and $\overrightarrow{OR} = \frac{3}{5}\overrightarrow{OQ}$

Find $\overrightarrow{RS}, \overrightarrow{PR}$ and $\overrightarrow{QS}$ in terms of **p** and **q.**

Solution



$\overrightarrow{RS} = \overrightarrow{RQ} + 0\overrightarrow{S} = \overrightarrow{RO} + \overrightarrow{OS}$

$= -\frac{3}{5}q + \frac{2}{3}p$

$= \frac{2p}{3} - \frac{-3q}{5}$

$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR}$

$= \mathbf{-p} + \frac{3}{5}\overrightarrow{OQ}$

$= -p + \frac{3}{5}q$

$= \frac{3}{5}\mathbf{q} - \mathbf{p}$

$\overrightarrow{QS} = \overrightarrow{OQ} + \overrightarrow{OS}$

$= -q + \frac{2}{3}\overrightarrow{OP}$

$= \mathbf{-q} + \frac{3}{5}\mathbf{p}$

$= \frac{2}{3}\mathbf{p} - \mathbf{q}$

Question 5

Given that

$\mathbf{a} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$ and

c = $\begin{pmatrix}18\\14\end{pmatrix}$

i) Find the vector **m** such that 4**b**- 2**a** = **m**.

ii) If x**a** = y**b** +**c**. Find the values of x and y.

Solution

i) **m**= 4b- 2a

= $4\begin{pmatrix}-4\\6\end{pmatrix}$ -2$\begin{pmatrix}4\\6\end{pmatrix}$

= $\begin{pmatrix}-24\\16\end{pmatrix}$-$\begin{pmatrix}8\\12\end{pmatrix}$

= $\begin{pmatrix}-32\\4\end{pmatrix}$

∴ **m** = $\begin{pmatrix}-32\\4\end{pmatrix}$

ii) x $\begin{pmatrix}4\\6\end{pmatrix}$ = y $\begin{pmatrix}-6\\4\end{pmatrix}$ + $\begin{pmatrix}14\\18\end{pmatrix}$

4x = -6y + 18 (i)

6x = 4y + 14 (ii)

$\begin{pmatrix}4x\\6x\end{pmatrix}$ = $\begin{pmatrix}6y+18\\4y+14\end{pmatrix}$

4x +6y = 18 (1) x 4

6x -4y = 14 (2) x 6

16x + 24y = 72

36 x -24y = 84)

$\frac{\cancel{52}x}{\cancel{52}} = \frac{\overset{3}{\cancel{156}}}{\cancel{52}}$

x= 3

From (1) 4 (3) +6y = 18

$12 + 6y = 18$

$6y = 18 - 12$

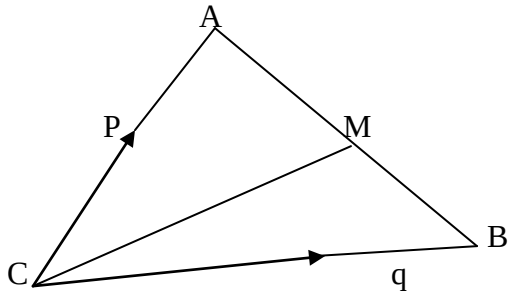
$\frac{\cancel{6}y}{\cancel{6}_1} = \frac{1}{\cancel{6}_1}$

$y = 1$

$\therefore$ x = 3 and y = 1

Question 6

In the triangle ABC, M Divides the line AB in the Ratio AM: MB= 5:3
If $\overrightarrow{CA}$ = **p** and $\overrightarrow{CB}$ = **q**, Express $\overrightarrow{CM}$ in terms of **p** and **q**.



Solution

$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

= **--p**+ **q**

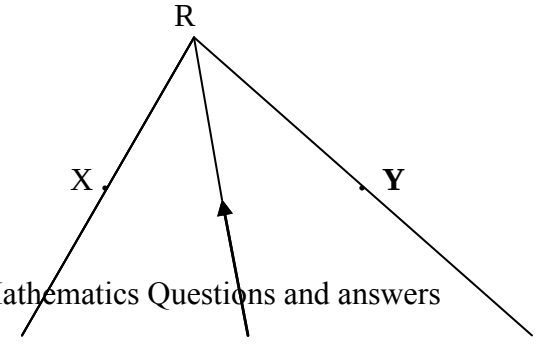
= **q** - **p**

$\overrightarrow{AM} = \frac{5}{8}\overrightarrow{AB}$

= $\frac{5}{8}$**(q-p)**

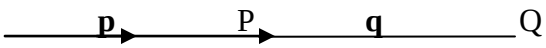
$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM}$

Question 7



$= \mathbf{p} + \frac{5}{8}(\mathbf{q} - \mathbf{p})$

$= \mathbf{p} + \frac{5}{8}(\mathbf{q} - \mathbf{p})$



$\therefore \frac{3}{8}\mathbf{p} + \frac{5}{8}\mathbf{q}$

X and Y are mid-points of OR and RQ respectively. It is given that $\overrightarrow{OP}$ =,**p**

$\overrightarrow{PQ}$ = **q** and $\overrightarrow{PR}$ = 2**p**. Express in terms of **p** and **q.**

a) $\overrightarrow{OR}$ b) $\overrightarrow{RQ}$ c) $\overrightarrow{XY}$

Solution 7

a) $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$

$= \mathbf{p} + 2\mathbf{p}\text{-}\mathbf{q}$

$= 3\mathbf{p} - \mathbf{q}$

b) $\overrightarrow{RQ} = \overrightarrow{RP} + \overrightarrow{PQ}$

$= -(2\mathbf{p}\text{-}\mathbf{q}) + \mathbf{q}$

$= -2\mathbf{p} + \mathbf{q} + \mathbf{q}$

$= -2\mathbf{p} + 2\mathbf{q}$

$= 2\mathbf{q} - 2\mathbf{p}$

$= \underline{2(\mathbf{q}\text{-}\mathbf{p})}$

c) $\overrightarrow{XY} = \overrightarrow{XR} + \overrightarrow{RY}$

$= \frac{1}{2}\overrightarrow{OR} + \frac{1}{2}\overrightarrow{RQ}$

$= \frac{1}{2}(3\mathbf{p}\text{-}\mathbf{q}) + \frac{1}{2}(2\mathbf{q}\text{-}2\mathbf{p})$

$= \frac{3}{2}\mathbf{p} - \mathbf{q} + \underline{\mathrm{q}} - \mathrm{p}$

$= \frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$

$\therefore \quad \overrightarrow{XY} = \frac{1}{2}(\mathbf{p} + \mathbf{q})$

Question 8:

A (-2,-1), B (0;2) C, D (4;2)
are vertices of a quadrilateral. Find the co-ordinates of C such that ABCD is a parallelogram. Find the coordinates of the point of intersection of its diagonals.

Solution

Since $ABCD$ is a parallelogram then $BD \parallel AC$ and $AB \parallel CD$
Let the co-ordinates of C be (x, y)

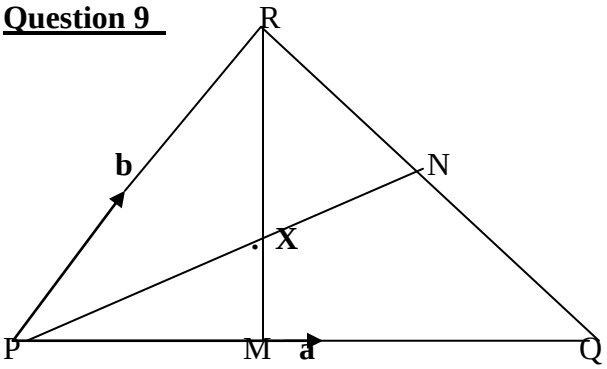
$\overrightarrow{BD} = \overrightarrow{AC}$

i.e $\begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} x+2 \\ y+1 \end{pmatrix}$

x + 2 = 4
y + 1 = 0
i.e. x = 2 and y = -1
The co-ordinates of C are (2;1)
Mid- point = $\left(\frac{4-2}{2}; \frac{2+1}{2} \right)$ = (1; 0,5)

Question 9



In the diagram above $\overrightarrow{PR}$ = **b,** $\overrightarrow{PQ}$ = **a** and M and N are midpoints of PQ and RQ respectively PX: XN= 4:3, Express the following in terms of a and b.

a) $\overrightarrow{QR}$
b) $\overrightarrow{QN}$
c) $\overrightarrow{PN}$
d) $\overrightarrow{PX}$

Solution

a) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$

$= \quad \mathbf{-a} + \mathbf{b}$

$= \mathbf{b - a}$

b) $\overrightarrow{QN} = \frac{1}{2}\overrightarrow{RQ}$

$= \frac{1}{2}(\mathbf{a-b})$

c) $\overrightarrow{PN} = \overrightarrow{PQ} + \overrightarrow{QN}$

$= \mathbf{a} + \frac{1}{2}(\mathbf{a} - \mathbf{b})$

$= \mathbf{a} + \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

$= \frac{3}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

d) $\overrightarrow{PX} = \frac{4}{7}\overrightarrow{PN}$

$= \frac{4}{7}(\frac{3}{2}\mathbf{a} - \frac{1}{2}\mathbf{b})$

$= \frac{12}{14}\mathbf{a} - \frac{4}{14}\mathbf{b}$

CHAPTER 10

FACTORISATION AND SIMPLIFICATION

Question 1:

Factorise completely
a) $y^2 – 12y + 27$
b) $lm – mn -2lp +2np$

Solution 1

a) $y^2 – 12y +27$
= $y^2 - 3y -9y +27$
= $y(y-3)-9 (y-3)$
= $(y-9)\ (y-3)$

b) $lm- mn – 2lp + 2np$
$m(l-n) -2p (l-n)$
$(m-2p) (l-n)$

Question 2:

Factorise completely:
a) $3mp +np – 6mq-2nq$

b) $16- 9r^2$

Solution 2

a) $3mp + np -6mq -2nq$

$= p(3m+n) -2q (3m +n)$

$= (p-2q) (3m +n)$

b) $16-qr^2$

Difference of Two Squares.

$a^2- b^2 = (a +b) (a –b)$

hence $16- 9r^2 = (4 + 3r) (4 -3r)$

Question 3

Factorise completely
a) $ab^2 – bc$
b) $6x^2 + 25x -9$
$5x^2 -45y^2$

a) $ab^2 – bc$
$b (ab-c) = b (ab-c)$

b) $6x^2 + 25x -9$

$= \quad 6x^2 - 2x + 27x - 9$
$= \quad 2x(3x-1) + 9(3x-1)$
$= \quad (2x+9)(3x-1)$

Question 4

Factorise completely
$5x^2 - 45y^2$

Solution

$5(x^2 - 9y)^2$ DIFFERENT OF TWO SQUARES

$5x^2 - 45y^2 = 5(x^2 - 9y^2)$

$=5(x-3y)(x+3y)$

Question 5

Factorise completely

a) $4m^2 - mn - 4dm + nd$

b) $9 - 36h^2$

Solution

a) $4m^2 - mn - 4dm + nd$

$= 4m(m-d) - n(m-d)$

$= (4m-d)(m-d)$

b) $9 - 36h^2$
$= 9(1-4h^2)$
$= \underline{9(1+2h)(1-2h)}$

Question 6
Factorise completely

a) $30 - y - y^2$
b) $8a^2 - 5ab^2$

Solution 6

Question 7:
Factorise completely

$y^2 + py - qy - pq$

Solution 7

$y^2 + py - qy - pq$
$= y(y+p) - q(y+p)$
$= (y-q)(y+p)$

b) $30 - y - y^2$
$= 30 - 6y + 5y - y^2$
$= 6(5 - y) + y(5 - y)$
$= (6 + y)(5 - y)$

b) $8a^2 - 50b^2$
$= 24a^2 - 25b^2$
Difference of two squares

$= 2(2a + 5b)(2a - 5b)$

Question 8: Factorise Completely

a) $mn - n^2 + np - mp$
b) $2x^2 y^2 + 7xy - 15$

Solution 8

a) $mn - n^2 + np - mp$
$= mn - n^2 - mp + np$
$= n(m-n) - p(m-n)$
$= \underline{(n-p)(m-n)}$

b) $2x^2 y^2 + 7xy - 15$
$2xy^2 + 10xy - 3xy - 15$
$2xy(xy + 5) - 3(xy + 5)$
$(2xy - 3)(xy + 5)$

Question 9:
Factorise completely
a) $2pq^2 - 4p^2q$
b) $x^2 - 6x + 9$

Solution 9

a) $2pq^2 - 4p^2q$
$= 2pq(q - 2p)$

b) $x^2 - 6x + 9$
$= x^2 - 3x - 3x + 9$
$= x(x - 3) - 3(x - 3)$
$= (x - 3)(x - 3)$
$= \underline{(x - 3)^2}$

Question 10

Factorise completely
a) $x^2 + 2xy - 8y^2$
b) $P^2 + 4pq - 21q^2$

Solution 10

a) $x^2 + 2xy - 8y^2$
$= x^2 + 4xy - 2xy - 8y2$
$= x(x + 4y) - 2y(x + 4y)$
$= (x - 2y)(x + 4y)$

b) $p^2 + 4pq - 21q^2$
$= p^2 + 7pq - 3pq - 21q^2$
$= p(p + 7q) - 3(p + 7q)$
$= \underline{(p - 3q)(p + 7q)}$

Question11:
Simplify

a) $\frac{a}{2}+\frac{2a}{3}+\frac{a}{4}$ b) $\frac{a-1}{3}-\frac{a-3}{4}$

Solution 11

a) $\frac{a}{2}+\frac{2a}{3}+\frac{a}{4}$

Find the L.C.M of 2, 3 and 4

$\frac{a}{2}+\frac{2a}{3}+\frac{a}{4}$ =

$\frac{6a+8a+3a}{12}$

$=\frac{17a}{12}$

b) $\frac{a-1}{3}-\frac{a-3}{4}$

L.C.M of 3 and 4 is 12

$\frac{4(a-1)-3(a-3)}{12}$

$=\frac{4a-4-3a+9}{12}$

$=\frac{a+5}{12}$

Question 12:
Simplify

a) $\frac{x^2-4xy+3y^2}{x^2-y^2}$

b) $\frac{2}{x^2-x-2}-\frac{4}{x-2}+\frac{6}{x+1}$

Solution 12

a) $\frac{x^2-4xy+3y^2}{x^2-y^2}$

NB: Factorise the numerator and the denominator first

$$\frac{x^2-4xy+3y^2}{x^2-y^2}$$

$$-=\frac{(x-3y)(x-y)}{(x+y(x-y)}$$

$$=\frac{x-3y}{x+y}$$

b) $\frac{2}{x^2-x-2}-\frac{4}{x-2}+\frac{6}{x+1}$

Factorise $x^2 - x - 2$ First find the L.C.M. of Denominators

$x^2 - x-2 = (x + 1)(x- 2)$

$$\frac{2}{(x+1)(x-2)}-\frac{4}{x-2}+\frac{6}{x+1}$$

$$=\frac{2-4(x+1)+6(x-2)}{(x+1)(x-2)}$$

$$=\frac{2-4x-4+6x-12}{(x-2)(x+1)}$$

$$\frac{2x-14}{(x-2)(x+1)}$$

Question 13

Simplify

a) $\frac{mn-n^2}{(m-n)^2}$ b) $\frac{a^2-ab-ac+bc}{a^2-ab+ac-bc}$

Solution 13

a) $\frac{mn-n^2}{(m-n)^2}$

$=\frac{n(m-n)}{(m-n)^2}$

$= \frac{n}{m-n}$

b) $\frac{a^2-ab-ac+bc}{a^2-ab+ac-bc}$

$= \frac{a^2-ab-ac+bc}{a^2-ab+ac+bc}$

$= \frac{(a-b)(a-c)}{(a+c)(a-b)}$

$$= \frac{a-c}{a+c}$$

Question 14

Express as a single fraction in its lowest terms.

a) $\frac{1}{m} - \frac{2}{m+3}$

b) $n + \frac{2n}{n+5}$

Solution

a) $\frac{1}{m} - \frac{2}{m+3}$ L.C.M = m (m +3)

hence $\frac{1}{m} - \frac{2}{m+3} = \frac{m+3-m(m+3)}{m(m+3)}$

$= \frac{m+3-2m}{m(m+3)}$

$= \frac{3-m}{m(m+3)}$

b) $n + \frac{2n}{n+5}$

$= \frac{n(6n+5)+2n}{6n+5}$

$= \frac{6n^2+5n+2n}{6n+5}$

$= \frac{6n^2+7n}{6n+5}$

Question 15

Simplify

$$\frac{a+1}{a^2-7a+12} \times \frac{8-a}{a^2+2a+1}$$

Solution 15 Factorise the denominators first.

$a^2 - 7a + 12 = a^2 - 4a - 3a + 12$

$= a(a-4) - 3(a-4)$

$= (a-3)(a-4)$

$a^2 + 2a + 1 = a^2 + a + a + 1$

$= a(a+1) + 1(a+1)$

$= (a+1)^2$

Hence

$$= \frac{a+1}{(a-3)(a-4)} \times \frac{8-2a}{(a+1)^2}$$

$$= \frac{a+1}{(a-3)(a-4)} \times \frac{2(4-a)}{(a+2)^2}$$

$$= \frac{a+1}{(a-3)(a-4)} \times \frac{(-2)(a-4)}{(a+1)(a+1)}$$

$$= \frac{\cancel{a+1}}{\cancel{(a-4)}(a-3)} \times \frac{(-2)\cancel{(a-4)}}{(a+1)\cancel{(a+1)}}$$

$$= \frac{-2}{(a-3)(a+1)}$$

Question 16:

Simplify

$$\frac{2x^2-8}{15x^2y} \times \frac{9x^2y^2}{3x+6}$$

Solution16

$$\frac{2x^2-8}{15x^2y} \times \frac{9x^2y^2}{3x+6} = \frac{2(x^2-4)}{15x^2y} \times \frac{9x^3y^2}{3x+6}$$

$$= -\frac{2(x+2)(x-2)}{15x^2y} \times \frac{9x^2y}{3(x+2)}$$

$$= \frac{2(x-2)}{5} \times 2y$$

$$= \frac{2xy(x-2)}{5}$$

$$= \frac{2x^2y-4}{5}$$

Question 17

Simplify

$$\frac{x^2-16}{x^2-x}\times\frac{x^2-3x+2}{x^2+2x-8}$$

Solution 17

$$\frac{x^2-16}{x^2-x}\times\frac{x^2-3x+2}{x^2+2x-8}$$

$$=\frac{(x-4)(x+4)}{x(x-1)}\times\frac{(x-1)(x-2)}{(x-2)(x+4)}$$

$$=\frac{x-4}{x}$$

Question 18:

Simplify:

$$\frac{y^2-4}{y^2-3y+2}\div\frac{y}{y-1}$$

Solution 18

$$\frac{y^2-4}{y^2-3y+2}\div\frac{y}{y-1}$$

$$\frac{(y+2)(y-2)}{(y-1)(y-2)}\div\frac{y}{y-1}$$

$$=\frac{(y+2)(y-2)}{(y-1)(y-2)}\times\frac{y-1}{y}$$

$$=\frac{y+2}{y}$$

Question 19

Simplify:

$$\frac{m^2-mn}{n^2-np}\div\frac{n^2-mn}{mn-mp}$$

Solution 19

$$\frac{m^2-mn}{n^2-np}\div\frac{n^2-mn}{mn-mp}$$

$$\frac{m^2-mn}{n^2-np}\div\frac{n^2-mn}{mn-mp}$$

$$=\frac{m(m-n)}{n(n-p)}\div\frac{n(n-m)}{m(n-p)}$$

$$=\frac{m(m-n)}{n(n-p)}\times\frac{m(n-p)}{n(n-m)}$$

$$=\frac{(-)m(n-m)}{n(n-p)}\times\frac{m(n-p)}{n(n-m)}$$

$$=\frac{-m^2}{n^2}$$

Question 20

Simplify

$$\frac{v^2-3v-4}{v^2-4v} \div \frac{v^2-4v+4}{v^2-4}$$

Solution 20

$$\frac{v^2-3v-4}{v^2-4v} \div \frac{v^2-4v+4}{v^2-4}$$

$$= \frac{(v-4)(v+1)}{v(v-4)} \div \frac{(v-2)(v+2)}{(v-2)(v-2)} \quad \text{(factorisation of quadratic terms)}$$

$$= \frac{(v-4)(v+1)}{v(v-4)} \times \frac{(v+2)(v-2)}{(v-2)(v-2)}$$

$$= \frac{(v+1)}{v} \times \frac{v+2}{v-2}$$

$$= \frac{(v+1)(v+2)}{v(v-2)}$$

CHAPTER 11

THE SINE RULE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

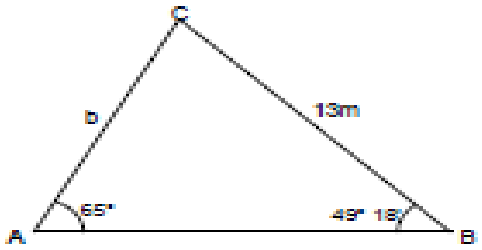
Alternatively

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

It is used for solving triangles which are not right-angled and in which either two angles and any side are given or two sides and the angle opposite to them are given.

Question 1:
Find the size of the side b in a triangle in which A= 65%, B= 49^0 18^1 and a = 13m.

Solution 1



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{13}{\sin 65^\circ} = \frac{b}{49^\circ 18'}$$

$$\frac{13 \times \sin 49^\circ 18'}{\sin 65^\circ} = \frac{b \cancel{\sin 65^\circ}}{\cancel{\sin 65^\circ}}$$

$$b = \frac{13\sin 49^{\circ}18'}{\sin 65^{\circ}}$$

13	1.1139
Sin 49° 18′	$+\bar{1}.8797$
	0.9936
Sin 65°	$-\bar{1}.9573$
10.87	1.0363

∴ b=10,87m to 2.d.p

⇒ Use the log of sines for angles

⇒ Where there is multiplication you add the Logs. And where there is divisions, subtract the log

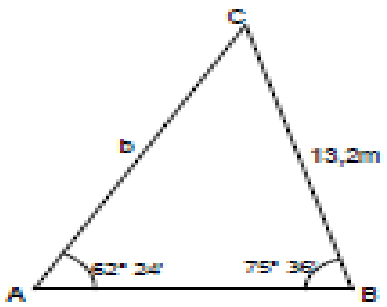
.

⇒ Check, 0363 on the antilog table to get the answer

Question 2

In △ ABC, A= 62°24′, B= 75°36′
a=13,2m.
Find b

Solution



$$\frac{a}{\sin A} = \frac{b}{SinB}$$

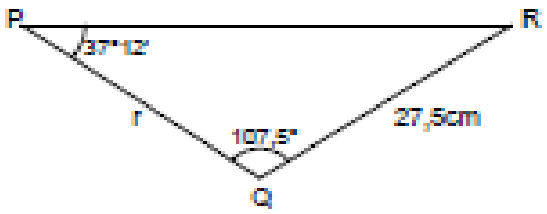
$$\frac{13,2 \times \sin 75^{\circ}36'}{\sin 62^{\circ}24'} = \frac{b \times \sin 62^{\circ}24'}{\sin 62^{\circ}24'}$$

$$b = \frac{13 \times \sin 62^{\circ}24'}{\sin 62^{\circ}24'}$$

No	Log
13,2	1,1206
Sin $75^{\circ}\ 36'$	$+\bar{1}$, 9861
	1, 1067
Sin $62^{\circ}23'$	$\bar{1}$, 9475
14, 43	1, 1592
$\therefore$ **b = 14,43m**	

Question 3

In $\triangle$ PQR, Q = $107,5^{\circ}$,P= $37^{\circ}12'$, p = 27,5cm



First Calculate P

1800- ($107,5^{\circ} + 37^{\circ}12'$)

= $35,3^{\circ}$

$$\frac{r}{\sin 37^{\circ}12'} = \frac{27,5}{\sin 35,3^{\circ}}$$

$$\therefore r = \frac{27,5 \times \sin 37^{\circ}12'}{\sin 35,3^{\circ}}$$

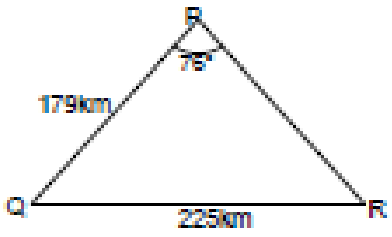
No	Log
27, 5	1,4393
Sin $37^{\circ}12'$	$+\bar{1}$, 7815
	1, 2208
Sin 35, 3°	$-\bar{1}$,7618
28, 77	1, 4590

$$r = \frac{21,5 \times 1.7815}{1,7618}$$
$$= 28,77$$

Question 4

In $\triangle$ PQR P = 76°, P = 225km and r = 179km. Calculate R.

Solution 4



$$\frac{\sin R}{r} = \frac{\sin P}{p}$$

$$\frac{\sin R}{179} = \frac{\sin 76^\circ}{225}$$

$$\text{Sin R} = \frac{179 \times \sin 76}{225}$$

No	Log
179	2,2529
Sin 76°	$\bar{1}$, 9869
	2,2398
225	2,3522
50,53°	$\bar{1}$,8876

:- R= 50,530 or 129470

But r < p → R< P

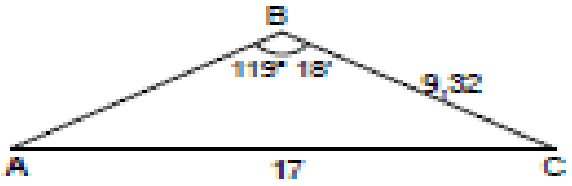
∴ R= 50,5° to 1.d.p

∴ R= 50,5° to 1. d.p

Question 5

Calculate the values of angles A and C of □ ABC where b= 17m, a= 9,32m and B= 119° 18′

Solution 5



$$\frac{\sin A}{9,32} = \frac{\sin 119^\circ 18'}{17}$$

$$\text{Sin A} = \frac{9,32 \times \sin 119^{\circ}18'}{17}$$

N0.	Log
9,32	0,9694
Sin $119^{\circ}18'$	$\bar{1}$, 9406
	0,9100
17	1,2304
$28^{\circ}34'$	$\bar{1}$,6796

a < b so A < B also

$\therefore$ A = $28^{\circ}34'$

= 180^{0}- ($119^{\circ}18'^{1}$ + $28^{\circ}34'$)

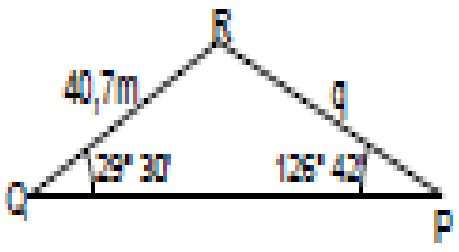
= $32^{\circ}8'$

$\therefore$ C = $32^{\circ}8'$

Question 6:

Solve the triangle completely in $\triangle$PQR, given that Q = $29^{\circ}30'$, R= $126^{\circ}42'$, r = 40,7m

Solution 6



$$P = 180^\circ - (126^\circ 42' + 29^\circ 30')$$
$$= 23^\circ 48'$$

$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{q}{\sin 29^\circ} = \frac{40,7}{\sin 126^\circ 42'}$$

$$\frac{q \times \sin 126^\circ 42'}{\sin 126^\circ 42'} = \frac{40,7 \times \sin 29^\circ 30'}{\sin 126^\circ 42'}$$

$$q = \frac{40,7 \times \sin 29^\circ 30'}{\sin 126^\circ 42'}$$

** $\text{Sin } 126^\circ 42' = \text{Sin } 53^\circ 18'$

N0.	Log
40,7	1,6096
Sin $29^\circ 30'$	$\bar{1}$, 6923
	1,3019
Sin $53^\circ 18'$	$\bar{1}$, 9041
25,00	1,3978

q = 25m

$$\frac{p}{\sin P} = \frac{r}{\sin R}$$

$$\frac{p}{\sin 23^\circ 48'} = \frac{40,7}{\sin 126^\circ 42'}$$

$$\frac{q \times \sin 126^\circ 42'}{\sin 126^\circ 42'} = \frac{40,7 \times \sin 29^\circ 30'}{\sin 126^\circ 42'}$$

$$p = \frac{40,7 \times \sin 23^{\circ}48}{\sin 126^{\circ}42'}$$

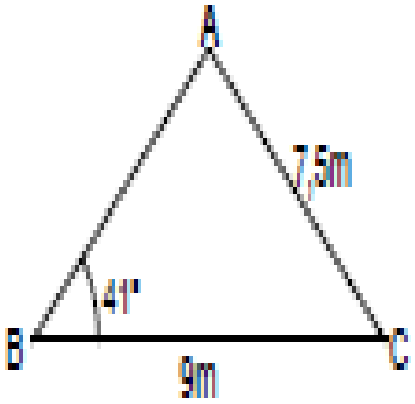
No	Log
40,7	1,6096
Sin $23^{\circ}48'$	$\bar{1}$,6059
	1,2155
Sin $53^{\circ}18'$	$\bar{1}$,9041
20,48	1,3114

$\therefore$ p= 20,5m to 1d.p

Question 7:

In triangle ABC, B= 41°, b= 7,5m, a= 9m. Find the value of the missing angles

Solution 7



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{9} = \frac{\sin 41^\circ}{7,5}$$

$$\text{Sin A} = \frac{9 \times \sin A}{7,5}$$

N0	**Log**
9	0,9542
Sin 41°	$\bar{1}$,8169
	0,771.1
7.5	0,8751
51°55′	1, 8960

** if a > b then A > B

A = 51°55′ or 180° - (51°55′)
= 128°5′

When A= 51°55′

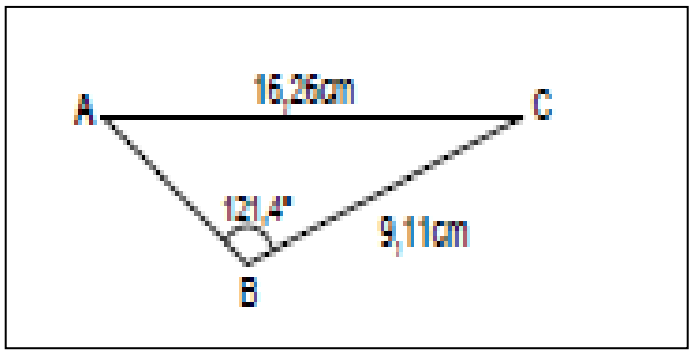
C= 180° – (51°55′ +41°)
= 87°5′

When A= 128°5′
C= 180° – (128°5′ +41°)
= 10°55′

Question 8

Calculate the values of angles A and C of Triangle ABC, where b = 16, 24cm
A= 9,11cm and B= 121,4°

Solution 8



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\text{Sin A} = \frac{9{,}11 \times \sin 121{,}4^{\circ}}{16{,}24}$$

since Sin $121{,}4^{\circ}$ = Sin $58{,}6^{\circ}$

Solution 9

N0	Log
9, 11	0,9595
Sin $58{,}6^{\circ}$	1, 93 12
	0,8907
16,24	1, 2106
28.6°	1, 6801

N0.	Log
10, 5	1,0212
Sin 73^{0}	1,9806
	1,0018
Sin 42^{0}	1,8255
15,01	1,1763

a < b A< B

x = 15,01cm

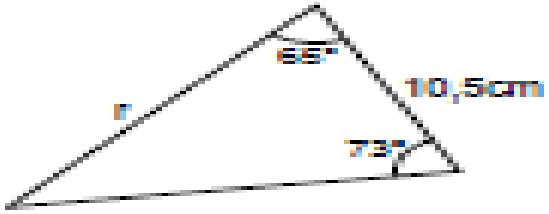
:- A=28.6°

C= 180° – (28.6° +$21{,}4^{\circ}$)
= 30°

Question 9:

Calculate the values of x in the diagram below:

DIAGRAM



$180^\circ - (73^\circ + 65^\circ) = 42^\circ$

$$\frac{x}{\sin 73^\circ} = \frac{10,5}{\sin 42^\circ}$$

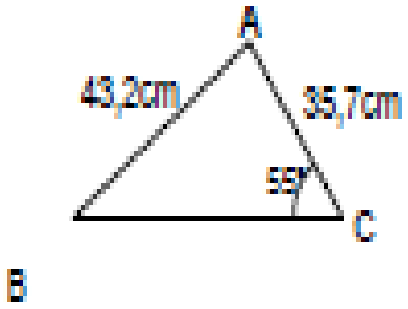
$$x = \frac{10,5 \times \sin 73^\circ}{\sin 42^\circ}$$

x = 15cm using a calculator

Question 10:

Calculate the angle B in triangle ABC, where C= 55°
b= 35,7cm c = 43,2cm

Solution 10



N0.	Log
35,7	1,5527
Sin 55°	$\bar{1}$,9134
	1,4661
43,2	1,6355
42°37′	$\bar{1}$,8306

B = 42°37′ or 180° - 42°37′ = 137°23′

$$\frac{\sin B}{35,7} = \frac{\sin 55^\circ}{43,2}$$

$$\therefore \text{Sin B} = \frac{35,7 \times \sin 55^\circ}{43,2}$$

Since b < c then B < C

4 $\therefore$ B= 42°37′

CHAPTER 12

CONSTRUCTION AND LOG

Notes

a) Always use a sharp pencil
b) Always show the construction line and arcs but lightly
c) It is often a good idea to draw a rough sketch first.

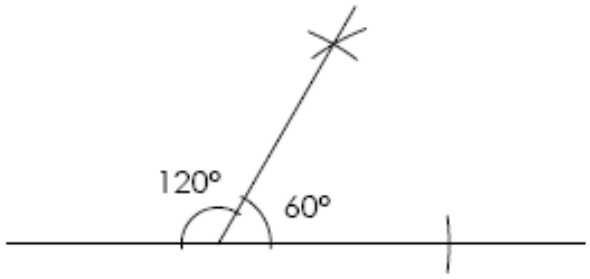
** Use Ruler and compasses only

Question 1

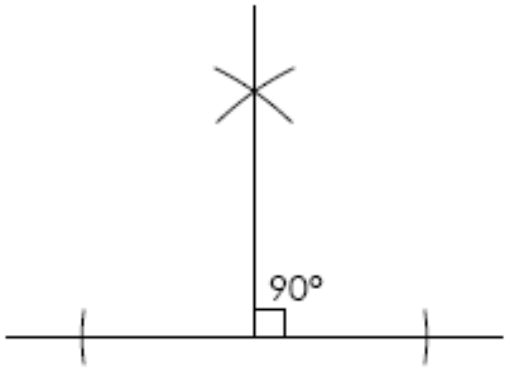
a) Construct angles of 60^0 and 120^0
b) Construct a 90^0 angle

Solution

a)



b)

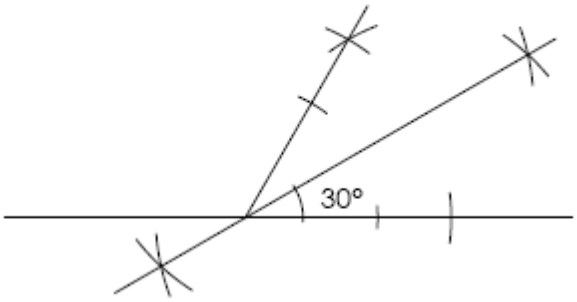


Question 2:

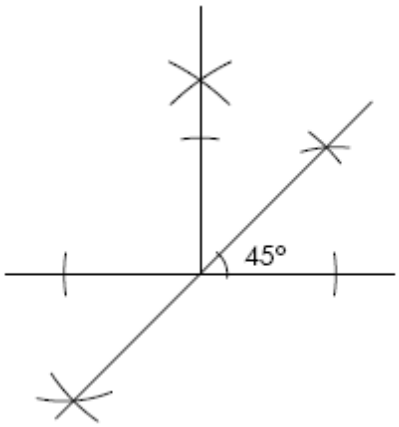
a) Construct a 30^0 Angle
b) Construct a 45^0 Angle
c) Construct a 15^0 Angle

Solution

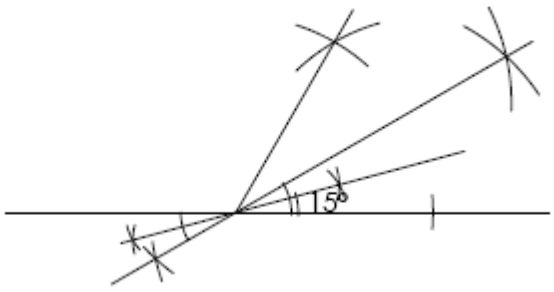
a) To construct a 30˚ angle, bisect a 60˚ angle



b) To construct a 45˚ angle, bisect a 90˚ angle



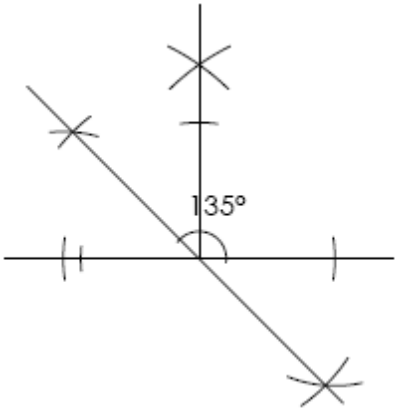
c) To construct a 15˚ angle, bisect a 30˚ angle



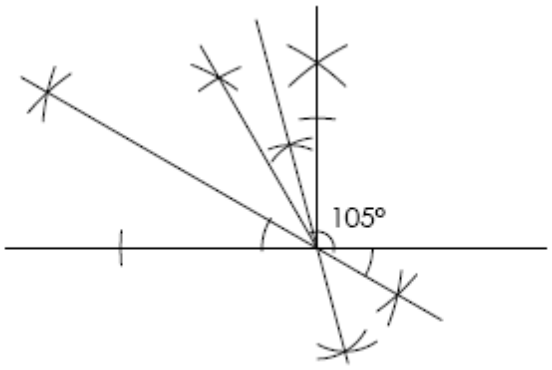
Question 3:

a) Construct a 135^0 Angle
b) Construct a 105^0 Angle

a) To construct a 135˚ angle, construct a 90˚ angle then bisect the other 90˚ angle & add the 45˚ to 90˚



b) To construct a 105˚ angle, construct a 30˚ angle on the other side of 90˚ angle & bisect the 30˚ angle then add the 15˚ to 90˚

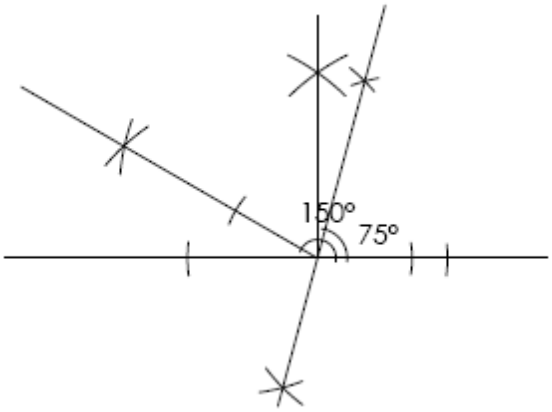


Question 4

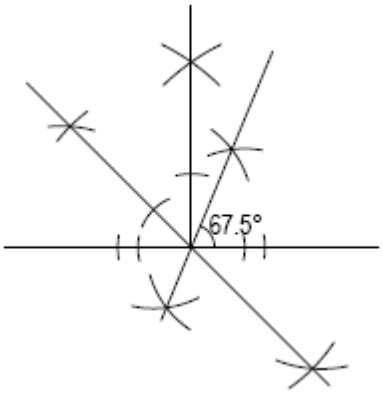
a) Construct a 150^0 Angle
b) Construct a $67{,}5^0$ Angle

Solution

a) To construct a 150˚ angle, first construct a 90˚ & then a 60˚ angle on the other side of the 90˚ angle. **NB** To construct a 75˚ angle, bisect a 150˚ angle



b) To construct a 67,5˚ angle, bisect a 135˚ angle

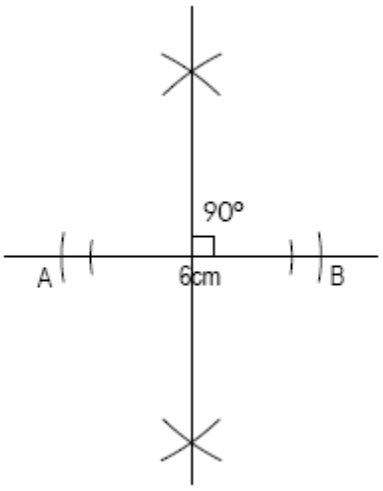


Question 5

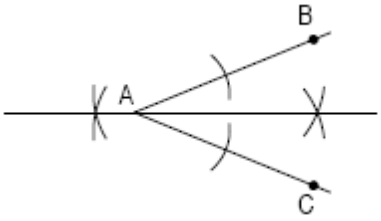
a) Construct a perpendicular Bisector of line AB which is 6cm long
b) Construct the locus of points equidistant from AB and AC

Solution

a) Perpendicular bisector of line A-B which is 6cm or locus of points which are equidistant from A and B.



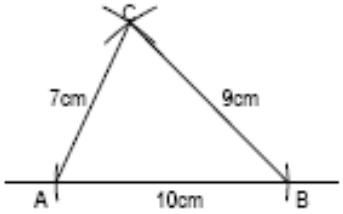
b) The angle bisector or the locus of points which are equidistant from AB and AC



Question 6

Construct a Triangle ABC, where AB= 10cm, BC= 9cm and AC= 7cm, AB being the Base of the triangle

Solution



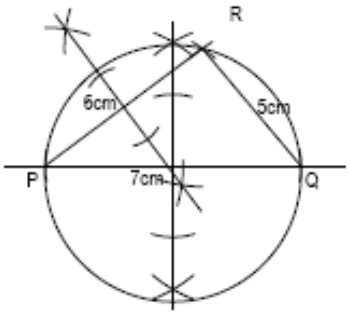
To construct a triangle,

1) draw straight line
2) open your compass to the required length and draw the arcs on the straight line
3) open your compass to the next length and draw an arc and then join the arcs together

Question 7:

Construct a circum-circle/ circumscribed circle outside a triangle PQR where PQ = 7cm, QR= 5cm and PR= 6cm.

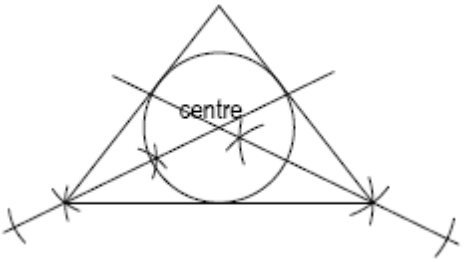
Solution
To construct a circum-circle or circumscribed circle, you bisect any two sides and where the two bisectors meet that is the centre of the circle.



Question 8

Construct an inscribed circle or encircle inside an equilateral triangle measuring 10cm.

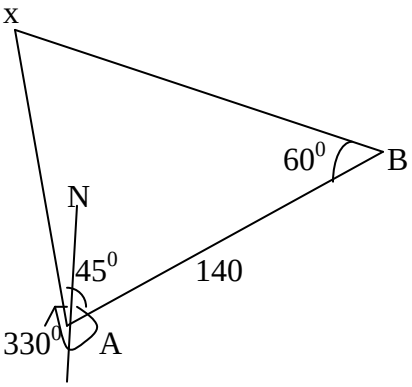
Solution
To construct an inscribed circle/in-circle, you have to bisect the angles and where the bisectors meet is the centre of the circle



Construction and Loci

Question 9:

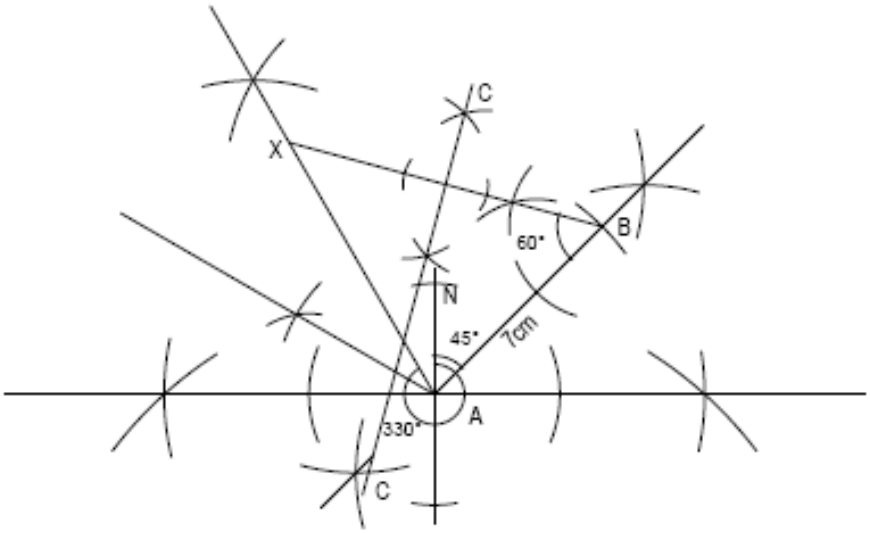
Answer the whole of this question on a sheet of plain paper.
Use ruler and compasses only and show all construction lines and arcs



In the diagram, A, B and X are three towns. The distance between A and B is 140km and ABX= 60^0. From A, the bearing of B is 045^0 and that of X is 330^0.

a) Using a scale of 1cm to represent 20km, construct an accurate scale drawing of ΔABX
b) Measure the length AX and state the distant between A and X in Kilometre
c) Construct the locust of powers equipment from X and B.

Solution



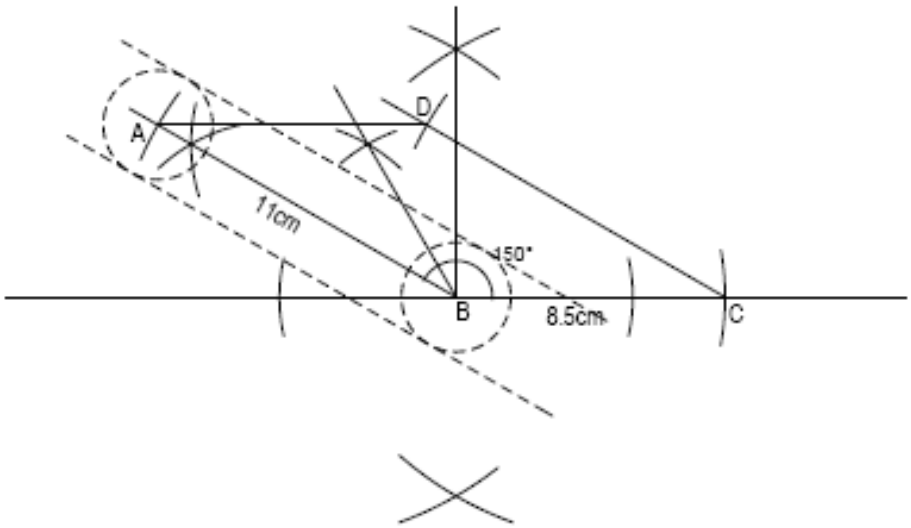
Question 10

Use ruler and compass only for all construction and show clearly all the construction lines and arcs.

a) Construct on a single diagram
i) Parallelogram ABCD in which AB= 11cm, BC= 8,5cm and ABC = 150^0
ii) Measure and write down the length of the perpendicular from B to DC.

iv) Locus of points which are cm from AB.

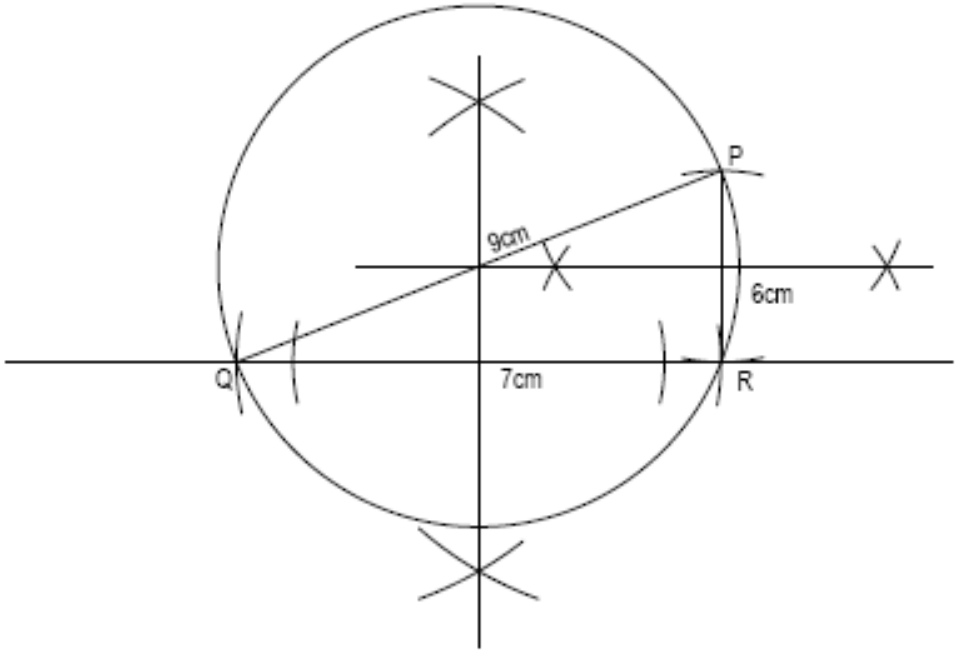
Solution



Question 11:

Construction PQR where PQ = 9cm, QR= 7cm and RP = 6cm. Draw the circumcircle of this triangle and measure its radius.

Solution



Question 12:

Answer this part of the question on a sheet of plain paper. Use ruler and compasses only.

i) Construct A triangle PQR in which PQ = 7,5cm, QR = 6cm and PQR= 900

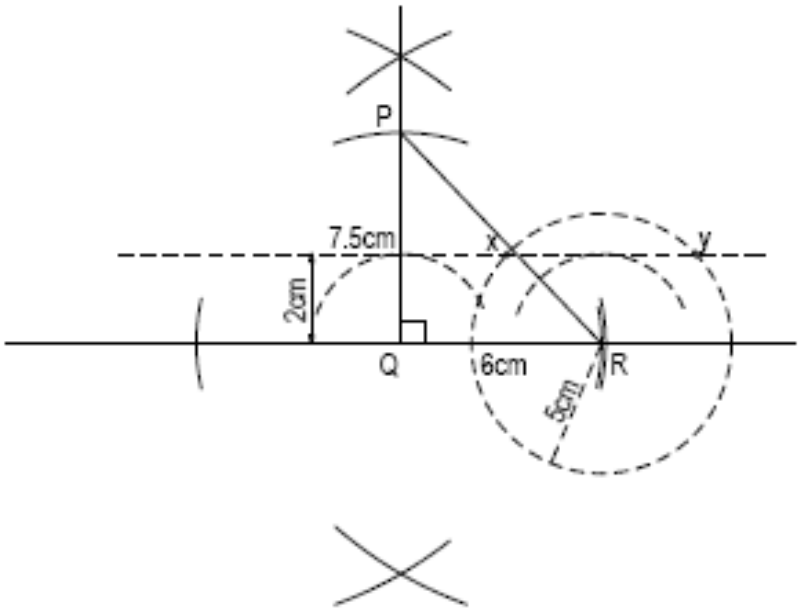
ii) Measure and write down the length of PR

iii) Draw the Locus of points which are 5cm from the point R

i) Draw the Locus of points which are 2cm from the line QR and on the same side of QR as P.

ii) Mark the two points, x inside the triangle and y outside the triangle which are 5cm from R and 2cm from QR.

Solution



Question 13:

i) Construct A triangle XYZ such that XY= 7cm, YZ= 9,5cm and XZ = 6cm.

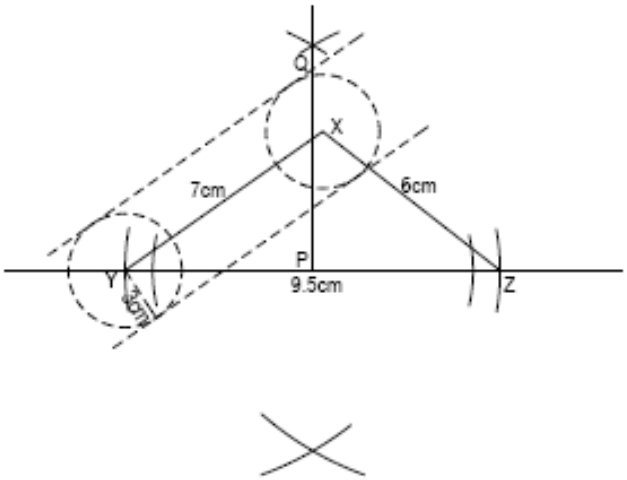
ii) Measure and state the size of the largest angle

iii) Construct the perpendicular Bisector of YZ

iv) Construct the lows of points 3cm from XY

v) Mark two points and label them P and Q which are 3cm from XY and equipment from Y and Z.

Solution



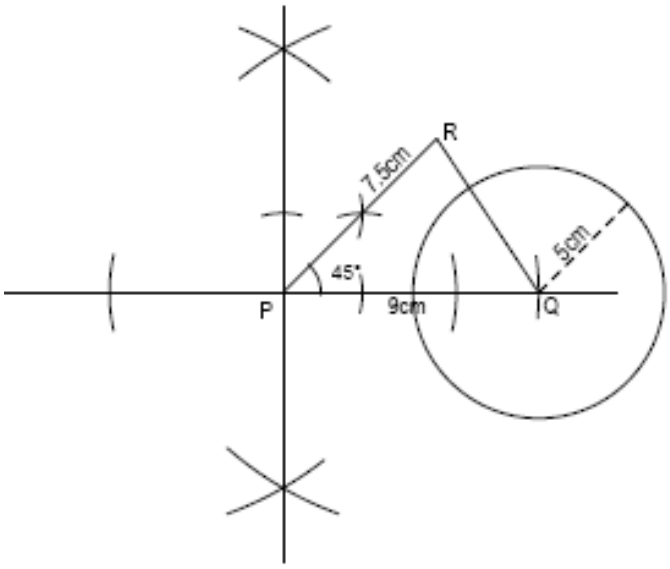
Question 14

Construct the triangle PQR in which PQ= 9cm, PR= 7,5cm, RPQ= 45^0

i) Measure and write down the length of QR

ii) Construct the locus of points which are 5cm from Q.

Solution



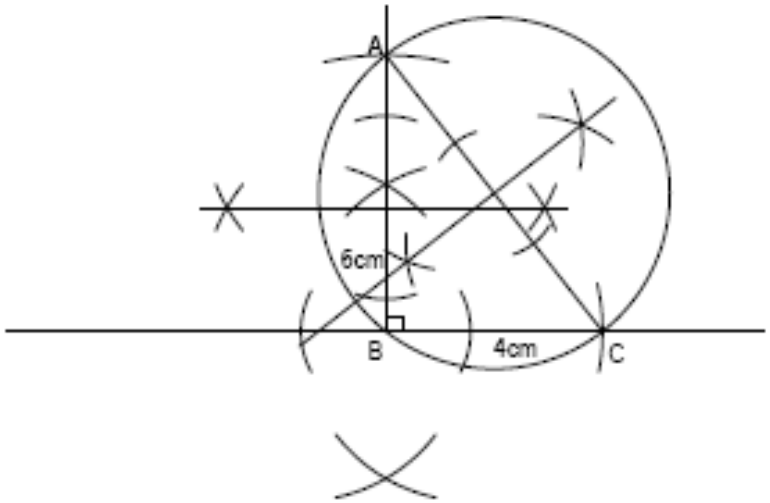
Question 15

i) Construct the triangle ABC such that AB= 6cm, BC= 4cm and ABC= 90^0

ii) Measure and write down the angle BCA and the Length of AC

iii) Construct the circum-circle of the triangle and measure its radius.

Solution



Question 16

Use Ruler and compasses only and clearly show all construction lines and arcs on a single diagram.

a) Construct a quadrilateral ABCD in which AB= 4cm, BC= 6cm CD= 5cm, ABC= 135^0 and BCD = 120^0

b) Measure and write down:

i) The length of AD

ii) BAD

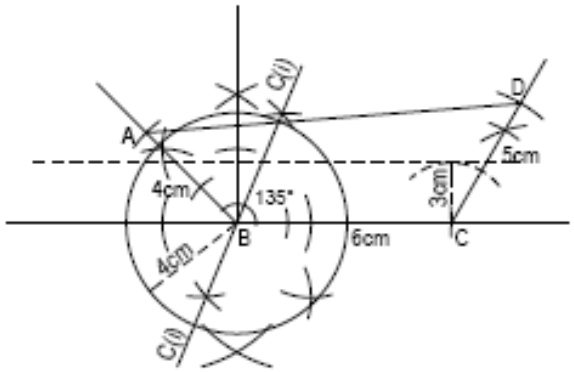
c) Construct the locus of points

i) Equidistant from AB and BC

ii) 3cm from BC and on the same side of BC as A

iii) 4cm from B

Solution



Question 17

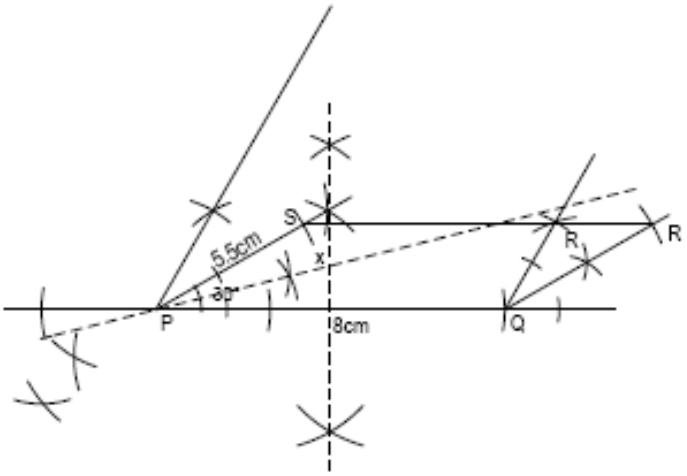
i) Construct A parallelogram PQRS with PQ= 8cm, PS= 5,5cm and SPQ= 30^0

ii) Construct from PQ and PS.

iii) Construct locus of points which are equidistant from P and Q

iv) Label X the point which is equidistant from PQ and PS and also equidistant from P and Q

Solution



Question 18:

Use ruler and compass only for all constructions. Use A plain paper to answer this question.

Construct the triangle LMN in which LM= 12cm, LMN= 600 and MN = 10cm

i) Measure and write down the length of LN

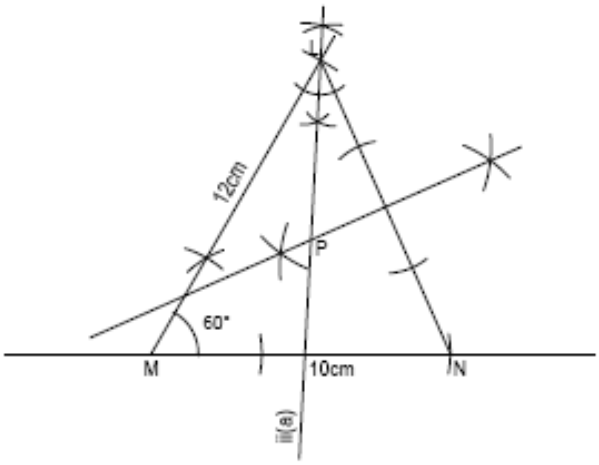
ii) On the same diagram

a) Draw the locus of points equidistant from LM and LN

b) Draw the locus of points equidistant from L and N

iii) Mark on your diagram, the point P which I equidistant from L and n and from LM and LN.

Solution



Question 19

Answer the whole of this question on A plain paper

a) Construct A triangle PQR in which PQ= 7,2cm PR = 10cm and RPQ = 135^0

b) Measure and write down:

i) The length of RQ

ii) The size of angle PQR

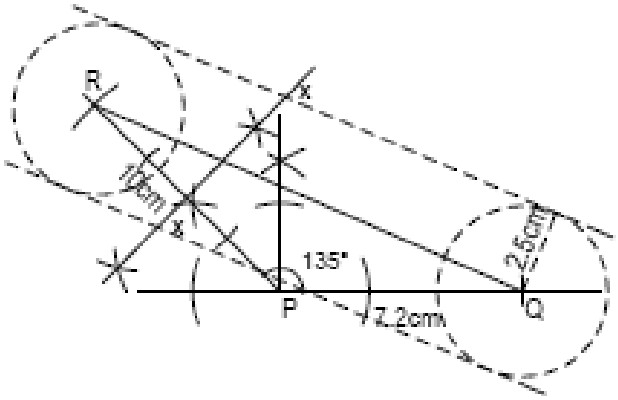
c) Construct the locus of points

i) 2,5cm from QR

ii) Equipment from P and R

d) Label X and Y the locus of points which are 2,5cm from QR and equidistant from P and R.

Solution



Question 20

Construct on a single diagram

a) Triangle XYZ with base
XY = 12cm, XZ = 10cm and YZ = 8,5 cm

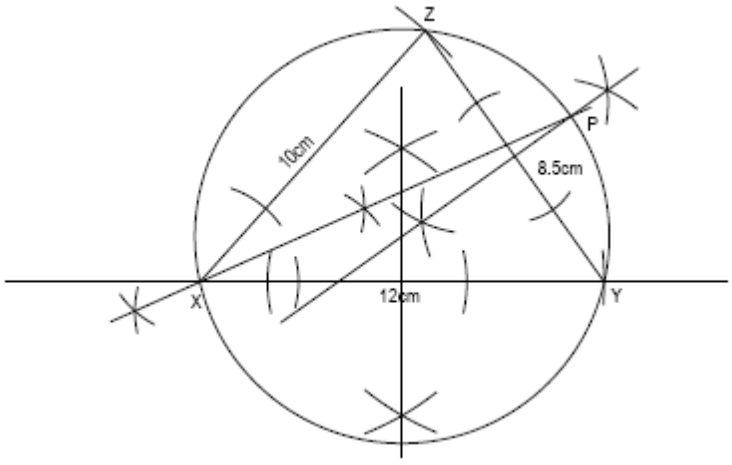
b) The locus of points equidistant from XY and XZ.

c) The circumcircle of the Triangle XYZ

d) Label The point P which is on the circumcircle and also equidistant from XY and YZ.

e) Measure and write down the length of PY

Solution



CHAPTER 13

COSINE RULE

The Cosine Formula for finding the sides :-

1. $a^2 = b^2 + c^2 - 2bc$ Cos A
2. $b^2 = a^2 + c^2 - 2ac$ Cos B
3. $c^2 = a^2 + b^2 - 2ab$ Cos C

Also For the angles:-

1. **Cos A =** $\dfrac{a^2 + b^2 + c^2}{2bc}$
2. **Cos B =** $\dfrac{a^2 + c^2 - b^2}{2ac}$
3. **Cos C =** $\dfrac{a^2 + b^2 - c^2}{2ab}$

Cos θ = -Cos (180° - θ)
Sin Q=Sin (180^0-θ)

Q1. Find the values of the following
a) Cos 87°
b) Cos 155°

Solution

a) Cos 87° = 0,0523 :-
b) Cos 155° = - Cos (180° - 155°)
= - Cos 25°
= -0,9063

Q2, Five O° ≤ θ≤ 180°, Find θ in degrees & Minutes if :-

a) Sin θ= 0,8725
b) Cos θ= -0,3227

Solution 2

a) Since= 0,8725
0,8725= 60° 45^1 or
180° - (60° 45^1)
= 119° 15^1

:- 0,8725 = 60° 45′ or 119° 15′

b) Cos θ= -0,3227

This is an obtuse angle, so you check the 0,3227 the cosine table ignoring the-ve sign.

0,3227= 71° 10′
180° - (71° 10′)= 108° 50′
θ= 108^0 50^1

Question 3:
Give your answers correct to the nearest degree. Find
α If :-

a) Cos α = 0, 9397
b) Cos α = 0,9397

Solution 3

Cos α = 0,9397
From Table 0,9397= 20^0

$\alpha = 20^\circ$

b) Cos α = - 0,9397
α is an obtuse angle
0,9397= 20^0
$180^0 - 20^0 = 160^0$
:- $\alpha = 160^0$

Question 4:

Solve the following Equation for values of θ between 0 and 180°

4 Cos θ + 3 = O

Solution 4

4 Cos θ +3 = 0

$$\frac{4Cos\theta}{4} = \frac{-3}{4}$$

$$\text{Cos } \theta = \frac{-3}{4}$$

θ is an obtuse angle from tables $0{,}7500 = 41^\circ\ 25'$
$180^\circ - (41^\circ\ 25') = 138'\ 35'$
:- $\theta = 138^\circ\ 35^1$

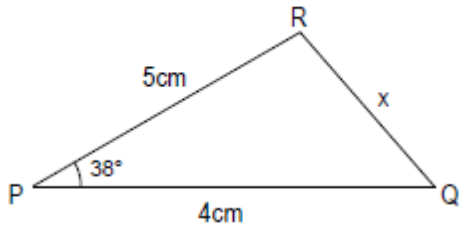
Question 5: if Cos 163° = P
State the value of Cos 17°

Solution 5

From the fact that Cos θ= - Cos $(180^\circ - \theta)$
If Cos 163° = P This means that Cos 17^0 = -p

Question 6

Find the marked side correct to 3 S.F



Solution 6

$\times^2$ = $PR^2 + PQ^2 - 2PR \times PQ \text{ Cos } 380$

$\times^2$ = $5^2 + 4^2 - 2(4 \times 5) \text{ Cos } 380$

$\times^2$ = $25 + 16 - 20 \times 0{,}7880$

$\times^2$ = $41 - 15{,}76$

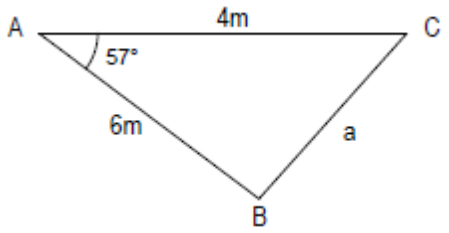
$\sqrt{\times^2}$ = $\sqrt{25.24}$

$\times$ = $\sqrt{25.24}$

$\times$ = 5,024

$\therefore \times$ = **<u>5,02cm</u>**

Q7: Calculate the length of the Third Side of A Triangle giving your To 1.d.p.



<u>Solution 7</u>

$a^2 = b^2 + c^2 - 2bc \text{ Cos A}$

$a^2 = 4^2 + 6^2 - 2(4 \times 6) \text{ Cos } 57^\circ$

$a^2 = 52 - 48 \text{ Cos } 57^\circ$

$a^2 = 52 - 48 \times 0{,}5446$

$a^2 = 52 - 26{,}1408$

$a^2 = 25{,}8592$

$a^2 = \sqrt{25{,}8592} \quad = \sqrt{25{,}86}$

From tables $\sqrt{25{,}86}$ =5,085

$\therefore$ <u>a = 5,1m</u>

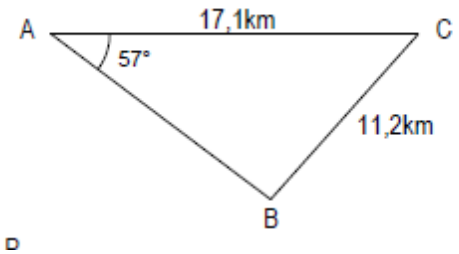
<u>Question 8:</u>

Turn-up College “O” Level Mathematics Questions and answers

Calculate the length of the side opposite the given angle in Triangle ABC. Give your answer correct to 3 S.F.

C= 45,7°, a = 11,2km, b= 17,1km

Solution 8



$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = (11,2)^2 + (17,1)^2 - 2(11,2 \times 17,1) \cos 45,7^\circ$

$c^2 = 125,44 + 292,41 - 383,04 \cos 45,7^0$

$c^2 = 417,85 - 383,04 \times 0,6984$

$c^2 = 417,85 - 267,52$

$c^2 = 150,33$

$c = \sqrt{150.33}$

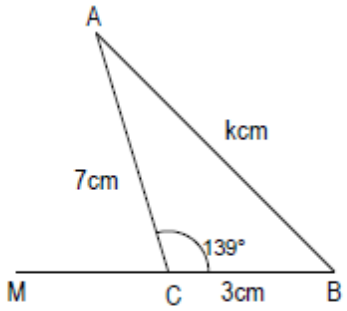
$c = 12,26\ldots$

:- C= 12,3km to 3.S.F

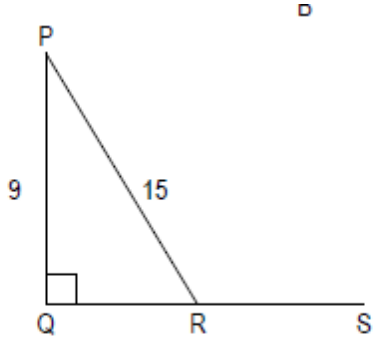
Question 9

In the Diagram below, ABC is A Triangle in which BC= 3cm, AC= 7cm, AB = kcm and ACB= 139^0. The Side BC is produced to M. Given that Cos 139^0 = -0,75,

a) State the value of Cos MCA
b) Calculate the value of k



Question 10



Solution 9

a) Cos MCA = 0,75

b) $k^2 = 7^2 + 3^2 - 2(7 \times 3)\cos C$

$k^2 = 49 + 9 - 42 \cos 139^\circ$

$k^2 = 58 - 42 \times (-0,75)$

$k^2 = 58 + 42 \times 0,75$

$k^2 = 58 + 31,2$

$k^2 = 89,5$

$k = \sqrt{89.5}$

:- **k= 9,46cm to 3 S.F**

PQR is A right- angled Triangle in which PQR=90^0 PQ = 9m and PR= 15cm. The point S lies on QR QR produced

a) Calculate QR

b) Write down, as a fraction the value of:-

i) Tan PRQ ii) Sin PRS iii) Cos PRS

Solution 10

$PR^2 = QR^2 + PQ^2$

$15^2 = QR^2 + 9^2$

$QR^2 = 15^2 - 9^2$

$QR^2 = 225 - 81$

$QR^2 = 144$

QR = $\sqrt{144}$

QR = 12

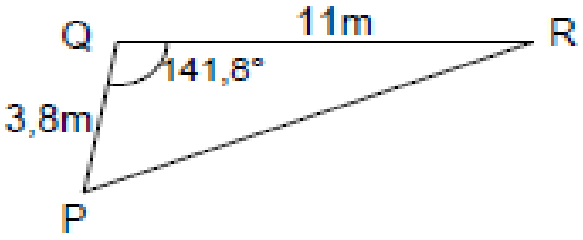
bi) Tan PRQ= $\frac{9}{12}$

ii) Sin PRS = $\frac{9}{15}$

iii) Cos PRS= $\frac{-12}{15}$

Question 11

Calculate the unknown side. Give the Final Answer Correct to 3.S.F



Solution 11

PR^2 = PQ^2 + QR^2 - 2 (PQ x QR) Cos Q

PR^2 = $3,8^2$ + 11^2 - 2 (3,8 x 11) Cos 141,8°

= 14,44 +121- 83,6 (-Cos 38,2°)

= 135, 44-83, 6 x (-07859)

= 135, 44 + 83, 6 x 0, 7859

= 135, 44 + 65, 70124

= 201, 14124

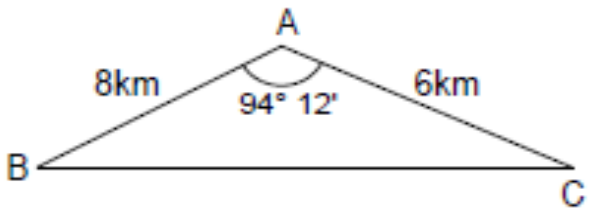
PR= $\sqrt{201.1}$

PR= 14,2m

Question 12

In the Triangle ABC, AB= 8km, Ac= 6km $\hat{BAC}= 94^0\ 12^1$. Calculate.

a) The Length of the third side
b) The Area of Triangle ABC to 3 S.



Solution

a) $a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 6^2 + 8^2 - 2(6 \times 8) \cos 94^0\ 12^1$

$a^2 = 36 + 64 - 96 \times \cos 94^0\ 12^1$

$a^2 = 100 - 96 \times (-\cos 85^0\ 48^1)$

$a^2 = 100 + 96 \times 0{,}0732$

$a^2 = 100 + 7{,}0272$

$a^2 = 107{,}0272$

$a = \sqrt{107{,}0}$

: -

$a = 10{,}3km$

b) Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 8 \times 6 \times \sin 94^\circ 12'$

$= \frac{1}{2} \times 8 \times 6 \times \sin 85^0 48^1$

$\frac{1}{2} \times 8 \times 6 \times 0{,}9973$

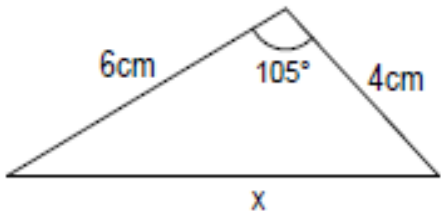
$= 24 \times 0{,}9973$

= 23,9352

= 23,9km2

Question 13

Find x in the Triangle below



Solution 13

$x^2 = 6^2 + 4^2 - 2 \times 6 \times 4 \times \text{Cos } 105^\circ$

$x^2 = 36 + 24 - 48 \times (-\text{Cos } 75)$

$x^2 = 60 + 48 \times 0{,}2588$

$x^2 = 60 + 12{,}4244$

$x^2 = 72{,}4224$

$x^2 = 72$, 42 to 4 SF

$x = \sqrt{72.24}$

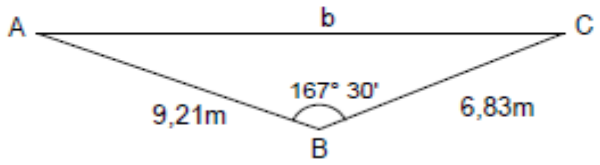
$x = 8{,}511$

$\therefore$ x= 8,51cm to 3.S.F

Question 14

In Triangle ABC.
C= 9,21m, a= 6,83m and
B= 1670 301, Calculate AC

Solution 14



$b^2 = 6{,}83^2 + 9{,}21^2 - 2\,(9{,}21 \times 6{,}83)\,\text{Cos}\ 167^\circ\ 30'$

$= 46{,}65 + 84{,}82 - 125{,}8086 \times (\text{Cos}\ 167^0\ 30^1$

$= 131{,}47 - 125{,}8086 \times (-\text{Cos}\ 12^0\ 30^1)$

$= 131{,}47 + 125{,}8086 \times 0{,}9763$

$= 131{,}47 + 122{,}83$

$= 254{,}3$

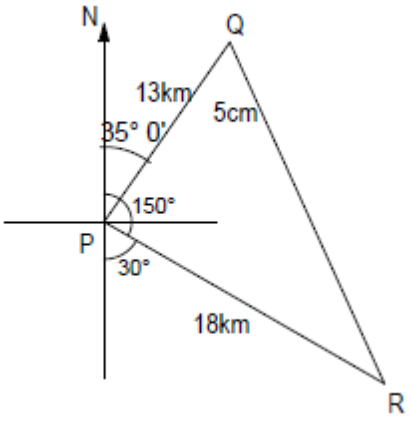
$b = \sqrt{254.3}$

$b = 15{,}95$

:- b= 16,0m

Question15:
Peter and Tom leave from the same point P. Peter
Travels on a bearing of 0350 for 13km to point Q. Tom Travels on A bearing of 1500 to point R, 18kmfrom P. Find QR.

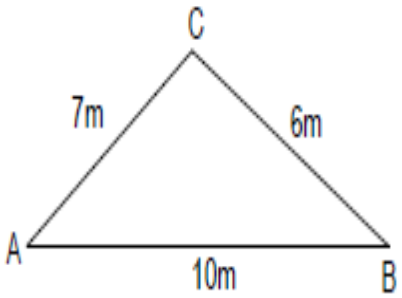
Solution 15



$P^2 = 18^2 + 13^2 - 2 \times 18 \times 13 \text{ Cos } 175^\circ$
$= \quad 324 + 169 - 468 \times (-\text{Cos } 65)$
$= \quad 493 + 468 \times 0{,}4226$
$= \quad 493 + 197{,}78$
$= \quad 690{,}78$
$= \quad \sqrt{690.8}$
$= \quad 26{,}28$

Question 16:
Calculate the largest Angle in triangle ABC with sides of length 6m, 7m and 10m leaving your answer in degrees

Solution 16



$$\text{Cos C} = \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{6^2+7^2-10^2}{2\times 6\times 7}$$

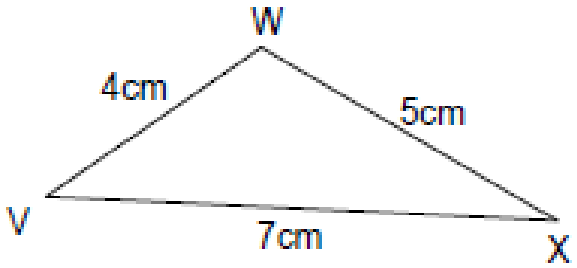
$$= \frac{85-100}{84}$$

$$= \frac{-15}{84}$$

$$\text{Cos c} = \frac{-15}{84}$$

Cos C= -0,1786
Since Cos c is negative, c is obtuse
0,1786 = $79,7^0$
C = 1800- $79,7^0$ = $100,3^0$
:- C = $100,3^0$

Question 17 In the diagram, VWX is a triangle in which VW= 4cm WX= 5cm and VX= 7cm calculate VWX.



Solution

Cos W= $\frac{X^2+V^2-W^2}{2\times V\times X}$

= $\frac{4^2+5^2-7^2}{2\times 4\times 5}$

= $\frac{41-49}{40}$

Question 18

Calculate the angles of the triangle ABC.mos sides are given in meters

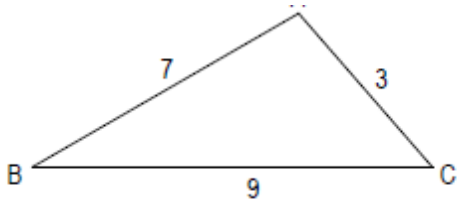
$= \frac{-8}{40}$

= -0,2

0;2= 780,50

W= $180^0 - 78, 5^0$

= $\underline{101,5^0}$



Solution 18

Cos A = $\frac{b^2 + c^2 - a^2}{2bc}$

= $\frac{3^2 + 7^2 - 9^2}{2 \times 3 \times 7}$

= $\frac{\underline{58 - 81}}{42}$

= $\frac{-23}{42}$

= -0,5476

A= $180^0 - 56,8^0$

= $\underline{123,2^0}$

Cos B = $\frac{a^2 + c^2 - b^2}{2ac}$

= $\frac{92 + 72 - 32}{2 \times 9 \times 7}$

$= \frac{81+49-9}{12b}$

$= \frac{121}{126}$

$= 0{,}9603$

B= $\underline{16{,}2^0}$

Cos C= $\frac{a^2+b^2-c^2}{2ab}$

$= \frac{9^2+3^2-7^2}{2\times 9\times 3}$

$= \frac{81+9-49}{54}$

$= \frac{41}{54}$

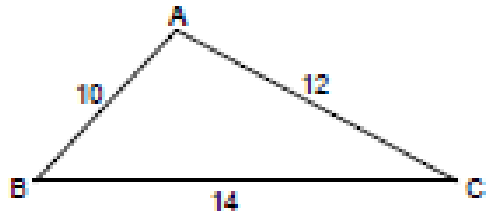
$= 0,\ 7593$

C= $\underline{40,\ 6^0}$

Question 20

Three Triangulation villages A,B and C are such that AB= 10km, BC= 14km and AC= 12km Respectively. Find ABC.

Solution 20



Cos B= $\frac{a^2+c^2-b^2}{2ac}$

= $\frac{1424+102-122}{2\times14\times10}$

= $\frac{152}{280}$

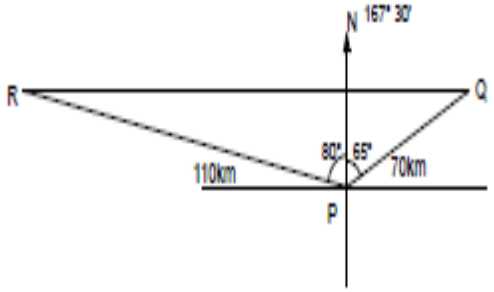
= 0,5429

:- **B= $57^0\ 7'$**

Question 21

The points P,Q and R are situated so that PQ = 70km, PR = 110km. The bearing of Q from P= 0650 and the bearing of R from P is 2800. Calculate the distance QR.

Solution 21

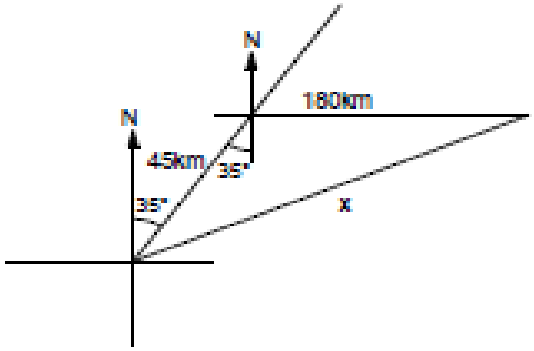


QR^2 = $110^2 + 70^2$ -2x 110× 70 (03 1450
= 12 100+ 4 900 – 15400 × (-Cos35˚)
= 1 7000 + 15400 × 0,8192
= 17 000 + 12 615, 68
= 29 615,68
= 29616
QR = $\sqrt{29616}$
QR = 1 72,09
= <u>172km</u> to 3S.F

<u>Question 22</u>

A Boy cycles 45km on a bearing 35^0 and then 180km due east. How far is he from his starting point?

<u>Solution 22</u>



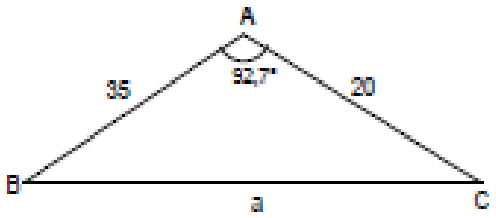
900 + 350 = 1250To correct

X^2 $=45^2 + 150^0 - 2 \times 45 \times 180 \text{ Cos } 125^0$
$=2025 + 32400 - 16200 \times -(\text{Cos } 55^0)$
=34425+ 16200 × -,5736
=34425 + 9 292, 32
=43717,32
=43717
× $=\sqrt{43717}$

× = 209,08
∴ ×<u>= 209km</u>

Question 23:
A triangular zinc sheets has two sides 20m and 35m respectively and the angle between these sides is 92,70. How long is the third side?

Solution 23



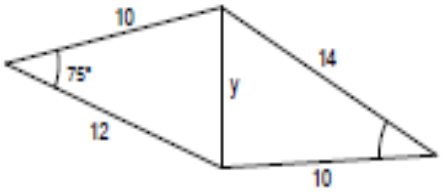
$A^2 = 35^2 + 20^2 - 2\times 35\times 20(0392,70)$

$A^2 = 1225 + 400 - 1400\times(-Cos87.30)$

= 1625 + 1 400 × 0,0471
= 1625 + 65,95
= 1690,94

a = $\sqrt{1690,94}$
a = 41,12
:-a = 41, 1km to 3.S.F

Question 24
Find y and & in the Diagram below. All lengths are given in metres



$\text{Cos } \alpha = \dfrac{14^2 + 10^2 - 13,5^2}{2\times 14\times 10}$

$$= \frac{196 \div 100 - 182,25}{280}$$

$$= \frac{296 - 182,25}{280}$$

Solution

y2 =122 +102-2 x 12 x 10 (Cos 7500

=144 +100- 240 x 0,2588
=244- 62,112
=181,888

$$= \frac{113,75}{280}$$

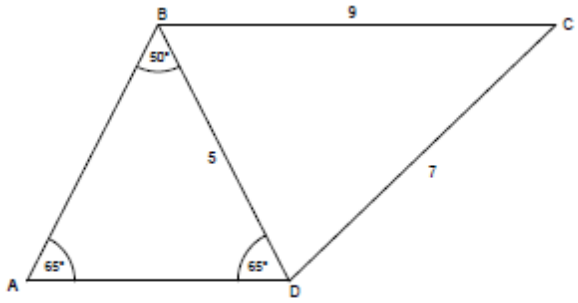
y $=\sqrt{181,89}$ $= 0,4063$

:- y= 13,5cm to 3. S.F :- $a = 66^{\circ}2^{1}$

Question 25

ABCD IS A Quadrilateral where BC= 9cm, CD= 7cm BD= 5cm, $\hat{ABD} = 50^{0}$ and $\hat{BDA}$= 650. Calculate

a) AD
b) BDC



b) Cos D= $\frac{7^2}{2} = \frac{5^2}{7} = \frac{9^2}{5}$

$$= \frac{74 - 81}{70}$$

Solution

$$= \frac{-7}{70}$$

180° $-(65^{\circ} + 50^{\circ}) = 65^{\circ}$

$= -0,1$

So BAD = 65°

D = $180^{\circ} - (84^{\circ}\ 16^{1})$

a) $\frac{AD}{Sin50^{\circ}} = \frac{5}{Sin65^{\circ}}$

$$\frac{AD \times Sin65^\circ}{Sin65^\circ} = \frac{5 \times Sin5^\circ}{Sin65^\circ}$$

$\therefore$ D =95° 44′

$$AD = \frac{5 \times Sin50}{Sin65^\circ}$$

$$AD = 423$$